Experimental Tests of Weinberg's Theory of Leptons*

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We discuss some experimental consequences of Weinberg's unified theory of weak and electromagnetic interactions of leptons, which may be renormalizable. We comment on the extant experimental data bearing on this theory (such as the data on the processes $\nu_\mu+e\to\nu_\mu+e$, $\bar\nu_e+e\to\bar\nu_e+e$), and on possible future refinements of these data. A bound is obtained on a parameter of the theory.

Some years ago, Weinberg[1] discussed a theory of leptons in which electromagnetic and weak interactions are mediated by Yang-Mills gauge vector bosons[2] (to be precise, by four of them, two neutral and two charged), and in which the masses of the charged and neutral vector bosons mediating weak interactions are due to the spontaneous breakdown of the gauge symmetry. Weinberg pointed out that the theory may be renormalizable since the equations of motion of this theory are formally identical to those of a Yang-Mills theory. Recently, t'Hooft[3] examined the renormalization question of theories of this kind and came to a conclusion which supports such optimism. A simpler model in which an Abelian gauge group is spontaneously broken is now known to be renormalizable in a physically satisfactory way[4] (in the sense that the physical S matrix is unitary despite the indefinite-metric nature of the Hilbert space used to construct the Green's functions). More recently, Weinberg[5] computed a number of higher-order weak processes in the lowest nontrivial order, and found them to be finite.

An equally important aspect of this theory is that it makes specific predictions on physically observable processes which differ from those of a conventional theory [say, the current×current theory of Feynman and Gell-Mann[6] (F-G)]. With the accumulation of more data that is envisaged today and by various experiments that are either in progress or in planning, the theory may be disproved in a few years. Furthermore, assuming that the theory will pass these preliminary tests, predictions of the theory can be tested in detail in the foreseeable future. It is the purpose of this note to comment on the extant experimental data bearing on this theory and on possible future refinements of these data.

A possible extension of the theory to the hadronic world has been discussed.[5] However, because of the uncertainty due to the strong interactions of hadrons, it seems wise to discuss, at this stage, only the predictions of the theory on purely leptonic processes. To lowest order, the predictions of this theory are identical to those of the conventional theory if the process in question does not allow the intervention of the massive neutral vector boson Z_μ. Let B_μ^3 and C_μ be the neutral gauge bosons which belong to the leptonic isospin-1 and -0 multiplets, respectively. Define the mixing angle θ by[7]

$$Z_\mu=B_\mu^3\cos\theta-C_\mu\sin\theta,\qquad A_\mu=B_\mu^3\sin\theta+C_\mu\cos\theta. \tag{1}$$

We shall make our comments in four parts.

I. SEARCH FOR $W^\pm$ AND Z

The theory gives

$$\frac{G_W}{\sqrt2}=\frac{e^2}{8M_W{}^2\sin^2\theta},\qquad \frac{G_W}{\sqrt2}=\frac{e^2}{8M_Z{}^2\cos^2\theta\sin^2\theta} \tag{2}$$

to the lowest order in the renormalized coupling constant of the theory where e and G_W are the conventional electric charge and the μ-decay constant, respectively: $e^2/4\pi=(137.036)^{-1}$, $G_W=1.026\times10^{-5}\,m_p{}^{-2}$, and M_W and M_Z are the physical masses of the $W^\pm$ and Z mesons. These equations lead to Weinberg's prediction that $M_W\gtrsim40$ GeV, $M_Z\gtrsim80$ GeV.[8] We wish to emphasize that the prediction refers to the physical masses of these bosons, and the uncertainty in the estimate cannot alter them by more than a few percent. A discovery of $W^\pm$ with mass <30 GeV, therefore, will disprove this theory.

II. SEARCH FOR A SCALAR MESON

Weinberg's theory predicts a scalar meson whose coupling to a lepton is proportional to the mass of the lepton in question. None of the first-order weak processes depends on the mass of this particle. For this reason, no bounds can be given for the mass of this meson. Its couplings to the leptons are extremely weak; they are given by

$$\mathcal{L}_{s\bar{l}l} = +(\sqrt{2}\, G_W)^{1/2} s (m_e \bar{e} e + m_\mu \bar{\mu}\mu) . \tag{3}$$

