# Conjecture on the Approach to Scaling in Deep-Inelastic Electron Scattering and Some Comments on Electromagnetic Mass Differences*

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We conjecture that if the equal-time commutator of the electromagnetic current with its third time derivative exists as defined by Bjorken's asymptotic expansion, then the approach to scaling in deep-inelastic electron scattering is like $1/q^2$, where $q^2$ is the square of the mass of the virtual photon. We show that in the scaling region one can expect the structure functions for charged pions to be the same as those for neutral pions. If the longitudinal structure functions vanish in the nonscaling as well as in the scaling region, then the pion mass difference, as calculated via the Cottingham formalism, converges. For nucleons, on the other hand, a similar assumption for an "almost" longitudinal structure function does not guarantee convergence. The condition that does, however, may be consistent with experiment.

## I. BACKGROUND

In the one-photon-exchange approximation the hadronic contribution to inelastic scattering of electrons from an unpolarized target can be described by the tensor $W_{\mu\nu}$:

$$W_{\mu\nu}=\tfrac{1}{2}\sum_n \langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle(2\pi)^3\delta^{(4)}(p+q-n). \tag{1}$$

Here $q$ is the four-momentum of the virtual photon and $p$ that of the target; $j$ is the electromagnetic current operator, and an average over the target spin is to be understood. The most general form for $W_{\mu\nu}$ consistent with general invariance principles is[1]

$$W_{\mu\nu}=-W_1(q^2,\nu)\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+\frac{W_2(q^2,\nu)}{M^2}\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{\nu}{q^2}q_\nu\right), \tag{2}$$

where $M$ is the mass of the target and $\nu\equiv p\cdot q$. The $W$'s can be related to scattering cross sections due to longitudinal ($L$) and transverse ($T$) photons by the following equations:

$$W_1(q^2,\nu)=\left(\frac{q^2+2M\nu}{2M}\right)\frac{\sigma_T(q^2,\nu)}{4\pi^2\alpha}, \tag{3a}$$

$$W_2(q^2,\nu)=\left(\frac{q^2+2M\nu}{2M}\right)\left(\frac{-q^2}{\nu^2-q^2}\right)\frac{\sigma_L(q^2,\nu)+\sigma_T(q^2,\nu)}{4\pi^2\alpha}. \tag{3b}$$

Here $\alpha$ is the fine-structure constant ($\simeq\frac{1}{137}$). The $W$'s can also be thought of as absorptive parts of the forward virtual Compton amplitude $T_{\mu\nu}(p,q)$ defined by

$$T_{\mu\nu}(p,q)=\int d^4x\, e^{iq\cdot x}\langle p|T[j_\mu(x)j_\nu(0)]|p\rangle \tag{4}$$

$$=-T_1(q^2,\nu)\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+\frac{T_2(q^2,\nu)}{M^2}\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{\nu}{q^2}q_\nu\right). \tag{5}$$

When nucleons are used as the target it is found experimentally[2] that for $-q^2\gtrsim 1\ (\text{GeV}/c)^2$, the dimensionless structure functions $F_1\equiv MW_1(q^2,\nu)$ and $F_2\equiv\nu W_2(q^2,\nu)$ scale; that is, they both become nontrivial functions of a single variable $\omega\equiv-2\nu/q^2$. Such a behavior was shown by Bjorken[3] to follow from the existence of certain equal-time commutators (ETC) of the electromagnetic current as defined by the behavior of $T_{\mu\nu}$ in the limit $q^0\to i\infty$ with $\vec{q}$ and $p$ fixed. One result of the present paper will be to show that Bjorken's original argument can be naively extended to conjecture that *the approach to scaling behaves like* $1/q^2$; that is to say, the asymptotic series for the $F_i(\omega,q^2)$ can be expected to be of the form

$$F_i(q^2,\omega)\underset{q^2\to-\infty}{\sim}F_i(q^2,\omega)+(1/q^2)G_i(q^2,\omega)+\cdots.$$

There are at the moment several empirical fits to the data which are consistent with this behavior[4]; however, it is far from being verified. Should it eventually be found that the data are inconsistent with this simple expansion, we would conclude that the naive assumption of the existence of ETC's as defined in the manner of Bjorken has broken down.

