CALT-68-297, 1971 (unpublished).

[6]J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971); D. J. Gross and S. B. Treiman, *ibid.* 4, 1059 (1971).

[7]Gross and Treiman, see Ref. 6.

[8]D. J. Gross and S. B. Treiman, Phys. Rev. D 4, 2105 (1971).

[9]J. Iliopoulos and E. A. Paschos, a contributed paper to the 1971 International Symposium on Electron and Photon Interactions at High Energies, Cornell (unpublished).

[10]N. Arteaga-Romero, A. Jaccarini, and P. Kessler, C. R. Acad. Sci. Ser. B269, 153, 1129 (1969); V. E. Balakin, V. M. Budnev, and I. F.Ginsburg, ZhETF Pis. Red. 11, 559 (1970) [JETP Lett. 11, 388 (1970)]; S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters 25, 972 (1970); Phys. Rev. D 4, 1532 (1971), and many references therein.

[11]T. F. Walsh, Phys. Letters 36B, 121 (1971); S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters 27, 280 (1971); C. E. Carlson and W. K. Tung, Phys. Rev. D 4, 2873 (1971).

[12]For the kinematical region where $-k_1{}^2$ and $-k_2{}^2$ are fairly large but still small compared to P^2, see the following two works: A. V. Efremov and I. F. Ginzburg, Phys. Letters 36B, 371 (1971), and R. Kingsley, Cambridge University Report No. DAMTP 71/29, 1971 (unpublished).

[13]The author is indebted to Professor K. Kikkawa for suggesting this point.

[14]Notice that our definiton of $M_{\alpha\beta;\mu\nu}$ differs from that in Ref. 8 by $(2\pi)^4$. We assume that the electromagnetic current commutator does not have any q-number Schwinger term.

[15]Our result is by $2(2\pi)^4$ smaller than that given in Eq. (68) of Ref. 8. See Ref. 13. The author thanks Professor D. J. Gross and Professor S. B. Treiman for correspondence on this point.

[16]To make the definition of ϕ clear, find

$$P^2=4(E-E_1')(E-E_2')-2E_1'E_2'(1-\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos\phi).$$

[17]This conjecture has a good chance to become true because the SLAC-MIT experiments revealed Bjorken's predicted scaling at surprisingly low momentum transfers and because the threshold values of P^2 for continuum states are much smaller than those in deep-inelastic electroproductions.

[18]The angles θ_1, θ_2, and ϕ are 21°, 9.4°, and 111°, respectively.

[19]For $E=3$ GeV, $E_1'=E_2'=2.5$ GeV, $q^2=-1.5$ GeV^2, $\omega=1.8$, and $P^2=1$ GeV^2, the effective cross section is of the order of 10^{-38} cm^2 for $\langle Q^4\rangle=1$. Therefore, if the scaling does not start at $-q^2$ less than 1 GeV^2 as it *does* in deep-inelastic electroproduction, it is hard to perform this test in the near future. Even in this case, however, this test is much more feasible than the test of Gross and Treiman because their effective cross section is estimated to be less than 10^{-42} cm^2 for the same kinematical values given above.

[20]V. L. Auslander *et al.*, Phys. Letters 25B, 433 (1967); J. E. Augustin *et al.*, *ibid.* 28B, 508 (1969).

PHYSICAL REVIEW D VOLUME 5, NUMBER 9 1 MAY 1972

Chiral and Dilational Symmetry Breaking and the $\eta' \to \eta\pi\pi$ Decay Rate*

P. Weisz, Riazuddin, and S. Oneda†

Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

(Received 9 December 1971)

Assuming the Gell-Mann ansatz for the structure of the hadronic energy density $\Theta_{00}=\overline{\Theta}_{00}+u+\delta$ with $u=\epsilon_0(S_0+cS_8)$ belonging to a $(3,3^*)+(3^*,3)$ representation of SU(3)⊗SU(3), $c\approx-1.25$, and the additional ansatz $\delta=$ constant, we obtain an estimate for the decay rate, $\Gamma(\eta'\to\eta\pi\pi)\lesssim 0.2$ MeV, assuming that the $\eta'(958)$ is the ninth pseudoscalar meson.

