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Symmetry Breaking and the Pionic Decays of *K* Mesons

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Symmetry-breaking corrections to the current-algebra formulas for $K \to 2\pi$ and $K \to 3\pi$ decays are calculated in the framework of a general form of the linear SU(3) σ model.

I. INTRODUCTION

The first generation of workers on "current algebras" produced some interesting results[1] on the $K \to 2\pi$ and $K \to 3\pi$ decays. Nevertheless these are still among the most mysterious processes in all of particle physics. One of the associated problems is to find the symmetry-breaking corrections to the current-algebra formulas. This has been attacked by many authors with different kinds of results. The basic difficulty is that the situation is rather complicated so that a fairly large number of assumptions must be brought into the picture. In the present paper we shall calculate the symmetry-breaking corrections in the framework of an SU(3) σ model of spin-0 mesons. The advantage of this model[2] is that, while it is realistic enough to give all the current-algebra formulas in the appropriate limits, it is simple enough so that we can perform the calculations in a self-consistent way without introducing extra assumptions. Specifically, we will consider corrections to the $K \to 3\pi$ amplitudes resulting from the SU(3) noninvariance of the "vacuum," and also corrections to the $K \to 2\pi$ amplitudes resulting both from the SU(3) noninvariance and the SU(2) (electromagnetic) noninvariance of the "vacuum." These effects are similar to the so-called "tadpole" effects but *not* to the strangeness-changing tadpoles. In our work we shall assume that the weak nonleptonic interaction is of current-current form and can be effectively represented by a pure octet in SU(3) space.

The main results in this model are as follows:

(1) For $K^+ \to \pi^+\pi^0$ the "tadpole" contribution is most likely much too small to explain the entire decay rate.

(2) For $K \to 3\pi$ the effect of symmetry breaking is possibly in the right direction to improve the agreement with experiment.

(3) A comparison of $K \to 3\pi$ and $\eta \to 3\pi$, which has been discussed in the present framework elsewhere,[3] shows that the predicted spectrum shape is the same even though $K \to 3\pi$ arises from a current-current interaction and $\eta \to 3\pi$ arises from a tadpole-type interaction. Thus the apparent experimental similarity of these two spectra need not indicate that both arise from effective inter-

actions of the same general form.

II. STATEMENT OF MODEL

The model[2] is defined by the Lagrangian density

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu \phi \partial_\mu \phi) - \tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu S \partial_\mu S) - V_0 - V_{sb} - V_{cc}\,, \qquad (1)$$

where ϕ and S are, respectively, the 3×3 matrices of pseudoscalar and scalar fields. We allow V_0 to be the most general nonderivative SU(3)×SU(3)-invariant interaction. V_{sb} is taken to be the simplest possible symmetry breaking term which belongs to the $(3,3^*)+(3^*,3)$ chiral representation, namely

$$V_{sb} = -2(A_1 S_1^1 + A_2 S_2^2 + A_3 S_3^3)\,, \qquad (2)$$

where the A_i are three real constants analogous to the quark masses in the quark model. Finally, V_{cc} is the current-current weak interaction with octet dominance assumed:

$$V_{cc} = (G/\sqrt{2}\,) X\,\mathrm{Tr}\,(J_\mu J_\mu U)\,. \qquad (3)$$

In (3) $G \simeq 1.026\times 10^{-5} M_p^{-2}$ is the universal Fermi constant, X is an adjustable parameter, U is the matrix

$$U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad (4)$$

while J_μ is the weak hadronic current constructed from the Noether currents of the Lagrangian (1) without V_{cc}. Specifically,

$$J_\mu = V_\mu + P_\mu - \tfrac{1}{3}\,\mathrm{Tr}(P_\mu)\,, \qquad (5a)$$

$$V_\mu = i(\phi \overleftrightarrow{\partial}_\mu \phi + S \overleftrightarrow{\partial}_\mu S)\,, \qquad (5b)$$

$$P_\mu = S \overleftrightarrow{\partial}_\mu \phi - \phi \overleftrightarrow{\partial}_\mu S\,. \qquad (5c)$$

The adjustable parameter, X, is expected to be $\frac{3}{5}\cos\theta_C \sin\theta_C$, where θ_C is the Cabibbo angle, from taking the octet part of the usual local theory of weak interactions. However, several authors[4] have noted that it comes out to be around unity from fitting the $K_S \to 2\pi$ rate to experiment. This discrepancy is quite mysterious and its resolution may provide a valuable clue to further development. Actually the value of X is immaterial for the present calculation, since it cancels out when we consider the amplitude ratios $(K^+ \to \pi^+\pi^0)/(K_S \to 2\pi)$ and $(K\to 3\pi)/(K\to 2\pi)$.