Most of this paper will be concerned with the problem of electromagnetic masses and mass differences. Cottingham[5] was the first to show how this problem can be directly related to inelastic electron scattering. He showed that, to first order in $\alpha$, the electromagnetic self-energy of a hadron is given by

$$\Delta M=\frac{\alpha}{(2\pi)^2}\int_0^{-\infty}\frac{dq^2}{q^2}\int_0^{\sqrt{q^2}}d\nu\,(-q^2-\nu^2)^{1/2}T_\mu{}^\mu(i\nu,q^2). \tag{6}$$

If dispersion relations are written for the $T_i$, then $\Delta M$ can be directly expressed in terms of integrals over the measured quantities $W_i(q^2,\nu)$. It is well known[5,6] that the low-energy contributions to the integrals lead to the wrong sign for the neutron-proton mass difference, whereas for the pion they lead not only to the correct sign but also to the correct magnitude. Harari[6] has suggested that the reason for this is that the isospin exchanged in the $t$ channel is different in the two cases, and according to Regge theory, this will give rise to different convergence properties. More specifically, in the nucleon case ($\Delta I=1$) one can expect $T_1$ to require a subtraction but $T_2$ to be unsubtracted. On the other hand, in the pion case ($\Delta I=2$) there are no known high-lying trajectories so we can expect both $T_1$ and $T_2$ to be unsubtracted. The extra convergence in the $\Delta I=2$ case is, according to Harari, the reason why these mass differences can be calculated using only the low-energy contributions. In the $\Delta I=1$ case, however, the presence of a subtraction term presumably indicates the importance of high-energy contributions. When the high-energy data became available, however, it was soon discovered that the scaling hypothesis led, in general, to divergent mass differences.[7,8] In fact, a straightforward calculation shows that the coefficient of the logarithmic divergence for the self-mass is

$$\Delta \equiv \lim_{q^2\to-\infty} q^2T_1(q^2)+\int_0^1 dx[2xF_1(x)+F_2(x)], \tag{7}$$

where $x\equiv 1/\omega$ and $T_1(q^2)\equiv T_1(q^2,0)$. This shows explicitly that

$$\lim_{q^2\to-\infty} q^2T_1(q^2)$$

must be a constant in order to avoid quadratic divergences, and that without some special relationship or assumption the self-masses and mass differences are logarithmically divergent. Below we shall discuss the possibility for ensuring both the finiteness and correct sign of the mass differences. Our conclusions in both cases will be rather unsatisfying: In particular, we confirm the general folklore that electromagnetic mass differences as calculated in the manner of Cottingham generally diverge.

## II. NAIVE CORRECTIONS TO SCALING

Let us first discuss the corrections to scaling. Bjorken[9] has suggested the following asymptotic series for $T_{\mu\nu}$ ($\vec{q}$ and $p$ fixed):

$$T_{\mu\nu}\underset{q_0\to i\infty}{\sim}\sum_{n=1} C_{\mu\nu}^{(n)}\left(\frac{1}{q_0}\right)^n , \tag{8}$$

where

$$C_{\mu\nu}^{(n)}\equiv\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\langle p|[j_\mu(\vec{x},0),\partial_0{}^{n-1}j_\nu(0)]|p\rangle . \tag{9}$$

In the frame where $\vec{q}=\vec{0}$, crossing symmetry eliminates the odd powers in this expansion.[10] The conventional fixed-$q^2$ dispersion relations for the $T_i$,

$$T_1(q^2,\nu)=T_1(q^2)+\nu^2\int_0^\infty \frac{W_1(q^2,\nu')d\nu'^2}{\nu'^2(\nu'^2-\nu^2)} \tag{10a}$$

and

$$T_2(q^2,\nu)=\int_0^\infty\frac{W_2(q^2,\nu')d\nu'^2}{\nu'^2-\nu^2}, \tag{10b}$$