I. INTRODUCTION

In a recent paper,[1] two of the present authors (R. and S.O.) presented a calculation of the rate Γ for the decay $\eta'(958)\to\eta\pi\pi$, under the assumption A that the SU(3)⊗SU(3) symmetry is of the Goldstone-Nambu type and that the symmetry-breaking Hamiltonian density u belongs to a $(3,3^*)+(3^*,3)$ representation of SU(3)⊗SU(3).[2,3] An anomalously small value for Γ was obtained and it was concluded in paper I that assumption A is probably invalid. It could be argued that such a conclusion can only be regarded as heuristic, since, of course, a prediction of an extremely small width Γ cannot rule out the theory involved, at least until experiment proves otherwise. The main purpose of this paper is, however, to point out that by modifying some of the at first sight seemingly harmless additional assumptions made in paper I, but retaining assumption A, that a value of ~0.2 MeV can be obtained as the approximate upper limit for $\Gamma(\eta'\to\eta\pi\pi)$, which is an order of magnitude larger than that obtained in paper I. The value $\Gamma\lesssim 0.2$ MeV is still surprisingly small for a strong decay and, if confirmed by future experiment, this could be proclaimed as a success of the model under consideration.

There are two main assumptions which were made in paper I that we modify here. The natures of these assumptions are somewhat distinct and hence we can consider the changes to the estimate for Γ in two independent steps.

The first modification concerns the behavior of the transition amplitude T over the Dalitz plot. In paper I it was assumed that T was approximately constant over the Dalitz plot. Here (in Sec. II), however, we allow for some variation which is experimentally indicated and find that by taking this into account a factor of about 5 is gained for Γ.

The second change is considered in Sec. III and concerns the matrix elements $\langle M_l|S_0|M_j\rangle$. In paper I it was assumed that $\langle M_l|S_0|M_j\rangle=\delta_{lj}\times$ (constant). We now prefer to drop this assumption which fails in general if dilation invariance is spontaneously broken, and in its place we assume the Gell-Mann decomposition for the energy density $\Theta_{00}=\overline{\Theta}_{00}+u+\delta$. We assume, moreover, that δ is a constant; this assumption is the most elegant and so far there is no evidence against it.[4] The magnitude of the amplitude T turns out to be quite sensitive to the value of the dimension d of u. For $d=2$ we recover the result for T obtained in paper I with $m_0=0$. However, for $d=3$ (as suggested by a quark model) the value for T is approximately double that obtained in paper I. In Sec. IV we add some comments concerning the physical implication of the present estimate.

II. DALITZ-PLOT CONSIDERATIONS

To make this paper self-contained we repeat some of the steps performed in paper I. We are considering the decay $\eta'(p')\to\pi_i(k_1)+\pi_j(k_2)+\eta(p)$, and define the variables

$$\begin{aligned}\nu&=-\frac{(k_2-k_1)\cdot(p+p')}{4m_{\eta'}}\\&=\nu_0+\frac{k_1\cdot(p+p')}{2m_{\eta'}}\\&=-\nu_0-\frac{k_2\cdot(p+p')}{2m_{\eta'}}\,,\\ \nu_0&=\frac{(m_{\eta'}{}^2-m_\eta{}^2)}{4m_{\eta'}}\,,\\ \nu_B&=-\frac{k_1\cdot k_2}{2m_{\eta'}}\,,\\ t&=-(p'-p)^2\,.\end{aligned}\tag{1}$$

