which is the minimally coupled nonlinear model.

To conclude, we have shown that the minimal-coupling and nonlinearization procedures are commutative, that is, the minimally coupled nonlinear model is indeed the infinite-chiral-scalar-mass limit of the linear model. Therefore, just as is the case without the electromagnetic interaction, any results obtained from the linear models, independent of the chiral scalar mass, should be uniquely related to the corresponding results obtained from the nonlinear models. This conclusion can also be straightforwardly generalized to models concerning the chiral $SU(3)\otimes SU(3)$ symmetry.

The author thanks Professor E. S. Abers for suggesting the problem and encouragements. Some illuminating discussion with Professor C. J. Goebel is also greatly appreciated.

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[1]See, for example, E. S. Abers and S. Fels, Phys. Rev. Letters 26, 1512 (1971); R. Aviv, N. D. Hari Dass, and R. F. Sawyer, *ibid.* 26, 591 (1971); T. F. Wong, *ibid.* 27, 1617 (1971).

[2]S. Weinberg, Phys. Rev. Letters 18, 188 (1967).

[3]J. Schwinger, Ann. Phys. (N.Y.) 2, 407 (1957); M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

[4]We use the three-vector notation $\vec{a}$ as vectors in the isospin space.

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η and Pion Decays into Lepton Pairs*

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A recent measurement of the branching ratio $\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to 2\gamma)$ yielded the value $(5.9\pm2.2)\times10^{-5}$. We examine here the implication of this result on theoretical predictions for the real part of the $\eta\to\mu^+\mu^-$ decay amplitude and conclude that the branching ratio is uncomfortably large for most of the models proposed so far. We then evaluate the $\eta\to\mu^+\mu^-$ decay rate in the baryon loop model. Predictions are also made for the associated decays $\eta\to\gamma\mu^+\mu^-$, $\pi^0\to e^+e^-$, and $\pi^0\to\gamma e^+e^-$.

I. INTRODUCTION

Recently, measurements have been reported by Hyams *et al.*[1] on the decay $\eta\to\mu^+\mu^-$ and by Clark *et al.*[2] on the decay $K_L^0\to\mu^+\mu^-$. Since these decays can proceed electromagnetically through a two-photon state it is instructive to give the experimental numbers as branching ratios, i.e., $\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to2\gamma)=(5.9\pm2.2)\times10^{-5}$ and $\Gamma(K_L^0\to\mu^+\mu^-)/\Gamma(K_L^0\to2\gamma)<0.31\times10^{-5}$ (90% confidence level). To the extent that neutral currents are neglected the only other physical states permissible in these decays are $2\pi\gamma$, 3π, and $3\pi\gamma$. We will assume for the moment that these states can be neglected completely. Then the decay amplitudes for $\eta\to2\gamma$ and $K_L^0\to2\gamma$ are real and can be taken from experiments. If we further assume CP invariance and standard quantum electrodynamics for the muon interaction then the above branching ratios should be almost identical because the masses of the η and K mesons are approximately equal. Numerically one finds[3] $\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to2\gamma)\geq1.07\times10^{-5}$ and[4] $\Gamma(K_L^0\to\mu^+\mu^-)/\Gamma(K_L^0\to2\gamma)\geq1.17\times10^{-5}$. These values, the so-called "unitarity bounds," are expected to be very reliable. There will be of course a small variation due to interferences with other physical states in the imaginary parts of the amplitudes. However, the addition of the real parts of the amplitudes can only increase the branching ratios given above.

It is obvious that the experimental results are significantly different from the unitarity bounds. The result for η decay is compatible with its bound whereas the result for K_L^0 decay is not. As regards the latter, several attempts have been made to explain the discrepancy by allowing interference with other channels, i.e., the $2\pi\gamma$, 3π, and $3\pi\gamma$. Original estimates made by Martin, de Rafael, and Smith[5] have been improved by Farrar and Treiman,[6] Gaillard,[7] Aviv and Sawyer,[8] and Adler, Farrar, and Treiman.[9] These states are now known to allow a variation of approximately 10% in the unitarity bound for K_L^0 decay. Such calculations (for the CP-conserving part of the decay amplitudes) are immediately applicable to η decay. In fact we have checked that the $\pi\pi\gamma$ intermediate state can only change the unitarity bound for η decay by 2%. Another possible explanation of the K_L^0 result has been proposed by Christ and Lee,[10] namely, that a CP violation is necessary to produce a destructive interference and thereby lower the unitarity bound. Such a mechanism has no implications for the η meson because it decays electromagnetically. In this paper we do not address ourselves to the problem of the K_L^0 decay but would like to examine the implication of the η result on models for the real part of the $\eta\to\mu^+\mu^-$ decay amplitude. Note that models for η decay also yield predictions for the decay $\pi^0\to e^+e^-$ for which there are no experimental results.

If we continue to assume that the decay is purely through the 2γ state then the real part of the decay amplitude must come from a dispersion integral over the two-photon cut. Let us take the coupling of $\eta(P)\to\gamma(k_1)+\gamma(k_2)$ to be of the form

$$A(\eta\to 2\gamma)=F_\eta(k_1^{\,2},k_2^{\,2})\epsilon_{\alpha\beta\gamma\delta}\epsilon_1^\alpha\epsilon_2^\beta k_1^\gamma k_2^\delta\,, \qquad (1.1)$$

so that the on-mass-shell decay $\eta\to 2\gamma$ measures $F_\eta(0,0)$. If $F_\eta(k_1^{\,2},k_2^{\,2})$ is taken to be a constant then the real part of the decay amplitude for $\eta\to\mu^+\mu^-$ is logarithmically divergent. It would therefore seem at first sight to be easy to produce a model for $F_\eta(k_1^{\,2},k_2^{\,2})$ which produces a large real part and thereby explains the experimental number given above. However this is not so easy. If we want to produce a real part of the amplitude which is twice as large as the imaginary part then the cutoff mass Λ in the logarithmic factor $\ln(\Lambda/m_\eta)$ must be chosen around eight times the η-meson mass. Clearly this is only a rough estimate because there are other coefficients involved as well as finite terms. A cutoff mass which is so large is rather peculiar; in fact, one would expect to be able to use a mass around the vector-meson mass if these states explain the decay rate. Let us now review the models proposed so far to see what actually happens.

