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Bi-Regge-Pole Duality

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Following an earlier suggestion that local duality be applicable to high-energy wide-angle scattering, a duality equation for amplitudes is presented and discussed. Solutions are found in both equal- and unequal-mass cases.

I. INTRODUCTION

At present, one of our best hopes of understanding the strong interaction of elementary particles at high energy lies in the general approach of Regge theory. The literature abounds with various fits of forward- and backward-angle high-energy data. And, of course, the subject has been well-developed theoretically since Regge's original observation.

Early pion-nucleon scattering Regge-theory fits to experimental data were performed essentially beyond the region of resonances. Thus, the theory was applied asymptotically, where the simple s^{α} form was expected to be most valid. More recent work suggested that the theory is valid at lower energies in the sense that the extrapolated asymptotic fit should equal lower-energy resonant curves on the average. Though still questioned by theorists, this concept of local, i.e., Dolen-Horn-

Schmid, duality has wide acceptance.

Shortly after the discovery of the concept, it was proposed[1] that local duality, in this same sense, have a place in our attempts to understand high-energy large-angle scattering using Regge theory. We certainly have no real theory of such strong-interaction processes, even though a variety of models has been constructed in efforts to gain some insight into what is happening. Because of the nature of the energy dependence that it gives, and its successes in phenomenological analyses, it is, indeed, quite tempting to think that at least part of the amplitudes in this region contain Regge-pole terms. In fact, Pinsky's paper[1] showed that recent differential cross-section measurements of proton-proton scattering are characterized by the Regge form quite well. The earlier work of Abarbanel *et al.*[2] illustrates one way in which they could enter.

It is the purpose of this paper to consider further the possibility that local duality is applicable to a Regge-theory approach to high-energy large-angle scattering. Regge-theory amplitudes have a definite mathematical form, after all, and it is of interest to explore how well it lends itself theoretically to the requirement of duality. It is amplitudes, rather than cross sections, that we shall examine, and throughout the paper, it is locally averaged Dolen-Horn-Schmid duality which is under study. We shall conclude that it is applicable in principle, though many unsolved questions remain.

II. ANALYTICAL STUDY

Thus, let us confine our attention to high energies, beyond any resonances, and to intermediate scattering angles where the forward and backward peaks run into each other. Therefore, the duality under consideration is that between two Regge-theory sums. Though similar, this is quite a different matter from resonance–Regge-theory duality, and it is most strongly emphasized that no resonance considerations are relevant here.[3] No statement of the kind "Regge pole a is dual with resonance b" is meant or implied in the present work. Only the asymptotic presence of Regge poles in the direct channel is examined.

Let us briefly review what is involved. Conventional Mandelstam variables and Regge-theory functions will be employed throughout the discussion. Consider, for example, a scattering process occurring in the direct (s) channel at high energy. Its forward peak may be thought to be influenced by Regge-pole exchanges from one (the t) crossed channel, its backward peak by those from another (the u) crossed channel. In the region between the peaks, therefore, it is expected that the part of the amplitude which is made up of Regge poles involves those from both the t and u channels. We consider further the idea that the separate contributions to an amplitude from the two crossed channels be regarded as dual to each other, and proceed to explore this possibility mathematically. It means some restriction of the form of the terms, and it is appealing.

A. Equal-Mass Reaction

There is one kind of reaction in which this idea has a simple, natural place, namely the elastic scattering of two identical particles. Thus, we also consider elastic proton-proton scattering in the region around 90° in the center-of-mass system. The experimental cross section must be symmetric about 90°, which is a minimum point, and various mathematical forms[4,5] have been suggested to represent the scattering data. We suppose that Regge poles contribute here, at least in part, and concentrate our attention upon them. Though one might well expect Regge cuts to be present as well, it is naturally proper to confine consideration to just pole terms at this stage. Branch-cut contributions will be considered briefly in the last part of the paper. In this case, local duality may be fully expressed as

$$\sum_i \beta_i(t)\xi_i(t)s^{\alpha_i(t)} = \sum_i \beta_i(u)\xi_i(u)s^{\alpha_i(u)} \qquad (1)$$

in an averaged sense. In this equation, ξ is the conventional signature factor, and the same Regge poles are on both sides, though associated with two different crossed channels. It is conceptually simplest to first think of this as one in energy at a fixed scattering angle. We shall also consider it below as having two independent variables, such that the scattering angle is near 90°. Of course, the third Mandelstam variable is determined by these two. Actually, (1) is two equations, because it is meant to hold in both real and imaginary parts. We might call such a relationship a bi-Regge-pole duality equation, because two Regge sums only are facing each other. Resonances are not involved in any way. The common value is the total Regge-pole contribution at some (s, t, u) point, and the contributions from the two crossed channels do not interfere. In particular, any t-channel term does not interfere with its brother from the u channel of the same name.

