Thus

$$\mathfrak{D}^{(S,0)}(A)_{\lambda'\lambda}=[(S+\lambda')!(S-\lambda')!(S+\lambda)!(S-\lambda)!]^{1/2}\sum_{a,b,c,d}{}'(a!b!c!d!)^{-1}(A_{++})^a(A_{+-})^b(A_{-+})^c(A_{--})^d, \tag{C5}$$

where the sum includes all those values of a, b, c, d in the range $0, 1, \ldots, 2S$ which satisfy (C4).

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*Work supported by the National Science Foundation under Grant No. GP-30738X.

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PHYSICAL REVIEW D VOLUME 5, NUMBER 8 15 APRIL 1972

Mixing Angle in Renormalizable Theories of Weak and Electromagnetic Interactions*

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(Received 27 December 1971)

It is suggested that the weak and electromagnetic interactions be incorporated into a theory based on an SU(3)⊗SU(3) gauge-invariant and parity-conserving Lagrangian, in which the lepton fields form a Konopinski-Mahmoud triplet μ^+, ν, e^-. The unobserved effects which would be produced by 10 of the 12 charged vector bosons in this theory are suppressed if the spontaneous breaking of SU(3)⊗SU(3) down to SU(2)⊗U(1) is much stronger than the spontaneous breaking of SU(2)⊗U(1) down to electromagnetic gauge invariance. The resulting theory is for most purposes equivalent to the previous SU(2)⊗U(1) model, but with mixing angle now fixed at 30°. In consequence, the mass of the charged vector boson which mediates the known weak interactions is now predicted to be 74.6 GeV. This model also provides a natural mechanism for producing an electron mass of order αm_μ.

Several years ago it was suggested[1] that a renormalizable theory of the weak and electromagnetic interactions might be constructed from a gauge-invariant Lagrangian by allowing a spontaneous breakdown of the gauge symmetry. This proposal has now been revived by a number of theoretical studies,[2] which tend to confirm the renormalizability of models of this general class.

There are many possible presumably renormalizable models, based on different underlying symmetries and different patterns of symmetry breaking. However, particular attention has been given to a simple model[1] of the weak and electromagnetic interactions, based on a previously suggested[3] SU(2)⊗U(1) gauge group, under which the left-handed leptons transform as two independent doublets $(\nu_\mu, \mu^-)_L$ and $(\nu_e, e^-)_L$, while the right-handed leptons transform as two independent singlets μ_R^-, e_R^-. Although reasonably economical, this model has a major disadvantage, in that it is based on a nonsimple gauge group, so that it involves *two* independent coupling constants g and g'. For this reason, it has been necessary to introduce a "mixing angle"

$$\tan\theta = g'/g \tag{1}$$

and to express all predictions in terms of this unknown parameter. For instance, the masses of the charged and neutral intermediate vector bosons are given by[1]

$$m_W = \frac{37.3 \text{ GeV}}{|\sin\theta|}, \tag{2}$$

$$m_Z = \frac{74.6 \text{ GeV}}{|\sin 2\theta|}. \tag{3}$$

In addition, this simple model is aesthetically unpleasing, because it offers no rationale for the presence of both electron-type and muon-type leptons, and because it deals in qualitatively different ways with the left- and right-handed parts of the lepton fields.

In this note I would like to propose a more attractive model, which does away with these disadvantages. It will be assumed that there exist three fundamental four-component lepton fields, and that the Lagrangian is invariant under a gauge group SU(3)$\otimes$SU(3), which transforms the lepton multiplet $l(x)$ according to the rule

$$l_L(x) \to U_L(x) l_L(x), \quad l_R(x) \to U_R(x) l_R(x)$$

where

$$l_L(x) \equiv \tfrac{1}{2}(1+\gamma_5) l(x), \quad l_R(x) \equiv \tfrac{1}{2}(1-\gamma_5) l(x),$$