can be transformed to the variables $x, q^2$ to obtain

$$T_1(q^2,x)=T_1(q^2)+\int_0^1\frac{W_1(q^2,x')dx'^2}{x'^2-x^2}, \tag{11a}$$

$$T_2(q^2,\nu)=x^2\int_0^1\frac{W_2(q^2,x')dx'^2}{x'^2-x^2}. \tag{11b}$$

In the $\vec{q}=\vec{0}$ frame, where $q_0{}^2=q^2$ and $x=-q_0/2p_0$, we need only consider $T_{ij}$; its asymptotic expansion in the $q_0\to i\infty$ limit is

$$T_{ij}\sim\left[T_1(q^2)-\frac{4p_0{}^2}{q^2}\int_0^1 W_1(q^2,x')\sum_{n=0}^\infty\left(\frac{2p_0x'}{q_0}\right)^{2n}dx'^2\right]\delta_{ij} -\left[\int_0^1 W_2(q^2,x')\sum_{n=0}^\infty\left(\frac{2p_0x'}{q_0}\right)^{2n}\frac{dx'^2}{x'^2}\right]p_ip_j . \tag{12}$$

This series can be compared with the Bjorken form, Eq. (8); the $C_{ij}$ are of the form

$$C_{ij}^{(2)}(\vec{p})=(A_2+B_2p_0{}^2)\delta_{ij}+D_2p_ip_j , \tag{13a}$$

$$C_{ij}^{(4)}(\vec{p})=(A_4+B_4p_0{}^2+C_4p_0{}^4)\delta_{ij}+(D_4+E_4p_0{}^2)p_ip_j , \tag{13b}$$

where $A_2, B_2, \ldots, D_4, E_4$ are constants. Comparing the two series we see that to this order in $1/q^2$,

$$\lim_{q^2\to-\infty}\int_0^1 W_1(q^2,x)dx^2\sim-\tfrac{1}{4}\left(B_2+\frac{B_4}{q^2}\right) \tag{14}$$

and

$$\lim_{q^2\to-\infty}\int_0^1 q^2W_2(q^2,x)\frac{dx^2}{x^2}\sim-\left(D_2+\frac{D_4}{q^2}\right). \tag{15}$$

Following Bjorken we can deduce from these that the limits

$$\lim_{q^2\to-\infty} W_1(q^2,x) \text{ and } \lim_{q^2\to-\infty} q^2W_2(q^2,x)$$

(with $x$ fixed) exist provided $B_2$ and $D_2$ exist. The argument can clearly be carried one step further, to conjecture that if $B_4$ and $D_4$ also exist then the scaling functions approach their limits like $1/q^2$. Should this prove to be untrue experimentally we would have a direct indication that the naive assumption that $C_{ij}^{(4)}$ exists is not valid.

Two remarks are worth making here: (a) Callan and Gross[11] have pointed out that in specific models $C_{ij}^{(2)}$ may actually be simpler than the general form. For example in the quark model[9] $B_2=-D_2$, in which case Eqs. (14) and (15) lead to $F_1(x) = 2xF_2(x)$, which is equivalent to the vanishing of $q^2\sigma_L$ in the scaling limit. On the other hand in the algebra of fields[12] $B_2=0$, which leads, from (14), to $F_1(x)=0$; this is equivalent to the vanishing of $q^2\sigma_T$. Present experimental data[2] seem to favor the former model. Although $C_{ij}^{(4)}$ may not exist in general, it is nevertheless unfortunate that even in simple models it is too complicated an object to evaluate. For example, it would be nice to have a prediction for at least the sign of the nonscaling contribution. Indeed, as we shall see below, these contributions play a crucial role in the discussion of the mass differences. (b) From the $\delta_{ij}$ term in Eq. (12) we can also deduce that

$$\lim_{q^2\to-\infty} T_1(q^2) \sim \frac{A_2}{q^2}+\frac{A_4}{q^4}, \tag{16}$$

*unless*, of course, there is an operator Schwinger term, in which case it would be given by[8]