III. THE PROCESS $\bar{\nu}_e + e \to \bar{\nu}_e + e$

The matrix element for this process may be obtained from the effective Lagrangian[9]

$$\mathcal{L}_{\rm eff} = \frac{G_W}{\sqrt{2}} [\bar{\nu}_e \gamma^\alpha (1+\gamma_5)\nu_e][\bar{e}\gamma_\alpha (C_V + \gamma_5 C_A) e] ,$$

where (4)

$$C_V = \tfrac{1}{2} + 2\sin^2\theta , \quad C_A = \tfrac{1}{2} .$$

These are to be compared with the values of the Feynman–Gell-Mann theory, $C_V = C_A = 1$. The differential cross section for this process per unit recoil-electron energy in the laboratory system is given by

$$\frac{1}{m_e}\frac{d\sigma}{dT} = \frac{G_W^2}{2\pi}\left[(C_V - C_A)^2 + (C_V + C_A)^2\left(1 - \frac{T}{\omega}\right)^2 - (C_V^2 - C_A^2)\frac{m_e T}{\omega^2}\right] , \tag{5}$$

where T is the recoil-electron kinetic energy, and ω is the incident neutrino energy. We have reanalyzed the Reines-Gurr experiment[10] for $\bar{\nu}_e + e \to \bar{\nu}_e + e$ in terms of the interaction (4).[11] The expected rates of events, normalized to the F-G-theory prediction, are plotted as a function of x ($= \sin^2\theta$) in Fig. 1. We have used the $\bar{\nu}_e$ spectrum of Avignone.[12] Furthermore the recoil-electron kinetic energy has been integrated over the range appropriate for the particular experimental situation. It is seen that the value

$$\frac{\sigma_{\rm exp}}{\sigma_{\rm F\text{-}G}} = 1.1 \pm 1.2 \tag{6}$$

in Reines's status report[13] implies the bound

$$x = \sin^2\theta \leq 0.35 . \tag{7}$$

This limit on x implies that $M_W \gtrsim 65$ GeV. The limit (6), i.e., $\sigma_{\rm exp}/\sigma_{\rm F\text{-}G} \leq 2.3$, can also be translated into a curve which restricts the allowable C_V and C_A regions. This is shown in Fig. 2.

Further improvement on the bound (6) will be extremely useful. If the experimental cross section is smaller than $\frac{1}{4}$ of the F-G value, the present theory, as well as the F-G theory, will be ruled out.

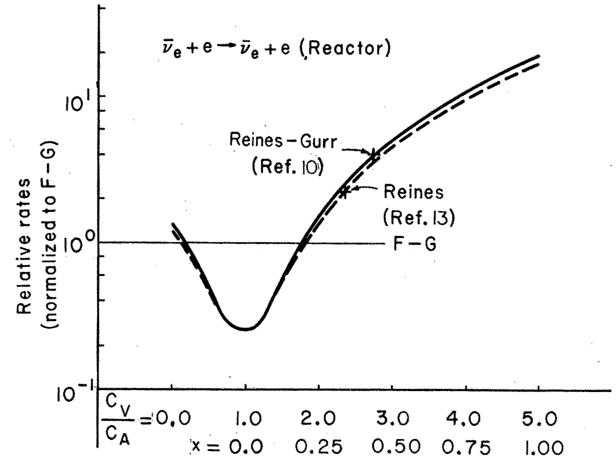

FIG. 1. The expected rates for $\bar{\nu}_e + e \to \bar{\nu}_e + e$, normalized to the Feynman–Gell-Mann theory prediction, are given as a function of x ($= \sin^2\theta$). The ratio C_V/C_A is related to x by $C_V/C_A = 1 + 4x$. The $\bar{\nu}_e$ spectrum of Avignone (Ref. 12) is used. The solid curve results from integrating the recoil-electron kinetic energy between 3.8 and 5.0 MeV, following Reines and Gurr (Ref. 10). Their limit gives $x \leq 0.44$. The dashed curve between 3.4 and 5.0 MeV follows Reines (Ref. 13). The limit, Eq. (6), gives $x \leq 0.35$.