and $U_L(x)$ and $U_R(x)$ are independent unimodular unitary matrices. The generators of this group are represented on the lepton fields by traceless Hermitian matrices, so since we want the charge to be among these generators, there must exist 3 leptons (of each helicity), with charges +1, 0, −1. [The requirement that the charge matrix be traceless rules out the possibility of an SU(2) $\otimes$ SU(2) gauge group with lepton charges 0 and ±1.] As shown by Konopinski and Mahmoud,[4] it is possible to put all the observed leptons into just such a triplet

$$l(x) = \begin{pmatrix} \mu^+(x) \\ \nu(x) \\ e^-(x) \end{pmatrix}$$

(and its adjoint), with the four-component neutrino formed out of the electron-type neutrino ν_e and the muon-type antineutrino ν_μ^c:

$$\nu_L \equiv \nu_e, \quad \nu_R \equiv \nu_\mu^c.$$

Finally, both for aesthetic reasons and in order to avoid the appearance of two independent coupling constants, it will be assumed here that *the Lagrangian conserves parity.*

It follows that the theory must involve 16 real Yang-Mills fields A_M^μ, the index M running over the values $L\alpha$ and $R\alpha$, with α running from 1 to 8. Their interaction with the leptons will be of the form

$$\mathcal{L}'_{lA} = if \sum_M (\bar{l}\gamma_\mu t_M l) A_M^\mu, \tag{4}$$

where f is a single dimensionless coupling constant, and

$$t_{\alpha L} = \tfrac{1}{2}(1+\gamma_5) t_\alpha, \quad t_{\alpha R} = \tfrac{1}{2}(1-\gamma_5) t_\alpha,$$

where t_α are the usual Hermitian SU(3) matrices,[5] normalized so that $\sum t_\alpha t_\alpha$ is an SU(3) invariant. In addition, we must suppose that this theory involves a multiplet ϕ_i of scalar fields which transform according to some reducible or irreducible representation of SU(3)$\otimes$SU(3), and whose vacuum expectation values break the gauge group and give the vector mesons their masses.[6] It follows from the Hermiticity of the mass matrix that we can find real canonically normalized fields $\bar{A}_N^\mu$ (with N running from 1 to 16) which describe vector bosons of definite mass as orthogonal linear combinations

$$\bar{A}_N^\mu = \sum_M C_{NM} A_M^\mu, \tag{5}$$

$$C^T = C^{-1}, \quad C^* = C. \tag{6}$$

The detailed structure of the C_{NM} matrix depends on the specific choice of the multiplet ϕ_i and on the values of its vacuum expectation values. However, there is one important constraint on C_{NM}, which we shall be using repeatedly in what follows, and which does not depend on the details of the symmetry-breaking mechanism. If we find a set of generators $\bar{t}_a$ which are not broken (or relatively weakly broken) by some spontaneous symmetry mechanism, and we write these as orthonormal linear combinations of the t_M,

$$\bar{t}_a = \sum_M C_{aM} t_M,$$

$$\sum_M C_{aM} C_{bM} = \delta_{ab},$$

then there must exist an equal number of vector bosons of zero mass[7] (or relatively small mass), described by real canonically normalized fields formed as parallel linear combinations of the A_M^μ:

$$\bar{A}_a^\mu = \sum_M C_{aM} A_M^\mu.$$

The rest of the C_{NM} elements are at least partly determined by the requirement that the whole C_{NM} matrix should be orthogonal. Once we complete the C_{NM} matrix in this way, we can write the interaction (4) in the form

$$\mathcal{L}'_{lA} = if \sum_N (\bar{l}\gamma_\mu \bar{t}_N l)\bar{A}_N^\mu , \tag{7}$$