$$\lim_{q^2\to-\infty} T_1(q^2).$$

An alternative way of seeing this is to work in the frame where $\vec{p}=\vec{0}$ and look at the $n=1$ term of the $T_{0i}$ component of Eq. (8). If

$$\langle p|[j_i(0,\vec{x}),j_i(0)]|p\rangle = iQ\partial_i\delta^{(3)}(\vec{x}), \tag{17}$$

then according to Eq. (8) $T_{0i}\to Qq_i/q_0$. But from the explicit representation, Eqs. (5) and (11), we have $T_{0i}\to T_1(\infty)q_i/q_0$, which verifies the statement that

$$Q=\lim_{q^2\to-\infty} T_1(q^2).$$

We see therefore from Eq. (7) that the removal of a possible quadratic divergence in $\Delta M$ requires the absence of operator Schwinger terms.[8,13]

### III. ELECTROMAGNETIC MASS DIFFERENCES

We shall now concentrate on the problem of the mass differences. It is clear that without an assumption which allows us to calculate

$$\lim_{q^2\to-\infty} q^2T_1(q^2)$$

we cannot say anything further. Such an assumption was suggested by Jackiw, van Royen, and myself,[8] and I shall now present a generalization of that argument here. Consider the amplitude

$$T_3(q^2,\nu)\equiv T_1(q^2,\nu)+\beta(q^2,\nu)\left(\frac{\nu^2-q^2}{q^2}\right)T_2(q^2,\nu), \tag{18}$$

where $\beta(q^2,\nu)$ is an arbitrary analytic function of $q^2$ and $\nu$ which scales. Its imaginary part is easily seen to be

$$W_3(q^2,\nu)=\left(\frac{q^2+2\nu}{2M}\right)\frac{\sigma_3(q^2,\nu)}{4\pi^2\alpha}, \tag{19}$$

where $\sigma_3\equiv(1-\beta)\sigma_T-\beta\sigma_L$. We can clearly choose $\beta$ in such a way that $\sigma_3$ vanishes in the Regge limit (i.e., $\nu\to\infty$ for arbitrary $q^2$). If, further, this $\beta$ makes $\nu\sigma_3\to 0$ we can write an unsubtracted dispersion relation for $T_3(q^2,\nu)$ from which we can deduce a sum rule for $T_1(q^2)$:

$$T_1(q^2)=\int_0^\infty \frac{d\nu^2}{\nu^2}[W_3(q^2,\nu)+\beta(q^2,\nu)W_2(q^2,\nu)]. \tag{20}$$

Taking the $q^2\to-\infty$ limit of this equation leads to a sum rule for $Q$:

$$Q=2\int\frac{dx}{x}F_3(x), \tag{21}$$

where $F_3(x)=F_1(x)-\beta(x)F_2(x)/2x$. The work in Ref. 8 was essentially equivalent to the choice $\beta=1$ in which case $\sigma_3=-\sigma_L$ and $F_3$ is purely longitudinal. Equation (21) is a simple generalization of the result given there and is similar in spirit to that of Corrigan, Cornwall, and Norton.[14] These latter authors go somewhat further than we have here and consider an amplitude analogous to $T_3$ which, by construction, satisfies the unsubtracted hypothesis. This is equivalent to taking $\beta$ to have the cut structure of the canonical asymptotic Regge form. As we have already remarked, the absence of a quadratic divergence in $\Delta M$ requires setting $Q=0$. This can be accomplished if $2xF_1(x) = \beta(x)F_2(x)$. As an example, suppose $R$ ($\equiv\sigma_L/\sigma_T$) is independent of $q^2$ and $\nu$ in both the asymptotic Regge and scaling regions (which for the proton is consistent with experiment); then the choice $\beta=1/(1+R)$ is consistent with our Regge assumptions and with $Q=0$. Now, recall that the coefficient of the logarithmic divergence in $\Delta M$ involves

$$\lim_{q^2\to-\infty} q^2T_1(q^2);$$

this can be determined from Eq. (20):

$$\lim_{q^2\to-\infty} q^2T_1(q^2) = 2\int_0^1\frac{dx}{x}\left[\lim_{q^2\to-\infty} q^2W_3(q^2,x)-2\beta(x)xF_2(x)\right]. \tag{22}$$

