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Similarly we expect a falloff in the timelike region to take place in lepton pair production via the weak interaction and we describe such a falloff by a phenomenological form factor. A model for the photoproduction of single spin-one W bosons based on the parton picture has been examined by K. O. Mikaelian, *Phys. Rev. D* **5**, 70 (1972).

¹¹See, for example, D. H. Perkins, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN, Geneva, 1969), p. 1.

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Triangle Graphs, Current Algebra, and K_{l3} Decay

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We develop a new low-energy theorem for K_{l3} decay as the momentum transferred to the lepton pair vanishes. Saturating the axial-vector channel with the pion and the baryon-antibaryon intermediate states, we obtain the result $\xi(0) \simeq -1$ while at the same time preserving the Callan-Treiman relation.

I. INTRODUCTION

It has been known for some time¹ that the triangle graph for π^0 decay leads to reasonable agreement with experiment, both in magnitude and in sign.² Adler³ has recently given a justification of this triangle approximation from the viewpoint of current algebra.

Here we show that the current-algebra triangle approximation for K_{l3} decay also leads to good agreement with experiment. Because of the change of the over-all spin-parity (normality) of the K_{l3} vertex as compared to the $\pi^0 \rightarrow 2\gamma$ vertex, the K_{l3} current-algebra triangle analysis must proceed differently from the π^0 analysis and does not have the perturbation-theory basis as does Adler's work.³

Our approach divides the problem into two parts. In the first part, we obtain the axial-vector Ward identity of current algebra and then formally solve it (Sec. II). In the second part, we saturate the axial-vector channel with a complete set of on-shell intermediate states (Sec. III), and then develop a new soft-pion theorem at zero momentum transferred to the lepton pair so as to enhance the importance of the triangle graph in the unitarity saturation (Sec. IV). The result of this theorem is the ratio $\xi = f_-(0)/f_+(0) \simeq -1$, which seems to be the accepted experimental value.⁴ Both the magnitude and sign of the triangle graph are important in

reaching this conclusion.

The large negative value of ξ is hard to understand by saturating the vector (lepton-pair) channel,^{4,5} which casts doubt upon the Callan-Treiman relation.^{6,7} Since the latter relation also follows from the axial-vector Ward identity (in the soft-pion limit), we believe it important to verify that this Ward identity follows from very general assumptions—other than the local commutation relations of current algebra.⁸ Following the ideas outlined in a previous paper,⁹ we show in the Appendix that the axial-vector Ward identity is a consequence of axial-vector current conservation. Given the validity of this Ward identity, it is then the triangle graph which resolves the apparent conflict between the Callan-Treiman relation and $\xi \simeq -1$.

II. SOLUTION OF THE WARD IDENTITY

Following the massless-pion approach to partial conservation of the axial-vector current (PCAC),^{9,10} we consider the process of Fig. 1, $K^+(k) \rightarrow A_\mu^0(q) + V_l^+(\Delta)$, related to the K_{l3} decay $K^+(k) \rightarrow \pi^0(q) + l^+ + \nu$. The normal (no $\epsilon_{\mu\nu\alpha\beta}$) three-point vertex function $M_{\mu\nu}$ is related to the two-point K_{l3} decay amplitude $i\sqrt{2}f_K k_\nu$ (where $f_K \simeq 120$ MeV) via the "axial-vector Ward identity,"^{11,12}

$$q^\mu M_{\mu\nu} = \frac{-1}{\sqrt{2}} i f_K k_\nu, \quad (1)$$

with $k = q + \Delta$ and $V^+ = V^4 - iV^5$.

Usually one derives Eq. (1) from the chiral commutation relation of Gell-Mann,⁶⁻⁸ symbolically written as $[A, V] = A$. We prefer to think of Eq. (1) as a consequence of axial-vector current conservation in a world of massless pions, first-order weak processes with massless leptons, and the isotopic structure (charge algebra) of lepton currents.⁹ We carry out the derivation of (1) in the Appendix.