Basically the models fall into two classes. There are authors who assume an *ad hoc* form for $F_\eta(k_1^{\,2},k_2^{\,2})$, i.e., Drell[11] and Berman and Geffen.[12] Then there are authors who use some form of vector-meson-dominance model with $\eta\rho\gamma$ or $\eta\rho\rho$ couplings, i.e., Young,[13] Quigg and Jackson,[14] Litskevitch and Franke,[15] and Litskevitch.[16] The model of Berman and Geffen,[12] when applied to η decay, gives a very small real part to the decay amplitude. Indeed a cutoff mass approximately 20 times the η mass is required to satisfy the branching ratio reported in Ref. 1. Drell[11] introduced a discontinuity in his form factor which gives a more singular real part, so that, when applied to η decay, it gives the correct value for the branching ratio with a cutoff mass eight times the η mass. The functional forms introduced by these authors do not seem to have any physical significance. Quigg and Jackson[14] used a model with direct couplings of the type $\eta\rho\rho$ etc., and found an even smaller real part than that of Berman and Geffen. With the second choice of coupling of the type $\eta\rho\gamma$ they found a slightly higher value but still require a vector-meson mass of approximately 18 times the mass of the η to fit the experimental result. In fact they find it difficult to imagine a branching ratio lying outside the interval from one to two times the unitarity bound. Their results seem to be in disagreement, however, with those of Litskevitch and Franke[15] who predict higher values both for $\pi^0\to e^+e^-$ decay and $\eta\to\mu^+\mu^-$ decay. Indeed Litskevitch[16] gives a result for the above branching ratio of 3.2×10^{-5} based on the physical vector-boson masses. This result clearly contradicts the conclusions of Ref. 14 and we have reason to believe it to be incorrect. Difficulties with the standard form of the vector-dominance model[17] are already known for these decays because the $\pi\rho\gamma$ coupling[18] has been found consistent with zero from photoproduction analysis. Only the $\omega\to\pi^0\gamma$ decay has been measured and a small upper limit is known for the decay $\rho\to\pi^0\gamma$. The model of Young,[13] which is a combination of different vector-meson-dominance models together with an *ad hoc* form for the high-energy behavior of $F_\eta(k_1^{\,2},k_2^{\,2})$, has a large number of parameters. It is possible to accommodate a branching ratio of 5×10^{-5} in his model, but one would require more data, especially on $\eta\to\mu^+\mu^-\gamma$ and $\pi^0\to e^+e^-$, before any conclusions can be drawn. Clearly the measurement reported above is disturbingly large for most of these models, and does not decisively rule out the presence of a neutral current coupling the η directly to a muon pair.

In view of the limitations of the models listed above, it seems worthwhile to investigate yet another model for the real part of the $\eta\to\mu^+\mu^-$ decay amplitude. We have therefore carried out a calculation based on the nucleon loop model to see what happens when higher-mass states are included. This model is actually much older than the models given above but its implications for this decay have not been worked out. Steinberger,[19] in his original paper, proposed this model to explain the π^0 lifetime and it is well known that the prediction is very good (assuming pseudoscalar pion-nucleon coupling). Lautrup and Olesen[20] generalized the original model to discuss baryon loop contributions with different SU(3) coupling constants. They found a reasonable prediction for $\eta\to 2\gamma$ decay. An additional reason for considering such a model comes from recent work on soft-pion limits.[21] It has been shown by Adler[22] that the Ward identity connecting the divergence of the axial-vector current to the pseudoscalar current contains an anomalous term. In the soft-pion limit of the decay $\pi^0\to 2\gamma$, only this anomalous term survives and yields the same result as that from the pseudoscalar nucleon loop model. This indicates that the pseudoscalar nucleon loop model should also give the dominant contribution in the associated decays $\pi^0\to\gamma e^+e^-$ and $\pi^0\to e^+e^-$, as well as the corresponding η decays $\eta\to 2\gamma$, $\eta\to\gamma\mu^+\mu^-$, and $\eta\to\mu^+\mu^-$, even if we do not take a soft-pion limit.

In Sec. II we give the basic formulas for the two-photon decays of the π^0 and η mesons. Our results are identical to those given by Lautrup and Olesen.[20] Section III gives the corresponding results for the decays $\eta\to\gamma\mu^+\mu^-$ and $\pi^0\to\gamma e^+e^-$. Previous calculations of these modes have been made by Geffen and Young[3] and by Jarlskog and Pilkuhn.[23] There are experimental results available for the slope of the form factor[24–26] in $\pi^0\to\gamma e^+e^-$ decay but they are rather inconclusive. At present the form factor is so small that radiative corrections[27] to this decay will have to be considered before any experimental result can be given. The decay $\eta\to\gamma\mu^+\mu^-$ should not suffer from this problem because the muon mass is much larger than the electron mass.

The decays $\eta\to\mu^+\mu^-$ and $\pi^0\to e^+e^-$ are considered in the nucleon loop model in Sec. IV. We do not use direct Feynman parametrization because this leaves a five-dimensional integral after we integrate over the two loop momenta and its evaluation is rather complicated due to the presence of singularities. Instead we use the unitarity equation for $\eta\to\mu^+\mu^-$ decay in the same spirit as Martin, de Rafael, and Smith[5] in their consideration of the decay $K_1^0\to\mu^+\mu^-$. Fortunately there are various simplifications in η decay as compared to K decay, because there are no infrared divergences. After calculating the absorptive part of the amplitude we compute numerically an unsubtracted dispersion integral to find its real part. Our conclusions are given in Sec. V where we also discuss the modifications due to other baryon-antibaryon exchanges with SU(3) values for the coupling constants.

In Appendix A we give the absorptive part of the $\eta\to\mu^+\mu^-$ amplitude with the $\gamma\pi\pi$ intermediate state because the relevant formula does not seem to be available in the literature. We also give a numerical estimate of the effect of this state on the two-photon unitarity bound.

II. THE DECAY $\eta\to 2\gamma$

The model we are proposing is based upon the Feynman diagram for $\eta(P)\to N(q)+\overline{N}(q')\to\gamma(k_1)+\gamma(k_2)$ shown in Fig. 1. Using m_η for the η mass, M for the baryon mass, and q for the loop integration momentum, we find the following expression for the amplitude:

$$\begin{aligned}A(\eta\to 2\gamma)&=-2ge^2\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\left(\gamma_5\frac{\not q-\not k_1+M}{(q-k_1)^2-M^2}\not\epsilon_1\frac{\not q+M}{q^2-M^2}\not\epsilon_2\frac{\not q+\not k_2+M}{(q+k_2)^2-M^2}\right)\\&=\frac{\alpha}{\pi}\frac{g}{M}\left(\frac{\sin^{-1}(\frac{1}{2}\xi)}{\frac{1}{2}\xi}\right)^2\epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu k_1^\rho k_2^\sigma\\&=\frac{H}{m_\eta}\epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu k_1^\rho k_2^\sigma\,.\end{aligned}\tag{2.1}$$

Here the $\eta N\overline{N}$ vertex is defined by $A(\eta\to\overline{N}(q')N(q))=g\overline{u}(q)\gamma_5 v(q')$ and $\xi=m_\eta/M$ is a small parameter. The decay rate is given by

$$\begin{aligned}\Gamma(\eta\to 2\gamma)&=\frac{m_\eta}{64\pi}|H|^2\\&=m_\eta\left(\frac{\alpha}{\pi}\right)^2\frac{g^2}{4\pi}\frac{\xi^2}{16}\left[\frac{2}{\xi}\sin^{-1}\left(\frac{\xi}{2}\right)\right]^4\\&=1.227\times10^{-7}m_\eta\frac{g^2}{4\pi}\,.\end{aligned}\tag{2.2}$$

If we expand this equation in terms of ξ, then the rate is given by

$$\Gamma(\eta\to 2\gamma)=m_\eta\left(\frac{\alpha}{\pi}\right)^2\frac{g^2}{4\pi}\frac{\xi^2}{16}\left(1+\tfrac{1}{6}\xi^2+\tfrac{7}{240}\xi^4+\cdots\right).\tag{2.3}$$

Equations (2.1), (2.2), and (2.3) agree with the results of Lautrup and Olesen[20] (who define the $\eta N\overline{N}$ vertex with the extra phase factor $-i$). The term in ξ^4 is very small and can safely be neglected.