Now, we have shown from the existence of $C_{ij}^{(4)}$ that the $F_i(x,q^2)$ satisfy an asymptotic power series in $1/q^2$ (for fixed $x$). Hence if $F_3(x)$ vanishes, then

$$\lim_{q^2\to-\infty} q^2W_3(q^2,x)\equiv G_3(x)$$

exists. Using this in Eq. (7) we find

$$\Delta = \int_0^1 \frac{dx}{x}\{2G_3(x) - [3\beta(x) - 1]xF_2(x)\}. \tag{23}$$

This explicitly demonstrates the apparently capricious relationship that must hold between the nonscaling and scaling contributions in order that $\Delta$ vanishes.[15] In Ref. 8 we considered the possibility that $G_3(x)=0$ and $\beta(q^2, \nu)=1$; in that case it is clear that although $\Delta$ does not vanish [since, in general, $F_2(x)\neq 0$] it does have a definite sign (<0). It should be noted that for a mass difference it is of course understood that we must use $F_2^{(p)} - F_2^{(n)}$ when we write $F_2$. Experimentally[2] this difference appears to be positive, which implies that the possible logarithmic divergence at least has the correct sign. This point has recently been emphasized by Lee,[16] who has pointed out that in his version of quantum electrodynamics the photon propagator in Eq. (6) has an extra convergence factor which makes the mass difference both convergent and of the correct sign if $G_3^{(p)}(x)=G_3^{(n)}(x)$. A judicious choice of the cutoff can, of course, lead to the correct magnitude. An extra term like $\Lambda^2/(\Lambda^2 - q^2)$ in the photon propagator could be thought of as a small scale-breaking term in the $F_i(x, q^2)$ which only becomes important for $-q^2 \gtrsim \Lambda^2$. A typical value for $\Lambda$ is ~50 GeV, so it is unlikely that such a term can be detected in these experiments. A different way of introducing a scale-breaking term which makes the mass difference convergent has recently been discussed by Moffat and Wright.[17]

Experimentally the data appear to indicate that $R$ is nonzero in the asymptotic limits. We shall therefore now examine this more realistic possibility, keeping the assumption that $G_3^{(p)}(x)=G_3^{(n)}(x)$. If we demand that the self-masses of both nucleons not be quadratically divergent, then we must require $\beta=(1+R)^{-1}$ for each nucleon in the asymptotic regions. The mass difference is then convergent if $G_3^{(p)}(x)=G_3^{(n)}(x)$ and

$$\left(\frac{2-R^{(n)}(x)}{1+R^{(n)}(x)}\right)F_2^{(n)}(x) = \left(\frac{2-R^{(p)}(x)}{1+R^{(p)}(x)}\right)F_2^{(p)}(x). \tag{24}$$

Experimentally, it appears that $F^{(n)}(x) < F^{(p)}(x)$, so for the mass difference to be convergent we must have $R^{(n)} < R^{(p)}$. Now, since it is extremely difficult to measure $R^{(p)}$ accurately it will be even more difficult to measure $R^{(n)}$. Hence it might appear that for a long time into the future (24) will always be consistent with experiment. As an example suppose $F_2^{(n)}/F_2^{(p)} \sim 0.9$ and $R^{(p)} \sim 0.2$; then for (24) to be satisfied we require $R^{(n)} \sim 0.12$, which would be very difficult to distinguish from zero experimentally. However, a word of caution should be added: If for some value of $x$ $R^{(p)}=0.2$, then (24) requires $F_2^{(p)}/F_2^{(n)} < \frac{4}{3}$, since $R^{(n)}$ must remain positive. This upper limit decreases with decreasing $R^{(p)}$. The data on this ratio are still rather crude, but there is some indication that the condition might be violated. The important point here is that even though $R^{(p)}$ and $R^{(n)}$ may be difficult to measure one might still be able to draw definite conclusions on the validity of condition (24) from the ratio $F_2^{(p)}/F_2^{(n)}$. It should be noted that if the mass difference *is* convergent then it is almost impossible to say anything about the sign of the high-energy contribution, since this would then involve only the nonscaling contributions. On the other hand, if we are willing to use a real cutoff (in the manner of Lee), then (24) indicates that the sign and magnitude of the high-energy contribution require accurate knowledge of $R^{(n)}$ and $R^{(p)}$ as well as $F_2^{(n)}$. In short, then, we would conclude that the problem of the $n$-$p$ mass difference is not a happy one; even if we are willing to make various simplifying assumptions as we have here, both its convergence and sign still depend upon experimental quantities like $G_3(x)$ and $R^{(n)}(x)$ which are extremely difficult to measure.

