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Nova Model of Inclusive Reactions*

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A model for particle production is presented in which the features recently observed in inclusive distributions can be reproduced. The calculation emphasizes the role of resonance-like objects which are diffractively produced and decay mainly through cascade emission of pions. The cross section for producing these objects which we call novas is diffractive and does not depend on energy. As a function of nova mass it rises to a low mass maximum (below 2 GeV) and falls off in a way which is controlled by Regge behavior and duality. We then construct a zero-parameter model for the inclusive distributions which also gives many predictions about exclusive reactions. Among the many features which can be obtained in terms of this picture are: the rapid approach of scaling for secondaries with relatively large center-of-mass momentum; a slower approach for soft secondaries and the pseudo-scaling observed at present machine energy in certain reactions; a simple connection between the transverse-momentum distribution, the longitudinal-momentum distribution, and the average multiplicity of secondaries; charge effects which should not vanish asymptotically; and a marked difference between meson and baryon distributions. The approach to scaling is connected to the rise of the diffractive cross section as inferred from the behavior of the K^+p total cross section. One also obtains an understanding for the "quark frame" results and the center-of-mass isotropy of slow secondaries.

I. INTRODUCTION

Recently much effort has been devoted to obtaining systematic surveys of inclusive cross sections.[1] Enough reactions have now been analyzed over the entire kinematic region to allow a comprehensive analysis.[2-7] For pions, which are by far the dominant secondaries, the key features of their inclusive distributions appear to be a strong maximum around zero center-of-mass momentum, and a rapid approach to a scaling limit for some medium values of $|x|$,[7] the Feynman scaling variable.[8] The distributions of secondary baryons differ from the meson distributions. Here the invariant cross section is much flatter, or even rises for increasing values of $|x|$, whereas it drops sharply with $|x|$ for pions. The kaons exhibit an intermediate behavior. In the cases where the observed secondary is identical to one of the initial particles, a strong leading-particle effect is observed, and has been known for a long time. These and other prominent features of inclusive cross sections have already been discussed and reviewed in several papers.[9]

One feature of these distributions which has attracted much attention is their slow variation with energy. In fact, some reactions appear energy-independent around $x=0$, which could be taken as evidence that an asymptotic scaling limit[8,10] has already been reached there at present machine momenta[11] (10 to 30 GeV/c). For example, the invariant cross section[12] for $\pi^+p(\pi^+)$ varies little between 8 and 16 GeV/c at $x=0$,[2] and $\pi^-p(\pi^-)$ shows little change between 8, 18, and 24 GeV/c.[3] The observed ratio of the 24-GeV/c to the 18-GeV/c distributions around $x=0$ is 1.10 ± 0.05. However, often the data show marked energy dependence and other behavior which indicate that the asymptotic limit has not yet been reached. For example, the $\pi^+p(\pi^-)$ cross section at $x=0$ rises by a factor of 1.2 between 8 and 16 GeV/c.[2] Moreover, this cross section is a factor of 0.6 times that of $\pi^+p(\pi^+)$ at $x=0$. Similarly, the ratio of $\pi^-p(\pi^-)$ to $\pi^-p(\pi^+)$ is 1.4 at 18 GeV/c and $x=0$.[3] However, one expects asymptotically the average multiplicities of π^+'s and π^-'s to become equal in π^-- or π^+-induced reactions, which implies that these ratios of invariant cross sections go to one, provided the distributions do not vanish at $x=0$.

Another important fact is the great energy dependence observed in the topological cross sections at present machine energies. Although the strong 4-prong cross section is fairly flat in the 10–30-GeV/c region, the inelastic 2-prong one decreases, while the 6-prong cross section rises. This can lead to obvious compensating effects in reactions like $\pi^-p(\pi^-)$ or $\pi^+p(\pi^+)$, and an apparent scaling may result which might fail at higher energies. This phenomenon, which we call pseudoscaling, will be further discussed. The 2-prong events cannot contribute to $\pi^-p(\pi^+)$ or $\pi^+p(\pi^-)$, and for these reactions energy dependence at $x=0$ is observed.

On the other hand, for intermediate values of

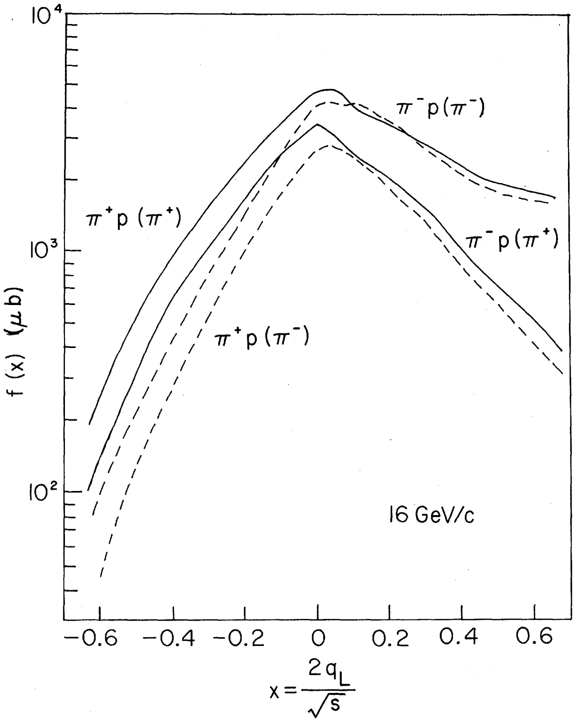

FIG. 1. Experimental π^+ and π^- distributions,

$$f(x) = \int dk_T^2 \frac{2\omega}{\pi\sqrt{s}} \frac{d^2\sigma}{dx dk_T^2},$$

obtained from π^+-p and π^--p collisions at 16 GeV/c, integrated over all k_T^2. Units are μb. The curves which are drawn through the histogram structure of Ref. 2 illustrate the similarities and differences obtained when the charge of an incident or observed particle is changed.

$|x|$ some reactions show an impressive scaling property which may indicate a low-energy onset of limiting fragmentation.[10] One finds striking correlations between the initial particles and the observed one. Even though the x distributions associated with secondary particles of the same isomultiplet show similar shapes, their values sometimes differ by a factor of order 2. For example, $\pi^- p(\pi^+)$ is about 1.5 times larger than $\pi^- p(\pi^-)$ at $x \approx -0.4$ (baryon fragment), even though $\pi^- p(\pi^-)$ and $\pi^+ p(\pi^-)$ are compatible within error bars here. Similarly for positive x values around 0.4, the $\pi^- p(\pi^-)$ cross section is twice the one observed for $\pi^- p(\pi^+)$. (This ratio must, of course, be taken below the onset of the leading-particle peak.) These charge ratios in the intermediate-x regions do not seem to change with energy. The data showing these systematics are gathered together in Fig. 1, which sketches the available results at 16 GeV/c for pion production in pion-induced reactions.[2,3] This connection between one of the initial particles and the observed particle is also found in baryon distributions.

The theoretical pictures used to interpret these data have so far been extrapolations of asymptotic models to present machine energies. Perhaps the two most popular models have been the multiperipheral model[13] with its multi-Regge–exchange generalization,[14] and the limiting-fragmentation diffraction model.[10] These models have both been rederived as different asymptotic limits of a common Regge-like theory.[15] The multiperipheral model is intended to give a description of particle production at low center-of-mass momentum, whereas limiting fragmentation is appropriate for secondaries which follow either the beam or target particle.

One major success of the multiperipheral model is its prediction of a logarithmic increase of average multiplicity[13] with energy, which appears to be correct experimentally.[16] Nevertheless, around $x = 0$ the ratio of the invariant distributions for $\pi^+ p(\pi^-)$ and $\pi^+ p(\pi^+)$ should be very close to unity, since the correlation of the middle of the multiperipheral chain with the end particles vanishes. Moreover, the rapidity distribution implied by this model is much flatter than that which is observed experimentally and which shows rather a Gaussian shape.[17] A way out of these difficulties is to group the secondaries into clusters which are themselves produced in a multiperipheral way. However, the number of clusters inferred from the observed multiplicity is much too small not to violate the spirit of the model when applied to inclusive analyses at present machine energies.[9] Other modifications intended to explain the observed structure function are very complicated.[18] Such features suggest that the model, if applicable asymptotically, is not quite suitable for present energies.

At the same time, a purely fragmentation picture seen also as an asymptotic one taken literally would predict a dip at zero center-of-mass rapidity, when present pion data show a maximum. Nevertheless, the charge ratios shown in Fig. 1 strongly suggest that a picture which emphasizes the *diffractive* excitation of the beam and (or) target particles should be relevant at presently accessible energies.[18a] A superposition of both particle fragmentation and multiperipheral pionization does explain qualitatively most of the observed features,[9,19] but this is so flexible that decisive checks or quantitative predictions have not been possible so far. This situation is somewhat frustrating in view of the fact that inclusive experiments involve so much averaging that one might expect a simple picture to hold as long as the demand for accuracy is not too stringent, which is certainly the case at present.

The purpose of this paper is to propose a detailed answer to this tantalizing problem. We develop a simple model, intended to work at present machine energy, which accounts for most aspects of the data very easily. The model leads to many predictions about inclusive and exclusive distributions which appear satisfied experimentally, or await check. The model has been briefly described in a letter.[20] We proceed here with a more systematic account, together with a survey of many predictions and experimental checks.

Present machine momentum is considered to range from 10 to 30 GeV/c. In this momentum range, we suggest that the dominant inelastic process is the diffractive excitation of *either* the beam *or* target particle.[20a] As a result, the main features of the inclusive distributions are determined by the diffraction excitation spectra (and by kinematics), and a strong leading-particle effect is expected. These excited states, or combination of states, will decay mainly through pion emission, and after a bright mesonic flare will return to a hadronic ground state. We call this flare a *nova.* Although the exact nature of the nova and its decay mechanisms are not really needed at the present level of accuracy, there is one feature which is extremely important. Due to the weakness of the triple-Pomeranchon coupling,[21] the diffractively excited novas will *behave as resonances.*[22] A nova is a very special "fireball," and its resonance nature will lead to specific properties regarding the energy and the multiplicity of the decay pions which almost determine the shape of the inclusive distributions. The quantum numbers of the beam or target particle are furthermore kept by the nova. This has many direct consequences. Thus, we see the nova as a combination of resonances decaying through cascade emission of pions.[23] Of course other decay modes are present, but it is well known that pion production dominates by nearly an order of magnitude.

The two primary features of diffractive excitation in this model are, first, the cross section for the production of a nova of mass M is independent of energy as long as it is kinematically allowed,[24] and second, it is a process which does not change internal quantum numbers and consequently yields strong constraints on the branching ratios of the novas as long as the multiplicity of secondaries is not too large.[25] This second feature leads to a simple explanation of the charge ratios of Fig. 1, including the impressive cross-over effect. We suggest then that the main inelastic process is the diffractive excitation of either the beam or target particle into a nova corresponding respectively to the two graphs of Fig. 2. Of course this is not the only production process.

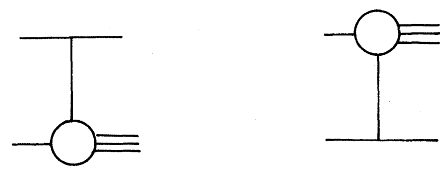

FIG. 2. Single-nova excitation of either beam or target particle. Owing to the intricate nova decay, the two cross sections may be simply added up incoherently to obtain the inclusive cross section.

However, as the dominant process (about $\frac{2}{3}$ of the inelastic cross section), it should be adequate for understanding most of the features of the inclusive distributions.[26]

It is obvious at this stage that a crucial point in this approach is specifying the cross section for producing a nova of mass M. This is just the diffractive excitation spectrum of resonances, and the features of nova production are just those of the production of single diffractive resonances.[27] The cross sections for each individual resonance are well known to be small (<1 mb), so we have to consider the excitation of relatively high-mass objects. Then the number of states may compensate the low individual yield and consequently build up the observed inelastic cross section.[28] However, the production cross section of very heavy novas should decrease with mass in order to give a bounded total cross section. As later discussed, the absence of triple-Pomeranchon coupling leads to a M^{-2} decrease of the excitation cross section. The mass spectrum we expect should show a sharp rise, jagged with resonance peaks, followed by a decrease with mass, the maximum being around 2 GeV for baryons and somewhat less for mesons. The shape of the spectrum is further discussed in Sec. II, but the presence of this maximum is an important feature of the model. Because the most prominent nova masses are so high, we are led to consider single excitation as the dominant effect, rather than double excitation, at least at present machine energies. Indeed, there is a kinematical preference for single-nova production if one follows the usual notion that diffraction excitation is associated with low momentum transfer squared. For single excitation the minimum value of $|t|$ goes down with s as s^{-2}, but decreases only as s^{-1} for double excitation. If one 2-GeV nova is produced, the momentum transfer can be kept small only for quasielastic scattering of the other particle.[29] Of course a *low*-mass resonant state could be produced,[26] but this cross section should be relatively small.[27] If the momentum-transfer argument does not apply, and double excitation of two relatively heavy novas is dominant, then as estimated later, one would expect an average multiplicity at present energies which would be higher than the observed one.

There appears to be increasing evidence[29,30] for the importance of single diffractive excitation, found in the exclusive analysis of particular final states. Single diffractive excitation is by far the most prominent mechanism in producing 4-particle final states in πp and Kp collisions around 10 GeV/c. These data also provide evidence for relatively strong cross sections for diffractively producing high-mass novas. However, it should be noted that these final states total at most 20% of the possible configurations. Furthermore, an analysis of 4-prong rather than 4-particle final states is really needed to ascertain the dominance of single-nova production, since double diffractive excitation would then be generally possible when it is not possible for 4-particle states studied in meson-baryon collisions. Evidence for the dominance of single excitation has indeed been reported for pp collisions,[29] at least for some configurations.