III. THE DECAY $\eta\to\gamma\mu^+\mu^-$

Now we proceed to calculate the probability that one photon is virtual and converts into a lepton

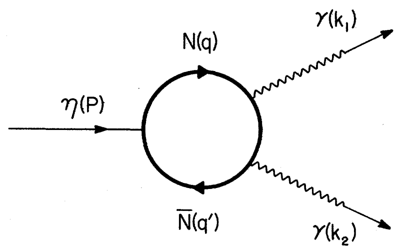

FIG. 1. Feynman diagram describing the decay $\eta \to 2\gamma$.

pair. Following the notation in Fig. 2, we find the amplitude

$$A(\eta \to \gamma\mu^+\mu^-)$$
$$= -8iMe^3 g\bar{u}(p_2)\gamma^\nu v(p_1)\frac{1}{k_2^2}\epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu k_1^\rho(p_1+p_2)^\sigma$$
$$\times \int \frac{d^4q}{(2\pi)^4}\frac{1}{(q-k_1)^2-M^2}\frac{1}{q^2-M^2}\frac{1}{(q+k_2)^2-M^2}. \tag{3.1}$$

The integral is the same as the one in Sec. II but now $k_2^2 \neq 0$. Feynman parametrization leads to the following expression:

$$\int_0^1 d\alpha \int_0^{1-\alpha} d\beta \frac{1}{[1-\alpha\beta\xi^2-\lambda^2\alpha(1-\alpha-\beta)]}$$
$$= 2\left(\frac{[\sin^{-1}(\frac{1}{2}\xi)]^2-[\sin^{-1}(\frac{1}{2}\lambda)]^2}{\xi^2-\lambda^2}\right)$$
$$= 2[1+\tfrac{1}{12}(\xi^2+\lambda^2)+\tfrac{1}{90}(\xi^4+\xi^2\lambda^2+\lambda^4)+\cdots], \tag{3.2}$$

where again $\xi = m_\eta/M$ and $\lambda = (k_2^2)^{1/2}/M$. It is obvious that the fourth-order term is again so small that it can be neglected because λ^2 is bounded by $4m^2/M^2 \leq \lambda^2 \leq m_\eta^2/M^2$, where m is the lepton mass. Retaining only the quadratic term leads to the prediction that the slope of the form factor in η decay is $\frac{1}{12}\xi^2$, i.e., if $F_\eta(k_1^2, k_2^2)$ is the general off-mass-shell form factor, then with $m_\eta^2 x = k_2^2 = \lambda^2 M^2$,

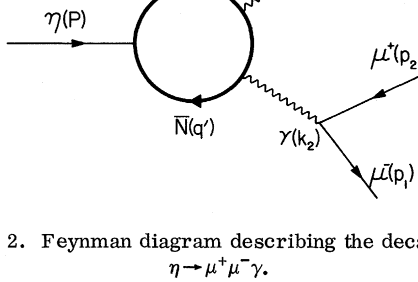

FIG. 2. Feynman diagram describing the decay $\eta \to \mu^+\mu^-\gamma$.

$$F_\eta(0, k_2^2) = F_\eta(0,0)\left[1+\left(\frac{m_\eta}{M}\right)^2\frac{x}{12}\right] = F_\eta(0,0)(1+\tfrac{1}{12}\xi^2 x). \tag{3.3}$$

Thus the slope of the form factor is predicted to be $a = \frac{1}{12}\xi^2 = 0.03$ for η decay and 0.002 for π^0 decay. These results are significantly different from those of the vector-meson-dominance model which, based on the ρ meson alone, give a value of $a_\eta = (m_\eta/m_\rho)^2 = 0.6$ for η decay and $a_\pi = (m_\pi/m_\rho)^2 = 0.03$ for π decay. Unfortunately the experimental data on this slope parameter are not yet good enough to distinguish between the various models. A recent experiment by Devons *et al.*[24] yielded the value $a_\pi = 0.01 \pm 0.11$ while older experiments[23,24] gave negative values. The fact that the slope is much larger for the decay $\eta \to \gamma\mu^+\mu^-$ gives some hope that this parameter can be measured in the near future.

IV. THE DECAY $\eta \to \mu^+\mu^-$

We now consider the more difficult calculation of the amplitude for $\eta \to \mu^+\mu^-$. The Feynman diagram is shown in Fig. 3, where we now use k and k' for the virtual-photon momenta. It is a relatively simple matter to write down the amplitude as a double-loop integral and reduce it to a five-dimensional integral over Feynman parameters. However the resulting integrations become rather complicated due to the presence of various branch cuts. We chose therefore to follow the unitarity calculation given by Martin, de Rafael, and Smith[5] for $K_1^0 \to \mu^+\mu^-$ decay, and, having found the absorptive part of the amplitude for $\eta \to \mu^+\mu^-$ decay, evaluate a dispersion integral for the real part. We define the decay amplitude by

$$A(\eta \to \mu^+(p')\mu^-(p)) = i\bar{u}(p)\gamma_5 F(t) v(p'), \tag{4.1}$$

where $t = (p+p')^2 = P^2$. The decay rate is therefore given by

$$\Gamma(\eta \to \mu^+\mu^-) = \frac{m_\eta}{8\pi}\left(1-\frac{4m^2}{m_\eta^2}\right)^{1/2}|F(m_\eta^2)|^2. \tag{4.2}$$

Assuming CP conservation in this decay, the muon pair is in a singlet S state. The projection operator for this state has been given in Ref. 5. We

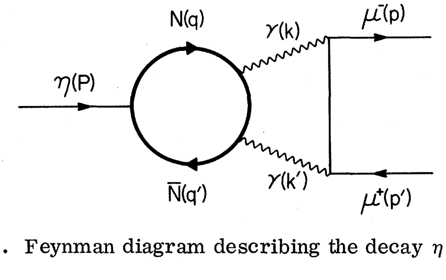

FIG. 3. Feynman diagram describing the decay $\eta \to \mu^+\mu^-$.

denote it by

$$P^{(0)}_{\rm out}(p,p') = \frac{1}{2(2t)^{1/2}}\left[-2m(p+p')_\mu\gamma^\mu\gamma_5 + \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}(p^\rho p'^\sigma - p'^\rho p^\sigma)\sigma^{\mu\nu} + t\gamma_5\right], \qquad (4.3)$$

so that the relation between $A(\eta\to\mu^+\mu^-)$ and $F(t)$ is

$$\mathrm{Tr}[P^{(0)}_{\rm out}(p,p')A(\eta\to\mu^+\mu^-)] = i(2t)^{1/2}F(t). \qquad (4.4)$$