Here we point out that the axial-vector Ward identity can be solved in the form

$$M_{\mu\nu} = \frac{-1}{\sqrt{2}} if_K [q^{-2} q_\mu (q_\nu + k_\nu) - g_{\mu\nu}] + \xi_1 (k \cdot q g_{\mu\nu} - k_\mu q_\nu) + \xi_2 (q^2 g_{\mu\nu} - q_\mu q_\nu), \quad (2)$$

where ξ_1 and ξ_2 are arbitrary scalar functions. Clearly multiplication of Eq. (2) by q^μ yields Eq. (1). This structure of the vertex function was suggested by $M_{\mu\nu}^{iP}$ of Ref. 9, or by $R_{\mu\nu}$ of Ref. 13, leading to the solution of the "factor-of-two problem" of pion photoproduction. The important point to note about Eq. (2) is that there is no $k_\mu k_\nu$ term. This is due to the vector Ward identity (knowledge of $M_{\mu\nu} \Delta^\nu$) in conjunction with the Callan-Treiman relation. It is clear that this solution, Eq. (2), uniquely determines only the $q_\mu k_\nu$ coefficient of $M_{\mu\nu}$. However, this is precisely the knowledge we need, as saturation of the axial-vector channel will now demonstrate. Our procedure will be to determine the coefficient of $q_\mu k_\nu$ and in particular to ignore the coefficient of $k_\mu k_\nu$ on the grounds that other diagrams will make its total coefficient vanish.

III. SATURATION OF THE AXIAL-VECTOR CHANNEL

First we pole-dominate the axial vector by a massless pion [Fig. 1(a)]. We define the $K^+ \rightarrow \pi^0 + l^+ + \nu$ amplitude as¹²

$$S_{fi} = -i(2\pi)^4 \delta^4(k - q - \Delta) H_{fi} = i(2\pi)^4 \delta^4(k - q - \Delta) (G/\sqrt{2}) \sin\theta M_\nu j^\nu, \quad (3)$$

where

$$H_{fi} = (G/\sqrt{2}) \sin\theta \langle \pi^0 | V_\nu^{4-i5} | K^+ \rangle j^\nu,$$

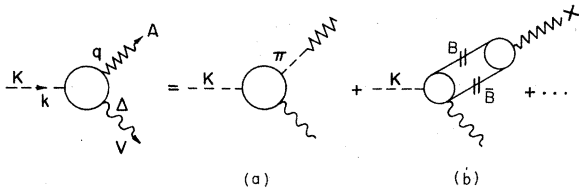


FIG. 1. The decay $K(k) \rightarrow A(q) + V(\Delta)$, with saturation of the axial-vector channel by the π (a) and $B\bar{B}$ (b) intermediate states. The x means g_A coupling.

$$M_\nu = -\langle \pi^0 | V_\nu | K^+ \rangle = (-1/\sqrt{2}) [f_+(t)(k_\nu + q_\nu) + f_-(t)(k_\nu - q_\nu)], \quad (4)$$

$$j^\nu = \bar{u}_{(l)} \gamma^\nu (1 - i\gamma_5) v_{(l)},$$

with θ the Cabibbo angle, $t = \Delta^2$ the square of the invariant momentum transferred to the lepton pair, and $f_\pm(t)$ the K_{13} form factors which become $f_+ \rightarrow 1$ and $f_- \rightarrow 0$ in the SU(3) limit. Further defining the pion decay amplitude by

$$\langle 0 | A_\mu^3 | \pi^0(q) \rangle = if_\pi q_\mu, \quad (5)$$

where $f_\pi \cong 83$ MeV, we can pole-dominate the $K^+ \rightarrow A_\mu^0 + V_\nu^+$ amplitude as in Fig. 1(a),

$$M_{\mu\nu}^\pi = if_\pi (q_\mu/q^2) M_\nu = \frac{-if_\pi}{\sqrt{2}} \frac{q_\mu}{q^2} \{ [f_+(t) + f_-(t)] k_\nu + [f_+(t) - f_-(t)] q_\nu \}. \quad (6)$$