One can use the available exclusive analyses[30] to obtain some information about the nova excitation spectrum. The observed baryon missing mass clusters around 1.7 or 1.8 GeV. Keeping single excitation for higher multiplicities, the distribution should shift slightly upward when all multiplicities are included. Another piece of evidence for the strength of single excitation is the strong leading-particle effect, which would be reduced to the extent that double-nova production and other processes are important.[31]

The relatively high mass of the most likely novas allows us to avoid a standard difficulty encountered in single diffractive models. In particular, single excitation seems to imply far too many low-momentum recoil protons (a leading particle effect) when the projectile particle is excited.[31] As later discussed, we do not find this, but are able to reproduce rather well some available data.

Let us again emphasize that single-nova production is not the only mechanism for particle production. At low energies, quantum-number exchange is of some importance,[26] although information from exclusive experiments still indicates diffraction to be stronger (at about 10 GeV/c).[30] Nevertheless, corrections to the basically diffractive picture will be necessary. For fixed nova mass, we further expect quantum-number exchange to decrease with energy. Quantum-number exchange is also known to give double as well as single excitation. Very high-energy corrections to the model may also appear, when the production of two relatively heavy novas is no longer unfavored kinematically.[32,33] These high-energy and low-energy corrections to the model appear to be closely related, as discussed in Sec. III. As the energy is even further increased, a multiperipheral picture might also become accurate. However, at present machine energies, single-nova excitation seems to be the dominant process, and the beautiful agreement with experiment[20] makes it worth extrapolating the model to National Accelerator Laboratory (NAL) and CERN Intersecting Storage Rings (ISR) energies, even though some modifications should, in principle, be necessary there.

Nova excitation may thus be justified as long as the nova mass is relatively small, $M \ll \sqrt{s}$. This includes masses around the maximum nova cross section, M about 2 GeV. As later emphasized, decay pions from novas around this mass dominate the intermediate values of $|x|$ ($0.2<|x|<0.6$), and consequently the distributions in this region are easily interpreted and found to scale quickly. However, as the nova mass approaches the limit of phase space (M about $\sqrt{s}$), the rationale for diffractive excitation in the usual sense is lost, and also there is no reason to prefer single over double excitation. Even so, essentially all the particles are slow in the center-of-mass frame, and in the limit, separation between beam and target novas becomes experimentally impossible. Therefore, we formally extrapolate the single-nova picture to the edge of phase space simply because this will give the correct slow increase in the average multiplicity. We then expect the predictions around $x\approx 0$ to be more general than the model itself. As energy increases, higher-mass novas contribute, which decay into large numbers of pions. These pions continue to build up the meson inclusive distribution around $x=0$, even though the nova cross section decreases with mass. The extension of diffractive excitation to high-mass objects leads to a peak at $x=0$,[34,35] rather than the dip usually associated with particle fragmentation. This peak eventually scales, but we expect an increase of the slope of the invariant cross section as a function of x for small $|x|$ values between conventional machine energies and ISR energies,[20] while the shape of the rapidity distribution remains almost flat. Although this result does not satisfy Feynman's scaling hypothesis, *if* applied at 30 GeV/c, one finds a scaled limit at higher energies to the extent that a logarithmic increase of the multiplicity is imposed.

In summary, we propose that single-nova diffractive excitation is the dominant production mechanism at present machine energies. The pion inclusive spectra are determined by relatively low-mass novas, where the model is most solidly grounded. Although the extension of the model to high-mass novas is somewhat formal, it does reproduce the slow increase of multiplicity observed in inelastic collisions.[16] In any event, the model should have some phenomenological value in understanding the small-x behavior. Corrections to the proposed picture may be as much as 30% at 20 GeV. Nevertheless, in view of the complexity of these production processes, finding a dominant mechanism is at least useful for analyzing experimental information as it becomes available.

The model itself is presented in Sec. II, where we provide a detailed prescription for its practical application and a discussion of its basic features. Obvious corrections to the single-nova diffraction model are discussed in Sec. III. Section IV includes a survey of predictions and experimental comparisons for production of pions and nucleons.[36] The predictions of the model for NAL and ISR are also given there.

II. THE NOVA MODEL

This section is divided into four parts, dealing respectively with the nova excitation, nova decay, the calculation of the inclusive cross sections, and the determination of the model parameters. Our main emphasis in this section is on setting up the model for practical applications. We also discuss some qualitative features of the model.

A. Nova Excitation

The most important ingredient of the model is the excitation spectrum, $\rho(M)$, which is defined to be the cross section for producing a nova of mass M. We first separate the cross section for beam excitation from the one for target excitation, and write the total inelastic cross section as a sum of two terms,

$$\sigma_\alpha = \int_{M_\alpha}^{M'_\alpha} \rho_\alpha(M)dM\,, \tag{1}$$

where α labels beam or target, depending on which has been excited. The integration extends from the threshold for diffractive excitation to the edge of phase space (asymptotically $\sqrt{s}$). As defined, $\rho(M)$ includes an integration over momentum transfer whose limit depends on s and M. However, except for threshold factors which depend on the details of the excitation mechanism, $\rho(M)$ is practically independent of s.

We first focus on values of $M/\sqrt{s} \ll 1$, where we can neglect threshold effects. For each nova mass, we assume a *constant* cross section such as those observed for single-resonance diffractive production.[27,28] This is what we mean by diffractive excitation, and it should be contrasted to the more usual assumption that the total cross section remains constant with increasing energy. For low nova masses, $\rho(M)$ should be small, but increase

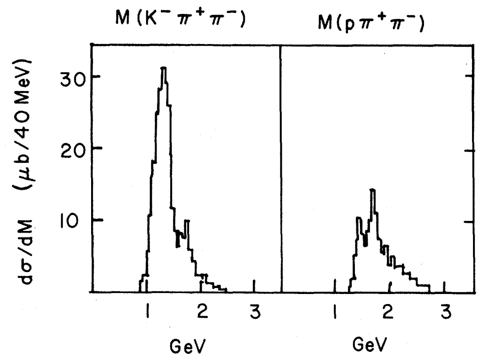

FIG. 3. Single diffractive excitation spectra observed in the channel $K^-p \to K^-\pi^+\pi^-p$ at 10 GeV/c, taken from Ref. 30.

rapidly with M as the density of states which can be diffractively excited increases. The more prominent diffractive resonances will be superimposed on this curve, but each resonance nevertheless contributes only a small fraction of σ_α. As an example, the missing-mass spectrum of an exclusive analysis shown in Fig. 3 provides evidence for this behavior of $\rho(M)$.[30]

If s is large enough, it is possible to explore $\rho(M)$ for larger M, but we still restrict $M/\sqrt{s} \ll 1$ in order to avoid threshold questions. Here, the spectrum should decrease with M, at least fast enough to avoid an unbounded total cross section, that is, faster than M^{-1}. However, a stronger bound on $\rho(M)$ and an interpretation of the nova can be obtained by appealing to the triple-Regge behavior which has been justified in this limit, M large with $M/\sqrt{s} \ll 1$.[37] To the leading power in s, the triple-Regge cross section for production of novas of mass M is

$$\frac{d\sigma}{dM\,dt} = \frac{\gamma(t)}{s^2}\left(\frac{s}{M^2}\right)^{2\alpha_P(t)} (M^2)^{\overline{\alpha}(0)+1/2}\,. \tag{2}$$

This contribution to σ_α is obtained by summing over all graphs of the type Fig. 4(a), which is equivalent to Fig. 4(b), which is approximated as Fig. 4(c). The leading trajectory, α_P, is the Pomeranchuk trajectory[38] which describes diffraction $[\alpha_P(t) \equiv 1]$, and $\overline{\alpha}(0)$ is the leading trajectory contributing to the "Pomeranchon-particle total cross section." We assume now that the triple-Pomeranchon coupling vanishes,[21] so that $\overline{\alpha}(0)$ is the intercept of the leading secondary trajectory with $\overline{\alpha}(0) \approx \frac{1}{2}$. We then integrate over t (still with $M/\sqrt{s} \ll 1$), and conclude that $\rho(M)$ decreases as M^{-2}. If we apply the ideas of duality, the nova, which is associated with the secondary Regge trajectories, should be considered as a superposition of resonance states.[22] Thus, as the energy is in-

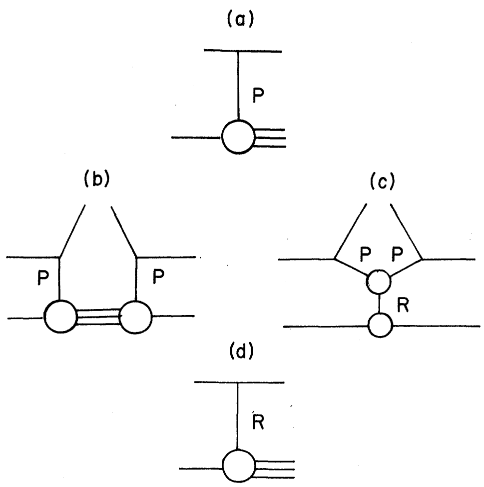

FIG. 4. Triple-Regge picture of the nova excitation spectrum: (a) inelastic amplitude for single-nova production; (b) cross section for single-nova production; (c) Regge approximation to single-nova production cross section for large M and $M/\sqrt{s}<<1$; (d) particle excitation with quantum-number exchange.

creased, it is possible to excite higher-mass resonances, but with a decreasing cross section. The resonant nature of the diffractively excited objects is extremely important. The resonance interpretation of the nova is directly connected to the absence of the triple-Pomeranchon coupling, whose existence, as is well known, would lead to many inconsistencies.[37]

We now attempt to extend this spectrum to larger values of M. We assume that the nova continues to be a sum of resonance states, which will usually cascade decay into one another through pion emission. The number of secondaries is therefore expected to be proportional to M, as contrasted to the slow (logarithmic) increase which should be expected from "background." The higher-mass novas should contribute to the multiplicity as M^{-1}. If we extend this behavior all the way to the edge of phase space ($M\approx\sqrt{s}$), we then obtain a logarithmic growth for the average total multiplicity of secondaries, a feature which we consider important. The M^{-2} decrease of $\rho(M)$, justified for lower M values, can be extended to the edge of phase space if we accept this phenomenological motivation.

Some further comments on the high-mass novas are needed. Since the minimum value of the square of the momentum transfer, $t_{\min}$, suddenly increases from a very small value as the nova mass approaches the limit of phase space, one might expect threshold effects to distort strongly the excitation spectrum near this limit. However, whether or not $\rho(M)$ should be cut off by, for example, $\exp(-Bt_{\min})$ is an open question. This again reflects the incomplete picture offered by diffraction analogies for high-mass nova production. Having there only the multiplicity increase as a guide, we assume that the M^{-2} behavior is effective up to the limit of phase space, and disregard the analogies with ordinary diffraction scattering which are no longer valid there.

Of course the high-multiplicity events could result from the production of two moderately heavy novas, rather than one very heavy nova. Aside from very detailed measurement of leading-particle effects, it is impossible to distinguish the two cases, since one or two slow novas in the center-of-mass frame will give essentially the same distributions at present machine energies.

Production by Regge-trajectory exchange, Fig. 4(d), as opposed to diffractive excitation requires further discussion, which is found in Sec. III. In summary, we interpret the increase of multiplicity with energy as due to the weak production of high-mass novas, which subsequently decay into a large number of particles. Thus the multiplicity continues to rise with s, while the total cross section is bounded. Of course, since we have neglected extra logarithmic factors in this analysis, the low-mass-nova contribution might eventually decrease like $\ln s$. However, for present machine energies, we take $\rho(M)$ to be an energy-independent function.

At intermediate values of nova mass, between its increase and M^{-2} decrease, the nova spectrum shows a maximum jagged with individual resonance peaks. Information from quasielastic scattering[39] and from available exclusive analyses[30] leads us to locate this peak at about 1.9 GeV for nucleons and 1.3 GeV for pions. Since the individual resonances have such small cross sections, and the inclusive cross section involves averaging over $\rho(M)$, we can attempt to approximate the spectrum by a smooth analytic form. All calculations in this paper have been carried out using an *ad hoc* formula[40]:

$$\rho_\alpha(M)=c_{\alpha,s}\frac{\exp[-\beta_\alpha/(M-M_\alpha)]}{(M-M_\alpha)^2}, \qquad (3)$$

where M_α is the mass of the particle which becomes a nova, and $c_{\alpha,s}$ also depends on the spectator particle. The parameter β_α is determined by the location of the maximum at $M=M_{\max}$ in $\rho(M)$ by

$$\beta_\alpha=2(M_{\max}-M_\alpha). \qquad (4)$$

A more accurate form of $\rho(M)$, including particularly strong resonances, and a more accurate behavior for very small M could be used. However, the rough form, Eq. (3), turns out to be adequate for reproducing the present inclusive data.

Once $\rho(M)$ is determined, the model immediately connects the total inelastic cross section, the multiplicity of pions, and the average Q values of the pions in nova decay. The numbers obtained are reasonable, but more experimental information could help refine $\rho(M)$ or the model itself.