The unitarity equation for the amplitude $A(\eta\to\mu^+\mu^-)$ is

$$\mathrm{Abs}A(\eta\to\mu^+\mu^-) = \frac{(2\pi)^4}{2}\int d\rho_\lambda \delta^{(4)}(p+p'-\textstyle\sum p_\lambda) \times A^*(\mu^+\mu^-\to\lambda)A(\eta\to\lambda),$$

so from Eq. (4.4) we find

$$\mathrm{Abs}F(t) = \frac{-i}{(2t)^{1/2}}\frac{(2\pi)^4}{2}\int d\rho_\lambda\delta^{(4)}(p+p'-\textstyle\sum p_\lambda) \times\langle\lambda|T|(\mu^+\mu^-)_{^1S_0}\rangle^* A(\eta\to\lambda), \qquad (4.5)$$

where the integration is taken over the phase space of the intermediate states λ. Because we assume CP conservation the absorptive part of the amplitude $F(t)$ is equal to the imaginary part of $F(t)$. Direct calculation of the absorptive part sometimes involves both real and imaginary terms on the right-hand side of Eq. (4.5) but the imaginary parts always cancel when we add all the contributions which are of the same order in the fine-structure constant. The amplitude

$$\langle\lambda|T|(\mu^+\mu^-)_{^1S_0}\rangle^*$$

is the Hermitian conjugate of the transition amplitude for $(\mu^+\mu^-)_{^1S_0}$ to go into the state λ. The projection operator which selects the singlet S state of the incoming muon pair is also given in Ref. 5, namely,

$$P^{(0)}_{\rm in}(p,p') = \frac{-1}{2(2t)^{1/2}}\left[2m(p+p')_\mu\gamma^\mu\gamma_5 + \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}(p'^\rho p^\sigma - p^\rho p'^\sigma)\sigma^{\mu\nu} + t\gamma_5\right]. \qquad (4.6)$$

After calculating the absorptive part of the decay amplitude we find its real part from an unsubtracted dispersion integral for the amplitude $F(t)$. For a physical η particle the only states allowed in Eq. (4.5) are the 2γ, $2\pi\gamma$, and 3π states. We know that the contributions of the states $2\pi\gamma$ and 3π are very small compared to those of the 2γ states. An explicit value is given for the $2\pi\gamma$ contributions in Appendix A, and we will henceforth neglect such states. However, as our model for $\eta\to2\gamma$ includes the $N\bar N$ states, it is convenient to assume that the η mass is so large that physical decays into $N\bar N$ states are allowed, as well as $\gamma N\bar N$ states. We therefore calculate the right-hand side of Eq. (4.5) for these states. Then we evaluate the real part of the amplitude by a dispersion integral at $t=m_\eta^2$ taking into account the 2γ, $N\bar N$, and $\gamma N\bar N$ states, using analytic continuation where necessary to have real functions. The decay rate is then calculated from this dispersive part and the physically allowed 2γ absorptive part. These possible states are drawn in Fig. 4.

A. The Two-Photon Contribution

When we take the intermediate state λ in Eq. (4.3) to be the two-photon state, we need the amplitude for $\eta\to2\gamma$ with the photons on their mass shells and the amplitude for $(\mu^+\mu^-)_{^1S_0}\to2\gamma$. The first term here is already known and given in Eq. (2.1). As we are considering $t=m_\eta^2>4M^2$, then we can also use unitarity techniques on Fig. 1 to find the absorptive part of the amplitude and evaluate a dispersion integral to find its real part. The imaginary part is canceled later by the corresponding contribution from the $N\bar N$ cut so we do not write it here. Our result is

$$A(\eta\to2\gamma) = \frac{\alpha Mg}{\pi t}\left[\pi^2 - \ln^2\left(\frac{1+\beta_N}{1-\beta_N}\right)\right]\epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu k_1^\rho k_2^\sigma, \qquad (4.7)$$

where

$$\beta_N^{\,2} = 1-4M^2/t. \qquad (4.8)$$

This expression coincides with the result given in Eq. (2.1) for the physical η meson because the analytic continuation of $\pi^2-\ln^2[(1+\beta_N)/(1-\beta_N)]$ to the case $t=m_\eta^2$ yields $4[\sin^{-1}(\frac{1}{2}\xi)]^2$ with $\xi=m_\eta/M$. The $(\mu^+\mu^-)_{^1S_0}\to2\gamma$ amplitude is found from the

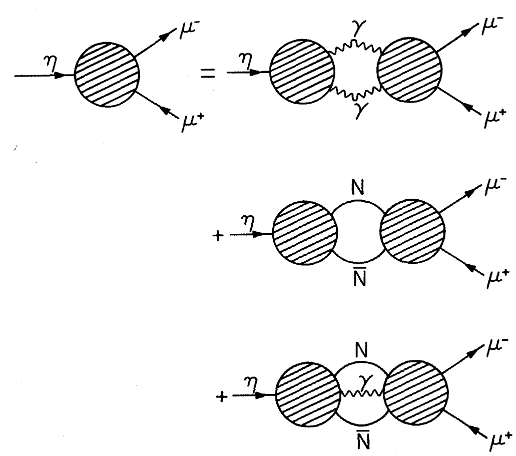

FIG. 4. Unitarity diagrams corresponding to the 2γ, $N\bar N$, and $N\bar N\gamma$ contributions to $\mathrm{Abs}A(t)$ in Eq. (4.5).

Born diagrams in Fig. 5. If we use the projection operator (4.6) then the singlet S transition amplitude is only a function of the scattering angle in the muon center-of-mass frame. The integration over λ can therefore be written as an integration over the cosine of this angle. Hence we find the following result for the absorptive part of the form factor $F(t)$:

$$\mathrm{Abs}F^{(2\gamma)}(t)=\frac{g\alpha^2}{4\pi}\,\frac{mM}{t}\,\frac{1}{\beta_\mu}\ln\left(\frac{1+\beta_\mu}{1-\beta_\mu}\right)\times\left[\pi^2-\ln^2\left(\frac{1+\beta_N}{1-\beta_N}\right)\right]\theta(t-4M^2), \tag{4.9}$$

where

$$\beta_\mu{}^2=1-4m^2/t.$$

When $4m^2\leqslant t\leqslant 4M^2$, then β_μ is still real but β_N becomes imaginary. Equation (4.9) then becomes

$$\mathrm{Abs}F^{(2\gamma)}(t)=g\,\frac{\alpha^2}{\pi}\,\frac{mM}{t}\,\frac{1}{\beta_\mu}\,\ln\left(\frac{1+\beta_\mu}{1-\beta_\mu}\right)\left[\sin^{-1}\left(\frac{\sqrt{t}}{2M}\right)\right]^2 \tag{4.10}$$

which is the expression to be used in the physical decay rate. However the dispersion integral is taken from $t=0$ to $t=\infty$ so we still need the absorptive part from $0\leqslant t\leqslant 4m^2$, i.e.,

$$\mathrm{Abs}F^{(2\gamma)}(t)=g\,\frac{\alpha^2}{\pi}\,\frac{mM}{t}\,\tan^{-1}\left(\frac{4m^2}{t}-1\right)^{1/2}\left[\sin^{-1}\left(\frac{\sqrt{t}}{2M}\right)\right]^2. \tag{4.11}$$