IV. THE PROCESS $\nu_\mu + e \to \nu_\mu + e$

The detection of this process is important in the present theory since the conventional theory does not allow it in the first order. The matrix element for this process is obtained from the effective Lagrangian

$$\mathcal{L}_{\rm eff} = -\frac{G_W}{\sqrt{2}} [\bar{\nu}_\mu \gamma^\alpha (1+\gamma_5)\nu_\mu][\bar{e}\gamma_\alpha (C_V' + \gamma_5 C_A') e] ,$$

where (8)

$$C_V' = \tfrac{1}{2} - 2\sin^2\theta , \quad C_A' = \tfrac{1}{2} .$$

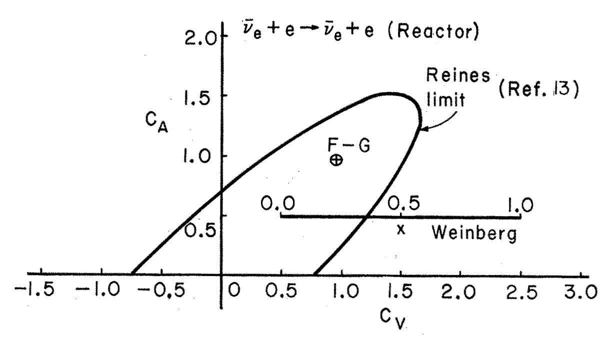

FIG. 2. The experimental limit on $\bar{\nu}_e + e \to \bar{\nu}_e + e$, Eq. (6), reported by Reines (Ref. 13), is translated into a curve which restricts the allowable C_V and C_A regions. The F-G point and the Weinberg line are also shown.

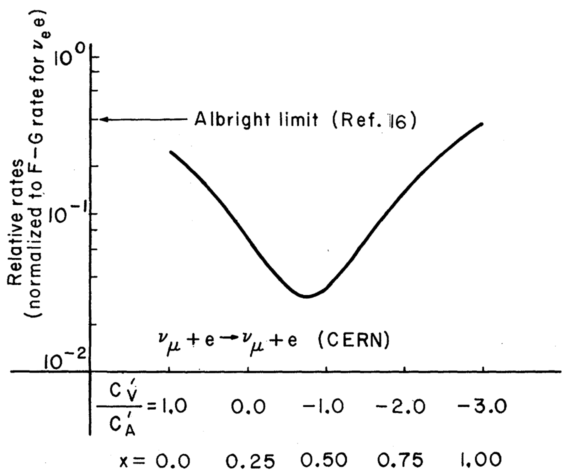

FIG. 3. The expected rates for $\nu_\mu + e \to \nu_\mu + e$, normalized to the Feynman–Gell-Mann theory prediction for $\nu_e + e \to \nu_e + e$, are given as a function of x $(=\sin^2\theta)$. The ratio C_V'/C_A' is related to x by $C_V'/C_A' = 1 - 4x$. The ν_μ spectrum of Holder *et al.* (Ref. 14) is used. Minimum energy of the recoil electron is taken to be 1 GeV, following Steiner (Ref. 15). The limit, from Albright (Ref. 16), sets no constraint on x.

The recoil-electron spectrum for this process is also given by Eq. (5), but with C_V (C_A) replaced by C_V' $(-C_A')$. The expected rates of events, normalized to the F-G-theory prediction for $\nu_e + e \to \nu_e + e$, are plotted as a function of x $(=\sin^2\theta)$ in Fig. 3. The ν_μ spectrum of Holder *et al.*[14] is used, and the minimum recoil-electron energy is set at 1 GeV (following Steiner[15]). Using the limit given by Albright,[16] i.e.,

$$\sigma_{\text{exp}}(\nu_\mu + e \to \nu_\mu + e) \leq 0.4\,\sigma_{\text{F-G}}(\nu_e + e \to \nu_e + e), \quad (9)$$

one sees from Fig. 3 that x is not constrained. The limit (9) has also been translated into a curve which restricts the allowable C_V' and C_A' regions. This is shown in Fig. 4.

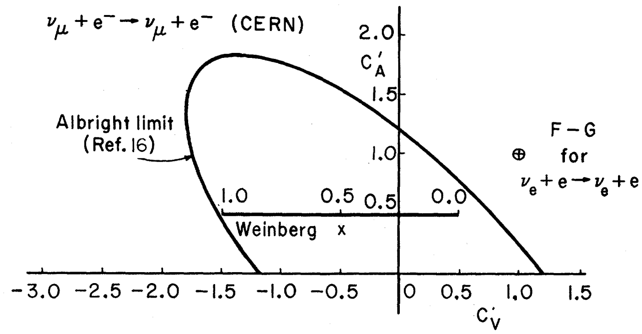

FIG. 4. The limit on $\nu_\mu + e \to \nu_\mu + e$, Eq. (9), given by Albright (Ref. 16), is translated into a curve which restricts the allowable C_V' and C_A' regions. The F-G point for $\nu_e + e \to \nu_e + e$ and the Weinberg line are also shown.