The presence of the maximum in $\rho(M)$ is an important feature of the nova model, as it contains enough information to interpret many features of the meson inclusive distributions at intermediate x values $(0. \lesssim |x| < 0.6)$, where Feynman-Yang scaling is observed at surprisingly low energies at least in some reactions.[2] On the other hand, our predictions for $x \approx 0$ are controlled by the high-mass novas which merely reflect the logarithmic increase of multiplicity. The importance of high-mass novas implies a relatively slow approach to the scaling limit.[20]

An interesting prediction which follows from Eqs. (1) and (3) concerns the contribution of diffractive processes to the total cross section. Ignoring momentum-transfer cutoffs, we integrate Eq. (3) to the limit of phase space. This yields a constant asymptotic value for the total inelastic cross section which is approached from *below* as $s^{-1/2}$:

$$\sigma_{\text{inel}}(s) = \sum_{\alpha} c_{\alpha,s}\beta_\alpha^{-1}\exp[-\beta_\alpha/(\sqrt{s} - M_s - M_\alpha)], \tag{5}$$

where M_s is the mass of the spectator particle. This energy behavior, which depends on our interpretation of diffraction, may well partially compensate a decreasing Regge behavior. This fast increase is just a consequence of the nova excitation spectrum. It could explain the energy behavior of the K^+p and K^-p total cross sections recently observed at Serpukhov.[41] The compensation of this increase and the decrease of the Regge contribution would hold for the K^-p total cross section, and the K^+p cross section would be predominantly diffractive between 20 and 50 GeV/c (Ref. 42) with a 5% increase over this range if we use the model parameters computed below.[43]

B. Nova Decay

We have already seen that the nova should be considered as a superposition of resonances, since it is dual to the leading secondary trajectories. The problem of nova decay is therefore that of resonance decay, and therefore one might expect to observe a sequential decay process. The decay chain may be described in the nova rest frame, where the nova (or rather the resonances it is made of) performs a random walk as it cascade decays through a series of resonant states. (This model of the decay is intended to describe in a rough way most of the decays. Although less common mechanisms are easily included in the model, they will be neglected in this paper.) If the number of secondaries is large enough, asymmetries due to polarization average themselves out and the decay distribution in the nova rest frame is basically isotropic.[44,45] The random-walk picture of the nova suggests a Gaussian[46] distribution in which the mean value of momentum K is related to the typical Q value of the cascade decay by

$$Q = Q_0 - \mu,$$

with (6)

$$Q_0{}^2 = \tfrac{3}{2}K^2 + \mu^2,$$

where μ is the pion mass. We have neglected the recoil energy of the heavy nova. The key point is that reasonable Q values lie between 300 and 400 MeV, which give similar values for K.[47] It follows that, on the average, 2.5 pions are emitted per GeV of nova mass. If we then use $\rho(M)$ as given in Eq. (3), we obtain the observed average multiplicity of 4 to 5 secondary pions at 16 GeV/c. We also reproduce the asymptotic $\ln s$ increase of the multiplicity,[16] though this is not a specific feature of the model.

From these statistical considerations, an approximate pion distribution in the nova rest frame can be written as

$$\frac{d^3D_i}{d^3q} = \frac{2d^3D_i}{dq_T{}^2\, dq_L\, d\phi}$$

$$= N_i \exp[-(q_L{}^2 + q_T{}^2)/K(M)^2]. \tag{7}$$

This distribution will be normalized to the average number, $N_i(M)$, of observed pions (π^+ or π^-) into which the nova of mass M decays. The transverse (q_T) and longitudinal (q_L) momenta are separated, and q_L is measured along the nova momentum as seen in the center-of-mass frame. One crucial feature of Eq. (7) is that the average pion rapidity is the same as the nova rapidity.[9] This alone determines much of the behavior of the inclusive distributions.

This decay picture may be contrasted with thermodynamic descriptions,[23] even though there are statistical features in our distribution. In contrast to equilibriumlike states, we view the nova decay as a highly correlated process, where the momentum distribution is inferred from a cascade-decay picture. This picture can be used in the absence of triple-Pomeranchon coupling.[48]

The simplest assumption for the M dependence of K is that K is constant. Although this allows

d^3D/d^3q to be factored, the factorization is only approximate when the distribution is transformed to the center-of-mass frame. (The center-of-mass longitudinal momentum depends on q_T.) For constant K, the q_T^2 distributions are sharpened around $x=0$. There are two compensating effects that suggest that K constant may be a relatively good approximation at least for the x distribution. On one hand, the higher level density of high-mass novas might suggest a sharper q_T^2 distribution, since $K(M)$ might decrease slightly at large M. On the other hand, high-mass novas include high-spin resonances which in the single-excitation approximation would be polarized normal to their production plane. These novas would therefore tend to give more pions in the beam direction, even though they would depolarize during the decay chain and give relatively isotropic distributions in their rest frame. This effect would tend to widen the x distribution, at least if one tries to obtain $K(M)$ from an analysis of the transverse-momentum distributions of inclusive spectra. This suggests, however, that the decay of low-mass novas is not quite isotropic in the nova rest frame, although we neglect this effect in this paper.

This picture eliminates coherence effects in the region where the distributions of the beam and target novas overlap. We simply add the two distributions after calculating separately the two-nova formation and decay process.

The simple picture of nova decay holds only for the largest fraction of decay pions. Smaller effects will be due to long jumps, which will contribute large-q_T pions no longer satisfying Eq. (7),[49] and to rarer decay modes. In particular, higher-mass novas can lead to strange-particle production. As an example, high-mass nucleon novas will obviously favor decay into K^+ over K^-, which is a feature clearly seen in the inclusive K distributions.[6] Many relations between inclusive cross sections for different strange particles, induced by non-strange-hadron collisions, can be derived following our considerations.

A nucleon nova "contains" a baryon left over as a hadronic ground state. It can furthermore be uniquely specified to the extent that $B\bar{B}$ pair production is small. Thus, during the cascade-decay process, the nova nucleus performs a random walk, collecting momentum in a statistical fashion. The average momentum squared should increase with the number of decay steps as

$$\langle q^2\rangle \approx NK^2 . \tag{8}$$

One then expects that the mean proton transverse momentum should increase with multiplicity and nova mass, whereas the pion transverse momentum remains approximately constant. This trend is at least compatible with present data.[50] The mean pion transverse momentum should not depend on the multiplicity while the mean proton momentum should increase with increasing multiplicity, this of course as long as they are not cut off by obvious phase-space limits.

C. Inclusive Distributions

For fixed nova mass, the momentum distribution of pions in the nova rest frame follows Eq. (7). To find the contribution of the decay pions of this nova to the inclusive cross section, it is necessary simply to Lorentz transform the distribution from the nova rest frame to the reaction center-of-mass frame.

As viewed from the center-of-mass frame, we expect the cross section for nova production to have a sharp (exponential) t dependence[27,28] since this is a key feature of diffractive processes. Therefore, the mean transverse momentum of both the nova and of the recoil or leading particle is around 300 MeV/c. Since the emitted pion has the same average transverse velocity as the nova, its average transverse momentum is only slightly corrected by the transverse motion of the nova. (The correction is about 10% for π novas of 3 GeV.) Consequently the transverse motion of the nova can be neglected for computing the distribution of the decay pions, and the nova cross section can be immediately integrated over t. This implies that $k_T = q_T$ and the k_T distribution in the center-of-mass frame is given approximately by the q_T behavior in Eq. (7). The choice made for K gives distributions in agreement with present experimental results, and K^{-2} is typically 9 $(\mathrm{GeV}/c)^{-2}$.[2,49] However, for the leading-particle effect and for the inclusive distributions of heavier particles, the transverse motion of the nova should be included. Nevertheless, in the latter case the center-of-mass distribution in k_T is not tremendously different from the q_T distribution in the nova rest frame.

In sharp contrast to the transverse momentum, the longitudinal distribution is greatly modified when transformed to the center-of-mass frame. Thus, to compute the center-of-mass distribution, we must write the center-of-mass momenta k_L and k_T of the observed particle in terms of the nova-rest-frame momenta q_L and q_T for fixed nova mass M.

A simple convention for the orientation of the nova rest frame is the helicity frame where q_T is perpendicular to the nova direction as seen in the center-of-mass frame. Thus, the nova rest frame is reached by a pure boost from the center-of-mass frame. A short calculation gives

$$k_T = [Q_N(q_T^2 + q_L^2 + M^2)^{1/2} + (Q_N^2 + M^2)^{1/2} q_L] \sin\theta_N / M + q_T \cos\theta_N ,$$

$$k_L = [Q_N(q_T^2 + q_L^2 + M^2)^{1/2} + (Q_N^2 + M^2)^{1/2} q_L] \cos\theta_N / M - q_T \sin\theta_N . \tag{9a}$$

The inverse relations, q_L and q_T in terms of k_L and k_T are also very useful,

$$q_T = k_T \cos\theta_N - k_L \sin\theta_N ,$$

$$q_L = (Q_N^2 + M^2)^{1/2}(k_L \cos\theta + k_T \sin\theta)/M - Q_N(k_L^2 + k_T^2 + \mu^2)^{1/2}/M , \tag{9b}$$

where Q_N is the center-of-mass momentum of the nova,

$$Q_N = [s^2 - 2(M^2 + M_s^2)s + (M^2 - M_s^2)^2]^{1/2}/2\sqrt{s} , \tag{10}$$

M_s is the mass of the spectator particle, the one which does not form the nova in our single-excitation approximation, θ_N is the angle at which the nova (or spectator particle) scatters in the center-of-mass frame, and μ is the mass of the observed secondary. Also needed for the transformation is the Jacobian

$$\frac{d^3q}{d^3k} = \frac{(q_L^2 + q_T^2 + \mu^2)^{1/2}}{(k_L^2 + k_T^2 + \mu^2)^{1/2}} . \tag{11}$$

It is now straightforward to write down the contribution to the inclusive distribution coming from nova production. One simply weights each M and $\cos\theta_N$ value with the nova production cross section $\rho(M, \cos\theta_N)$. Combining Eqs. (1), (3), (7), (9b), and (11), we obtain

$$\frac{d^3\sigma_i}{d^3k} = \sum_\alpha \int_{M_\alpha}^{M'_\alpha} dM\, d(\cos\theta_N)\, \rho(M, \cos\theta_N) \times \frac{d^3D_i(q(k))}{d^3q} \left(\frac{d^3q}{d^3k}\right) . \tag{12}$$

As already discussed, for the pion distributions, $\cos\theta_N$ can be set equal to 1, and the integration over $\cos\theta_N$ ignored. However we shall include this integration for the baryon distributions. The distribution from the leading-particle effect will be discussed later.

Energy and momentum are conserved in Eq. (12) only on the average.[51] Although messy in practice, one could impose energy-momentum conservation on each decay of the nova. Instead, we have imposed simple θ-function cutoffs in Eq. (12) when energy and momentum conservation are manifestly violated. The two conditions are that the maximum invariant energy of the nova decay products must not exceed the nova mass, and the maximum value of the sum of the center-of-mass energies of the nova products and the recoil particle do not exceed $s^{1/2}$. The first of these conditions is important only for large k_T. This is accurate enough for pion distributions, and if necessary better threshold factors can be easily introduced for baryon and strange-particle distributions,[52] where larger $|x|$ values are more important. The averaging over M in Eq. (12) tends to correct this crude treatment of phase-space constraints, especially for pions.

It is easy to evaluate Eq. (12) and the steps leading to it on a computer.[53] Results of such calculations are given in Sec. IV, using the choice of parameters given later in this section. There are, however, some prominent features of Eq. (12) which are obtained in detailed calculations. These can already be seen from an approximate form of Eq. (12) which is valid for intermediate values of $|x|$. These are characteristic features of the model which provide an understanding of some of the observed properties of the data which have only been noticed so far. This approximation holds for large $\sqrt{s}$, where the nova center-of-mass rapidity, z, is given by $\ln(\sqrt{s}/M)$, and where k_L is a finite fraction of $\sqrt{s}$. Then the relation between q_L and k_L, Eq. (9a), reduces to

$$q_L = k_L \exp(-z) = k_L M/\sqrt{s} = \tfrac{1}{2} x M . \tag{13}$$

We have also neglected the transverse motion of the nova, so that $q_T = k_T$, and assumed that intermediate x values correspond to novas emitted along the nova center-of-mass momentum.

The pions obtained from the decay of a nova of mass M give a distribution that peaks at $|x| \approx m/M$, where m is the average transverse mass of the pion, and independent of energy. For smaller $|x|$, the distribution falls off very sharply. The distribution is energy-dependent here and falls off even more sharply as s increases. However, for $|x|$ greater than m/M, the distribution is energy-independent. From Eqs. (7) and (13), the x distribution in the center-of-mass frame associated with a nova of mass M is

$$\frac{dD}{dx\, dk_T^2} = \pi N_i M \exp(-x^2M^2/4K^2) \exp(-k_T^2/K^2) , \tag{14}$$

where we have used the appropriate approximate form for the Jacobian. For these $|x|$ values the k_T^2 dependence factors out,[54] and the contribution to the invariant distribution of a nova of mass M is $(x' = 2\omega/\sqrt{s})$

$$f_M(x) = x' \frac{d\sigma}{dx}$$
$$\approx \sum_{\alpha} \pi N_i K^2 |x| M \rho_\alpha(M) \exp(-x^2 M^2/4K^2) . \tag{15}$$

This is suitable for intermediate positive x for the beam nova, and intermediate negative x for the target nova.

It is easy to deduce several important properties of the inclusive pion distributions from these approximate formulas.

(i) Higher-mass novas produce meson distributions which peak toward lower values of $|x|$. With the Gaussian decrease of Eq. (15), only low-mass novas can contribute significantly at intermediate values of $|x|$. In most situations, phase space restricts the pions from low-mass novas from larger values of $|x|$. Consequently, the high-mass novas concentrate their pions around $x=0$, and also give a diffractive contribution near $|x| \approx 1$ which, however, must compete with quantum-number-exchange peripheral-collision contributions. Since the nova mass is closely related to the mean number of secondaries into which the nova decays, we expect the inclusive distributions restricted to a definite number of prongs to fall off more steeply with $|x|$ as the prong number is increased. This effect is clearly present in the 28-GeV/c data shown in Fig. 5.[4]

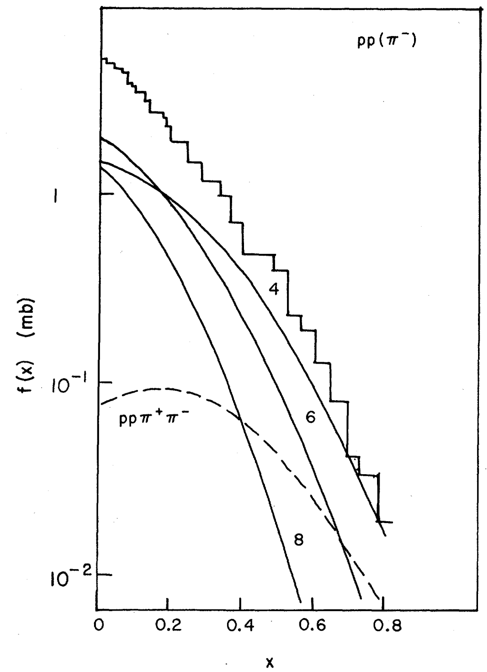

FIG. 5. Total and topological invariant distributions for $pp(\pi^-)$ at 28 GeV/c taken from Ref. 4. The histogram is the total invariant distribution for $pp(\pi^-)$; the solid curves are the topological cross sections (distributions for π^-) for 4, 6, and 8 prongs, respectively; the dashed curve is the distribution of π^- in $pp \to pp\pi^+\pi^-$. Note the faster falloff with x as the prong number increases. Units are mb.