B. Contribution from the $N\bar{N}$ States

Equation (4.3) can again be reduced to an integration over the center-of-mass angle of the muon pair. First of all we need to discuss the general amplitude and the sum over the spin states of the baryon-antibaryon pair. The amplitude for η decay is

$$A(\eta\to N\bar{N})=g\bar{u}_N(q)\gamma_5 v_N(q'). \tag{4.12}$$

Now we need the S-wave projection of the amplitude for $\mu^+\mu^-\to N\bar{N}$ in the two-photon-exchange approximation. Using the notation shown in Fig. 6 where l and $P-l$ are now the photon momenta, we find that the amplitude for diagram (a) is given by

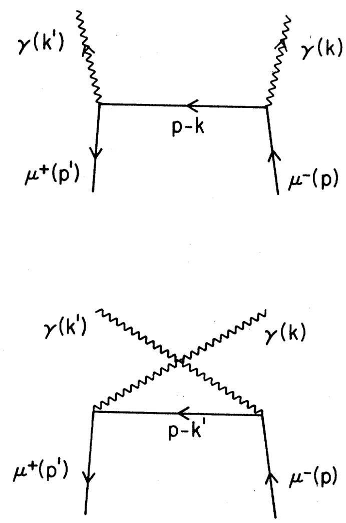

FIG. 5. Feynman diagrams describing the $\mu^+\mu^-\to 2\gamma$ transition amplitude.

$$A(\mu^+\mu^-\to N\bar{N})=e^4\int\frac{d^4l}{(2\pi)^4}\bar{u}_N(q)\gamma_\mu(\not{q}-\not{l}+M)\gamma_\nu v_N(q')\times\frac{g^{\mu\alpha}g^{\nu\beta}\bar{v}_\mu(p')\gamma_\beta(\not{p}-\not{l}+m)\gamma_\alpha u_\mu(p)}{l^2[(p-l)^2-m^2][(q-l)^2-M^2](P-l)^2}. \tag{4.13}$$

At this stage we perform the sum over the possible spins of the nucleon-antinucleon pair. This gives us the following trace for the nucleon loop:

$$\sum_{\text{spins}}A(\eta\to N\bar{N})A^*(\mu^+\mu^-\to N\bar{N})=\mathrm{Tr}[(\not{q}+M)\gamma_5(\not{q}'-M)\gamma_\nu(\not{q}-\not{l}+M)\gamma_\mu]=-4iMg\epsilon_{a\nu b\mu}p^a l^b. \tag{4.14}$$

Now we take the projection of the final $\mu^+\mu^-$ pair in the singlet S state,

$$\mathrm{Tr}[P_{\text{in}}^{(0)}(p,p')\gamma_\beta(\not{p}-\not{l}+m)\gamma_\alpha]=-\frac{4mi}{(2t)^{1/2}}\epsilon_{\alpha\nu\beta\mu}P^\mu l^\nu.$$

The product of Eq. (4.13) and Eq. (4.14) gives the simple result

$$\epsilon_{a\nu b\mu}P^a l^b\epsilon_{\alpha\nu\beta\mu}P^\alpha l^\beta=-2[P^2l^2-(P\cdot l)^2]. \tag{4.15}$$

In terms of the center-of-mass angle ϕ the unitarity equation for the $N\bar{N}$ intermediate state is

$$\mathrm{Abs}F^{(N\bar{N})}(t)=\frac{-i\beta_N}{16\pi}\,\frac{1}{(2t)^{1/2}}\,\frac{1}{2}\int_{-1}^{+1}d\cos\phi\sum_{\text{spins}}A(\eta\to N\bar{N})P_{\text{in}}^{(0)}A^*(\mu^+\mu^-\to N\bar{N}). \tag{4.16}$$

The expression to be integrated can now be written as

$$\sum_{\text{spins}}A(\eta\to N\bar{N})P_{\text{in}}^{(0)}A^*(\mu^+\mu^-\to N\bar{N})=-\frac{16\alpha^2}{\pi^2}\,\frac{mMg}{(2t)^{1/2}}I(t), \tag{4.17}$$

with

$$I(t)=\int d^4l\ \frac{2[P^2l^2-(P\cdot l)^2]}{l^2(P-l)^2[(p-l)^2-m^2][(q-l)^2-M^2]}\ . \tag{4.18}$$

The evaluation of this integral has been done using the Mandelstam representation.[28] Its real part cancels with a corresponding contribution coming from the two-photon intermediate state. The evaluation of the imaginary part is straightforward. It is convenient to define the function

$$F(y,t)=2\ln y\ln(t/m^2)-4\ln y+\ln a\ln\left(\frac{a-y}{1-ay}\right)+\ln y\ln\left(\frac{ay(a-y)(1-ay)}{(1-y^2)^2}\right)$$
$$-\mathrm{Li}_2\left(\frac{1-ay}{1-y^2}\right)+\mathrm{Li}_2\left(\frac{y(ay-1)}{a(y^2-1)}\right)+\mathrm{Li}_2\left(\frac{y(a-y)}{1-y^2}\right)-\mathrm{Li}_2\left(\frac{a-y}{a(1-y^2)}\right)\ , \tag{4.19}$$

where

$$y=\frac{[(M+m)^2-s]^{1/2}-[(M-m)^2-s]^{1/2}}{[(M+m)^2-s]^{1/2}+[(M-m)^2-s]^{1/2}}\ , \tag{4.20}$$

$a=m/M$, and $s=(p-q)^2$. Our $\mathrm{Li}_2(x)$ functions are defined as in the book of Lewin.[29] The integral is now given by

$$I(t)=-\frac{i\pi^2}{2}\,\frac{ty}{Mm(1-y^2)}\,F(y,t). \tag{4.21}$$

The integration over the center-of-mass angle can now be written as an integration over y, i.e.,

$$\frac{1}{2}\int_{-1}^{+1}d\cos\phi=\frac{mM}{t\beta_\mu\beta_N}\int_{x_1}^{x_2}\frac{(1-y^2)}{y^2}\,dy \tag{4.22}$$

with limits given by

$${x_1}^2=\frac{(1-\beta_\mu)(1-\beta_N)}{(1+\beta_\mu)(1+\beta_N)} \tag{4.23}$$

and

$${x_2}^2=\frac{(1-\beta_\mu)(1+\beta_N)}{(1+\beta_\mu)(1-\beta_N)}\ . \tag{4.24}$$

The contribution of the crossed diagram in Fig. 6(b) only multiplies the above result by two, so after some simplification we find the final result,

$$\mathrm{Abs}F^{(N\bar N)}(t)=\frac{g\alpha^2}{2\pi}\,\frac{mM}{t\beta_\mu}\left(\int_{x_1}^{x_2}\frac{dy}{y}F(y,t)\right)\theta(t-4M^2), \tag{4.25}$$

where $F(y)$ is given in Eq. (4.19). The analytic evaluation of this integral is possible in terms of trilogarithmic functions. However it would take too much time so we evaluated Eq. (4.25) numerically.