Using standard current-algebra and PCAC (partial conservation of axial-vector current) techniques[5] and taking $k_2=0$, $k_1{}^2=0$ and $k_1=0$, $k_2{}^2=0$, respectively, we obtain

$$\begin{aligned}\delta_{ij}T(\nu=-\nu_0,\,\nu_B=0,\,k_1{}^2=0,\,k_2{}^2=0)\\&=\delta_{ij}T(\nu=\nu_0,\,\nu_B=0,\,k_1{}^2=0,\,k_2{}^2=0)\\&=-\frac{2}{f_\pi{}^2}\langle\eta(p')|\Sigma_{ij}(0)|\eta'(p)\rangle|_{t=0}\,,\end{aligned}\tag{2}$$

where $T(\nu,\nu_B,k_1{}^2,k_2{}^2)$ is the weak off-shell amplitude with normalization

$$\begin{aligned}\delta_{ji}T(\nu,\nu_B,k_1{}^2=-m_\pi{}^2,k_2{}^2=-m_\pi{}^2)\\&=-i\langle\eta(p)\pi_j(k_2)\pi_i(k_1)|S|\eta'(p')\rangle\\&=T(\nu,\nu_B)\delta_{ji}\,,\end{aligned}\tag{3}$$

and Σ_{ij} is the usual Σ commutator

$$\Sigma_{ij}(0)=i[F_i^5(0),\,\partial A_j(0)].\tag{4}$$

In addition to the above constraints we also have the Adler zeros[5]

$$\begin{aligned}T(\nu=\nu_0,\,\nu_B=0,\,k_1{}^2=0,\,k_2{}^2=-m_\pi{}^2)\\&=0\\&=T(\nu=-\nu_0,\,\nu_B=0,\,k_1{}^2=-m_\pi{}^2,\,k_2{}^2=0).\end{aligned}\tag{5}$$

With assumption A, that the SU(3)$\otimes$SU(3)-symmetry-breaking part u of the Hamiltonian belongs to a $(3,3^*)+(3^*,3)$ representation of SU(3)$\otimes$SU(3),[3] we have

$$u=\epsilon_0(S_0+cS_8),\tag{6}$$

and Σ_{ij} becomes

$$\Sigma_{ij}=\delta_{ij}\tfrac{1}{3}\epsilon_0(c+\sqrt{2})(\sqrt{2}\,S_0+S_8).\tag{7}$$

Bose statistics require T to be an even function in ν, and thus the conditions (2) and (6) can be written

$$T(\nu^2=\nu_0{}^2,\,\nu_B=0,\,k_1{}^2=0,\,k_2{}^2=0)=-\frac{2}{f_\pi{}^2}\sigma_{\eta\eta'}\tag{8}$$

and

$$\begin{aligned}T(\nu^2=\nu_0{}^2,\,\nu_B=0,\,k_1{}^2=0,\,k_2{}^2=-m_\pi{}^2)\\&=0\\&=T(\nu^2=\nu_0{}^2,\,\nu_B=0,\,k_1{}^2=-m_\pi{}^2,\,k_2{}^2=0),\end{aligned}\tag{9}$$

respectively, where

$$\sigma_{\eta\eta'}=\tfrac{1}{3}\epsilon_0(c+\sqrt{2})\langle\eta(p)|\sqrt{2}\,S_0+S_8|\eta'(p')\rangle|_{t=0}\,.\tag{10}$$

For the off-shell amplitude T we shall assume an approximate linear variation in the variables ν_B, $k_1{}^2$, $k_2{}^2$ in the region $0\gtrsim k_1{}^2\gtrsim-m_\pi{}^2$, $0\gtrsim k_2{}^2\gtrsim-m_\pi{}^2$ and ν_B and ν in the neighborhood of the physical region,

$$T(\nu,\nu_B,k_1{}^2,k_2{}^2)\approx a+b(k_1{}^2+k_2{}^2)+e\nu_B\,.\tag{11}$$

Then using the conditions (8) and (9) we have for the on-shell amplitude in the neighborhood of the physical region

$$T(\nu, \nu_B) = \frac{2}{f_\pi^2}\sigma_{\eta\eta'} + e\nu_B\,. \tag{12}$$

One may ask, what is the intrinsic error introduced by the linear assumption of linear variation of T in the region of interest? This is certainly hard to estimate, but we doubt whether this assumption introduces errors for the on-shell amplitude much greater than 20% (and hence errors for Γ much greater than 50%). Our argument is simply that the first nonsmoothness can be expected to be introduced by some enhancements in the π-π channel, i.e., for $t \approx m_\sigma^2$. But if one accepts a rather large value $\approx$700 MeV for m_σ then (t/m_σ^2) reaches a maximum value $(m_{\eta'} - m_\eta)^4/m_\sigma^4 \approx 0.1$ in the physical region for $\eta' \to \eta\pi\pi$ and is much smaller in the bulk of phase space. This we consider supports our statement above that our assumption of linear variation probably introduces errors for Γ less than 50%. Furthermore, we eventually take e from a linear fit to experimental data and this consideration is also obviously relevant to our theoretical procedure.