One might try to argue that the clustering of pions from high-multiplicity collisions around $x=0$ is due to the lack of available energy. However, these results represent a slow increase of multiplicity compared to the available energy, and the average center-of-mass energy of these pions could easily have been larger than that found experimentally. Also there is no variation of the mean transverse momentum with increasing multiplicity except close to the phase-space limit. It is also interesting to note the difference between the 4-particle and 4-prong distributions in Fig. 5. The 4-particle distribution shows a typical nova distribution with a mass readily associated with the diffractive excitation of the $N^*(1470)$, but the 4-prong distribution gets contributions from higher-mass novas, which build up the distributions down to $x=0$. Indeed, the $N^*(1470)$ production cross section is small compared to the contributions of the higher-mass novas (around 2 GeV). These novas typically yield three or four pions including π^0's, in accordance with our cascade-decay picture, and can also contribute many 4- and 6-prong events.

(ii) The intermediate $|x|$ region is dominated by the pions from relatively low-mass novas, the pions from higher-mass novas being sharply suppressed according to Eq. (15). The low-mass novas can already be excited at relatively low energies, and their diffractive cross sections remain constant. Thus, integrating Eq. (15) over mass, we find that the great majority of pions contributing at intermediate $|x|$ already contribute at relatively low energy. As a result, the distribution should approach a scaled limit very quickly, neglecting secondary trajectory effects which are discussed in Sec. III. Even though the upper limit of integration over the nova spectrum increases as $\sqrt{s}$, the pions from the high-mass novas so rarely diffuse to intermediate $|x|$ values that the approach to scaling is extremely fast. No specific high-energy features are important in order to understand the x behavior and rapid scaling for these x values.

A crude estimate of the x behavior of the invariant cross section can already be obtained if one assumes that $\rho(M)$ is very sharply peaked at about $M=2$ GeV for baryons and $M=1.3$ GeV for pions, which are the values inferred from available missing-mass spectra. This gives Gaussian-like distributions which fall like $x\exp(-10x^2)$ in the baryon fragmentation region and $x\exp(-5x^2)$ for meson fragmentation, which are already reasonable estimates of the observed values.[11,32] When the integrals over the mass spectrum of Eq. (3) are performed (Sec. IV), one obtains a distribution with more exponential appearance and the rapidity plot is more of a Gaussian shape. Such a distribution is shown as an example in Fig. 6.[55,56]

(iii) A rationale for the quark-frame result[2,56a] follows from the feature that the nova excitation spectra peak at different values of nova mass for mesons and baryons. In this result, pion-induced inclusive spectra are approximately symmetrical if measured in a Lorentz frame where the meson momentum is $\frac{2}{3}$ of the baryon momentum, so that the magnitude of the momenta of the quarks composing the hadrons would all be equal in this frame. We obtain this result simply by noting that the most symmetrical distribution is found in a frame where zero rapidity is near the halfway point between the rapidities of the peak values of the meson and baryon distributions. The rapidity of a beam nova of mass M_B is $Z_B = \ln(\sqrt{s}/M_B)$. Thus, to transform the rapidity from the center-of-mass frame to the frame of symmetry, we must subtract $\frac{1}{2}(Z_B - Z_T) = \frac{1}{2}\ln(M_B/M_T)$ from the beam-nova rapidity and add the same amount to the target-nova rapidity, where M_B and M_T are

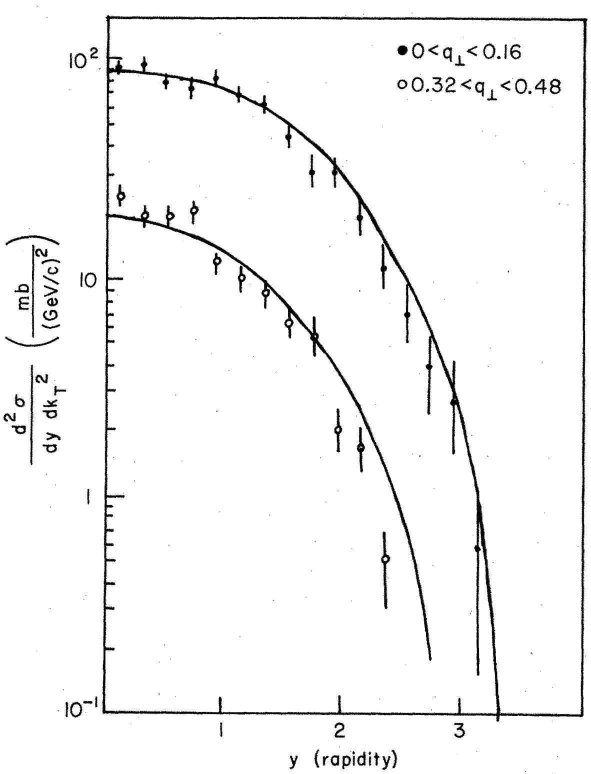

FIG. 6. Rapidity distributions for $pp(\pi^-)$ at 19 GeV/c. The solid curves are the zero-parameter prediction of the model. The data are taken from Ref. 55.

the values of the nova mass where the beam- and target-nova cross sections have their peak values, respectively. Converting the rapidities back to momenta, the ratio of the nova momenta in this frame is

$$\frac{q_T}{q_B} = \frac{M_T}{M_B}. \tag{16}$$

The momenta of the incoming particles are the same in this frame if $\sqrt{s}$ is much larger than either of these typical nova masses. This ratio is 1.5 for the values $M_T = 2$ GeV and $M_B = 1.3$ GeV used for the maxima of the two distributions.

As discussed below, the production cross section for meson novas seems to be about double that for nucleon novas. If taken into account, the frame of maximum symmetry will move somewhat faster in the center-of-mass frame than the one computed above. This effect gives a value for q_T/q_B a little larger than 1.5.

This ratio is decreased if we measure the distribution due to the high-mass novas, for example, by requiring large prong number in the measurement. This effectively raises M_T and M_B by the same amount, so that their ratio is decreased. We do not expect the ratio q_T/q_B to remain constant for distributions due to different prong numbers, but to go to unity as the prong number is increased. This is exactly the effect which is observed experimentally.

(iv) Although the intermediate $|x|$ part of the invariant cross section reaches its asymptotic form very quickly, the low $|x|$ ($x \approx 0$) and large x ($|x| \approx 1$) are expected to approach their asymptotic values much more slowly. The calculations of Sec. IV show a definite increase of f at $x = 0$ between 30 and 200 GeV/c, where it is already near its asymptotic limit. As the energy increases, it becomes kinematically possible to create heavier novas which contribute many pions around $x \approx 0$ without appreciably changing the total cross sections. The pions build up the distribution around $x = 0$, which becomes more peaked as the energy is increased. The asymptotic limit is approached from below. Contrary to the previous discussions, this prediction is based on the behavior of high-mass novas which involve a phenomenological extrapolation from the better-motivated lower-M behavior. Even so, this effect, which is localized in the $0 < |x| < 0.1$ region, is worth checking. As already mentioned a single- or double-nova picture is not easily distinguished in this limit, and so the effect is likely to be more general than a single-nova model, provided the multiplicity continues to increase.[16,56] This could even be associated with an entirely new production mechanism. However this peaking is not expected in the multiperipheral model, where the x distributions flatten at $x=0$ as energy increases, or in the double-Pomeranchon-exchange scaling limit,[15] where the asymptotic value can be approached either from above or below, depending on the reaction.

Scaling should also be slow for $|x|\approx 1$, if present at all, in those cases where diffraction might provide a dominant contribution there. Heavy novas contribute very little [see Eq. (15)], but the pions from low-mass novas are restricted from $|x|$ near 1 by energy-momentum conservation. However, this is the domain where secondary trajectory exchange is likely to dominate, and therefore, diffractive processes may be quite irrelevant. Of course these considerations are greatly modified when a leading-particle effect is possible, which we discuss in (vii) below.

(v) The pseudoscaling behavior in the 15- to 25-GeV/c region which was mentioned in the Introduction can also be partially understood using simplified kinematics. Equation (15) was derived under the assumption that the pion is emitted in the direction of the nova motion. If the pion goes against the nova motion another approximate formula holds,

$$f_M(x)=\pi N M|x|\rho(M)\exp(-x^2s^2/4K^2M^2). \qquad (17)$$

This expression fails near $x=0$, but it does give the general trend of the small $|x|$ side of the distribution of pions from a nova of mass M. We see that near $x=0$, the contribution of a low-mass nova decreases as s is increased. This effect slows the increase of $f(x)$ near $x\approx 0$ due to the increasing contribution of high-mass novas. The partial compensation of the increase of pions from high-mass novas and decrease from low-mass novas is largest when the lower-mass novas can contribute most strongly, such as in reactions like $\pi^-p(\pi^-)$ or $\pi^+p(\pi^+)$ where all prong configurations are collected. In reactions such as $\pi^\mp p(\pi^\pm)$, numerous 2-prong reactions are excluded and low-mass novas are discriminated against. The effect is also borne out by the quantitative calculations of Sec. IV using Eq. (12). The effect is further enhanced by the decrease of charge-exchange processes, which is especially apparent in the 2-prong inelastic events.

(vi) The charge ratios observed in Fig. 1 can be understood in terms of the masses of the novas which give pions dominating certain x regions, and the knowledge that the excitation is diffractive.[25] With the Q values of the nova decay previously discussed, an average of 2 to 3 pions per GeV of excited nova mass is expected. This is also the value inferred from the branching ratios of high-mass resonances. The very high-mass novas should then decay into approximately equal numbers of π^+, π^0, and π^-, since there are many decay pions and total charge must be conserved. Therefore $N_i(M)$ should tend to $R(M-M_\alpha)$ where $R\approx 0.8$/GeV, and i refers to the charge state of the observed pion. However lower-mass novas decay into few pions, so that one expects a strong correlation between the pion charges and the charge of the nova. Low-mass proton novas yield more π^+ than π^-, and meson fragmentation should show a strong charge asymmetry even before the leading-particle effect sets in around $x\approx 0.6$. Simple estimates typically lead to ratios of π^+/π^- of order 2 (or $\frac{1}{2}$) for charged novas producing 2 to 3 pions. This asymmetry slowly vanishes at lower $|x|$ and high energy, where high-mass novas become important[57a]; but at machine energies the low-mass novas still make an important contribution of pions around $x=0$, which diminish as s increases according to Eq. (17).

A simple example of these considerations is the crossover effect of $\pi^-p(\pi^-)$ and $\pi^-p(\pi^+)$ at $x\approx -0.1$.[2,3] On the proton side ($x<0$), the cross section for low-mass proton novas to produce π^+ is larger than it is to produce π^-. However, for positive x, the probability that a π^- nova produces a π^- exceeds that of producing a π^+. Therefore, the $\pi^-p(\pi^-)$ should rise above the $\pi^-p(\pi^+)$ for positive x. The crossover is at negative x simply because the meson nova couples more strongly than the proton nova, as we discuss later. A quantitative calculation, shown in Fig. 7, reproduces the data quite well. This is a good piece of evidence for the dominance of diffractive processes.

Several other examples are that $pp(\pi^+)$ at intermediate $|x|$ should be about a factor of 1.5 greater than $pp(\pi^-)$, and the $pp(K^+)$ distribution should be much larger than the $pp(K^-)$ distribution.[6]

(vii) The dominance of single-nova production implies the presence of a strong leading-particle effect. When a relatively low-mass nova is produced ($M\ll\sqrt{s}$), then the quasielastically scattered particle is quite fast in the center-of-mass frame. Besides the obvious implications for exclusive reactions, the inclusive distributions in which the observed particle is identical to one of the initial particles are also affected. For example, in the reaction $\pi^-p(\pi^-)$ the formation of a proton nova leaves a fast forward π^- corresponding to the incident π^-. This mechanism will provide the large majority of π^- near $x\approx 1$. The normalization of this quasielastic peak is determined by the distribution of the recoil nova, and a comparison of the distribution of, for example, $\pi^-p(\pi^-)$ at $x\approx 0.9$ and $x\approx -0.2$ gives a definite check on the importance of single-nova excitation. Although the experimental determination of this peak for leading beam

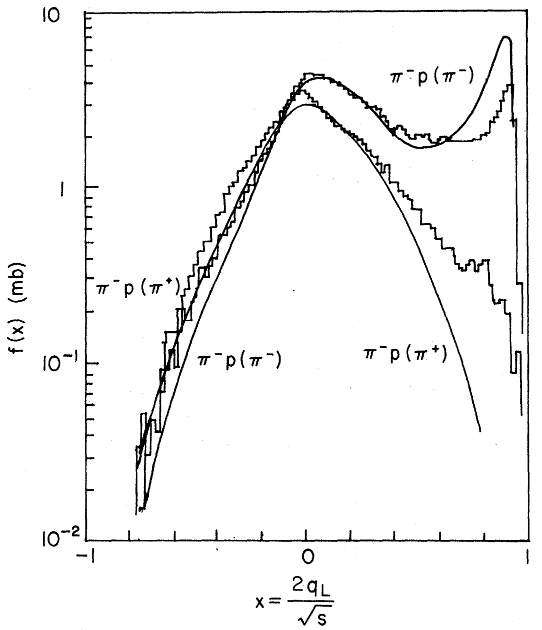

FIG. 7. The invariant distributions for $\pi^- p(\pi^\pm)$ at 16 GeV/c compared with the data of Ref. 2. This is a zero-parameter calculation, with the normalization obtained from total and total-inelastic cross-section data. Charge-exchange corrections have been neglected in the calculation, but they would raise $\pi^- p(\pi^+)$ on the proton fragmentation side. The slightly better fit of Ref. 20 used a slightly higher pion multiplicity and an arbitrary normalization. The large-x contribution of the $\pi^- p(\pi^+)$ distribution can be traced to ρ production.

particles is difficult, recoil-proton spectra can be measured and provide a substantial check on the approximation. It should be stressed though that even a sizable fraction of double excitation would not drastically change our result for the part of the inclusive distribution not corresponding to the leading-particle peak. This is in particular the case for nova resonance (double excitation) production as opposed to quasielastic scattering. This is why the model works so well.