Next we saturate $A_\mu^0(q)$ by the $B\bar{B}$ ($B = \text{baryon}$) state according to Fig. 1(b). In line with the approach to $\pi^0 \rightarrow \gamma\gamma$,¹³ we expect the $N\bar{N}$ state to become important at low momentum where the Λ pole controls the $K^+ \rightarrow p + \bar{p} + V^+$ amplitude. When the $p\bar{p}$ state is "tied" to the axial vector, the resulting "triangle" diagram is given by Fig. 2, with neutral coupling $\frac{1}{2} ig_A \gamma_\mu \gamma_5$. In analogy with the nucleon axial-vector vertex function, the $N\bar{N}$ state accounts for the g_A term, whereas the pion pole accounts for h_A , the induced pseudoscalar term.

There is one important difference between the $\pi^0 \rightarrow \gamma\gamma$ and K_{13} decays. Strangeness conservation rules out the "exchange" K_{13} triangle graph and this indicates that the hyperon $\Sigma\bar{\Sigma}$, $\Xi\bar{\Xi}$, etc. intermediate states must also be taken into account. There are ten such triangle graphs of which nine approximately cancel because of SU(3) $D + \frac{2}{3}F$ couplings. The remaining hyperon triangle graph is indicated in Fig. 3, and its magnitude is essentially equal to the nucleon triangle graph so that it is the effective exchange diagram for this process.

Using the Feynman rules we then compute the contribution of the $B\bar{B}$ triangle graphs to be

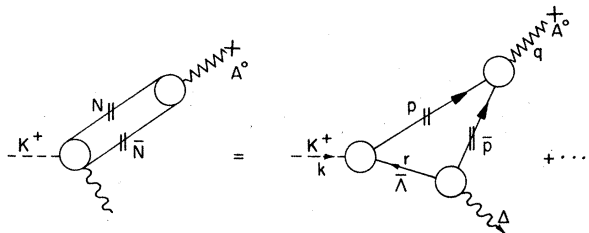


FIG. 2. Domination of the $N\bar{N}$ intermediate state by the $p\bar{p}\Lambda$ triangle graph as $\Delta \rightarrow 0$.

$$M_{\mu\nu}^{\bar{B}\bar{B}} = \frac{1}{2}g_A\zeta(2\pi)^{-4} \int d^4r \text{Tr} \gamma_\mu \gamma_5 [\gamma \cdot (r+k) - m]^{-1} \gamma_5 [\gamma \cdot r - m]^{-1} \gamma_\nu [\gamma \cdot (r+\Delta) - m]^{-1}, \quad (7)$$

with the SU(3) coupling factor ζ given by¹⁴

$$\zeta = \frac{g_{\rho\Delta K^+} G_{\Delta\rho}}{G} + \frac{g_{\Sigma^+ \Xi^0 K^+} G_{\Xi^0 \Sigma^+}}{G} \approx 2\sqrt{2}g, \quad (8)$$

where $g^2/4\pi \cong 15$. The loop integral in Eq. (7) diverges logarithmically, yet the coefficient of the $q_\mu k_\nu$ covariant is finite. This coefficient we extract from either the Feynman rules or dispersion-theory rules, and with the help of the Goldberger-Treiman relation $m_{G_A} = f_\pi g$ we find

$$M_{\mu\nu}^{\bar{B}\bar{B}} \cong \frac{-if_\pi g^2}{\sqrt{2}} \frac{q_\mu k_\nu}{4\pi^2 m_N^2} + \dots \quad (9)$$

In order that these triangle graphs completely dominate the $\bar{B}\bar{B}$ intermediate states, the momentum-transfer variable Δ must be small (see Sec. IV) in the sense that

$$\Delta^2 - (m_\Lambda^2 - m_N^2) < m_N^2 \text{ and } \Delta^2 - (m_\Xi^2 - m_\Sigma^2) < m_N^2.$$

Besides the $\bar{B}\bar{B}$ intermediate states one should include higher-mass single-particle states, $\bar{B}\bar{B}\pi$ states, and $N\bar{\Delta}$, $\Delta\bar{\Delta}$, $\Sigma^* \bar{\Sigma}^*$ states, etc. In the $\pi^0 \rightarrow \gamma\gamma$ problem, Adler and Bardeen¹⁵ have shown that the σ model justifies ignoring all but the simplest nucleon triangle graphs. We believe there exists a current-algebra-preserving field-theory model which justifies ignoring such states in the K_{13} problem as well, and we shall simply ignore them.