Note that since the Σ term (10) is itself of order m_π^2 an extrapolation of the form (11) is at least necessary; however, even with this linear extrapolation to go from zero-mass pions for which current-algebra constraints hold to the mass-shell pions, the slope parameter e is not fixed. In paper I the assumption that T is approximately constant over the Dalitz plot was made, i.e., $e = 0$; however, experimentally this does not appear to be the case. Writing

$$T(\nu, \nu_B) = M(1 + \alpha y), \tag{13a}$$

where

$$y = -1 + \frac{2(m_\eta + 2m_\pi)}{m_\pi(m_{\eta'} - m_\eta - 2m_\pi)} \times \left[\frac{(m_{\eta'} - m_\eta)^2 - 2m_\pi^2}{4m_{\eta'}} - \nu_B\right], \tag{13b}$$

we have

$$M = \frac{(2/f_\pi^2)\sigma_{\eta\eta'}}{(1 + 2.0\alpha)}\,. \tag{14}$$

Osborn and Wallace[6] have calculated the phase-space integrals relativistically and find

$$\Gamma = 3(1.00 + 0.24\alpha + 0.27\alpha^2)|M|^2 \text{ keV}. \tag{15}$$

We see, therefore, from (14) and (15) that our estimate for Γ is very sensitive to the slope parameter α.

For $\alpha = 0$ as assumed in paper I, the result is essentially the same as one would obtain for a nonrelativistic calculation and one had

$$\Gamma \approx 3.0\left|\frac{2}{f_\pi^2}\sigma_{\eta\eta'}\right|^2 \text{ keV}.$$

However, the experimental estimate for α is $\alpha = -0.28 \pm 0.06$,[7] representing a quite appreciable enhancement in the number of events for higher dipion invariant mass values and thus probably reflecting the presence of the σ-meson resonance. This gives, using (14) and (15),

$$\Gamma \approx 14.8\left|\frac{2}{f_\pi^2}\sigma_{\eta\eta'}\right|^2 \text{ keV}, \tag{16}$$

which is larger than the above-mentioned estimate in paper I by a factor ~5.

III. ESTIMATE FOR $\sigma_{\eta\eta'}$

There already exist many calculations[1,6,8,9] of the $\eta' \to \eta\pi\pi$ decay in the literature and here we first wish to comment briefly on some of them. If $\sigma_{\eta\eta'}$ is estimated as in paper I so that

$$\sigma_{\eta\eta'} = \frac{c+\sqrt{2}}{3c}p\cdot q[(1-\sqrt{2}\,c)(m_{\eta'}^2 + m_\eta^2 - 2m_K^2) + \sqrt{2}\,cm_0^2]$$

$$\approx \frac{c+\sqrt{2}}{3c}p\cdot q(2\text{ GeV}^2 - 1.8m_0^2)\,, \tag{17}$$

then a large value of the width Γ can be obtained only if m_0^2 is very large. Only if m_0^2 is of the order of 6 GeV^2 would one obtain the value of Γ from (16) and (17) with $c = -1.25$ to be of the order of 1 MeV as in Ref. 6. Thus although the calculation of Γ with $\sigma_{\eta\eta'}$ given in (17) is similar to that in Ref. 6 where a nonlinear chiral Lagrangian with $(3, 3^*) + (3^*, 3)$-symmetry breaking is used, paper I differs in the use of the value of m_0^2, for there it was thought that m_0^2 should lie in the range $0 \le m_0^2 \lesssim 1.25$ GeV^2 which is the natural scale for the pseudoscalar nonet. Certain other calculations[8] have been commented upon in Ref. 6 and concerning these we do not wish to add anything more. We just add that there also exists a calculation by Schechter and Ueda[9] who use a linear SU(3) σ model, but it is difficult to compare their results with paper I.