A detailed discussion of the present model without V_{cc} has been given elsewhere.[2] It has been found to yield the usual current-algebra results when the limit of infinite scalar-meson masses is taken. If this limit is not taken, it is possible to explain the η' mass, the isovector scalar particle mass, the electromagnetic mass splittings, and the $\eta \to 3\pi$ decay rate without running into any severe contradictions with experiment. The existence of isosinglet and κ-type scalar mesons has long been a subject of controversy, but the broad particles expected in this model may possibly be reconciled with the recently performed analyses of s-wave phase shifts in these channels.

A characteristic feature in our treatment of the present model is the recognition that the ground state need not be chiral SU(3)×SU(3) invariant, but must be determined as a possible solution of extremum equations:

$$\left\langle \frac{\partial V_0}{\partial \phi}\right\rangle_0 + \left\langle \frac{\partial V_{sb}}{\partial \phi}\right\rangle_0 = 0\,, \qquad (6a)$$

$$\left\langle \frac{\partial V_0}{\partial S}\right\rangle_0 + \left\langle \frac{\partial V_{sb}}{\partial S}\right\rangle_0 = 0\,, \qquad (6b)$$

where the notation $\langle\ \rangle_0$ means that the enclosed object is evaluated in the ground state. As explained in Ref. 2, we work in a semiclassical approximation and set

$$\langle S_a^b\rangle_0 = \alpha_a \delta_a^b\,, \qquad (7)$$

where the α_a are three real constants. The physical scalar fields $\tilde{S}$ are given as

$$\tilde{S} = S - \langle S\rangle_0\,. \qquad (8)$$

(A generalization to the situation where $\langle\phi\rangle_0 \neq 0$ so that the vacuum also violates parity and CP is given still elsewhere.[5]) For our present purposes it is crucial to note that the replacement (8) means that the vector and pseudovector currents of (5b) and (5c) must be rewritten as

$$V_\mu = i(\phi \overleftrightarrow{\partial}_\mu \phi + \tilde{S}\overleftrightarrow{\partial}_\mu \tilde{S}) + i[\langle S\rangle_0, \partial_\mu \tilde{S}]\,, \qquad (9a)$$

$$P_\mu = \tilde{S}\overleftrightarrow{\partial}_\mu \phi - \phi\overleftrightarrow{\partial}_\mu \tilde{S} + [\langle S\rangle_0, \partial_\mu \phi]_+\,. \qquad (9b)$$

Apart from the weak-interaction constant GX the parameters[6] which enter into our model are the three A_i and the three α_i. These have been determined from the masses of the spin-zero particles (taking into account electromagnetic mass differences) and the weak decay constants. In the isospin-invariant limit, $A_1 = A_2$ and $\alpha_1 = \alpha_2$.

FIG. 1. Diagrams for $K \to 2\pi$.

III. $K\to 2\pi$ AMPLITUDES

The two types of Feynman diagrams which are needed are shown in Fig. 1. To calculate these we first must find the 2- and 3-point weak vertices and the $K\kappa\pi$ strong vertex, including electromagnetic corrections. A slight complication in doing this is the existence of π^0-η-η' mixing. We take this into account [see Eq. (3.1) of Ref. 3] by expressing the matrix elements ϕ_1^1, ϕ_2^2, and ϕ_3^3 in terms of π^0, η, and η' as

$$\begin{pmatrix}\phi_1^1\\ \phi_2^2\\ \phi_3^3\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt2}+\psi_1 b+\psi_3 a & \frac{-\psi_1}{\sqrt2}+b & \frac{-\psi_3}{\sqrt2}+a\\ -\frac{1}{\sqrt2}+\psi_1 b+\psi_3 a & \frac{\psi_1}{\sqrt2}+b & \frac{\psi_3}{\sqrt2}+a\\ \sqrt2(-\psi_1 a+\psi_3 b) & -\sqrt2 a & \sqrt2 b\end{pmatrix}\begin{pmatrix}\pi^0\\ \eta\\ \eta'\end{pmatrix}, \tag{10}$$

where ψ_1 is the $\pi^0\eta$ mixing angle, ψ_3 is the $\pi^0\eta'$ mixing angle, and

$$a=(1/\sqrt6)(\sin\theta_p+\sqrt2\cos\theta_p),$$
$$b=(1/\sqrt6)(\cos\theta_p-\sqrt2\sin\theta_p), \tag{11}$$