IV. SOFT-MESON THEOREMS

We are now in a position to find soft-meson K_{13} theorems by equating the coefficients of the covariant $q_\mu k_\nu$. Usually the Callan-Treiman relation is found from the coefficient of k_ν in the axial-vector Ward identity, but in addition we will obtain new information by examining $M_{\mu\nu}$ itself. From the solution of the axial-vector Ward identity, Eq. (2), we see that only the coefficient of $q_\mu k_\nu$ is deter-

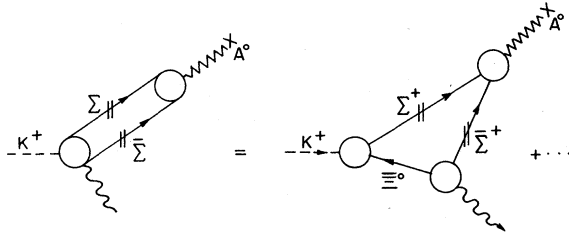


FIG. 3. Domination of the $\Sigma\bar{\Sigma}$ intermediate state by the $\Sigma^+\Sigma^+\Xi^0$ "exchange" triangle graph as $\Delta \rightarrow 0$.

mined,

$$M_{\mu\nu} = \frac{-if_K}{\sqrt{2}} \frac{q_\mu k_\nu}{q^2} + \dots, \quad (10)$$

whereas from saturation of the axial-vector channel we have

$$M_{\mu\nu} = M_{\mu\nu}^\pi + M_{\mu\nu}^{\bar{B}\bar{B}} + \dots \\ = \frac{-if_\pi}{\sqrt{2}} [f_+(t) + f_-(t)] \frac{q_\mu k_\nu}{q^2} - \frac{if_\pi}{\sqrt{2}} \frac{g^2}{4\pi^2} \frac{q_\mu k_\nu}{m_N^2} + \dots \quad (11)$$

In the soft-pion limit $q \rightarrow 0$, the leading q^{-2} terms of Eqs. (10) and (11) dominate, and we recover the Callan-Treiman relation,

$$f_+(m_K^2) + f_-(m_K^2) = f_K/f_\pi, \quad (12)$$

where $f_K/f_\pi f_+(0) \cong 1.28$ from a single-Cabibbo-angle theory of K_{12} and π_{12} decays. The $\bar{B}\bar{B}$ intermediate states do not enter the soft-pion $q \rightarrow 0$ limit, in contrast with π^0 decay where they lead to a resolution of the PCAC puzzle of Bell and Jackiw.¹³

If instead we look at the soft limit $\Delta \rightarrow 0$, i.e., $q \rightarrow k$ and $q^2 \rightarrow m_K^2$, the soft-"vector-photon" theorem of Low¹⁶ tells us that the triangle graphs of Figs. 2 and 3 dominate the intermediate process $K^+ \rightarrow B + \bar{B} + V^+(\Delta)$. In this limit we again equate the coefficients of $q_\mu k_\nu$ in Eqs. (10) and (11) to obtain a new soft theorem,

$$f_+(0) + f_-(0) \cong \frac{f_K}{f_\pi} - \frac{g^2}{4\pi^2} \frac{m_K^2}{m_N^2} \cong 1.3 - 1.3 = 0. \quad (13)$$