In this section, however, it is our main aim to present yet another estimate for $\sigma_{\eta\eta'}$ using presently acceptable ideas on dilation symmetry. We assume Gell-Mann's ansatz for the structure of the energy density

$$\Theta_{00} = \overline{\Theta}_{00} + u + \sum_i \delta_i\,, \tag{18}$$

where u, the only term breaking chiral symmetry, has scale dimension $d \neq 4$; $\overline{\Theta}_{00}$ has dimension 4 and δ_i are Lorentz scalars of dimension $d_i \neq 4$. We shall, however, make the more restrictive assumption that there is only one c-number δ present in Θ_{00}. As mentioned in the Introduction, this is certainly the most elegant assumption, and to the knowledge of the present authors, so far there is no evidence against it. Moreover, it seems that if there were a δ which played an important role for η-η' mixing, then it would be difficult to include its effect and at the same time justify a calculation based on similar assumptions to those we shall be making in the following.

The virial theorem is now

$$\Theta_{\mu\mu} = -(4-d)(u - {}_0\langle u \rangle_0), \tag{19}$$

and we have for any state $|M\rangle$

$$\langle M(p)|u|M(p')\rangle|_{t=0} = \frac{2m_M{}^2}{4-d} N, \tag{20}$$

where N is a state normalization factor which we choose equal to one for mesons and equal to $1/2m_M$ for baryons.

We define the physical states η, η' with obvious notation as

$$\eta = p\eta_8 + q\eta_0, \quad \eta' = q\eta_8 - p\eta_0,$$

with (21)

$$p^2 + q^2 = 1.$$

Then it follows immediately from (20) that

$$m_\eta{}^2 + m_{\eta'}{}^2 = m_{\eta_8}{}^2 + m_{\eta_0}{}^2. \tag{22}$$

For $i, j, l = 1, \ldots, 8$ we now assume that the following parametrization for the matrix elements of S_i and the vector currents $V_{i\lambda}$ between the pseudoscalar mesons,

$$\langle M_l(p)|S_i|M_j(p')\rangle|_{t=0} = \beta d_{ijl} \tag{23}$$

and

$$\langle M_l(p)|V_{i\lambda}|M_j(p')\rangle = if_{ijl}[f_+(t)(p'+p)_\lambda + f_-(t)(p'-p)_\lambda], \tag{24}$$

with $f_+(0) = 1$, are reasonably reliable. Then taking the divergence of (24), setting $t=0$, and using (25) and the relation

$$\partial V_i = \epsilon_0 c f_{i8m} S_m, \tag{25}$$

one obtains

$$f_{ijl}(m_j{}^2 - m_l{}^2) = \epsilon_0 c\beta f_{i8m} d_{mjl}. \tag{26}$$

Taking $M_j = K^-$, $i = 4 + i5$, and $l = 3$ and 8, respectively, one obtains first the Gell-Mann–Okubo mass formula

$$3m_{\eta_8}{}^2 - 4m_K{}^2 + m_\pi{}^2 = 0, \tag{27}$$

and second the matrix element

$$\epsilon_0\langle\eta_8|cS_8|\eta_8\rangle = -\epsilon_0 \frac{c\beta}{\sqrt{3}} = \tfrac{2}{3}(m_K{}^2 - m_\pi{}^2). \tag{28}$$