where now, for all N,

$$\bar{t}_N = \sum_M C_{NM} t_M . \tag{8}$$

Any model of the weak and electromagnetic interactions which is based on an SU(3)⊗ SU(3) gauge group risks a severe conflict with observation. As remarked some time ago by Salam and Ward,[8] the known weak interactions can be accounted for in a Konopinski-Mahmoud formalism as due to the exchange of charged vector bosons associated with the SU(3)⊗ SU(3) generators $t_{6L}+t_{1R}$ and $t_{7L}+t_{2R}$, while the photon is associated with the generator $\sqrt{3}\,t_{8L}+t_{3L}+\sqrt{3}\,t_{8R}+t_{3R}$. However, unlike Salam and Ward, we are now assuming gauge invariance under the whole of SU(3) ⊗ SU(3) so our model will include 10 additional charged vector bosons as well as 3 massive neutral vector bosons. Any one of the additional charged vector bosons would produce hitherto unobserved effects: The exchange of the vector bosons associated with the generators $t_{6L}-t_{1R}$ and $t_{7L}-t_{2R}$ would alter the relative strengths of the muon-decay, muon-type semileptonic, and electron-type semileptonic Fermi interactions; the exchange of the vector bosons associated with t_{1L}, t_{2L}, t_{6R}, or t_{7R} would introduce terms in the weak interaction involving the fields $(1-\gamma_5)e^-$ or $(1+\gamma_5)\mu^+$, violating the successful two-component-neutrino theories of muon decay and beta decay, and also violating the conservation of muon number in neutrino reactions; finally, the exchange of the doubly charged vector bosons associated with the generators t_{4L}, t_{5L}, t_{4R}, or t_{5R} would lead to the muon-absorption reaction $\mu^- + (A,Z) \to e^+ + (A, Z-2)$ in muonic atoms.[9] (However, we do not have to worry about processes like $\mu^+ \to e^+ + \gamma$, which are forbidden by "lepton" conservation.)

All these effects may in fact exist, but they must be considerably weaker than the known weak interactions in order to have escaped observation thus far. There is only one coupling constant f here, so in order to suppress these unobserved effects it is necessary to assume that most of the vector bosons have masses much larger than have the bosons associated with the generators $t_{6L}+t_{1R}$ and $t_{7L}+t_{2R}$, which mediate the known weak interactions. However, in the general class of presumably renormalizable models, the masses of the various vector bosons are directly related to the strengths of the spontaneous breaking of the corresponding generators. Hence, in order to account for two different scales of vector-boson mass, it is necessary to assume that there exist two different scales of spontaneous symmetry breaking: a *superstrong symmetry breaking*, which gives extremely large masses (say, >200 GeV) to most of the vector bosons, and an *ordinary symmetry breaking*, which gives smaller (though still quite large) masses to all the remaining vector bosons except the photon.

The vector bosons which do not acquire superheavy masses from the superstrong symmetry-breaking mechanisms are just those associated with that subalgebra $\mathfrak{a}$ of SU(3)⊗ SU(3) which is left unbroken by this mechanism. In what follows, it will be assumed that $\mathfrak{a}$ is the *smallest* subalgebra of SU(3)⊗ SU(3) which contains the generators $t_{6L}+t_{1R}$ and $t_{7L}+t_{2R}$ and the charge $\sqrt{3}\,t_{8L}+t_{3L}+\sqrt{3}\,t_{8R}+t_{3R}$. As already well known,[3] this minimal subalgebra is just the previously considered gauge algebra SU(2)⊗ U(1), with generators

$$T_1 \equiv t_{6L}+t_{1R}, \tag{9}$$

$$T_2 \equiv t_{7L}+t_{2R}, \tag{10}$$

$$T_3 \equiv \tfrac{1}{2}\sqrt{3}\,t_{8L} - \tfrac{1}{2}t_{3L} + t_{3R}, \tag{11}$$

$$Y \equiv T_3 - Q = -\sqrt{3}\,t_{8R} - \tfrac{1}{2}\sqrt{3}\,t_{8L} - \tfrac{3}{2}t_{3L}. \tag{12}$$

(The possibility of enlarging $\mathfrak{a}$ is considered below.) This gauge subgroup is powerful enough to prevent the leptons from acquiring any mass from the superstrong symmetry-breaking mechanism. The much weaker ordinary symmetry-breaking mechanism is supposed to give the leptons their masses, and also gives masses to three of the four vector bosons associated with the generators (9)–(12). Hence, to the extent that we can ignore the small effects due to exchange of superheavy vector bosons, the weak interactions will appear as if the fundamental gauge group were SU(2)⊗ U(1), spontaneously broken down to electromagnetic gauge invariance, rather than SU(3)⊗ SU(3).