C. Contribution from the $\gamma N\bar N$ State

The integration in Eq. (4.5) is now over three-body phase space, and we require the amplitude for the transitions $\eta\to\gamma N\bar N$ and $\mu^+\mu^-\to\gamma N\bar N$. To lowest order in the electromagnetic coupling constant the amplitude for $\eta\to\gamma N\bar N$ is determined by bremsstrahlung radiation from the nucleons. Hence following the notation in Fig. 7, with k' the photon momentum, we find

$$A(\eta\to\gamma N\bar N)=(-ie)gi\bar u_N(q)\left(\gamma_5\,\frac{(-\not q'-\not k'+M)}{(q'+k')^2-M^2}\,\not\epsilon+\frac{\not\epsilon(\not q+\not k'+M)}{(q+k')^2-M^2}\,\gamma_5\right)v_N(q'). \tag{4.26}$$

The amplitude for $\mu^+\mu^-\to\gamma N\bar N$ is similarly determined by the two Born diagrams in Fig. 8, where k is the virtual-photon momentum. If we take the singlet S state for the muons, then

$$P^{(0)}_{\mathrm{in}}A(\mu^+\mu^-\to\gamma N\bar N)=(-ie)^3\,\mathrm{Tr}\left[P_{\mathrm{in}}(p,p')\left(\not\epsilon\,\frac{i(\not p-\not k+m)}{(p-k)^2-m^2}\gamma_\lambda+\gamma_\lambda\,\frac{i(\not p-\not k'+m)}{(p-k')^2-m^2}\,\not\epsilon\right)\bar u_N(q)\left(\frac{-ig^{\lambda\nu}}{k^2}\right)\gamma_\nu v_N(q')\right] \tag{4.27}$$

$$=(-ie)^3\,\frac{(-4mi)}{(2t)^{1/2}}\,\epsilon_{\mu\alpha\beta\lambda}\epsilon^\alpha k^\beta k'^\mu\left(\frac{1}{k^2-2p\cdot k}+\frac{1}{k'^2-2p\cdot k'}\right)\bar u_N(q)g^{\lambda\nu}\gamma_\nu v_N(q'). \tag{4.28}$$

Our method of integration follows that of Ref. 5 so we only give a brief description here.

We use $k^2=s$ and let θ be the angle between the momentum of the virtual photon $\vec{k}$ and the μ^+ momentum $\vec{p}\,'$ in the c.m. frame of the $\mu^+\mu^-$ system. Then the scalar products in the denominator of Eq. (4.28) are easily expressed in terms of s, t, β_μ, and $\cos\theta$. Similarly, if we define θ' as the angle between the N momentum $\vec{q}$ and the direction of $\vec{k}$ in the c.m. frame of the $N\bar N$ system (ϕ' is the corresponding azimuthal angle), then scalar products in the denominator of Eq. (4.26) can be written in terms of s, t, β_N', and $\cos\theta'$ with $\beta_N'=1-4M^2/s$. The sum over the spins of the $N\bar{N}$ state now yields

$$\sum_{\text{spins}} A(\eta\to\gamma N\bar{N})P_{\text{in}}^{(0)}A^*(\mu^+\mu^-\to\gamma N\bar{N})=\frac{64ige^4mM}{(2t)^{1/2}}\,\frac{1}{1-\beta_\mu{}^2\cos^2\theta}\,\frac{1}{1-\beta_N'^2\cos^2\theta'}\,. \tag{4.29}$$

Equation (4.5) can now be written as an integration over s, $\cos\theta$, $\cos\theta'$, and ϕ', where the last integration yields a factor of 2π, i.e.,

$$\text{Abs}A^{(\gamma N\bar{N})}(t)=\frac{(2\pi)^4}{2}\int\frac{d^3q}{2q_0}\int\frac{d^3q'}{2q_0'}\int\frac{d^3k'}{2k_0'}\,\frac{1}{(2\pi)^9}\,\delta^{(4)}(P-q-q'-k')\sum_{\text{spins}}A(\eta\to\gamma N\bar{N})A^*(\mu^+\mu^-\to\gamma N\bar{N}). \tag{4.30}$$

So

$$\text{Abs}F^{(\gamma N\bar{N})}(t)=\frac{-i}{2\sqrt{2t}}\,\frac{1}{(2\pi)^5}\int_{4M^2}^{t}ds\,2\pi\frac{\lambda^{1/2}(t,0,s)}{4t}\,\frac{1}{2}\int_{-1}^{+1}d\cos\theta\,\frac{\lambda^{1/2}(s,M^2,M^2)}{4s}$$
$$\times\frac{1}{2}\int_{-1}^{+1}d\cos\theta'\int_0^{2\pi}d\phi'\sum_{\text{spins}}A(\eta\to\gamma N\bar{N})A^*((\mu^+\mu^-)_{^1S_0}\to\gamma N\bar{N}). \tag{4.31}$$

Substituting Eq. (4.29) into Eq. (4.31) yields a rather simple four-dimensional integral which can be evaluated analytically.

Integration over the angles requires only one integral, i.e.,

$$\int_{-1}^{+1}\frac{d\cos\theta}{1-\beta_\mu{}^2\cos^2\theta}=\frac{1}{\beta_\mu}\ln\left(\frac{1+\beta_\mu}{1-\beta_\mu}\right),$$

so we finally obtain the absorptive part of the form factor $F(t)$,

$$\text{Abs}F^{(\gamma N\bar{N})}(t)=\frac{g\alpha^2mM}{\pi t\beta_\mu}\ln\left(\frac{1+\beta_\mu}{1-\beta_\mu}\right)J(\beta_N)\theta(t-4M^2), \tag{4.32}$$

where

$$J(\beta_N)=\int_{4M^2}^{t}\frac{t-s}{ts}\ln\left(\frac{1+\beta_N'}{1-\beta_N'}\right)ds$$
$$=2\ln\left(\frac{1+\beta_N}{1-\beta_N}\right)\ln\left(\frac{2}{1-\beta_N}\right)+2\,\text{Li}_2\left(-\frac{1+\beta_N}{1-\beta_N}\right)+\frac{1}{6}\pi^2-\frac{1}{2}\ln^2\left(\frac{1+\beta_N}{1-\beta_N}\right)-\frac{1+\beta_N{}^2}{2}\ln\left(\frac{1+\beta_N}{1-\beta_N}\right)+\beta_N\,. \tag{4.33}$$

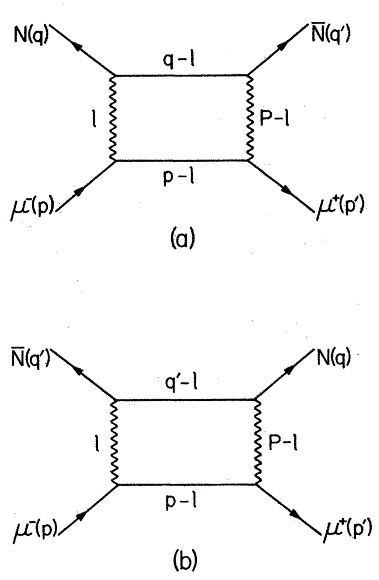

FIG. 6. Feynman diagrams describing the $\mu^+\mu^-\to N\bar{N}$ transition amplitude.