In order to compute the leading-particle (or recoil) spectrum, we write the nova cross section, $\rho(M)$, as

$$\rho(M) = \frac{1}{64\pi k^2 s} \int dt \, |F_M(t)|^2 , \tag{18a}$$

where k is the center-of-mass momentum of the incoming particle. This allows us to identify

$$\frac{d\rho(M,t)}{dt} = \frac{|F_M(t)|^2}{64\pi k^2 s} . \tag{18b}$$

Then, transforming from an M and t distribution to k_L and k_T, the center-of-mass momenta of the recoil particle, we obtain

$$\omega \frac{d\sigma}{dk_L dk_T^2} = \frac{\sqrt{s}\, k}{M} \frac{d\rho}{dt} , \tag{19}$$

where ω is the center-of-mass energy of the recoil particle, and M and t are written as

$$\begin{aligned} M^2 &= s - 2\sqrt{s}\,(k_T^2 + k_L^2 + \mu^2)^{1/2} + \mu^2 , \\ t &= 2\mu^2 - 2(k^2 + \mu^2)^{1/2}(k^2 + k_T^2 + \mu^2)^{1/2} + 2kk_L , \end{aligned} \tag{20}$$

where μ is the mass of the quasielastically scattered particle.

For the calculations of this paper, we have assumed

$$\frac{d\rho(M,t)}{dt} = B \exp[B(t - t_{\min})]\, \rho(M) , \tag{21}$$

where $\rho(M)$ is given by Eq. (3) and B is a constant whose value for diffractively produced resonances is measured to be between 3 and 8 $(\mathrm{GeV}/c)^{-2}$.

Thus for each value of k_L and k_T, the leading-particle cross section is immediately related to $\rho(M)$, and is therefore comparable to the spectrum of particles coming from the nova. Distribution (19) is simply added to the distribution in Eq. (12) when the leading-particle effect is possible.

The energy dependence of the leading-particle effect on the $x > 0$ side of the leading-particle peak is quite prominent. The higher-mass novas control this part of the distribution so $\rho(M)$ is controlled by its M^{-2} decrease. With $\rho_\alpha(M) = c_{\alpha,s} M^{-2}$ [see Eq. (3)], Eqs. (19) and (20) give

$$\omega \frac{d\sigma}{dk_L} = x' \frac{d\sigma}{dx} = \frac{c_{\alpha,s}}{\sqrt{s}} \frac{x}{(1-x)^{3/2}} . \tag{22}$$

In meson spectra, this leads to a deepening valley between the leading-particle peak and the peak around $x \approx 0$ as the energy is increased.[2] The peak itself is determined by the peak in $\rho(M)$, and as the energy increases the leading-particle peak becomes narrower and moves toward $x = 1$.

The leading-particle effect is more easily analyzed with a slow recoil proton. For instance, we may study $d\sigma/dp$, where p is the modulus of the laboratory proton momentum. With the nova excitation spectra in Eq. (3), we find a cross section peaked at around $p \approx 0.4$ GeV/c, and a negligible flux of protons with $p < 100$ MeV/c, Fig. 8. This reflects the mean value of the momentum transfer in a single-nova excitation. Furthermore, not too much emphasis is placed on the low-mass excitations, with a mass spectrum showing a maximum around 2 GeV instead of simply falling with M.

(viii) The Mueller analysis[15] of inclusive reactions is extremely general, and the model described here does not conflict with it. Nevertheless, to the extent that all secondaries are attached

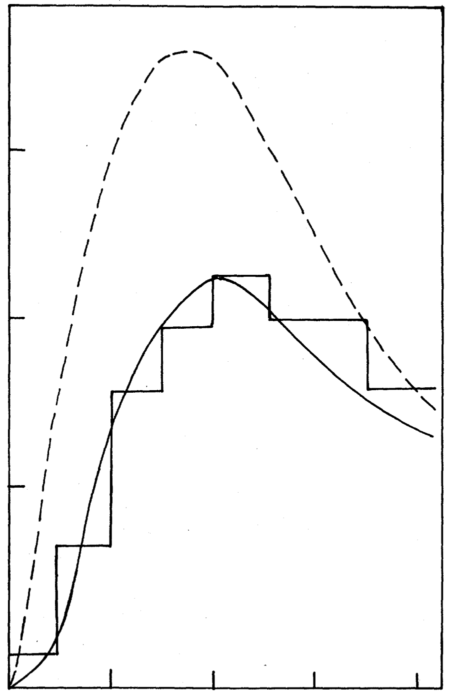

FIG. 8. Typical low-energy spectrum (in arbitrary units) of recoil protons computed for $K^-p(p)$ at 12.7 GeV/c and compared with some data from the Yale bubble-chamber group (Ref. 66). The shape of the proton spectrum in $\pi p(p)$ is given by the dashed line. The $K^-p(p)$ distribution is normalized to the experimental results. The histogram which contains about 3000 events includes only 4-prong events.

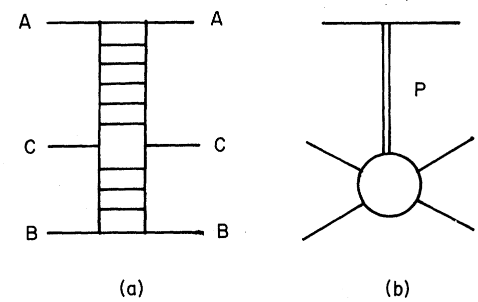

FIG. 9. (a) Ladder picture for single-Regge exchange. (b) Model of fragmentation of particle B. In the nova model at present machine energies most of the rungs of the ladder in (a) are clustered around C. Although this suggests that the single-Pomeranchon-exchange approximation, seen as the shadow of particle production, could not be accurate until very high energy, Mueller's analysis[15] could still apply. We thank Dr. L. L. Wang for clarifying this point.

to either a beam or target nova, one might not expect single-Pomeranchon exchange to be a good approximation to the absorptive part of the three-body elastic amplitude until very large energies. This is inferred as follows:

Consider as an example target fragmentation [Fig. 9(a)] where $s_{a\bar{c}}$ is large and $s_{b\bar{c}}$ is relatively small. Invoking a multiperipheral picture, one expects many secondaries to have rapidities between those of particles a and c, with only a few between c and b [Fig. 9(b)]. For the absorptive part, one readily introduces a Pomeranchon-exchange approximation for the shadow of the many secondary pions associated with the large $s_{a\bar{c}}$ energy [Fig. 9(b)]. The same approximation does not hold for $s_{b\bar{c}}$ unless $s_{b\bar{c}}$ is also large which then gives the double-Pomeranchon approximation.[15]

In a model which gives most secondary rapidities close to that of c, few secondaries span the $a\bar{c}$ rapidity interval. The single-Pomeranchon exchange might not be a good approximation to the absorptive part until energies are so large that many secondaries will in any event have rapidities between a and $\bar{c}$. This is, however, a specific mechanism for generating single-Pomeranchon exchange when the Mueller analysis applies to more complicated singularities. Following our picture the relevant contribution to the absorptive part of the three-body elastic amplitude could be calculated in terms of our single-Pomeranchon-exchange production amplitude. Pomeranchon exchange, as shown on Fig. 9(b), would then simply indicate the energy independence of the fragmentation cross section. Such Pomeranchon exchange should not be confused with the one shown on Fig. 2. They both summarize the same energy behavior. In any case, we obtain a quick scaling from the shape of the excitation spectrum which is theoretically connected to the weakness of the triple-Pomeranchon coupling.

D. The Model Parameters

The equations of the nova model have all been presented, and it remains to fix the values of the parameters appearing in these formulas. These parameters are the shape and strength parameters of the excitation cross section, β_α and $c_{\alpha,s}$ appearing in Eq. (3); the branching ratios or multiplicities of observed final particles, $N_i(M)$ found in Eq. (7); the parameter K in Eq. (7), which is also closely related to the average number of decay particles per GeV of nova mass; and B, the t dependence of the nova cross section, Eq. (7). Rather than attempting a best fit to the inclusive data, we stress that these parameters are already quite well determined from other experiments or by other considerations. As a result, we can approach the inclusive spectra with basically a *zero-parameter* model.

The parameter β determines the general shape of the distribution, Fig. 3, and in particular the location of the maximum of $\rho(M)$. Although there are not yet enough data to conclude positively, the exclusive analyses of diffractively produced final states give good indications that the maximum of the proton excitation should be around 1.9 GeV for protons and 1.3 GeV for pions. We have chosen $\beta_N = 1.8$ GeV and $\beta_\pi = 2.1$ GeV [Eq. (4)], which give the nova-excitation spectra shown in Fig. 10. Cross checks on these values are offered by the location of the leading-particle (or recoil) peak, and the rise of K^+p total cross section. As already stressed, the value of β is most important for fixing the intermediate $|x|$ behavior of the inclusive distributions.

The parameter $c_{\alpha,s}$ is the strength of the nova-formation cross section, and depends both on the particle which becomes a nova and the recoil particle. For reactions with identical beam and target particles, such as $pp(\pi)$, c only determines the over-all normalization of the distribution, and can be determined through the model from the total inelastic diffractive cross section. [See Eq. (5).] However, at machine momenta (~20 GeV/c) the diffractive part may be only ~70% of the total inelastic cross section, and one may wish to keep this over-all normalization as a free parameter for reasons discussed in Sec. III. If c is normalized by the

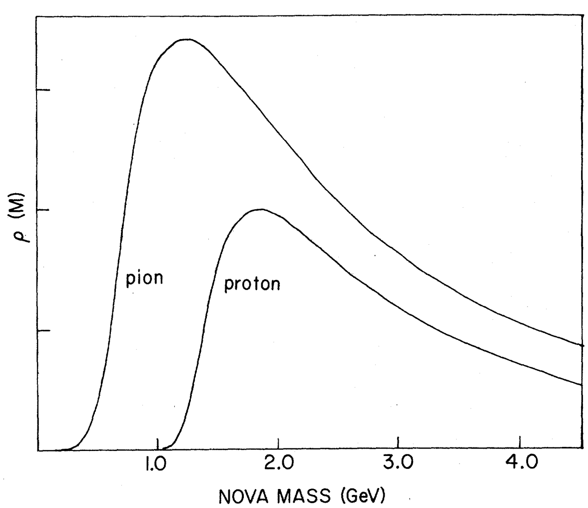

FIG. 10. The nova excitation spectrum (in arbitrary units), $\rho(M)$, for pion and nucleon novas, Eq. (3). The parameter β is adjusted so that the proton spectrum peaks at 1.85 GeV ($\beta = 1.8$ GeV) and the pion spectrum peaks at 1.3 GeV ($\beta = 2.1$ GeV). The ratio, C_π/C_p, is 2.3. Compare with Fig. 3 where only low-mass states are dominant in the exclusive analysis limited to 4-particle final states.

total inelastic cross section, then the multiplicity does take on the correct value. Therefore we shall normalize c in this manner.

For reactions with different targets and projectiles, the ratio of the two c's does affect the shape of the distribution, and is of importance in understanding the shape of the distribution in the region $-0.4 < x < 0.4$. In the $\pi p(\pi)$ reactions, best results are obtained when c_π/c_p is near 2. The dominance of meson nova production is already clear from Fig. 1. However, two other estimates are easily made. A rough estimate can be made from the exclusive analyses,[30] which show that excited meson production dominates by a factor 2. Or, we may assume that factorization[28] holds approximately for Pomeranchon exchange with nova production, and estimate these ratios by comparing total inelastic with total elastic cross sections. The total area under the excitation curve is $c_{\alpha,s}\,\beta_\alpha^{-1}$. For instance, using the rough values $\sigma_{pp}^{tot} = 40$ mb, $\sigma_{pp}^{el} = 10$ mb, $\sigma_{\pi p}^{tot} = 24$ mb, and $\sigma_{\pi p}^{el} = 4$ mb, and calculating the cross sections according to Eqs. (3) and (5), we find $c_{\pi p}/c_{p\pi} = 2.3$. The values used in the calculations of Sec. IV are $c_{pp} = 47$ mb GeV, $c_{p\pi} = 19$ mb GeV, and $c_{\pi p} = 44$ mb GeV.[57] Thus, the distributions are normalized to the total and total-inelastic cross-section data.

The value of K, which is a measure of the mean momentum of the pion in the nova rest frame has already been discussed. We take $K = 330$ MeV/c. Future refinements obtained from experimental information on resonance decay, or from an analysis of dual-resonance models, can be easily incorporated. This value of K further fixes the average charged multiplicity at 20 GeV/c to be 4 in πp collisions, which is in agreement with experiment. This value of K also determines the average multiplicity of decay pions per unit of nova mass to be 2.5/GeV, and together with $\rho(M)$ determines the correct x distribution.

Next, we need to divide this total multiplicity among the possible pion charge states, and eventually other decay particles. This should require detailed branching ratios not available for daughter and higher-mass states. However, for low-mass novas, the multiplicity is low and isospin conservation together with an equal average branching ratio for isospin-$\frac{3}{2}$ and isospin-$\frac{1}{2}$ baryons can be used to constrain $N_i(M)$. This leads to the charge effects at intermediate $|x|$ already discussed. For example the probabilities of obtaining a π^+ or a π^- from proton nova giving two pions and a nucleon are around 0.5 and 0.25, respectively. On the other hand, the probabilities of getting a π^+ or a π^- from a π^- nova decaying into three pions are 0.15 and 0.5, respectively. Now, as the mass of the nova is increased, the multiplicity of pions increases, so that statistically we expect the ratios of π^+ to π^0 to π^- to become unity. This eventually gives identical asymptotic π^+ and π^- distributions at $x=0$. For intermediate M values we join smoothly the different values. Of course this entails some arbitrariness, and some forms may fit better than others, although without any qualitative change in the fits.