The one qualitatively new feature of an SU(3) ⊗ SU(3) model, which persists even in the limit of negligible superheavy boson exchange effects, is the fixed ratio of the coupling constants. To derive this ratio, it is easiest to begin by neglecting the "ordinary" symmetry breaking. The canonically normalized real fields of zero mass are then given by linear relations parallel to (9)–(12):

$$A_1^\mu = (1/\sqrt{2})(A_{6L}^\mu + A_{1R}^\mu), \tag{13}$$

$$A_2^\mu = (1/\sqrt{2})(A_{7L}^\mu + A_{2R}^\mu), \tag{14}$$

$$A_3^\mu = (1/2\sqrt{2})(\sqrt{3}\,A_{8L}^\mu - A_{3L}^\mu + 2A_{3R}^\mu), \tag{15}$$

$$B^\mu = (1/2\sqrt{2})(A_{8L}^\mu + \sqrt{3}\,A_{3L}^\mu + 2A_{8R}^\mu). \tag{16}$$

Neglecting all superheavy fields, the interaction (7) now takes the form

$$\mathcal{L}'_{lh} = if\bar{l}\gamma_\mu[(1/\sqrt{2})\vec{T}\cdot\vec{A}^\mu - (1/\sqrt{6})YB^\mu]l. \tag{17}$$

Comparing this with earlier work,[1] we see that the coupling constants g and g' have the values

$$g = f/\sqrt{2}, \quad g' = -f/\sqrt{6}, \tag{18}$$

so the mixing angle defined by (1) is

$$|\theta| = 30°. \tag{19}$$

Of course, the algebra $SU(2)\otimes U(1)$ is in fact spontaneously broken. Without going into the details of the symmetry-breaking mechanism, we know that the one field which retains a zero mass is the electromagnetic field. Since the charge operator is $t_{3L}+\sqrt{3}\,t_{8L}+t_{3R}+\sqrt{3}\,t_{8R}$, the canonically normalized electromagnetic field is

$$A^\mu = (1/2\sqrt{2})(A^\mu_{3L}+\sqrt{3}\,A^\mu_{8L}+A^\mu_{3R}+\sqrt{3}\,A^\mu_{8R})$$
$$= \tfrac{1}{2}(A^\mu_3 + \sqrt{3}\,B^\mu). \tag{20}$$

Hence the neutral vector boson of nonzero (but nonsuperheavy) mass is described by the orthogonal linear combination

$$Z^\mu = \tfrac{1}{2}(\sqrt{3}\,A^\mu_3 - B^\mu). \tag{21}$$

The interaction (17) thus takes the form

$$\mathcal{L}'_{lA} = if\bar{l}\gamma_\mu[\tfrac{1}{2}(T_1 - iT_2)W^\mu + \tfrac{1}{2}(T_1+iT_2)W^{\mu\dagger}$$
$$+ (1/2\sqrt{6})(3T_3+Y)Z^\mu$$
$$+ (1/2\sqrt{2})(T_3 - Y)A^\mu]l, \tag{22}$$

where

$$W^\mu \equiv (1/\sqrt{2})(A^\mu_1 + iA^\mu_2).$$

This agrees with the interaction deduced in earlier work,[1] if g and g' are given the special values (18). In particular, the electronic charge is immediately seen to be

$$e = f/2\sqrt{2}, \tag{23}$$

so g and g' may be expressed in terms of e:

$$g = 2e, \quad g' = -2e/\sqrt{3}. \tag{24}$$

The strength of the known weak interactions, which are due to W exchange, is expressed in an effective Fermi coupling constant

$$G/\sqrt{2} = g^2/8m_W^2 = e^2/2m_W^2, \tag{25}$$

so m_W is now predicted unequivocally to have the value

$$m_W = (e^2/\sqrt{2}\,G)^{1/2} = 74.6 \text{ GeV} \tag{26}$$

in agreement with Eq. (2) for $\theta = 30°$.