D. Numerical Results for the Absorptive and Dispersive Parts of the Amplitude

For completeness we now check the unitarity bound and discuss briefly the evaluation of the dispersive part. For the physical η mass, the only contribution to Abs $F(t)$ is given by Eq. (2.10) so that the decay rate is

$$\Gamma(\eta\to\mu^+\mu^-)=m_\eta\left(\frac{g^2}{4\pi}\right)\left(\frac{\alpha^4}{2\pi^2}\right)\frac{m^2M^2}{m_\eta{}^4}\,\frac{1}{\beta_\mu}$$
$$\times\ln^2\left(\frac{1+\beta_\mu}{1-\beta_\mu}\right)\left(\sin^{-1}\frac{m_\eta}{2M}\right)^4\,. \tag{4.34}$$

However the corresponding decay rate for $\eta\to2\gamma$ involves the same coupling constant g, so that if we divide Eq. (4.34) by

$$\Gamma(\eta\to2\gamma)=m_\eta\left(\frac{g^2}{4\pi}\right)\left(\frac{\alpha^2}{\pi^2}\right)\frac{M^2}{m_\eta{}^2}\left(\sin^{-1}\frac{m_\eta}{2M}\right)^4$$
$$=1.226\times10^{-7}m_\eta\left(\frac{g^2}{4\pi}\right), \tag{4.35}$$

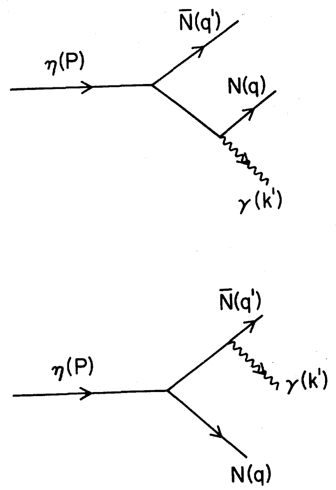

FIG. 7. Feynman diagrams contributing to $\eta \to \gamma N\bar{N}$.

we find

$$\frac{\Gamma(\eta\to\mu^+\mu^-)}{\Gamma(\eta\to 2\gamma)}=\alpha^2\left(\frac{m}{m_\eta}\right)^2\frac{1}{2\beta_\mu}\ln^2\left(\frac{1+\beta_\mu}{1-\beta_\mu}\right)$$

$$=1.07\times10^{-5}, \qquad (4.36)$$

which agrees with the usual unitarity bound for the decay.[3] For convenience, we give the numerical value of the absorptive part of $F^{(2\gamma)}(m_\eta{}^2)$ from Eq. (4.10),

$$\mathrm{Abs}F^{(2\gamma)}(m_\eta{}^2)=1.714\times10^{-6}g. \qquad (4.37)$$

To find the dispersive part of the amplitude, we write an unsubtracted dispersion integral for $F(t)$, and evaluate it at the mass of the η meson,

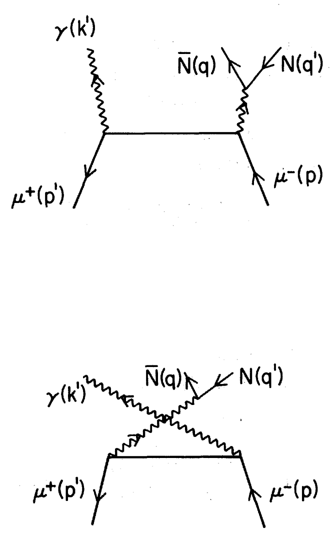

FIG. 8. Feynman diagrams contributing to $\mu^+\mu^-\to\gamma N\bar{N}$.

$$\mathrm{Re}F(m_\eta{}^2)=\frac{\mathrm{P}}{\pi}\int_0^\infty\frac{\mathrm{Abs}F(t)}{t-m_\eta{}^2}\,dt. \qquad (4.38)$$

The 2γ intermediate state yields an absorptive part given by Eq. (4.9) in the region $t\geq 4M^2$, Eq. (4.10) in $4m^2\leq t<4M^2$, and Eq. (4.11) in $0\leq t<4m^2$. Equation (4.38) is only a principal-value integral in the second region, and it converges very slowly in the first region. Combining all the terms, we find that

$$\mathrm{Re}F^{(2\gamma)}(m_\eta{}^2)=2.13\times10^{-6}g. \qquad (4.39)$$

The $N\bar{N}$ intermediate state similarly gives a contribution to Eq. (4.38). However we have to perform the integral over y in Eq. (4.25) and then the integral over t. The factor $\theta(t-4M^2)$ means that Eq. (4.38) is no longer a principal-value integral. Our numerical evaluation gives

$$\mathrm{Re}F^{(N\bar{N})}(m_\eta{}^2)=-0.78\times10^{-6}g. \qquad (4.40)$$

Similarly the $\gamma N\bar{N}$ contribution to Eq. (4.38) gives an ordinary integration over the absorptive part Eq. (4.32). Numerical integration yields the result

$$\mathrm{Re}F^{(\gamma N\bar{N})}(m_\eta{}^2)=1.32\times10^{-6}g. \qquad (4.41)$$

We see that the real part of the amplitude in our model has the final value

$$\mathrm{Re}F(m_\eta{}^2)=2.67\times10^{-6}g, \qquad (4.42)$$

which is not appreciably larger than the absorptive part given in Eq. (4.37). Hence our final result for the rate is

$$\Gamma(\eta\to\mu^+\mu^-)=4.52\times10^{-12}m_\eta\left(\frac{g^2}{4\pi}\right), \qquad (4.43)$$

so the branching ratio is

$$\frac{\Gamma(\eta\to\mu^+\mu^-)}{\Gamma(\eta\to2\gamma)}=3.6\times10^{-5}. \qquad (4.44)$$

This value is in reasonable agreement with the experimental number $(5.9\pm2.2)\times10^{-5}$.