For convenience we now first rewrite $\sigma_{\eta\eta'}$ defined in (10) as

$$\sigma_{\eta\eta'} = f(c)p\cdot q(\langle\eta_8|u|\eta_8\rangle - \epsilon_0\langle\eta_8|cS_8|\eta_8\rangle - \langle\eta_0|u|\eta_0\rangle) + \{(1/c)\langle\eta|u|\eta'\rangle + \epsilon_0(\sqrt{2} - 1/c)[p\cdot q\langle\eta_0|cS_8|\eta_0\rangle + (q^2 - p^2)\langle\eta_8|S_0|\eta_0\rangle]\}, \tag{29}$$

where

$$f(c) = \tfrac{1}{3}(c + \sqrt{2})(\sqrt{2} - 1/c). \tag{30}$$

It seems reasonable to impose the orthogonality condition

$$\langle\eta|u|\eta'\rangle|_{t=0} = 0 \tag{31}$$

simultaneously with

$$\langle\eta_0|S_8|\eta_0\rangle|_{t=0} \approx 0 \approx \langle\eta_8|S_0|\eta_0\rangle|_{t=0}. \tag{32}$$

Then we may neglect the terms in the curly brackets in Eq. (29). Note that we do not assume that $\langle M_l|S_0|M_j\rangle = \delta_{lj} \times (\text{constant})$ for $l, j = 1, \ldots, 8$ as was done in paper I, as this would only be consistent with our other assumptions in the case $d = 2$.

The orthogonality condition (31) gives

$$p^2 - q^2 = \frac{m_{\eta_0}{}^2 - m_{\eta_8}{}^2}{m_{\eta'}{}^2 - m_\eta{}^2}, \tag{33}$$

which together with (21), (22), and (27) gives

$$|p\cdot q| \approx 0.21. \tag{34}$$

We now have our final estimate

$$\sigma_{\eta\eta'} \approx -f(c)p\cdot q\left[(m_{\eta'}{}^2 + m_\eta{}^2 - 2m_K{}^2) + \frac{d-2}{4-d}(m_{\eta'}{}^2 + m_\eta{}^2 - 2m_{\eta_8}{}^2)\right] \approx \pm 0.12 f(c)\left(\frac{d-2}{4-d} + 0.64\right). \tag{35}$$

The function $f(c)$ is very sensitive to the value of c. However, if we take $c \approx -1.25$ [i.e., $f(c) \approx 0.12$] the "preferred" value of Gell-Mann, Oakes, and Renner,[3] one obtains using $f_\pi = 0.134$ (Ref. 10)

$$\left|\frac{2}{f_\pi{}^2}\sigma_{\eta\eta'}\right|^2 \approx 2.0 \text{ for } d=1$$
$$\approx 3.7 \text{ for } d=2$$
$$\approx 11.8 \text{ for } d=3. \tag{36}$$

Note that the result for $d=2$ reduces to (17) in the case $m_0=0$.

IV. DISCUSSION AND CONCLUSION

We see from the result (36) that the estimate for Γ is quite strongly dependent on the value of the dimension d. Combining the results (16) and (36) a "maximum value" for Γ of about 0.2 MeV is obtained for $d=3$ (incidentally the value suggested by a quark model). Recall the similar situation for πN scattering. There the assumption of just a c-number δ gives

$$\sigma_{NN}=\tfrac{1}{3}(c+\sqrt{2})\left[\frac{\sqrt{2}\,M_N}{4-d}+\epsilon_0\left(\frac{1}{c}-\sqrt{2}\right)\langle N|cS_8|N\rangle\right], \tag{37}$$

assuming $\epsilon_0\langle N|cS_8|N\rangle$ is of the order of baryon mass splittings, e.g., ≈ -0.21, then for $c=-1.25$ one has

$$\sigma_{NN}\approx 0.073\left(\frac{1}{4-d}+0.38\right)\text{GeV}, \tag{38}$$

and again one has a maximum Σ term for $d=3$ of about 0.1 GeV. The latter value was in fact obtained in an analysis of Cheng and Dashen[11] using the assumption of linear variation in the pion mass variables of the weak off-shell πN amplitude to relate its value at the on-shell point ($\nu=0$, $\nu_B=0$) to σ_{NN}. However, there are too many varying estimates for σ_{NN} in the literature[12] and we prefer not to trust the estimate of Cheng and Dashen which in the above analysis favors the value $d=3$; indeed if one had the above situation with $\epsilon_0\langle N|S_0|N\rangle$ large, $\approx M_N$, then one would expect a very large Σ term in KN scattering of about 1 GeV and the question arises – why didn't von Hippel and Kim "see" such a large effect in their analysis?[13]