To go further, we need to consider the mechanism which breaks $SU(2)\otimes U(1)$. Since this mechanism is responsible for the lepton masses, it is natural to consider a complex scalar multiplet ϕ_{ij} which transforms according to the $(3,\bar{3})$ representation and is coupled to the leptons through the $SU(3)\otimes SU(3)$-invariant interaction

$$\mathcal{L}'_{l\phi} = -G_\phi \bar{l}_{iR} l_{jL}\phi_{ij} + \text{H.c.}, \tag{27}$$

where i and j run over the values μ, ν, e, and G_ϕ is a new dimensionless coupling constant. Charge conservation allows nonvanishing vacuum expectation values only for the diagonal elements of ϕ_{ij}; by a suitable choice of phase for the three lepton fields, these vacuum expectation values as well as G_ϕ may be chosen to be real. On the basis of the observed values of the lepton masses, it is then a good approximation to neglect all these vacuum expectation values except for

$$\langle\phi_{11}\rangle_0 \equiv \lambda \simeq m_\mu/G_\phi. \tag{28}$$

But with respect to the group $SU(2)\otimes U(1)$ the field ϕ_{11} forms part of a doublet (ϕ_{11}, ϕ_{21}). Hence, to the extent that the nonvanishing vacuum expectation value of ϕ_{11} is the *only* mechanism which breaks $SU(2)\otimes U(1)$, all of the results of the previous theory,[1] in which $SU(2)\otimes U(1)$ was also broken by the nonvanishing vacuum expectation value of a doublet scalar field, will still apply. In particular, the mass of the neutral vector boson Z is again given by Eq. (3):

$$m_Z = m_W/\cos\theta = (2/\sqrt{3})m_W = 86.2 \text{ GeV}. \tag{29}$$

The formulas for neutrino-electron scattering derived by Chen and Lee[10] and by 't Hooft[11] also still apply, but with $|\theta|$ now fixed at 30°. This is safely within the upper bound $|\theta| < 36°$ derived by Chen and Lee[10] from the data of Reines[12] on the process $\bar{\nu}_e + e \to \bar{\nu}_e + e$. The emission and absorption of virtual W and Z bosons would change the gyromagnetic ratio of the muon by an amount which for $\theta = 30°$ has the value[13]

$$\Delta g_\mu = \frac{5Gm_\mu^2}{12\pi^2\sqrt{2}} = 0.4\times 10^{-8}.$$

Apart from terms of order m_e/m_μ, and the direct effects of superheavy vector bosons, the only difference between the present $SU(3)\otimes SU(3)$ model and the earlier $SU(2)\otimes U(1)$ model with $|\theta| = 30°$ is the presence of a richer spectrum of scalar bosons, now all with the same coupling constant G_ϕ. To evaluate G_ϕ, we note that m_W is again related to λ by

$$m_W = \tfrac{1}{2}\lambda g,$$

so the coupling constant here has the value

$$G_\phi = m_\mu/\lambda = m_\mu g/2m_W$$
$$= 2^{1/4} m_\mu G^{1/2} = 4.27\times 10^{-4}. \tag{30}$$

After eliminating the Goldstone boson fields ϕ_{12} and $\mathrm{Im}\phi_{11}$ by a gauge transformation, there are left 13 real scalar fields. The possibility of observing these fields depends critically on the unknown mass values of the corresponding bosons.[13]

Unfortunately, the extension of SU(2)⊗U(1) to SU(3)⊗SU(3) has only increased the difficulty of incorporating the hadrons in this sort of gauge model. Quite apart from any assumption as to the nature of the elementary quark fields (if any) the charge structure of the familiar baryon and meson octets shows that the approximate SU(3) symmetry of the strong interactions is not the same as the underlying gauge symmetry SU(3), because there is no automorphism of SU(3) (either inner or outer) which can transform $t_3+t_8\sqrt{3}$ into $t_3+t_8/\sqrt{3}$. However, if we simply close our eyes to these difficulties and assume that the proton and neutron behave approximately like an SU(2) ⊗U(1) doublet, then it is possible to make various useful predictions about semileptonic interactions. In this way, it has been shown[14] that for not-too-large values of the momentum transfer, the neutral and charge-exchange neutrino-nucleon scattering cross sections have the ratio

$$\frac{d\sigma(\nu+p\rightarrow\nu+p)/dq^2}{d\sigma(\nu+n\rightarrow\mu^-+p)/dq^2}=\frac{1}{4}\left(\frac{(1.2)^2+(1-4\sin^2\theta)^2}{(1.2)^2+1}\right).$$