It is now quite straightforward to change masses and calculate the same quantities in $\pi^0\to e^+e^-$ decay. From (4.10) we have

$$\mathrm{Abs}F^{(2\gamma)}(m_\pi{}^2)=2.58\times10^{-8}g', \qquad (4.45)$$

g' being the $\bar{N}N\pi$ coupling constant defined as before. The three contributions to the two-photon intermediate state give

$$\mathrm{Re}F^{(2\gamma)}(m_\pi{}^2)=3.13\times10^{-8}g', \qquad (4.46)$$

while the $N\bar{N}$ and $\gamma N\bar{N}$ states yield

$$\mathrm{Re}F^{(N\bar{N})}(m_\pi{}^2)=-0.77\times10^{-8}g', \qquad (4.47)$$

$$\mathrm{Re}F^{(\gamma N\bar{N})}(m_\pi{}^2)=1.43\times10^{-8}g'. \qquad (4.48)$$

Our final value for the real part of the $\pi^0 \to e^+e^-$ decay amplitude is therefore

$$\mathrm{Re}F(m_\pi^2) = 3.79\times10^{-8} g' , \qquad (4.49)$$

so that the decay rate is

$$\Gamma(\pi^0 \to e^+e^-) = 1.05\times10^{-15} m_\pi \left(\frac{g'^2}{4\pi}\right) . \qquad (4.50)$$

The corresponding two-photon rate from Eq. (2.2) is

$$\Gamma(\pi^0 \to 2\gamma) = 7.08\times10^{-9} m_\pi \left(\frac{g'^2}{4\pi}\right) , \qquad (4.51)$$

and hence our final branching ratio is

$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to 2\gamma)} = 1.4\times10^{-7}. \qquad (4.52)$$

If we only use the absorptive part of the amplitude in Eq. (4.45), we find the unitarity bound

$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to 2\gamma)} \geq 4.7\times10^{-8}.$$

The predictions of other models can be found in Ref. 14.

V. CONCLUSIONS

We have concentrated here on the calculation of the rates for $\eta \to \mu^+\mu^-$ and $\pi^0 \to e^+e^-$ using a nucleon loop model with the mass of the nucleon equal to that of the physical proton. Clearly it is possible to vary our predictions for the absolute decay rates by using different baryon intermediate states and SU(3) coupling constants at the meson-baryon vertices. Such modifications have indeed been discussed by Lautrup and Olesen.[20] The idea is to sum over the various states assuming an average mass and then use the SU(3) values of the coupling constants. Instead of simply having the strong-interaction coupling constant g, one finds instead $\sum_i g_i = (2/\sqrt{3})\alpha_s g$ for η decay and $\sum_i g_i = 2\alpha_s g$ for π^0 decay. The parameter α_s is the strong-interaction SU(3) mixing parameter which is determined experimentally to have the value 0.73. Hence the η decay rates are slightly reduced from the predictions in the previous sections and the π^0 decay rates are increased by approximately a factor of 2. We have avoided stressing the specific decay lifetimes. We rather believe that the branching ratios, which are independent of the strong-interaction coupling constants, have more reliability. Clearly the result for $\Gamma(\eta \to \mu^+\mu^-)/\Gamma(\eta \to 2\gamma)$ is in reasonable agreement with experiment. Whether the value for the branching ratio $\Gamma(\pi^0 \to e^+e^-)/\Gamma(\pi^0 \to 2\gamma)$ is compatible with experiment should be known in the near future. Experiments are possible at the Los Alamos Meson Physics Facility[30] and it will not be very difficult to lower the present upper limit on this branching ratio by several orders of magnitude.

APPENDIX

We outline the results for the $\gamma\pi\pi$ contribution to the absorptive part of the $\eta \to \mu^+\mu^-$ amplitude. Since $\eta \not\to 2\pi$, there are no bremsstrahlung graphs in the $\eta \to \gamma\pi\pi$ amplitude. We therefore define a structure-dependent amplitude for $\eta(P) \to \gamma(k') + \pi(q) + \pi(q')$, i.e.,

$$A(\eta \to \gamma\pi\pi) = \frac{eB}{m_\eta^3} \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu Q^\nu k^\rho k'^\sigma , \qquad (\mathrm{A1})$$

where $Q = q - q'$ and $k = q + q'$. The decay rate for $\eta \to \gamma\pi\pi$ is therefore given by ($t = m_\eta^2$)

$$\Gamma(\eta \to \gamma\pi\pi) = m_\eta |B|^2 \frac{\alpha}{384\pi^2 m_\eta^4} \int_{4m_\pi^2}^{t} s\left(1-\frac{s}{t}\right)^3 \left(1-\frac{4m_\pi^2}{s}\right)^{3/2} ds \qquad (\mathrm{A2})$$

$$= 1.453\times10^{-8} m_\eta |B|^2 \qquad (\mathrm{A3})$$

on the assumption that B is a dimensionless constant.

The evaluation of the contribution of the $\gamma\pi\pi$ intermediate states now parallels the calculation of the $\gamma N\bar{N}$ states. We find $[\beta_\mu = (1 - 4m^2/t)^{1/2}]$

$$\mathrm{Abs}F^{(\gamma\pi\pi)}(m_\eta^2) = \frac{\alpha^2 m B}{12\pi m_\eta^3 \beta_\mu} \int_{4m_\pi^2}^{t} \left(1-\frac{s}{t}\right)\left[\left(3-\frac{s}{t}\right)\ln\left|\frac{1+\beta_\mu}{1-\beta_\mu}\right| - 4\beta_\mu\right]\left(1-\frac{4m_\pi^2}{s}\right)^{3/2} ds$$

$$= 9.904\times10^{-8} B. \qquad (\mathrm{A4})$$

If we now take the branching ratio $\Gamma(\eta \to \gamma\pi\pi)/\Gamma(\eta \to \mathrm{all}) = 0.047$, then the value of B is 3.99 and

$$\mathrm{Abs}F^{(\gamma\pi\pi)}(m_\eta^2) = 3.95\times10^{-7} , \qquad (\mathrm{A5})$$

which is to be compared with the contribution from only the two-photon state. If we ignore for the moment the nucleon loop model and take a basic pseudoscalar coupling for this decay, namely,

$$A(\eta \to 2\gamma) = \frac{H}{m_\eta} \epsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu k_1^\rho k_2^\sigma, \quad \text{(A6)}$$

then

$$\Gamma(\eta \to 2\gamma) = \frac{m_\eta}{64\pi} |H|^2 . \quad \text{(A7)}$$

The corresponding contribution to the absorptive part of the $\eta \to \mu^+\mu^-$ decay amplitude is

$$\begin{aligned} \mathrm{Abs}F^{(\gamma\gamma)}(m_\eta{}^2) &= \frac{\alpha m}{4m_\eta} \frac{H}{\beta_\mu} \ln\left|\frac{1+\beta_\mu}{1-\beta_\mu}\right| \\ &= 1.224\times10^{-3} H . \end{aligned} \quad \text{(A8)}$$

Taking therefore the branching ratio $\Gamma(\eta \to 2\gamma)/\Gamma(\eta \to \text{all}) = 0.375$, we solve for H finding $H = 1.92\times10^{-2}$; so the numerical value of Eq. (A8) is

$$\mathrm{Abs}F^{(\gamma\gamma)}(m_\eta{}^2) = 2.35\times10^{-5} .$$

The effect of the $\gamma\pi\pi$ state is thus to change the $\gamma\gamma$ contribution by approximately 2%. The reason this ratio is lower in η decay than in K_L^0 is due entirely to the fact that the rate for $\eta \to \gamma\pi\pi$ is known, whereas only a large upper limit exists for $K_L^0 \to \gamma\pi\pi$.

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