We sum up our conclusions as follows: The $\eta'\to\eta\pi\pi$ is a strong decay which proceeds without angular momentum barrier effects. Therefore, naively we may expect $\Gamma(\eta'\to\eta\pi\pi)\gtrsim 1$ MeV. For example, the very similar (but G-forbidden) process, $\eta\to\pi\pi\pi$, has a partial width around 1 keV. If this decay were not G-forbidden, we might have expected a partial rate for this decay of the order 1–10 MeV. Indeed popular models[14] (e.g., the Veneziano model and quark model) usually predict $\Gamma(\eta'\to\eta\pi\pi)\simeq 1-10$ MeV.

In this paper we have obtained under the assumption A an approximate upper bound of ~0.2 MeV for Γ. Although this bound is lower than the above-mentioned naive expectation, it could not be considered totally unacceptable, i.e., the attractive assumption A is not disfavored by the present mere theoretical consideration on Γ. However, we remark that even accepting possible large errors it would be hard, within the present framework, to attain a width Γ as large as 1 MeV, keeping c reasonably close to -1.25.[15] It would thus be very important to see whether the present experimental upper limit of $\sim 4\times 0.65=2.6$ MeV for Γ can be lowered by an order of magnitude. Of course, our bound depends also very strongly on the slope parameter α and it will be interesting to see whether such a large value of $\alpha=-0.26$ is found in future analyses of experiments with higher statistics.

We also note another rather surprising possibility which is relevant to the present problem. The spin of $\eta'(958)$ is not yet established, while there is another strong candidate for the ninth pseudoscalar meson, i.e., $E(1422)$. $E(1422)$ is listed[16] as 0^-, although the assignment 1^+ is not completely ruled out. Theoretically, there are indeed various considerations[17] which favor E as the ninth pseudoscalar meson. If the E meson [instead of $\eta'(958)$] were the ninth pseudoscalar meson, the above-discussed difficulty related to the rate $\Gamma(\eta'(958)\to\eta\pi\pi)$ disappears. The E meson has a decay mode $E\to\eta\pi\pi$ but the decay is dominated by the process $E\to\pi_N(1016)\pi\to\eta\pi\pi$.

In this connection then, the clear determination of the spin-parities of $\eta'(958)$ as well as that of $E(1422)$ is certainly awaited.

ACKNOWLEDGMENTS

Two of us (R. and O. S.) would like to thank Dr. J. Schechter, Dr. Y. Ueda, Dr. H. Osborn, Dr. D. J. Wallace, and Dr. D. P. Majumdar for correspondence.

*Supported in part by the National Science Foundation, Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, under Grant No. NSF-GU -2061.

†Supported in part by the National Science Foundation under Grant No. NSF-GP-8748.

[1] Riazuddin and S. Oneda, Phys. Rev. Letters 27, 548 (1971); 27, 1250(E) (1971), hereafter referred to as paper I.

[2] M. Gell-Mann, Physics 1, 63 (1964); S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[3] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys.

Rev. 175, 2195 (1968).

[4]J. Ellis, P. Weisz, and B. Zumino, Phys. Letters 34B, 91 (1971).

[5]See, e.g., S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968); B. Renner, *Current Algebras and Their Applications* (Pergamon, New York, 1968).

[6]H. Osborn and D. J. Wallace, Nucl. Phys. B20, 23 (1970).

[7]J. P. Dufey *et al.*, Phys. Letters 29B, 605 (1969).

[8]J. A. Cronin, Phys. Rev. 161, 1483 (1967); J. Schwinger, *ibid.* 167, 1432 (1968); D. P. Majumdar, Phys. Rev. Letters 21, 502 (1968); V. S. Mathur, S. Okubo, and J. S. Rao, Phys. Rev. D 1, 2058 (1970).

[9]J. Schechter and Y. Ueda, Phys. Rev. D 3, 2874 (1971).