For baryon distributions, we assume that only one nucleon emerges from a proton nova. For low-mass proton novas, a proton is favored, but for high-mass novas, the probability of a proton or a neutron becomes equal. Rarer decays such as associated with strange-particle production will be discussed in a later paper.

Our description of nova decay conserves energy and momentum only in an average way. Of course energy-momentum conservation is, in principle, easily imposed at each step of the decay, but it appears not needed at the present level of accuracy for reasons previously mentioned. We have simply used θ-function cutoffs when energy and momentum conservation are manifestly violated.

The t dependence of the nova production cross section is controlled by B. We have taken $B=8$ GeV^{-2} in this paper, although apparently B does depend on the nova mass.

All the parameters being thus determined, the formulas which are given can be used to calculate any distribution and, in particular, those given on Figs. 6 and 7. The nova model may of course be looked at as a specific version of a fireball picture.[58] The key properties of the novas which distinguish them from "usual" fireballs are the presence of low-mass maxima in the excitation spectra and the resonancelike behavior of the excited objects which is related to their diffractive production.

III. CORRECTIONS TO THE NOVA MODEL

As is obvious from Fig. 1, the baryon fragmentation distribution depends on the charge of the incoming pion. If considered too large to be blamed on inaccurate normalization or on the lack of factorization of diffraction, it must be interpreted as evidence for quantum-number-exchange contributions to the proton excitation. Even if most of the diffractive contribution corresponds to single excitation, there is reason to expect sizable charge-exchange corrections, since Eq. (5), taken at 16 GeV/c, indicates that only 70% of the πp total cross section would be diffractive. Exclusive analyses which grant a dominant role to diffraction also leave a reasonable cross section for quantum-number exchange.[30]

For fixed and relatively low excited mass, we again consider the triple-Regge limit, Eq. (2), with α_P replaced by the leading secondary trajectory. This leads to a cross section which decreases with energy as $s^{2\alpha(t)-2}$, up to logarithmic terms.[38] This contribution therefore vanishes with increasing energy as $1/s$,[37] but it should affect the various distributions in a selective manner. In the proton fragmentation regions of $\pi^+p(\pi^+)$ and $\pi^-p(\pi^+)$, the former distribution is more strongly corrected than the latter since charge exchange in $\pi^+p(\pi^+)$ can produce baryon states with charge +2. Therefore, there should be more π^+'s from $\pi^+p(\pi^+)$ than $\pi^-p(\pi^+)$ due to the charge-exchange corrections. Similarly, the $\pi^-p(\pi^-)$ distribution should be above $\pi^+p(\pi^-)$ in the proton fragmentation region. At least a small charge-exchange contribution may be present in $\pi^-p(\pi^-)$. In our picture, the $\pi^+p(\pi^-)$ distribution should be the one least affected for negative intermediate x. Interestingly, this is just the channel with exotic $AB\overline{C}$ quantum numbers.[59]

In the meson fragmentation region, charge exchange leads to neutral mesonic states which will affect the π^+ and π^- distributions in the same way. Therefore, the equality between the distributions shown in Fig. 1 is not evidence for the absence of quantum-number exchange in meson excitation. In fact, the pion distributions extend all the way to $x\approx 0.8$, even when the leading-particle effect is absent. This implies appreciable ρ production, since only ρ's are light enough to produce a pion with large x, while the other decay pion has $x\approx 0$.[60]

We now examine the kinematical region where the mass of the excited object is large, but still much less than $\sqrt{s}$. The triple-Regge formula, Eq. (2), is valid, except α_P has to be replaced by the leading secondary trajectory, $\alpha(t)\approx\frac{1}{2}+\alpha' t$, and $\alpha(0)$ in Eq. (2) can now be the Pomeranchon intercept. For this kinematical region, the cross section for producing an excitation of mass M grows linearly with M. Therefore, if the missing mass is allowed to grow proportionally to $\sqrt{s}$, the inclusive distribution scales and the rationale for expecting quantum-number exchange to decrease with energy is lost.[61] At first sight it would appear that these processes, if present, would give a too-rapid rise in multiplicity. However, it should be stressed that the "excited" object in Fig. 4(d) is no longer a nova, since in the triple-Regge picture it may be dual to the Pomeranchon instead of the leading secondary trajectory. As background, its "decay" mechanism will be very different from the nova decay. Indeed, we can consider that this contribution corresponds to a Pomeranchon exchange between a Reggeon switched into a mass-shell meson and an actual nova [Fig. 11(a)], or to the Reggeon giving the nova, on a quasielastically scattered target particle [Fig.

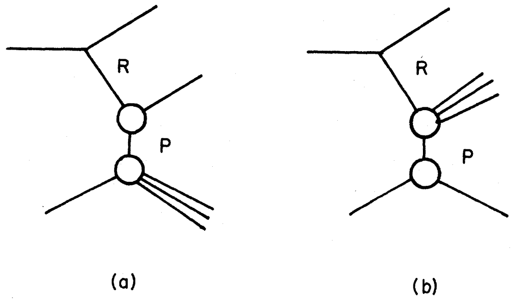

FIG. 11. Dual interpretation of charge-exchange excitation of high-mass objects. Diagram (a) is a charge-exchange contribution that can eventually lead to double-nova excitation. Diagram (b) simply gives back the single-nova excitation in the dual approximation.

11(b)]. Thus, we would conclude that this picture still leads to an over-all logarithmic increase of multiplicity, even if we carry the increase of cross section for producing an object of mass M all the way to the edge of phase space.

It should now be noted that the second process, Fig. 11(b), is nothing but single-nova production, provided that it is looked at in a dual way.[62] On the other hand, the first process is the beginning of a two-nova-excitation process, provided that double-Pomeranchon exchange is forbidden.[63] A more detailed analysis of this diagram could lead to a good understanding of the onset and importance of double-nova production at higher energies. Here, we emphasize that our picture of inclusive distributions is quite stable to quantum-number-exchange corrections, because of the kinematical properties of the model. In the large-M limits, the pion distributions will be little affected by the production mechanisms, provided that the over-all multiplicity increase is kept the same (i.e., logarithmic). It is just this stability which allows us to adjust the normalization of $c_{\alpha,s}$ to the total cross section (not just the diffractive cross section) and still obtain such good agreement. It should be stressed that we certainly do not forbid two-nova production as the energy increases. We even see how they might become important.

The rapid scaling at intermediate $|x|$ followed from the constant cross section for producing lower-mass novas. As the energy is increased, and double-nova production becomes possible, this stability remains only if the fragmentation of the target (or projectile) does not depend appreciably on whether or not the other particle scatters or fragments. With this attitude toward double-nova production, diffraction production cannot be a factorizable process, even though some properties derivable from factorization, such as the asymptotic equality of the $\pi^+p(\pi^-)$ and $\pi^-p(\pi^-)$ distributions in the proton fragmentation region or the proportionality between pion- and proton-induced inclusive distributions can easily be maintained. Instead of factorization, we have sum rules similar to Eq. (5) which require the sum of the total fragmentation cross sections to add up to the total inelastic cross section. Another feature of a factorized Pomeranchon in our model would be a sizable increase in the total cross section, which doe not seem to be present experimentally.

We have seen that the predictions for the importance of secondary trajectories in the $\pi p(\pi)$ reactions agree with those obtained from Regge analysis. This is *not* the case for $K^+p(\pi)$ in the proton fragmentation region for which both AB and $AB\overline{C}$ have exotic quantum numbers.[59] Just as for $\pi^+p(\pi^-)$, we expect negligible effects for $K^+p(\pi^-)$, as charge exchange gives doubly charged baryons with decays into π^- unfavored. On the other hand, such objects could contribute easily to $K^+p(\pi^+)$, and we expect $K^+p(\pi^+)$ to show an energy dependence similar to that observed for $\pi^+p(\pi^+)$ for intermediate negative x. We expect this violation of the "exoticity rule" to occur at relatively low energy (<20 GeV). It is readily connected to the opening of the ΔK^*-like channels (or with Z^*'s), which is known to be a large correction to diffractive excitation in K-induced reactions.

The dual analysis should, of course, become correct at very high energies, but secondary-trajectory effects might no longer be very relevant. Another type of example is the $\pi p(\overline{K})$ reactions at negative x. The production of $\overline{K}$'s from excited protons requires a very heavy nova mass, which implies energy dependence in the diffractive formation. At the same time, $\overline{K}$ production from Y^* decay is also energy-dependent.

In summary, we note that although diffractive excitation is not the only mechanism of particle production, it appears to be an interesting guideline for defining and studying secondary effects. This is particularly true for quantum-number exchange at relatively low momenta (<20 GeV/c).

IV. COMPARISON WITH EXPERIMENT

In Sec. II(d), we have estimated the parameters of the nova model from considerations other than the inclusive spectra. Although the values of these parameters can be further refined using a wider selection of data and the inclusive spectra, we leave detailed fits plus any improvements to the discretion of the reader. Our attitude here is only an exploratory one.

The simplest distribution from our point of view

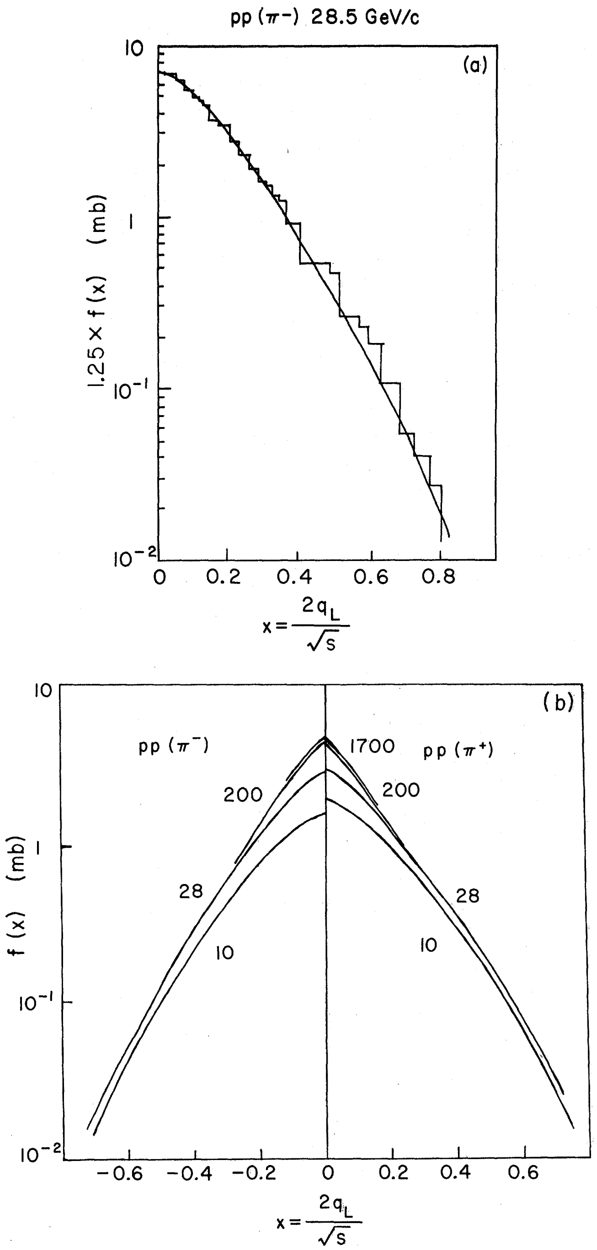

FIG. 12. Pion inclusive distribution in pp collisions. (a) The solid curve is the computed curve for $pp(\pi^-)$. The curve is normalized to the total and total-inelastic pp cross sections, so that this is a *zero-parameter calculation.* The histogram is experimental data from Ref. 4. The $pp(\pi^-)$ distribution is the simplest one from the point of view of our model. (b) Calculated distributions for 10, 28, 200, and 1700 GeV/c.

is $pp(\pi^-)$, not because it is *a priori* simpler theoretically, but simply because of the low mass of the pion and the identity of the initial particles. As discussed, even the corrections should be weak in this case. The parameters from Sec. II are β_p $=1.8$ GeV, $c_{pp}=47$ mb GeV, $K=330$ MeV/c, and the number of pions per GeV of nova mass is 2.5. c_{pp} was determined from the total inelastic cross section. In general, one might expect this procedure to give too large normalizations. However, charge exchange makes virtually no correction to $pp(\pi^-)$, so this contribution to the pp total cross section should be well described without any adjustable parameter. Of course, this last comment does not apply to relatively low-energy $pp(\pi^+)$.

In Fig. 12 we show the predictions of the model for both $pp(\pi^-)$ and $pp(\pi^+)$ at 10, 28, 200, and 1700 GeV/c. Quantum-number corrections have not been applied to $pp(\pi^+)$, nor are they made to any other predictions in this paper. $pp(\pi^+)$ is greater than $pp(\pi^-)$ at intermediate $|x|$ because π^+'s should dominate over π^-'s in proton nova decay. The scaling property at intermediate $|x|$ should be noted: There is little change between 10 and 1700 GeV/c. On the other hand the distribution varies much at $x=0$ between 10, 28, and 200 GeV/c, but shows approximate scaling between 200 and 1700 GeV/c. Thus, we predict scaling for all ISR energies compared to the energy variation between 30 and 200 GeV/c.