What kinds of solutions does such an equation have, which fit in well with what we already know? The Regge form is specific, and we shall now search for ones that obey (1) closely. Basically speaking, then, we have a mathematical problem before us, and only the simplest possibilities will be discussed. Solutions of (1) could be quite complicated, and it is not simple to find them. The easiest way to obtain further insight is to assume that the trajectories be regarded as dual also. This means that a t-channel trajectory naturally becomes its u-channel brother of the same name, along the lines in which the original duality suggestion was made.[1] This means the added stringent requirement that

$$\alpha(t)=\alpha(u) \tag{2}$$

in the region of interest. This is not necessary, but simplifies what would otherwise be a difficult search, and is sensible, because the two crossed channels contribute the same trajectories. Duality is a useful concept just "on the average," so it is not surprising that (2) cannot hold exactly. (A direct violation of analyticity would be required.) However, the form

$$\alpha(x)=\alpha_0+\epsilon x^{-1} \tag{3}$$

for x in the region of interest is dual to order ϵ, and with increasing absolute value of x. Actually, x^{-1} has been used here for simplicity only, and any function which is reasonable enough would seem to do, too. If we demand that the residues be approximately dual, too, the duality equation will be trivially satisfied, and the entire Regge-pole contribution will be essentially independent of angle. The experimental data do have zero slope at the symmetrical point, of course, but they curve around it, so all curvature would then necessarily have to be accounted for by other contributing mechanisms. It seems likely that one Regge-pole term with strongly varying trajectory and residue functions cannot satisfy the equation closely, but this is not completely clear. Perhaps a larger, but finite, number can. For simplicity, we work with trajectory functions of the form (3) in the region of interest. It will presently be seen that such an equation can be satisfied with just two or three terms which give curvature, too. The latter is *a priori* necessary if such solutions can also make contact with experiment without additional terms.

For the purpose of looking for specific solutions, let us take the proton-proton duality equation to be in the form

$$\sum_i \beta_i(t)\xi_i(t)(s-4M^2)^{\alpha_i(t)}=\sum_i \beta_i(u)\xi_i(u)(s-4M^2)^{\alpha_i(u)} \tag{4}$$

at a fixed scattering angle, where M is the proton mass. We are free to do this, and its convenience will now become clear. In all further work, the residues will be taken to be real. In this reaction, the Mandelstam variables relate to each other and the scattering angle as follows:

$$t=-\tfrac{1}{2}(s-4M^2)(1-\cos\theta),$$
$$u=-\tfrac{1}{2}(s-4M^2)(1+\cos\theta).$$

(a) Two trajectories of opposite signature:

$$\beta_1(x)=-\beta_0 x,\quad \beta_2(x)=\beta_0 x^2,\quad \alpha_2=\alpha_1-1.$$

(b) Two trajectories of the same signature:

$$\beta_1(x)=-\beta_0 x,\quad \beta_2(x)=\tfrac{4}{3}\beta_0 x^3,\quad \alpha_2=\alpha_1-2.$$

(c) Two leading trajectories of the same signature and a third having opposite value:

$$\beta_1(x)=\beta_0 x,\quad \beta_2(x)=-2\beta_0 x^3,\quad \beta_3(x)=\beta_0 x^4,$$
$$\alpha_2=\alpha_1-2,\quad \alpha_3=\alpha_1-3.$$

The quantities β_0, τ_1, and α_1 are arbitrary in these examples. In (a) and (c) the duality equation (4) is satisfied to order ϵ, but in (b), just to second power of the angle. It is suggested that the reader performs the substitutions and verifies this, as an aid in understanding the spirit of this approach. Remember that x is a general variable standing for either t or u. It will be used in this same way throughout the discussions below. In general, if an α_i is near an integral value, an appropriate factor must be included in its residue to cancel any singularity arising from its signature factor. At this point, let us reflect somewhat, trying to see what we are doing in perspective. In an equal-mass reaction such as this, the duality condition (1) is really just asking that the total Regge-pole contribution be symmetrical about the 90° ($t=u$) point. These examples accomplish this because they have only even powers of $\cos\theta$, as the duality condition thus requires.