θ_p being the $\eta\eta'$ mixing angle. All of ψ_1, ψ_3, and θ_p are given in terms of known quantities (see Refs. 2 and 3) so no new parameters are involved. The relevant part of the 2-point weak vertex is found from (3), (5a), (9a), (9b), and (10) to be

$$-\mathcal{L}_W(S\phi)=\frac{iGX}{\sqrt2}\bigg\{\tfrac{2}{3}(\alpha_3-\alpha_2)\left[\frac{2\alpha_1+\alpha_2}{\sqrt2}+\alpha\psi_1(b+\sqrt2\,aW)+\alpha\psi_3(a-\sqrt2\,bW)\right]\partial_\mu\pi^0\partial_\mu(\kappa^0-\bar\kappa^0)$$
$$+(\alpha_1+\alpha_2)(\alpha_3-\alpha_1)[\partial_\mu\kappa^-\partial_\mu\pi^+-\partial_\mu\kappa^+\partial_\mu\pi^-]+\cdots\bigg\}, \tag{12}$$

where for convenience we have used α instead of α_1 or α_2 when they multiply quantities of electromagnetic order. We have also introduced a quantity

$$W=\alpha_3/\alpha, \tag{13}$$

which would be unity if the vacuum were exactly SU(3)-invariant.

The relevant part of the 3-point weak vertex is similarly found to be

$$-\mathcal{L}_W(\phi^3)=GX(\tfrac{2}{3}\{-(\alpha_1+\tfrac12\alpha_2)+\alpha\psi_1[\sqrt2\,b+(3-W)a]+\alpha\psi_3[\sqrt2\,a-(3-W)b]\}\partial_\mu\pi^0(K_1\overleftrightarrow{\partial}_\mu\pi^0)$$
$$-(\alpha_1+\alpha_2)K_1\partial_\mu\pi^-\partial_\mu\pi^++\tfrac12(\alpha_1+\alpha_2)\partial_\mu K_1(\partial_\mu\pi^-\pi^++\partial_\mu\pi^+\pi^-))$$
$$+iGX\{(\pi^-\overleftrightarrow{\partial}_\mu\pi^0)\partial_\mu K^+(\alpha_1+\alpha_3)+\partial_\mu\pi^0(\pi^-\overleftrightarrow{\partial}_\mu K^+)\tfrac13\sqrt2\,[-\tfrac12\sqrt2(2\alpha_1+\alpha_2)-\alpha\psi_1(\sqrt2\,aW+b)+\alpha\psi_3(\sqrt2\,bW-a)]$$
$$+\partial_\mu\pi^-(\pi^0\overleftrightarrow{\partial}_\mu K^+)[\tfrac12(\alpha_1+\alpha_2)+\sqrt2\,\alpha\psi_1(b+\sqrt2\,a)+\sqrt2\,\alpha\psi_3(a-\sqrt2\,b)]\}, \tag{14}$$

where

$$K_1=\frac{\bar K^0-K^0}{\sqrt2 i},\quad K_2=\frac{\bar K^0+K^0}{\sqrt2}. \tag{15}$$

Finally from Eq. (4.2) of Ref. 2 we get the strong $K\kappa\pi$ vertex including electromagnetic corrections:

$$-\mathcal{L}(K\kappa\pi)=(g_{K^+\kappa^-\pi^0})K^+\kappa^-\pi^0+(g_{K^0\kappa^-\pi^+})K^0\kappa^-\pi^++(g_{K^+\bar\kappa^0\pi^-})K^+\bar\kappa^0\pi^-+(g_{K^0\bar\kappa^0\pi^0})K^0\bar\kappa^0\pi^0+\text{H.c.}, \tag{16a}$$

$$g_{K^0\kappa^-\pi^+}=\frac{m^2(\kappa^+)-m^2(\pi)}{\alpha_2+\alpha_3}, \tag{16b}$$