Further improvement on the bound (9) will begin to constrain the x near 0 (and x near 1) regions. One could eliminate the present theory by using a combination of improved limits from $\bar\nu_e + e \to \bar\nu_e + e$ and $\nu_\mu + e \to \nu_\mu + e$. For example, if the $\bar\nu_e + e$ limit were at the F-G level (implying $x \leq 0.20$), and the $\nu_\mu + e$ limit is improved by a factor of 5 (implying $x \leq 0.65$ and $x \geq 0.22$), then the entire allowable range of x would be covered.[17]

Another way of checking the interaction (8) is through e^+e^- pair production in the nuclear Coulomb field by ν_μ[18]:

$$\begin{pmatrix}\nu_\mu\\ \bar\nu_\mu\end{pmatrix}(k) + Z(P) \to \begin{pmatrix}\nu_\mu\\ \bar\nu_\mu\end{pmatrix}(k') + e^-(p) + e^+(p') + Z(P'). \quad (10)$$

The differential cross section in the laboratory system, applicable when $m_e^2 \ll |q^2| \ll M^2 \ll \omega^2$, where $\omega = k_0$ is the incident neutrino energy, M is the mass of the target nucleus Z, and $q = P' - P$ is the momentum transfer to the nucleus, is given by

$$\pi^7 \frac{d^9\sigma}{d^3p\,d^3p'\,d^3P'} = \frac{1}{M\omega}\left(\frac{G_W Z\alpha}{q^2}\right)^2 \left(\frac{M_z^2}{M_z^2 - \kappa^2}\right)^2 \left(\frac{p'\cdot P}{p'\cdot q} - \frac{p\cdot P}{p\cdot q}\right)^2$$
$$\times\{(C_V'^2 + C_A'^2)[(p\cdot k)(p'\cdot k') + (p\cdot k')(p'\cdot k) - 2(p\cdot p')(k\cdot k')]$$
$$\mp 2C_V' C_A'[(p\cdot k)(p'\cdot k') - (p\cdot k')(p'\cdot k)]\}, \quad (11)$$

where $\kappa = k - k'$ and the upper (lower) sign applies to the ν_μ $(\bar\nu_\mu)$-initiated reaction. In Eq. (11) terms of relative order m_e^2 and q^2 have been omitted.

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[7]In the notation of Ref. 1, $\cos\theta$ is defined as $g(g^2+g'^2)^{-1/2}$.

[8]See also S. L. Glashow, Ref. 2; J. Schechter and Y. Ueda, Phys. Rev. D 2, 736 (1970); T. D. Lee, Phys. Rev. Letters 26, 801 (1971).

[9]Equation (4) corrects a minor error in the last equation on p. 1265 of Ref. 1. We thank Professor S. Weinberg for verifying this error.

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[17]The new facility at Los Alamos (LAMPF) may be able to provide ≈100-MeV ν_μ's of sufficient purity to put a more meaningful bound than Eq. (9) on $\nu_\mu + e \rightarrow \nu_\mu + e$. The planned beam-dump neutrino facility at LAMPF has a high intensity of ν_e, $\bar{\nu}_\mu$ (from μ^+ decay, assuming the usual additive lepton-number conservation law), and ν_μ (from π^+ decay). The constraint on x, $x \leq 0.35$, can be used to show that recoil electrons from elastic scattering at this facility would come mostly from $\nu_e + e$, particularly for $E_e \geq 30$ MeV. See H. H. Chen, Univ. of California at Irvine Report No. UCI-10P19-64 (unpublished). The existence of the multiplicative lepton-number conservation law would complicate matters considerably.

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Erratum

Nonleptonic Hyperon Decays in a Current-Current Quark Model, Michael Gronau [Phys. Rev. D 5, 118 (1972)]. Equation (1) should read

$$\mathcal{H}(x) = \frac{G}{\sqrt{2}} : \{J_\mu(x), J^{\mu\dagger}(x)\} : . \tag{1}$$

Third line in Ref. 14 should read: "The SU(3)-invariant *VMM* coupling is purely antisymmetric,... ."