Now that we have seen that the duality equation has solutions, it is natural to wonder if ones can be found which can be related to the actual physical world. The experimental data have sometimes seemed to require amplitudes of the form[4,5]

$$e^{-ak\sin\theta},\quad k=\tfrac{1}{2}(s-4M^2)^{1/2},$$

for example. For the present discussion, we assume that this cross-section form transfers its imprint directly to the amplitudes. This is sufficient, simple, and clearly at least somewhat correct. Another solution shows that a dual sum of just four Regge poles can provide this approximately, for angles restricted to be near enough 90° that

$$e^{-ak\sin\theta}\approx e^{-ak}(1+\tfrac{1}{2}ak\cos^2\theta), \tag{5}$$

and a restricted enough energy region that the exponential may be replaced by a power of momentum. Take trajectories of the form (3) again, and

$$\beta_1(x) = -\tfrac{1}{4}\beta_0 + \epsilon x^{-1}, \quad \alpha_1, \quad \tau = +;$$

$$\beta_2(x) = \beta_1 + \epsilon x^{-1}, \quad \alpha_2 = \alpha_1 - \tfrac{1}{2}, \quad \tau = +;$$

$$\beta_3(x) = \beta_0 x, \quad \alpha_3 = \alpha_1 - 1, \quad \tau = -;$$

$$\beta_4(x) = -\beta_0 x^2, \quad \alpha_4 = \alpha_1 - 2, \quad \tau = +.$$

The quantities β_0, β_1, and α_1 are determined from the data form (5), and this set of terms satisfies the duality equation (4) to order ϵ. Notice that the trajectory order and signatures for the first three poles above correspond exactly with what is expected for the three known leading boson Regge poles on the basis of detailed proton-proton small-angle fits.[6] This is desirable if it is hoped to identify these sometime as continuations of the known poles. Different residue and trajectory forms and/or more poles will be needed, the wider the angular range and the closer the approximation to exponential energy behavior desired.

The results of experiment are not final yet, but other forms proposed to represent the scattering data[4,5] may be treated similarly. For example, the more universal form suggested by the experiment of Allaby *et al.*,[5]

$$e^{-as\sin\theta},$$

can be achieved by a similar solution. We stay close enough to 90° that

$$e^{-as\sin\theta} \approx e^{-as}(1 + \tfrac{1}{2}as\cos^2\theta), \tag{6}$$

and a restricted enough energy region to again use a power of momentum for the exponential. To the extent that these kinds of ideas are successful in a localized region around the symmetrical point, one might daringly try to describe a more appreciable part of the identical forward and backward peaks by bi-Regge-pole dual amplitudes.

The correctness of duality can only be surmised from specific numerical studies, of course, but we can examine its possible consistency in very general terms. Any form of the experimental cross section in energy and angle variables must be symmetrical about 90°, i.e., in the interchange of t and u. We assume again that an amplitude takes this same form.[7] Furthermore, let us assume that only Regge poles and cuts contribute in this region, or, equivalently, and as might be the case, that any other contributing mechanism is symmetrical about 90°. The duality approach suggests that all or part of an amplitude required by any experimental form may be written directly as a Regge sum of either t-channel contributions or of u-channel contributions. Such an attitude then implies, by itself, that these will be locally dual with each other in the sense of satisfying Eq. (1), independent of pole and cut function specifics. For example, the form we have been considering here, (5), may be written in terms of Mandelstam variables as

$$\exp\{-a[tu/(s-4M^2)]^{1/2}\}. \tag{7}$$

It is to be emphasized that we are localized in a small region around 90° ($t=u$) scattering angle. Let us suppose that a dual description is, indeed, preferable to an interfering one. We would regard this form as written out in terms of just a t-channel contribution, coming from the substitution in (7) of

$$u = 4M^2 - s - t,$$

or in terms of just a u-channel one, coming from the substitution

$$t = 4M^2 - s - u.$$

Because the reaction involves equal masses, the most natural possibility is that the parameters of the latter have exactly the same form as those of the former. Then, (4) is satisfied in the manner of the solutions above with common value (7), regardless of the details of individual pole and cut terms. We conclude that, in general, the equal-mass reaction variables allow a consistent duality interpretation.