$$g_{K^+\bar\kappa^0\pi^-}=\frac{m^2(\kappa^0)-m^2(\pi)}{\alpha_1+\alpha_3}, \tag{16c}$$

$$g_{K^+\kappa^-\pi^0}=\frac{1}{\sqrt2}\,\frac{m^2(\kappa^+)-m^2(\pi)}{\alpha_1+\alpha_3}[1+\sqrt2(b-\sqrt2\,a)\psi_1+\sqrt2(a+\sqrt2\,b)\psi_3], \tag{16d}$$

$$g_{K^0\bar{K}^0\pi^0} = -\frac{1}{\sqrt{2}}\frac{m^2(K^0)-m^2(\pi)}{\alpha_2+\alpha_3}[1-\sqrt{2}\,(b-\sqrt{2}\,a)\psi_1-\sqrt{2}\,(a+\sqrt{2}\,b)\psi_3]\ . \tag{16e}$$

With the above vertices we calculate the diagrams of Fig. 1 and find the T amplitudes[7]:

$$T(K_1\to\pi^+\pi^-)\equiv T_{+-}=-GX(\alpha_1+\alpha_2)\left[m^2(K^0)-m^2(\pi)+m^2(\pi)\left(\frac{\alpha_3-\alpha_1}{\alpha_3+\alpha_2}\right)\right], \tag{17}$$

$$\begin{aligned}T(K^+\to\pi^+\pi^0)\equiv T_{+0}=\tfrac{1}{6}i\,GX\Big[&[m^2(K)-m^2(\pi)]\{(\alpha_1-\alpha_2)-4\alpha\psi_1[\sqrt{2}\,b+(3-W)a]-4\alpha\psi_3[\sqrt{2}\,a-(3-W)b]\}\\ &+4\alpha m^2(\pi)\left(\frac{1-W}{1+W}\right)\{[\sqrt{2}\,b-(3+W)a]\,\psi_1+[\sqrt{2}\,a+(3+W)b]\,\psi_3\}+(\alpha_1-\alpha_2)m^2(\pi)\left(\frac{5+W}{1+W}\right)\Big],\end{aligned} \tag{18}$$

$$T(K_1\to\pi^0\pi^0)\equiv T_{00}=T_{+-}+2iT_{+0}\ . \tag{19}$$

Equation (19) is actually a well-known sum rule which expresses the fact that the net interaction (weak plus electromagnetic) satisfies $|\Delta I|\leq\frac{3}{2}$. This holds in our model since our weak interaction satisfies $|\Delta I|=\frac{1}{2}$ by construction while the electromagnetic symmetry breaking [which appears explicitly through the fact that $A_1\neq A_2$ in (2)] is a pure $|\Delta I|=1$ object. *CP*-violating effects and final-state interactions have, of course, been neglected in the present treatment. Furthermore, in the above calculations we always took $m(\pi^+)=m(\pi^0)$ wherever it appeared. The reason for this is that the pion electromagnetic mass splitting is a pure $|\Delta I|=2$ object and hence cannot come from the symmetry-breaking term of (2). Calculations involving one-photon emission and absorption generally give a good account of this pion mass difference. The class of diagrams involving one-photon emission and absorption also give vertex corrections in Fig. 2. For consistency, therefore, we have taken *none* of the one-photon diagrams into account, rather than taking just *some* of them into account by allowing $m(\pi^0)\neq m(\pi^+)$. If these effects were to be taken into account the sum rule (19) would no longer hold.

It is interesting to consider the limits of (17)–(19) where electromagnetic effects are neglected. It is sufficient to set $\alpha_1=\alpha_2=\alpha$ [SU(2)-invariant vacuum] and $\psi_1=\psi_3=0$. Then we get

$$\begin{aligned}&T_{+-}\to-2\alpha GX\left(m^2(K)-\frac{2}{W+1}\,m^2(\pi)\right),\\ &T_{00}\to T_{+-},\\ &T_{+0}\to 0\ .\end{aligned} \tag{20}$$

The experimental values are

$$\begin{aligned}&T_{+-}\simeq 29.9\times10^{-7}m(\pi^0),\\ &T_{00}\simeq 28.0\times10^{-7}m(\pi^0),\\ &T_{+0}\simeq 1.31\times10^{-7}m(\pi^0)\ .\end{aligned} \tag{21}$$