The π^- and π^+ distributions should eventually tend to the same $x=0$ limit at very high energy. The rapidity distributions for $pp(\pi^-)$, already shown in Fig. 6, have the typical Gaussian shape. It should flatten with energy. It should be stressed, however, that the only clue which we have about such a behavior is the increase of the multiplicity which has been built into the model (heavy-nova contribution). The rapidity distribution around zero center-of-mass rapidity may stay flat or develop a wide saddle behavior.[64] The model as so far built has but little predictive value there. The peaking of the x distribution around $x=0$ is, however, much more general than the exact behavior at $x=0$. Our results may be compared with the data in Ref. 17, which were taken at 19 GeV/c. The observed Gaussian shape should be contrasted to some versions of the multiperipheral model which predict a dip at $x=0$.[65]

Our predictions for $pp(K^\pm)$ will be reported elsewhere.[36] We briefly discuss their basic features. They scale more slowly than $pp(\pi^\pm)$ because it takes a heavier p nova to produce a K. The K distributions are much flatter around $x=0$ than the π distributions for 2 reasons: Proton novas very rarely decay into two K's of the same charge, whereas they can often decay into two π's of the same charge. The second effect is kinematical. Due to the heavier mass of the K, the nova must be very heavy before the K is likely to have a small $|x|$ value. These two effects far offset the suppression of the low-mass nova contribution.

Similar effects are found in the $pp(p)$ distributions analyzed below. Finally the K^+ distribution is expected to be much larger than the K^- distribution (except at $x=0$ at asymptotic energy.) This is a simple consequence of the dominance of diffractive excitation of the proton.

The pion-induced reactions involve more parameters, which we evaluate in Sec. II. The values $c_{p\pi}=19$ mb GeV and $c_{\pi p}=44$ mb GeV followed from a simple factorization argument and the πp and pp total and total-inelastic cross-section data. Although we do not expect the Pomeranchon-exchange term to factor, this should still be a fairly reliable way of estimating these parameters. The ratio $c_{\pi p}/c_{p\pi}=2.3$ is particularly important for understanding the shapes of the distributions. This is indeed the ratio also suggested by the exclusive analyses which are available.[30] We again emphasize that our model, as it is used to confront the inclusive distributions, has zero adjustable parameters. All its parameters can be determined or at least estimated from other sources. Many of the features of these distributions have already been discussed in Sec. II.

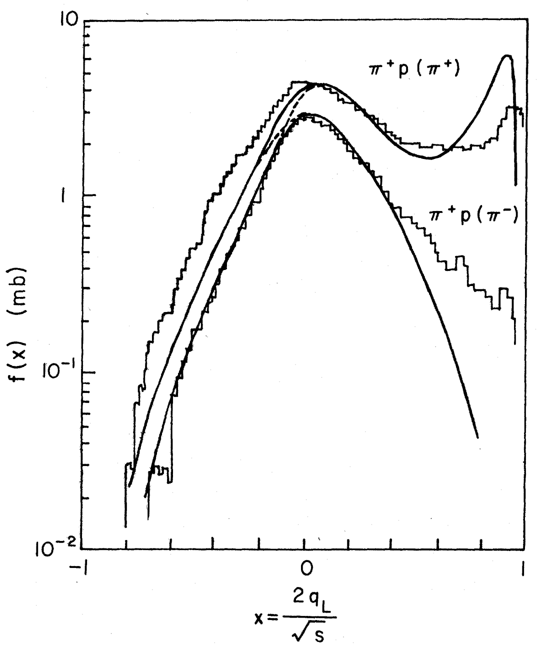

FIG. 13. Pion distributions in pion-induced reactions with proton targets at 16 GeV/c. The $\pi^+p(\pi^+)$ and $\pi^+p(\pi^-)$ data are from Ref. 2, and the solid lines are the model predictions for these distributions. Since quantum-number exchange is absent from our model, $\pi^+p(\pi^+)$ and $\pi^-p(\pi^+)$ are equal for $x<-0.2$, as are $\pi^+p(\pi^-)$ and $\pi^-p(\pi^-)$. On the other hand, for $x>0.1$, $\pi^+p(\pi^+)$ and $\pi^-p(\pi^-)$ are equal and above $\pi^+p(\pi^-)$ and $\pi^-p(\pi^+)$, which are also equal. The crossover region is shown with the dotted lines. Deviations of the predictions from the experimental results can be traced to quantum-number exchanges which are particularly important in the $\pi^+p(\pi^+)$ distribution in the proton fragmentation region.

Figures 7 and 13 give the results of the model for all four $\pi p(\pi)$ distributions evaluated at 16 GeV/c. These should be compared with the data sketched in Fig. 1. Present data are available in Refs. 2 and 3. As in Ref. 20, our prediction is too high in the leading-particle-peak region near $x\approx 1$. Further information would help to separate the dominant single excitation from the many other possible mechanisms leading to similar proton fragmentation.

The pseudoscaling phenomenon is shown in Fig. 14. In Fig. 14(a), the nova distribution for $\pi^+p(\pi^-)$ at 8 and 16 GeV/c is compared with the data,[2] and in Fig. 14(b), $\pi^-p(\pi^-)$ at 18 and 24 GeV/c is computed and compared with Ref. 3. It is seen that the remarks of Sec. II are borne out by explicit calculation. The pseudoscaling effect, which occurs in reactions with a leading particle (i.e., those in which the lowest-mass novas, or 2-prong events, contribute), such as $\pi^-p(\pi^-)$, is clearly present. The breakdown of this apparent scaling is discussed below. See Fig. 15.

The nova predictions for $\pi^+p(\pi^-)$ and $\pi^-p(\pi^+)$ distributions fall off much faster than the data for $x\gtrsim 0.5$, after the distributions have already fallen

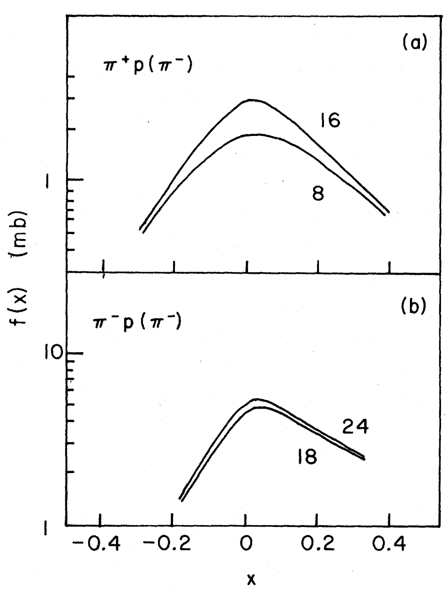

FIG. 14. Examples of selective pseudoscaling in $\pi^+p(\pi^-)$ (no pseudoscaling) and $\pi^-p(\pi^-)$ (pseudoscaling): (a) The $\pi^+p(\pi^-)$ distribution does not pseudoscale around $x=0$ between 8 and 16 GeV/c in the model. This is due to the suppression of lower-mass novas (2-prong events). The distribution does scale for $x>0.2$. (b) The $\pi^-p(\pi^-)$ distribution does seem to scale around $x=0$ between 18 and 24 GeV/c. However, the model predicts the value of $f(x)$ at $x=0$ to almost double between 24 and 200 GeV/c. Only at much higher energies does $f(x)$ scale around $x=0$.

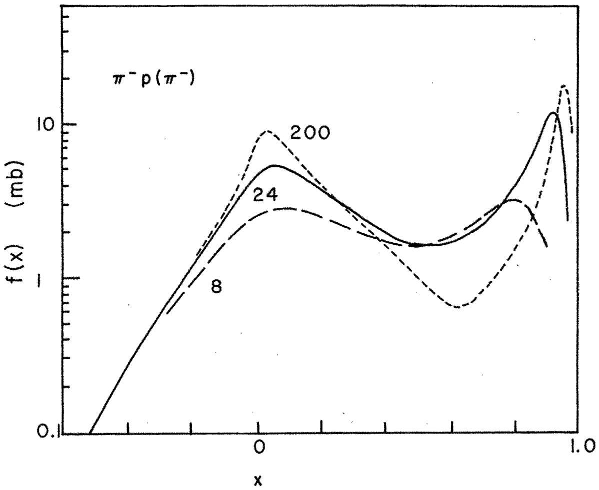

FIG. 15. Behavior of $\pi^- p(\pi^-)$ distribution as a function of energy. No correction for quantum-number exchange has been made. This could greatly modify the 8-GeV/c curve. The model calculations for 24 and 200 GeV/c are also given. Distributions for any fixed value of q_T^2 can be easily obtained from the formula and parameters given in the text.

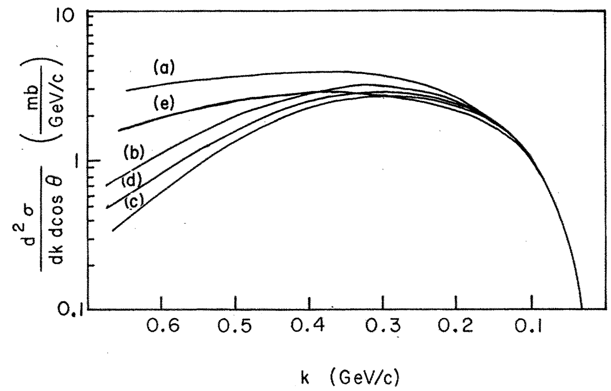

FIG. 16. Isotropy of the differential cross section in the reaction center-of-mass frame for $\pi^+ p(\pi^-)$ at 16 GeV/c. Curve (a) is the integral of $d^2\sigma/dk\, d\cos\theta_{c.m.}$ over $\cos\theta_{c.m.}$ from $\cos\theta_{c.m.}=1.0$ to $\cos\theta_{c.m.}=0.6$, (b) from 0.6 to 0.2, (c) from 0.2 to −0.2, (d) from −0.2 to −0.6, and (e) from −0.6 to −1.0. This is the same kind of behavior noted by Ko and Lander in $K^- p(\pi)$ distributions.[56a]

by about an order of magnitude. It is easy to associate these pions with a quantum-number-exchange effect ignored in our calculations. A short kinematical calculation shows that ρ mesons produced with large center-of-mass momentum can easily provide these pions. (The π distribution from longitudinally polarized ρ's peaks at large and small x when it peaks at intermediate $|x|$ values for transversely polarized ρ's.[45]) Quantum-number exchange can also be used to obtain better agreement in the proton fragmentation, as discussed in Sec. III. Nevertheless, diffraction still gives the leading contribution to the total cross section. The nova model, with further emphasis on single diffractive excitation, is able to reproduce the key features of these distributions. The obvious corrections agree in sign, and a semiquantitative analysis provides even better agreement with experiment.

Figure 15 shows the characteristic approach to scaling for $\pi^- p(\pi^-)$ at 8, 16, and 200 GeV/c. This is similar to the behavior observed for $pp(\pi)$ around $x=0$, but in $\pi^- p(\pi^-)$ we also predict the sharpening of the leading-particle peak, with the valley between the peaks deepening as $s^{-1/2}$ until it reaches the nova contribution or that of secondary effects. The latter effect appears present in the 8- and 16-GeV/c $\pi^+ p(\pi^+)$ data of Ref. 2, which nicely fit the decrease shown in Fig. 15.

It has recently been noted that the center-of-mass π distributions are isotropic in $\cos\theta$.[64] Although a thermodynamic role has been attributed to this behavior, we note that the nova model, with its highly correlated decay mechanism, also leads to such isotropy. Typically, the π distributions are isotropic out to 350 MeV/c. In our model this effect is, however, strongly connected to the relatively slow motion of novas, a feature which could be disputable as the energy increases. More data on the quasielastic peak would be very valuable, but will be difficult to collect. A model calculation for $\pi^+ p(\pi^-)$ at 16 GeV/c is shown in Fig. 16, as an example.

The normalization of the leading-particle peak provides a clean test of single-nova dominance. A precise determination of this peak in $\pi p(\pi)$ collisions would also allow access to the proton-nova distribution, and to the other excitation mechanisms, since varying K and N gives little flexibility in connecting the normalization of the proton fragmentation to the leading-particle peak. On the other hand, the behavior of the peak is quite sensitive to the exact form of $\rho(M)$.

Such effects are more easily analyzed in reactions like $Kp(p)$ or $\pi p(p)$, by looking at quasielastic p's which are slow (<1500 MeV/c) in the lab. We have compared the distribution of recoil protons in 12.7-GeV/c $K^- p$ collisions made available to us by the Yale bubble-chamber group[66] and find the satisfactory agreement in shape shown in Fig. 8. Further information on these distributions would be very interesting, since they provide a sensitive measure of secondary effects.

With this in mind, we present the $pp(p)$ distributions. Here we use Eq. (12), integrating over the momentum transfer to the proton nova. This is necessary because of the large mass of the proton. These distributions show a slow approach to scal-

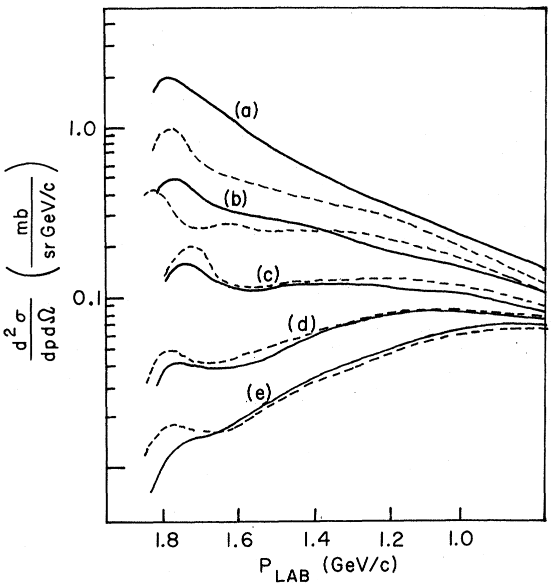

FIG. 17. The distributions for $pp(p)$ at 19 GeV/c. The dashed lines are the 12.5-, 20-, 30-, 40-, and 50-mrad data of Ref. 6. The model calculation is the average value of $d^2\sigma/dp\,d\Omega$ over the ranges: (a) 0 to 10 mrad; (b) 10 to 20 mrad; (c) 20 to 30 mrad; (d) 30 to 40 mrad; and (e) 40 to 50 mrad. It shows that the model correctly reproduces the trend of the data.