If we take this estimate seriously, then the experimental value[15] (or upper limit) 0.12 ± 0.06 for this ratio requires a value of $|\theta|$ close to the SU(3)⊗SU(3) value of 30°.

Finally, let us return to the superheavy vector bosons. In considering their properties, it is an adequate approximation to ignore the "ordinary" spontaneous breaking of SU(2)⊗U(1), taking into account only the superstrong breakdown of SU(3) ⊗SU(3) to SU(2)⊗U(1). Classifying all vector-boson fields by their SU(2)⊗U(1) transformation properties, we find two $Y=0$ triplets

$$(A_1^\mu,A_2^\mu,A_3^\mu) \tag{31}$$

and

$$\begin{aligned}A_9^\mu&=(1/\sqrt{2})(A_{6L}^\mu-A_{1R}^\mu),\\ A_{10}^\mu&=(1/\sqrt{2})(A_{7L}^\mu-A_{2R}^\mu),\\ A_{11}^\mu&=(1/2\sqrt{2})(\sqrt{3}A_{8L}^\mu-A_{3L}^\mu-2A_{3R}^\mu),\end{aligned} \tag{32}$$

plus two $Y=0$ singlets

$$A_8^\mu=B^\mu=(1/2\sqrt{2})(A_{8L}^\mu+\sqrt{3}A_{3L}^\mu+2A_{3R}^\mu) \tag{33}$$

and

$$A_{16}^\mu=(1/2\sqrt{2})(A_{8L}^\mu+\sqrt{3}A_{3L}^\mu-2A_{3R}^\mu), \tag{34}$$

plus two complex $Y=+\frac{3}{2}$ doublets

$$\begin{pmatrix}A_4^\mu+iA_5^\mu\\ A_6^\mu+iA_7^\mu\end{pmatrix},\quad\begin{pmatrix}A_{12}^\mu+iA_{13}^\mu\\ A_{14}^\mu+iA_{15}^\mu\end{pmatrix} \tag{35}$$

(and their antidoublets), where

$$A_4^\mu=A_{4R}^\mu,\quad A_5^\mu=A_{5R}^\mu,\quad A_6^\mu=A_{6R}^\mu,\quad A_7^\mu=A_{7R}^\mu,$$
$$A_{12}^\mu=A_{4L}^\mu,\quad A_{13}^\mu=A_{5L}^\mu,\quad A_{14}^\mu=-A_{1L}^\mu,\quad A_{15}^\mu=-A_{2L}^\mu.$$

We know that when only the superstrong symmetry breaking is taken into account, the triplet (31) and singlet (33) describe massless vector bosons. On the other hand, the general features of the weak interactions discussed above require that the vector bosons described by the triplet (32) and the two doublets (35) are superheavy. (This applies also to the neutral vector boson described by A_{11}^μ, because even if it would not itself have produced observable effects, it must be degenerate with the superheavy charged bosons described by A_9^μ and A_{10}^μ.) This leaves the neutral boson described by the singlet (34), which might or might not be superheavy. Thus there are just two possibilities for the subalgebra which is left unbroken at the superstrong level: If the boson described by A_{16}^μ is superheavy then $\mathfrak{A}$ is just the familiar SU(2)⊗U(1) algebra, while if this boson is not superheavy then $\mathfrak{A}$ must be SU(2)⊗U(1)⊗U(1), with the extra generator given as $t_{8L}+\sqrt{3}\,t_{3L}-2t_{8R}$.