B. Unequal-Mass Reaction and Regge Cuts

Duality between Regge poles as discussed here may be considered in unequal-mass reactions as well. For example, consider the region where the forward and backward peaks meet in pion-nucleon scattering. The duality condition may then be written as

$$\sum_i \beta_{T_i}(t)\xi_{T_i}(t)s^{\alpha_{T_i}(t)} = \sum_i \beta_{U_i}(u)\xi_{U_i}(u)s^{\alpha_{U_i}(u)-1/2}, \tag{8}$$

for the Regge poles in the invariant Mandelstam amplitudes. Now, the sums are over two completely different kinds of Regge poles and, at first sight, it seems as if the equation may be satisfied in a number of ways. We confine ourselves to pointing out one hypothetical example. Take the duality equation as

$$\sum_i \beta_{T_i}(t)\xi_{T_i}(t)[s-(2M^2+2m_\pi^2)]^{\alpha_{T_i}(t)} = \sum_i \beta_{U_i}(u)\xi_{U_i}(u)[s-(2M^2+2m_\pi^2)]^{\alpha_{U_i}(u)-1/2}, \tag{9}$$

in correspondence with what was done in the equal-mass case where $(s-4M^2)$ was used instead. Trajectories of the form (3) are used again. Then,

$$\beta_{T_1}(x) = \beta_{U_1}(x) = -\beta_0 x, \quad \text{signature } \tau$$

$$\beta_{T_2}(x)=\beta_{U_2}(x)=\beta_0 x^2\,,\quad \text{signature } -\tau$$

$$\alpha_{T_1}=\alpha_{U_1}-\tfrac{1}{2}\,,\quad \alpha_{T_2}=\alpha_{U_2}-\tfrac{1}{2}\,,\quad \alpha_{T_2}=\alpha_{T_1}-1\,,$$

where β_0 and α_{T_1} are free constants, satisfies the duality equation (9) to order ϵ. This can be readily verified by eliminating $[s-(2M^2+2m_\pi^2)]$ in favor of $-(t+u)$, and substituting in the residue and trajectory forms. One might tentatively identify the u-channel terms as the known nucleon and Δ poles, and those of the t channel as one of the Pomeranchons and the ρ. In this solution, a t-channel residue function is directly dual with, i.e., directly becomes, one of the u channel, but the residue functions are split apart by $\frac{1}{2}$. The latter has naturally arisen here, because boson spins are integral, whereas fermion spins are half-integral. Because of the complicating presence of a square root, it is difficult to find a solution in which one trajectory naturally becomes a different one of the other channel. In general, the unequal-mass case is more complicated, and much more effort seems to be required in order to construct examples which can be connected to experiment.

In principle, full poles involving $P_\alpha(\cos\theta_x)$ or $P_{\alpha+1/2}'(\cos\theta_x)$, instead of the simple asymptotic form, should be present in the duality equations (1) and (8). This is an inessential theoretical complication which can be considered later.

Regge cuts could be present in the amplitudes, too. We suppose that the cuts from the two channels be added to the respective poles in the duality equation, or maybe satisfy a separate one of their own. In the proton-proton scattering case, the cuts from the two cross channels will be the same, except for the variable (i.e., crossed channel) involved, just as the poles were. For definiteness, suppose we have[8]

$$\sum_i B_i(x)s^{\alpha_{ci}(x)}(\ln s)^{-\gamma_{ci}(x)} \tag{10}$$

for the leading contribution associated with a given crossed channel, where x is again either t or u, and γ_c depends upon the branch cut discontinuity. If α_c and γ_c have the form (3) in the region of interest, simple dual solutions can probably be found for cuts, just as they were for poles.

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