Comparing (20) and (21) we verify, as expected, that X is roughly 1.06. Equation (20) reduces to the current-algebra formula[8] in the limit when $W\to1$ [see Eq. (13)]. For a realistic value of W this is only a 2% effect. Thus the deviation of the vacuum from SU(3) gives only a small correction for $K\to2\pi$. The interesting effect results from the deviation of the vacuum from SU(2) and makes $K^+\to\pi^+\pi^0$ possible. Using the values[9] determined in Refs. 2 and 4 for the quantities in (18) we calculate

$$\left|\frac{T_{+0}}{T_{+-}}\right|\simeq 0.0089\ . \tag{22}$$

This is considerably smaller than the experimental value, which is seen from (21) to be around 0.044. Our conclusion from the present model is thus that of the following three possible contributions to $K^+\to\pi^+\pi^0$:

(a) "tadpole" electromagnetic corrections to the $|\Delta I|=\frac{1}{2}$ weak interaction,

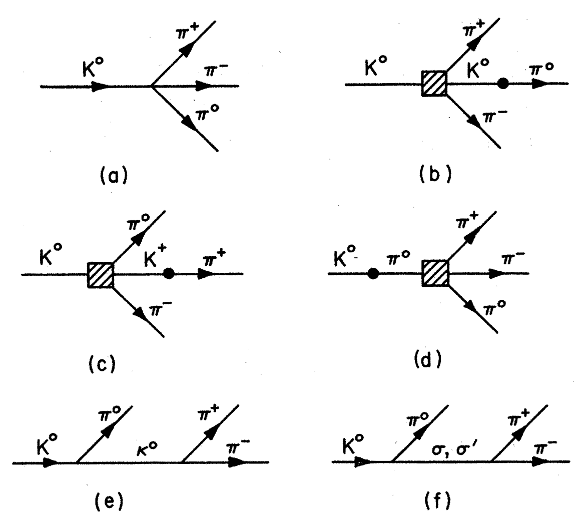

FIG. 2. Diagrams for $K\to3\pi$.

(b) one-photon emission and absorption corrections to the $|\Delta I|=\frac{1}{2}$ weak interaction,
(c) intrinsic $|\Delta I|=\frac{3}{2}$ weak interaction,
type (a) is not the dominant one.

Another prediction of our model from (17) and (19) is that the type (a) effect gives $|T_{+-}|>|T_{00}|$. This is seen from (21) to be in the right direction but too small.

Contributions of type (a) to $K^+\to\pi^+\pi^0$ have also been found to be small by Goyal and Li[10] and by Baće[11] in different models. If we were to allow $m(\pi^+)\neq m(\pi^0)$ in our calculation we would effectively pick up a contribution of type (b), which would add a term of order

$$\frac{m^2(\pi^+)-m^2(\pi^0)}{m^2(K)}$$

to

$$T_{+0}/T_{+-}\,.$$

This term is very small even compared to (22), as was noticed by Hara and Nambu.[8] Several authors[12] have pointed out that it is possible to construct theories in which the type (b) contribution is of order

$$\frac{m^2(\pi^+)-m^2(\pi^0)}{m^2(\pi)}\,.$$

This value actually gives fairly good agreement for the experimental $K^+\to\pi^+\pi^0$ rate. Contributions of type (c) are very hard to calculate reliably but they may in fact be the most important ones.

IV. $K\to3\pi$ AMPLITUDES

The Feynman diagrams for these decays are shown in Fig. 2. We shall calculate the amplitude for $K^0\to\pi^+\pi^-\pi^0$ (or $1/\sqrt{2}$ times the amplitude for $K_2\to\pi^+\pi^-\pi^0$). Since we shall not include electromagnetic corrections here, the $|\Delta I|=\frac{1}{2}$ rule holds, so it is not necessary to give the expressions for the other 3π-decay modes. The cross-hatched boxes in Fig. 2 stand for the off-shell $\pi\pi$ and πK scattering amplitudes.