[10]If we preferred, instead, to use the Goldberger-Treiman relation value for $f_\pi = Mg_A/g_{NN\pi} = 0.124$ we would enhance our estimate for Γ by 40%.

[11]T. P. Cheng and R. F. Dashen, Phys. Rev. Letters 26, 594 (1971).

[12]See, e.g., F. von Hippel and J. K. Kim, Phys. Rev. D 1, 151 (1970); G. Höhler, H. P. Jakob, and R. Strauss, Phys. Letters 35B, 445 (1971); M. Erikson and M. Rho, *ibid.* 36B, 93 (1971).

[13]For a more complete discussion see, e.g., B. Renner, Hamburg report, 1971 (unpublished).

[14]D. H. Dalitz, in *High Energy Physics, 1965 Les Houches Lectures*, edited by C. DeWitt and M. Jacob (Gordon and Breach, New York, 1966); I. Baacke, M. Jacob, and S. Pokorski, Nuovo Cimento 62A, 332 (1969); A. M. Harun-ar-Rashid, *ibid.* 64A, 985 (1969).

[15]Note, however, that if we take a small value, e.g., $c = -0.25$ (the value preferred by R. A. Brandt and G. Preparata), then for $d = 3$ we would obtain $\Gamma \approx 60$ MeV, thereby badly violating the experimental upper bound. However, we are prepared to concede that our present method of estimating Γ might not be valid at all if only weak PCAC holds.

[16]Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).

[17]J. Schwinger, Phys. Rev. Letters 12, 237 (1964); Riazuddin and K. T. Mahanthappa, Phys. Rev. 147, 972 (1966); D. Horn, J. J. Coyne, S. Meshkov, and J. C. Carter, *ibid.* 147, 980 (1966); A. N. Zaslavsky, V. I. Ogievetsky, and V. Tybor, JETP Lett. 6, 106 (1967); V. I. Ogievetsky, Phys. Letters 33B, 227 (1970); S. Oneda and Seisaku Matsuda, *ibid.* 37B, 105 (1971).

PHYSICAL REVIEW D VOLUME 5, NUMBER 9 1 MAY 1972

Lower Bound for the Hadronic Contribution to the Muon Magnetic Moment*

G. Auberson and Ling-Fong Li
The Rockefeller University, New York, New York 10021
(Received 2 February 1972)

A new lower bound is derived for the hadronic contribution to the anomalous magnetic moment of the muon. In spite of a careful treatment of the ρ-dominance region, the numerical improvement over a previous bound due to Langacker and Suzuki is not very significant.

I. INTRODUCTION

Recently, various interesting inequalities imposing restrictions on some vertex functions involving electromagnetic[1] or weak[2] hadronic currents have been derived by assuming cut-plane analyticity. Technically, the methods used in these derivations essentially rely on the Schwarz inequality or upon a quite different maximum principle originally introduced by Meiman[3] and recently improved by Okubo and the present authors.[4]

More specifically, Langacker and Suzuki,[5] and Palmer[6] have obtained in this way lower bounds for the hadronic contribution to the anomalous magnetic moment of the muon, $\Delta a_{(\mu)}$, in terms of the charge radius of the pion r_π. The bound derived in Ref. 6 by using Okubo's method is the best possible one with the total charge and charge radius of the pion given, and is free of any phenomenological input. On the other hand, the method of Ref. 5 assumes a once-subtracted dispersion relation for the form factor and requires in addition some knowledge of the p-wave $\pi\pi$ cross section, resulting in a substantial numerical improvement. Although the lower bound thus derived is a direct consequence of the Schwarz inequality, it can be shown that it is an optimal one if the pion charge radius and the p-wave cross section are the *only* input. However, the pion form factor which saturates this lower bound clearly violates Watson's theorem,[7] another condition which arises as soon as one decides to include the rather reliable experimental information on the low-energy p-wave $\pi\pi$ scattering, dominated by the ρ-meson contribution. Hence, one can expect a further improvement if the method can be modified in such a way that the phase of the pion form factor is enforced to have the correct behavior in the ρ region. The present paper is devoted to the solution of this problem. It turns out that the practical