Solving Eq. (13) for the ratio ξ gives¹⁷

$$\xi(0) = f_-(0)/f_+(0) \cong -1, \quad (14)$$

which is now the accepted experimental value.⁴

Finally we remark that our soft limits $q \rightarrow 0$ and $\Delta \rightarrow 0$ give form factors which are off their physical mass shell. Defining the K_{13} form factors as $f_\pm(t, q^2, k^2)$, the $q \rightarrow 0$ limit involves $f_\pm(m_K^2, 0, m_K^2)$ instead of $f_\pm(m_K^2, m_\pi^2, m_K^2)$, whereas the $\Delta \rightarrow 0$ limit involves $f_\pm(0, m_K^2, m_K^2)$ instead of $f_\pm(0, m_\pi^2, m_K^2)$. We assume the extrapolation in the q^2 variable is valid in either case, though the latter extrapolation is eight times the former,

$$f_\pm(m_K^2, 0, m_K^2) \cong f_\pm(m_K^2, m_\pi^2, m_K^2), \quad (15)$$

$$f_\pm(0, m_K^2, m_K^2) \cong f_\pm(0, m_\pi^2, m_K^2). \quad (16)$$

We stress that no extrapolation in $t = \Delta^2$ is necessary to obtain $\xi(0) \cong -1$; this is a new low-energy

theorem at $\Delta=0$ (and $q^2 = m_K^2$). This limit does not allow a replacement of m_K^2 by q^2 in (13), which would otherwise contradict (16). Furthermore, note that the kaon is always on its mass shell in our approach, so that PCAC applied to the kaon need not be considered.

V. CONCLUSION

We have treated K_{l3} decay by saturating the axial-vector channel with the π and $B\bar{B}$ intermediate states. The effective triangle graphs arising from the $B\bar{B}$ intermediate states give good agreement with experiment. The only remaining question in our on-shell approach is why other two-particle baryon states such as the $\Delta\bar{\Delta}$ do not contribute significantly to the axial-vector current triangle graphs of K_{l3} .

In particular we have developed a new soft-pion K_{l3} theorem to show that the triangle graph must be taken into account when $f_+(t) + f_-(t)$ is evaluated at $t=0$ rather than at $t=m_K^2$. The Callan-Treiman relation is then consistent with $\xi \simeq -1$ if we assume that the form factors vary slowly in q^2 for fixed t . If we were to extrapolate the form factors in t from $t=m_K^2$ to $t=0$, the triangle graph would indicate a rapid variation in t for fixed q^2 .

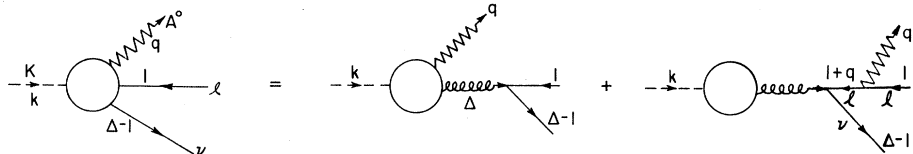
This picture complements the work of Banerjee,¹⁸ Kang,¹⁹ and Fubini and Furlan²⁰ who keep the pion on its mass shell and demonstrate that a rapid variation in t of the divergence form factor, dipping near $t=(m_K - m_\pi)^2$, is consistent with the Callan-Treiman relation, $\xi \simeq -1$, λ_+ small and positive, and the $SU(2) \times SU(2)$ symmetry of the hadronic Hamiltonian.²¹ Furthermore, Fubini and Furlan obtain a modified Callan-Treiman relation on the boundary of the physical region $t=(m_K - m_\pi)^2$.

$$M_\mu^0 = (G/\sqrt{2}) M_{\mu\nu}^{AW} j^\nu \sin\theta - (G/\sqrt{2}) \langle 0 | J_\nu^W | K^+ \rangle \bar{u}_{(l)} [\gamma^\nu (1 - i\gamma_5) (-\gamma \cdot l - \gamma \cdot q)^{-1} i\gamma_\mu \gamma_5] v_{(l)}, \quad (A1)$$

where A and W on $M_{\mu\nu}^{AW}$ indicate the axial-vector and weak [$W = (V - A)^{4-i5}$] nature of the currents. Note that the axial-vector current $A_\mu^{3+8/\sqrt{3}}$ tags only on the charged lepton line in Fig. 4. In (A1) we have used the notion of universality to replace the electromagnetic current $eJ^{3+8/\sqrt{3}}$ by the weak current $(G/\sqrt{2})J^{3+8/\sqrt{3}}$.