The needed two-point effective weak interaction is found from Eqs. (3), (5), and (9b) to be

$$-\mathcal{L}_W(\phi\phi)=\sqrt{2}\,GX\alpha^2(1+W)[\partial_\mu\pi^-\partial_\mu K^+-(1/\sqrt{2})\partial_\mu\pi^-\partial_\mu K^0]+\text{H.c.}+\cdots\,. \tag{23}$$

The three-point effective weak interaction is similarly

$$-\mathcal{L}_W(S\phi^2)=GX\alpha\{-\sqrt{2}\,\partial_\mu\kappa_0(\partial_\mu\pi^-\pi^++\tfrac{1}{2}\partial_\mu\pi^0\pi^0)+\sqrt{2}\,\kappa_0(\partial_\mu\pi^-\partial_\mu\pi^++\tfrac{1}{2}\partial_\mu\pi^0\partial_\mu\pi^0)$$
$$+(1+W)\partial_\mu K_0[b'(\pi_0\overleftrightarrow{\partial}_\mu\sigma)+a'(\pi_0\overleftrightarrow{\partial}_\mu\sigma')]+\partial_\mu\pi^0[(b'-\sqrt{2}\,a')(K_0\overleftrightarrow{\partial}_\mu\sigma)+(a'+\sqrt{2}\,b')(K_0\overleftrightarrow{\partial}_\mu\sigma')]\}+\text{H.c.}\,, \tag{24}$$

where in terms of the scalar mixing angle θ_S,

$$a'\equiv\frac{1}{\sqrt{6}}(\sqrt{2}\cos\theta_S+\sin\theta_S)\,,\quad b'\equiv\frac{1}{\sqrt{6}}(\cos\theta_S-\sqrt{2}\sin\theta_S)\,.$$

Finally the on-mass-shell four-point effective weak interaction is

$$-\mathcal{L}_W(\phi^4)=-GX[2(\pi^0\overleftrightarrow{\partial}_\mu\pi^-)\partial_\mu\pi^++(\pi^-\overleftrightarrow{\partial}_\mu\pi^+)\partial_\mu\pi^0]K^0+\text{H.c.}+\cdots\,. \tag{25}$$

The contributions to the T amplitude for $K^0(p)\to\pi^+(q^+)$, $\pi^-(q^-)$, and $\pi^0(q^0)$ of each diagram in Fig. 2 are listed below:

$$T_a=-GX[m^2(K)+3p\cdot q^0]\,, \tag{26a}$$

$$T_b=-GX\alpha^2(1+W)\frac{m^2(\pi)}{m^2(K)-m^2(\pi)}T(K^0(p)\to\pi^+(q^+)\pi^-(q^-)K^0(q^0))\,, \tag{26b}$$

$$T_c=\sqrt{2}\,GX\alpha^2(1+W)\frac{m^2(\pi)}{m^2(K)-m^2(\pi)}T(K^0(p)\to\pi^-(q^-)\pi^0(q^0)K^+(q^+))\,, \tag{26c}$$

$$T_d=GX\alpha^2(1+W)\frac{m^2(K)}{m^2(K)-m^2(\pi)}T(\pi^0(p)\to\pi^+(q^+)\pi^-(q^-)\pi^0(q^0))\,, \tag{26d}$$

$$T_e=GX\alpha g_{\kappa K\pi}\frac{m^2(\pi)-2q^+\cdot q^-}{m^2(\kappa)+(q^++q^-)^2}\,, \tag{26e}$$

$$T_f = GX\alpha\bigg(\frac{g_{\sigma\pi\pi}}{m^2(\sigma)+(p-q^0)^2}\{(1+W)b'[m^2(K)+2p\cdot q^0]-(\sqrt{2}\,a'-b')[m^2(\pi)+2p\cdot q^0]\}$$
$$+\frac{g_{\sigma'\pi\pi}}{m^2(\sigma')+(p-q^0)^2}\{(1+W)a'[m^2(K)+2p\cdot q^0]+(a'+\sqrt{2}\,b')[m^2(\pi)+2p\cdot q^0]\}\bigg)\,. \tag{26f}$$