We conclude this discussion with some remarks concerning the pion mass difference. As already mentioned above, the absence of any $T=2$ Regge trajectory suggests writing unsubtracted dispersion relations for the $I=2$ components of $T_1$ and $T_2$. We shall denote the charged-pion amplitudes by $T_i^{+}$ and the neutral-pion ones by $T_i^0$. Because $(T_1^+ - T_1^0)$ is unsubtracted we shall find upon carrying through the Bjorken procedure [as in Eqs. (8) and (12)] that $\nu(W_1^+ - W_1^0)$ scales. Since each $T_1$ separately is still subtracted we know that the separate $W_1$'s scale as before; this means then that *in the scaling limit* $W_1^+ = W_1^0$, i.e., $F_1^+(x)=F_1^0(x)$. Also consistent with this Regge hypothesis is the unsubtractedness of an amplitude such as $(T_3^+ - T_3^0)$. This leads to a sum rule of the form (e.g., take $\beta=1$)

$$\int_0^\infty d\nu^2[W_2^+(q^2,\nu) - W_2^0(q^2,\nu)] = 0. \tag{25}$$

Looking in the scaling region this says that *if the difference* $F_2^+(x) - F^0(x)$ *does not oscillate then it vanishes*. The equality of $F_i^+(x)$ with $F_i^0(x)$ is consistent with the notion introduced by Wilson[18] that in the scaling region the structure functions are sensitive to certain operators which occur in the light-cone expansion of the commutator in Eq. (4). These operators are supposed to have low dimension; now there are no known operators of low dimension with $I=2$, which is consistent with the result stated above, viz., that $F_i^+(x)=F_i^0(x)$. Now, in the algebra of fields[12] there are operators with $I=2$, so in that case these simple results would not follow. On the other hand this algebra is inconsistent with the nucleon data since it requires $F_1(x)$ to vanish and, as already remarked above, the data appear to be in much better agreement with the quark algebra which has no $I=2$ operators. It is well known that the pion mass difference is divergent in the algebra of fields.[19] It might be hoped that the simplicity of the results which follow from what we have done here (which is consistent with the quark algebra) might lead to a finite answer. Unfortunately this hope does not seem to be borne out in general. Since $F_i^+(x) = F_i^0(x)$ the coefficient of the logarithmic divergence in the pion mass difference will simply be

$$\Delta = \lim_{q^2 \to -\infty} q^2 T_1(q^2). \tag{26}$$

For this to vanish we require (as in the nucleon case) the nonscaling contributions to vanish [e.g., $G_3^+(x) = G_3^0(x)$].[20] Since this cannot be demonstrated even in a model, we are left with the rather frustrating result that this mass difference also diverges in general. If, on the other hand, we are willing to assume that $G_3^+(x) = G_3^0(x)$ (e.g., that the $q^2\sigma_L$ vanishes in the nonscaling as well as scaling region) then the pion mass difference is indeed convergent.

Clearly, nothing can be said about the sign of this divergent term. However, any reasonable value for a cutoff (~10 GeV) together with a reasonable choice for $G_3(x)$ [$\sim M^2 F_2(x)$] leads to a contribution of ~1 MeV to the mass difference. This is considerably smaller than that from the low-energy terms, which is presumably the reason why the low-energy calculations work. In the nucleon case, on the other hand, the high-energy contribution is comparable to that of the low-energy, so the problem is considerably more delicate.

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