There is one good reason for suspecting that the generator $t_{8L}+\sqrt{3}\,t_{3L}-2t_{8R}$ is in fact broken at the superheavy level. If the superstrong symmetry-breaking mechanism left both generators $t_{8L}+\sqrt{3}\,t_{3L}\pm2t_R$ unbroken, then the generator $t_{8L}+\sqrt{3}\,t_{3L}+4t_{8R}$ would be left unbroken by both this mechanism and by the vacuum expectation value $\langle\phi_{11}\rangle_0$ responsible for the muon mass. On the other hand, the vacuum expectation value $\langle\phi_{33}\rangle_0$ responsible for the electron mass does break this generator. If this is the *only* symmetry-breaking mechanism for $t_{8L}+\sqrt{3}\,t_{3L}+4t_{8R}$, then the corresponding vector boson must have an unacceptably low mass, of order $(m_e/m_\mu)m_W$.

Assuming then that the algebra $\mathfrak{A}$ left unbroken at the superstrong level is just SU(2)⊗U(1), we conclude that the superheavy bosons must form one degenerate real triplet with charges +1, 0, −1, plus two degenerate doublets with charges −1, −2 and their antidoublets, plus one singlet of charge 0. With $t_{8L}+\sqrt{3}\,t_{3L}-2t_{8R}$ broken, there is nothing to prevent a strong mixing of the two doublets (35), so that the fields which describe the superheavy doublet bosons of definite mass are orthogonal linear combinations of the doublet fields in (35).

The mixing of the two doublets (35) is potentially very interesting, because it offers a possible mechanism for generating an electron mass. Since the electron mass is so small, it is natural to suppose that the self-interaction of the ϕ_{ij} fields is such that ϕ_{11} is the only one of these fields to have a nonvanishing vacuum expectation value in lowest order. In this case, the muon is the only lepton to have a lowest-order mass, but the emission of a doubly charged superheavy boson by the current $\bar{\mu}^+\gamma_\lambda(1+\gamma_5)e^-$ and its subsequent absorption by the current $\bar{e}^-\gamma^\lambda(1-\gamma_5)\mu^+$ (and vice versa) will generate a second-order electron mass, of order αm_μ. There is no similar mechanism for generating a neutrino mass, and indeed, there is a combined reflection and SU(3)⊗SU(3) outer automorphism

$$\mu_L \to \mu_L, \quad \nu_L \to -\nu_L, \quad e_L \to -e_L,$$

$$\mu_R \to \mu_R, \quad \nu_R \to +\nu_R, \quad e_R \to -e_R,$$

$$A_M^\mu \to \pm A_M^\mu \quad \text{for} \quad M = \begin{cases} 3L, 6L, 7L, 8L, 1R, 2R, 3R, 8R \\ 1L, 2L, 4L, 5L, 4R, 5R, 6R, 7R \end{cases}$$

which is not broken by the superheavy boson mass matrix or by the muon mass, and which would protect the neutrino from acquiring a mass. The precise value of the electron mass produced by this theory depends on the details of the symmetry-breaking mechanism, and, in particular, on the mass ratios and mixing angles of the superheavy bosons.

It is difficult even to guess at how heavy the superheavy bosons might be. For the present, it might be worthwhile to look once again for weak violations of the two-component neutrino theory of beta decay or of the conservation of muon number.

Note added in proof. It should be emphasized that a calculation of the electron mass along the lines discussed in this paper would naturally give a *finite* result. The reason is that we are here supposing that the scalar fields introduced to break the gauge symmetry belong to such simple multiplets that there are not enough free parameters in the Lagrangian to allow us to adjust the zeroth-order electron mass to any value we like. In this case, there is no counterterm available to absorb an infinity in the electron self-energy, so if the theory is to be renormalizable at all, the self-energy must be finite. Its precise value will depend of course on the choice of the representation to which the scalar multiplet belongs; unfortunately there is as yet no compelling reason to choose one particular representation rather than any other.

I am grateful for valuable conversations with Roman Jackiw and Francis Low.

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*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-3069.

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