In the above the off-shell scattering amplitudes may either be computed explicitly or, since no derivative coupling is involved, read off from Eqs. (5.12) and (5.15) of Ref. 2. Explicitly,

$$T(K^0(p)\to\pi^+(q^+)\pi^-(q^-)K^0(q^0))=g_K^{(4)}-\left(\frac{g_{\kappa K\pi}{}^2}{m^2(\kappa)+(p-q^-)^2}+\frac{g_{\sigma\pi\pi}g_{\sigma KK}}{m^2(\sigma)+(p-q^0)^2}+\frac{g_{\sigma'\pi\pi}g_{\sigma'KK}}{m^2(\sigma')+(p-q^0)^2}\right), \tag{27a}$$

$$T(K^0(p)\to\pi^-(q^-)\pi^0(q^0)K^+(q^+))=\frac{1}{\sqrt{2}}(g_{\kappa K\pi})^2\left(\frac{1}{m^2(\kappa)+(p-q^0)^2}-\frac{1}{m^2(\kappa)+(p-q^-)^2}\right), \tag{27b}$$

$$T(\pi^0(p)\to\pi^+(q^+)\pi^-(q^-)\pi^0(q^0))=\tfrac{1}{2}g^{(4)}-\left(\frac{g_{\sigma\pi\pi}{}^2}{m^2(\sigma)+(p-q^0)^2}+\frac{g_{\sigma'\pi\pi}{}^2}{m^2(\sigma')+(p-q_0)^2}\right). \tag{27c}$$

Putting everything together gives the final result for $K_2\to\pi^+\pi^-\pi^0$:

$$T(K_2\to\pi^+\pi^-\pi^0)=-\frac{GX}{\sqrt{2}}\,m(K)[m(K)-2\omega_0]\bigg(1+\frac{(W-1)m^2(\pi)}{m^2(K)-m^2(\pi)}+\frac{m^2(K)[2m(K)\omega_0-m^2(\pi)]}{[m^2(K)-m^2(\pi)][m^2(\kappa)-m^2(K)-m^2(\pi)+2m(K)\omega_0]}$$
$$+\frac{2b'\{m^2(K)[m^2(\pi)-2m(K)\omega_0](\sqrt{2}\,a'-b')+b'm^2(\pi)[m^2(K)-2m(K)\omega_0](1+W)\}}{[m^2(K)-m^2(\pi)][m^2(\sigma)-m^2(K)-m^2(\pi)+2m(K)\omega_0]}$$
$$+\frac{2a'\{-m^2(K)[m^2(\pi)-2m(K)\omega_0](a'+\sqrt{2}\,b')+a'm^2(\pi)[m^2(K)-2m(K)\omega_0](1+W)\}}{[m^2(K)-m^2(\pi)][m^2(\sigma')-m^2(K)-m^2(\pi)+2m(K)\omega_0]}\bigg), \tag{28}$$

where ω_0 is the π^0 energy in the K_2 rest system.

The current-algebra result is obtained by setting the σ and σ' masses equal to infinity and also by putting $W=1$, which [see Eq. (3.9) of Ref. 2] sends the κ mass to infinity. This yields

$$T_{\rm ca}(K_2\to\pi^+\pi^-\pi^0)=-(GX/\sqrt{2})m(K)[m(K)-2\omega_0]\,. \tag{29}$$

Eliminating GX between this equation and the $W=1$ limit of Eq. (20) gives the well-known current-algebra relation[8] between the $K\to2\pi$ and $K\to3\pi$ amplitudes.

We note that the characteristic spectrum shape $\propto m(K)-2\omega_0$ predicted by (29) from the current-current form of the weak interaction is the same as the $\eta\to\pi^+\pi^-\pi^0$ spectrum shape $\propto m(\eta)-2\omega_0$ predicted from an effective electromagnetic tadpole interaction in the context of the present model.[3] Thus this similarity cannot be taken as evidence that both processes result from effective interactions with the same structure.

Now to evaluate (28) we need to know $m(\sigma)$, $m(\sigma')$, and the scalar mixing angle θ_S as well as W. However, since $m(\sigma)$, $m(\sigma')$, and θ_S are not well known we shall, for the purpose of getting an idea of the corrections, neglect the last two (σ and σ' pole) terms of (28). Then, the correction to the current-algebra result is, in linear approximation,

$$T(K_2\to\pi^+\pi^-\pi^0)\simeq T_{\rm ca}(1.22+0.19T_0)\,, \tag{30}$$

where $T_0=\omega_0-m(\pi^0)$. Equation (30) gives a spectrum to linear order in $|T|^2$ which agrees with experiment. Furthermore, the decay rate comes out to be larger than the current-algebra rate, which is about 15% too low.