In our massless-pion world, the principle of

FIG. 4. Axial-vector Ward-identity diagrams. The spiral line indicates the first-order weak interaction.



When this relation is linearly extrapolated back to $t=0$ using the experimental value of λ_+ , even ignoring symmetry-breaking effects, Fubini and Furlan show that ξ is altered from its usual current-algebra value⁵ of $\xi \sim 0$ to the range $\xi \sim -0.4$ to -0.7 . Their extrapolation is in t with $q^2 = m_\pi^2$; ours is in q^2 with $t=0$.

These combined viewpoints consistently give the Callan-Treiman relation, $\xi \sim -1$, and a negative slope for the divergence form factor [$\lambda = \lambda_+ + m_\pi^2 \times (m_K^2 - m_\pi^2)^{-1} \xi$], which would seem to rule out¹⁸ pole models of the divergence form factor, as well as "weak PCAC"²² with an $SU(3)$ -symmetry-breaking Hamiltonian.

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APPENDIX

We derive the K_{l3} axial-vector Ward identity from the assumptions of axial-vector current conservation and massless leptons. Consider the process $K^+ \rightarrow A_\mu^0 + l^+ + \nu$ described by the amplitude M_μ^0 . The superscript on the axial vector, 0, here refers to the entire charge state $I_3 + \frac{1}{2}Y = 3 + 8/\sqrt{3}$. Then expanding the amplitude according to Fig. 4, we write

axial-vector current conservation states that

$$q \cdot M^0 = 0. \quad (A2)$$

Note that $q \cdot M^3 \neq 0$ because it is the total²³ current that is conserved and $\Delta Y \neq 0$ in K_{l3} decay means that M_μ^3 has hypercharge current, leaving the diagram corresponding to Fig. 4. Applying (A2) to (A1) and using the Dirac equation for massless lep-

tons, we find

$$0 = (G/\sqrt{2}) j^\nu [q^\mu M_{\mu\nu}^{AW} \sin\theta + \langle 0 | J_\nu^W | K^+ \rangle]. \quad (\text{A3})$$

Since (A3) holds for any value of q , we can equate separately to zero the vector part of the term in brackets,

$$q^\mu M_{\mu\nu}^{AV} \sin\theta = \langle 0 | J_\nu^A | K^+ \rangle. \quad (\text{A4})$$

Then using²⁴

$$\begin{aligned} \langle 0 | J_\nu^A | K^+ \rangle &= -\sin\theta \langle 0 | A_\nu^{4-i5} | K^+(k) \rangle \\ &= -\sin\theta i\sqrt{2} f_K k_\nu, \end{aligned} \quad (\text{A5})$$

with $f_K \cong 120$ MeV corresponding to our normalization of (5) along with the Goldberger-Treiman relation $m_{G_A} = f_\pi g$, we find

$$q^\mu M_{\mu\nu}^{3+8/\sqrt{3}, A-i5} = -i\sqrt{2} f_K k_\nu. \quad (\text{A6})$$

In a similar manner we conserve the axial-vec-

tor current for the process $K^0 \rightarrow A_\mu^0 + \bar{\nu} + \nu$. Now the vector current carries off the SU(3) quanta $6 - i7$, and because the "charge-selecting" axial-vector current $3 + 8/\sqrt{3}$ does not tag on (neutral) neutrinos, we obtain

$$q^\mu M_{\mu\nu}^{3+8/\sqrt{3}, 6-i7} = 0. \quad (\text{A7})$$

The SU(3) solution of (A6) and (A7) for all components separates the isovector and isoscalar parts of the axial-vector current and is

$$q^\mu M_{\mu\nu}^{ij} = -if^{ijk}(-if_K k_\nu), \quad (\text{A8})$$

with the general K_{13} process defined by

$$\begin{aligned} M_{\mu\nu}^{ijk} &= -\langle \pi^i(q) | V_\nu^j | K^k(k) \rangle \\ &= if^{ijk} [f_+(t)(k_\nu + q_\nu) + f_-(t)(k_\nu - q_\nu)]. \end{aligned} \quad (\text{A9})$$

Equation (A8) includes Eq. (1) as a special case.

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