It may be worthwhile to reemphasize that the fine details of the experimental measurements are not at all settled. In particular, some experimental analyses[13] of the $K\to3\pi$ spectra give a best fit for a linear squared matrix element (i.e., zero quadratic term). However, the current-algebra results give a linear matrix element which, when squared, definitely predicts a nonzero quadratic term. Thus higher-order terms in T_0 may be important. Equation (28) actually gives terms of all orders since the energy denominators occurring in it must be expanded. When the accuracy of the experimental work increases so that the coefficients of higher-order terms (if any) become clearly delineated it may be interesting to attempt a more ambitious fit than the crude estimate of (30).

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[1]For references see R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969); S. L. Adler and R. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).

[2]The specific form of the σ model we are using is described in J. Schechter and Y. Ueda, Phys. Rev. D 3, 2874 (1971). Further references may be traced from here.

[3]J. Schechter and Y. Ueda, Phys. Rev. D 4, 733 (1971).

[4]J. J. Sakurai, Phys. Rev. 156, 1508 (1967); W. W. Wada, *ibid.* 138, B1488 (1965); J. A. Cronin, *ibid.* 161, 1483 (1967).

[5]J. Schechter and Y. Ueda, Phys. Rev. D (to be published).

[6]To relate the η and η' masses an additional parameter defined in Eq. (3.7) of Ref. 2 is also required.

[7]The decay widths are given by

$$\Gamma(K\to\pi^i\pi^j)=(1-\tfrac{1}{2}\delta_{ij})\frac{1}{16\pi m(K)}\times\left(1-4\frac{m^2(\pi)}{m^2(K)}\right)^{1/2}|T(K\to\pi^i\pi^j)|^2,$$

where i and j can be $+$, $-$, or 0.

[8]Y. Hara and Y. Nambu, Phys. Rev. Letters 16, 875 (1966).

[9]We take ψ_1 and ψ_3 from Table I of Ref. 3 corresponding to the entry for $\epsilon_+{}^2=80.5$.

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Photoproduction of Neutral Vector Mesons in a Regge-Pole Model with Cuts

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Photoproduction of vector mesons in $\gamma p\to\rho V^0$ reactions is considered in a model with P, P', A_2, and π trajectories and PP cuts. The contributions of cuts are described phenomenologically; SU(3) symmetry is used for vertices. The results obtained are in good agreement with all existing experimental data.

The photoproduction of ρ, ω, and ϕ mesons in reactions

$$\gamma N\to V^0 N \tag{1}$$

is considered in the model which takes into account P, P', A_2, and π Regge poles and phenomenologically parametrized PP cuts.

Reactions (1) are described by 12 helicity amplitudes. Following the standard procedure[1,2] we construct the kinematic-singularity-free t-channel helicity amplitudes and get three conspiracy relations

$$\bar f^-_{00}-i\bar f^-_{10}=O(\sqrt{t}\,), \tag{2a}$$

$$\bar f^-_{01}-i\bar f^+_{11}=O(\sqrt{t}\,), \tag{2b}$$

$$\bar f^-_{02}-i\bar f^+_{12}=O(\sqrt{t}\,). \tag{2c}$$

We have chosen an evasive solution to these equations for contributions of the Regge trajectories and a conspirative solution for the cut. Thus, the kinematical factors for the pole part of the amplitudes are[3]

$$\begin{aligned}
&K^+_{00}=2(t-\mu^2)^{-1}(t-4m^2)^{-1/2}, && K^-_{00}=\tfrac{1}{2}t^{1/2}(t-\mu^2)^{-1},\\
&K^+_{01}=1, && K^-_{01}=\tfrac{1}{2}t^{1/2}(t-4m^2)^{1/2},\\
&K^+_{02}=\tfrac{1}{2}(t-\mu^2)(t-4m^2)^{1/2}, && K^-_{02}=\tfrac{1}{2}(t-\mu^2)(t-4m^2)^{1/2},\\
&K^+_{10}=1, && K^-_{10}=\tfrac{1}{2}t^{1/2}(t-4m^2)^{1/2},\\
&K^+_{11}=t^{1/2}, && K^-_{11}=\tfrac{1}{2}(t-4m^2)^{1/2},\\
&K^+_{12}=\tfrac{1}{2}t^{1/2}(t-\mu^2)(t-4m^2)^{1/2}, && K^-_{12}=\tfrac{1}{4}(t-\mu^2)(t-4m^2),
\end{aligned} \tag{3}$$