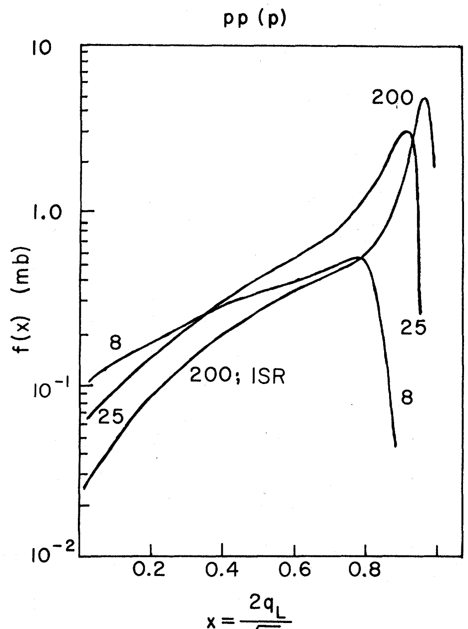

FIG. 18. Invariant distribution for $pp(p)$ as a function of energy. The slow scaling is due to the strength of the leading-proton peak. Because of the large proton mass, secondary effects at low energies and double, or more intricate, excitations at higher energies could change the shape of these distributions in a noticeable way. For intermediate values of x ($0.2 < x < 0.6$) the 200-GeV and ISR-energy predictions almost coincide. Distribution for any fixed value of $q_T{}^2$ can be obtained in terms of the formula and parameters given in the text.

ing because of the overwhelming leading-particle effect. As an example, we compare the nova distribution at 19 GeV/c with the results of Ref. 6 in Fig. 17. In Fig. 18 we show the invariant distributions at 10, 25, and 200 GeV/c.

The emphasis which we put on single-nova production leads to obvious constraints for exclusive distributions. We expect that the prominent contribution, at least for relatively low nova mass or relatively low multiplicity, should come from events where one particle is a leading particle and there is a cluster of secondaries which are expected to come from the nova. These should generally have momenta of order K in the nova rest frame, and the invariant mass of the cluster should be relatively small. Large invariant masses should be associated with the leading particle unless associated with nonoverlapping sets of particles, which would indicate multiple excitation. This behavior should help to separate single-nova production from other production processes. As previously emphasized, single diffractive excitation seems to dominate for the 4-particle final states which have been analyzed, but further information is, of course, needed.

In conclusion, we again note that distributions at intermediate $|x|$ mainly depend on low- and medium-energy features. Scaling is expected to be approached relatively quickly and the observed features appear to be understood. It would be interesting to study in more detail quantum-number exchange at energies less than 20 GeV but the detailed phenomenology is intricate. The most interesting aspects of the distributions at high energies should be looked for at $x \approx 0$ and $|x| \approx 1$ and, in particular, at low center-of-mass rapidity. The diffractive excitation of either the beam or target particle appears to be an important process which alone reproduces many features of the observed inclusive distribution. It may therefore be used as a guide in the analysis of many-particle phenomena. Having a leading process should greatly simplify the study of secondary effects and has led us to an understanding of many reported, but hitherto puzzling, phenomena.[9]

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It is a pleasure to thank our colleagues at Yale and in particular R. K. Adair, L. Ehrlich, J. Harte, T. Ludlam, P. Ramond, J. Rawls, and C. C. Wu for many stimulating conversations. One of us (M.J.) would like to thank the Yale Physics Department for its hospitality.

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[2]Aachen, Berlin, Bonn, CERN, Cracow, Heidelberg, and Warsaw Collaboration, in Proceedings of the Amsterdam Conference, 1971 (unpublished).

[3]W. D. Shephard *et al.*, Phys. Rev. Letters 27, 1164 (1971); 28, 260 (1972).

[4]W. H. Sims, R. S. Panvini, R. R. Kinsey, and T. W. Morris, Phys. Letters 38B, 55 (1972). We are grateful to Dr. R. S. Panvini for very stimulating discussions and information about his most recent data.

[5]N. N. Biswas *et al.*, Phys. Rev. Letters 26, 1589 (1971).

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[7]L. G. Ratner, R. J. Ellis, G. Vannini, B. P. Babcock, A. D. Krisch, and J. B. Roberts, Phys. Rev. Letters 27, 68 (1971).

[8]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969). $x = 2k_L/\sqrt{s}$ where k_L is the longitudinal component of the center-of-mass momentum.

[9]L. Van Hove, Phys. Reports 1C, 347 (1971); D. Horn, *ibid.* (to be published); W. Frazer *et al.*, Rev. Mod. Phys. (to be published); S. Gasiorowicz, in Erice lecture notes, 1971 (unpublished); E. Berger, in Proceedings of the Helsinki Conference, 1971 (unpublished); M. Jacob, in Les Houches lecture notes, 1971 (unpublished).

[10]J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

[11]N. F. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Rev. Letters 25, 557 (1970).

[12]We systematically write $AB(C)$ to refer to a reaction in which beam particle A collides with target B, giving particle C together with anything else.

[13]D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962); S. Fubini, in *Proceedings of the Fourth Scottish Universities Summer School, Edinburgh*, edited by R. G. Moorhouse (Oliver and Boyd, Edinburgh, Scotland, 1964).

[14]F. Zachariasen and G. Zweig, Phys. Rev. 160, 1322 (1967); 160, 1326 (1967).

[15]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[16]L. W. Jones *et al.*, Phys. Rev. Letters 25, 1679 (1970).

[17]H. Boggild, E. Dahl Jansen, M. Gavilas, K. H. Hansen, and H. Johnstad, in Proceedings of the Amsterdam Conference, 1971 (unpublished).

[18]M. Bishari, D. Horn, and S. Nussinov, Nucl. Phys. B36, 109 (1972).

[18a]It should be mentioned that asymptotic diffractive models have been a major approach to high-energy scattering for many years, started by the original work of M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960). Later versions of this model have been given by many people; here we only mentioned R. K. Adair, Phys. Rev. 172, 1370 (1968); J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, *ibid.* 188, 2159 (1969); and R. C. Hwa and C. S. Lam, Phys. Rev. Letters 27, 1098 (1971). These models have all emphasized an asymptotic validity for the diffraction picture. Our work on the nova model is directed toward identifying those specific features of diffraction dissociation models which are pertinent for directly understanding the presently available inclusive data, and to give an explicit and detailed analysis of the data.

[19]K. Wilson, Cornell report, 1971 (unpublished).

[20]M. Jacob and R. Slansky, Phys. Letters 34B, 408 (1971).

[20a]The importance of single excitation has also been recognized by others. See R. F. Panvini, in *High Energy Collisions*, Third International Conference held at State University of New York, Stony Brook, 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1970), p. 461.

[21]H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Letters 26, 937 (1971); J. M. Wang and L. L. Wang, *ibid.* 26, 1287 (1971); P. D. Ting and J. H. Yesian, Phys. Letters 35B, 321 (1971); H. D. I. Abarbanel, Princeton report, 1971 (unpublished).

[22]The hadron states produced through diffraction are dual to the leading secondary trajectories and should therefore be described as resonances. Their decay patterns should then show strong correlations. This general duality result has been emphasized by R. C. Hwa and C. S. Lam, Phys. Rev. Letters 27, 1098 (1971).

[23]We are, therefore, far from a thermodynamic equilibrium. Nevertheless, many features of the statistical treatment of the decay are similar to those obtained in the thermodynamical model: R. Hagedorn, Suppl. Nuovo Cimento 3, 147 (1965); 6, 311 (1968). We are grateful to Dr. J. Harte and Dr. J. Rawls for discussions of this point.

[24]It should be emphasized that it is the cross section for production of a nova which is here taken as constant and not the total cross section. An increase of incident energy allows more (heavier) novas to be produced and the diffractive contribution to the total cross section increases to its asymptotic value.

[25]The possible similarity of the Pomeranchon and P' coupling could enforce this at even lower energies.

[26]The key point in reproducing the observed inclusive distributions is that the nova excitation spectrum strongly peaks at a relatively low mass and that the step-by-step resonance decay controls the average energy of the secondaries. Our calculation of the distribution may therefore still apply for medium-energy reactions where a nova is produced together with a vector meson (vector meson or pion exchange), even though our exclusive picture of the reaction would not be correct.

[27]E. W. Anderson *et al.*, Phys. Rev. Letters 16, 855 (1966); 25, 699 (1970); K. T. Foley *et al.*, *ibid.* 19, 397 (1967); U. Amaldi *et al.*, Phys. Letters 34B, 435 (1971).

[28]F. Zachariasen, Phys. Reports 2C, 1 (1971).

[29]R. F. Panvini, in *High Energy Collisions*, (Ref. 20a), p. 461.

[30]W. Kittel, S. Ratti, and L. Van Hove, CERN Report No. 70-19, 1970 (unpublished); J. Beaupre *et al.*, CERN Report No. 71-30, 1971 (unpublished); Scandinavian Collaboration, Phys. Letters (to be published).

[31]The cleanest test for the importance of the leading-particle effect, interpreted here as single-nova excitation, should be obtained from the proton recoil spectrum.

Information on such distributions should be extremely useful. A large fraction of the final protons should be in a low-energy peak with maximum around 400 MeV/c. We are grateful to Professor R. K. Adair for discussions of this point.

[32]R. S. Hwa and C. S. Lam, Phys. Rev. Letters 27, 1098 (1971).

[33]When considering the highest kinematically possible excitation, it is practically irrelevant to distinguish between single- and double-nova excitation. Both pictures merge if the diffractive cross section is not required to factorize.

[34]This merely translates the fact that the diffraction excitation spectrum is allowed to extend in order to give an increasing multiplicity.

[35]R. K. Adair, Brookhaven-Yale report, 1971 (unpublished), and Phys. Rev. 172, 1370 (1968).

[36]Reactions involving strange particles and, in particular, hyperon production will be discussed separately.

[37]C. E. DeTar *et al.*, Phys. Rev. Letters 26, 675 (1971); P. Chliapnikov *et al.*, Phys. Letters 36B, 235 (1971); H. Abarbanel, G. Chew, M. Goldberger, and L. Saunders, Phys. Rev. Letters 26, 937 (1971).

[38]We phenomenologically describe diffraction as due to Pomeranchon exchange with a zero-slope trajectory. Taking the Pomeranchon small effective slope into account we are led to a logarithmic decrease of each nova cross section. This is an important feature which is, however, not relevant when discussing distributions at present machine energies.

[39]E. W. Anderson *et al.*, Phys. Rev. Letters 16, 855 (1966); 25, 699 (1970).

[40]Although this form seems to fall off rather slowly for intermediate nova mass, the multiplicity prediction is consistent with the data if we assume a nova decays into 2.5 pions per GeV of excitation.

[41]S. P. Denisov *et al.*, Phys. Letters 36B, 528 (1971).

[42]If the Freund-Harari duality argument applies to K^+p scattering, its extension to pp scattering has to meet all the difficulties encountered by a simple duality argument when applied to the two-baryon case.

[43]This rise of the diffractive contribution to the total cross section makes it conceivable that the Kp and πp total cross sections become equal at infinite energies. Extrapolation of the present data is consistent with such a conjecture, although one would then expect sizable nondiffractive contributions to K^+p scattering below 20 GeV/c.

[44]In certain cases, however, such as photoproduction of pions, the polarization of the ρ and ω should be closely followed. Transversely and longitudinally polarized vector mesons give very different inclusive distributions. This is discussed in detail in Ref. 45.

[45]M. Jacob, R. Slansky, and C. C. Wu, Phys. Letters 38B, 85 (1972).

[46]A sum of 2 Gaussians such as used in Ref. 4 could account phenomenologically for the long jump decays. Such refinements will be ignored in this paper. See Ref. 49.

[47]E. Yen and E. Berger, Phys. Rev. Letters 24, 695 (1970). We thank E. Berger for comments on this work.

[48]Similar decay distributions can be derived from bootstrap models. J. Harte, J. Rawls, and H. C. Yen, Yale report, 1972 (unpublished).

[49]The q_T^2 distribution is better described by the sum of two exponentials rather than by a single one. To the dominant large-slope term (9), which is the only one considered here, one should add a smaller-slope term (2.5), which we would associate with such long jump decays. To the extent that the x and q_T^2 distributions very roughly factorize at intermediate $|x|$ values, we can systematically neglect this effect when discussing inclusive distributions at 10% accuracy level.

[50]As reported by C. Quigg in *Particles and Fields-1971*, proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Mellissinos and P. F. Slattery (A.I.P., New York, 1971).

[51]The nova gives an average number of decay pions of well-defined average energy in its rest frame which is proportional to the total excitation energy. Such a conservation of energy "on the average" eliminates long Monte Carlo-type calculations and is sufficient for the calculation of inclusive distributions.

[52]Threshold for ΣK decay is such that centrifugal-barrier effects are important for reproducing the details of the inclusive Σ spectrum near its maximum edge as it sharply dips for $|x| > 0.7$. Since the heaviest nova produced in a 20–30-GeV collision will hardly give more than one single Σ (as opposed to many pions), the inclusive distribution should be almost flat in the interval $0 < |x| < 0.7$.

[53]We are grateful to J. Vitale for his advice concerning the program.

[54]Since the average secondary rapidity is the same as the the nova rapidity, the inclusive distribution will approximately factorize in terms of rapidity. It will, then, not factorize in terms of x at low $|x|$ values. It will peak less in x as q_T^2 increases.

[55]H. Boggild *et al.*, in Proceedings of the Amsterdam Conference, 1971 (unpublished).

[56]Reproducing the rapidity behavior at very high energy around zero center-of-mass rapidity is also outside the scope of the model. To the extent, again, that the multiplicity increases logarithmically we expect it to be flat. One has at present to turn to experiment to learn more. See Bombay, CERN, Cracow Collaboration, Phys. Letters 36B, 611 (1971).

[56a]W. Ko and R. L. Lander, Phys. Rev. Letters 26, 1284 (1971).

[57]This is given as a mere illustration since one has to give up factorization in order to merge the single-excitation picture into a more complicated one at large energy and still keep scaling at intermediate $|x|$ values. With factorization, inclusive distributions in a specific fragmentation region should scale according to the square of the elastic cross section. For instance, $f[pp(\pi)]/f[\pi p(\pi)] = (\sigma_{pp}^{el}/\sigma_{\pi p}^{el})^2$ in the proton fragmentation region. This is not incompatible with present results (with Pomeranchon approximation of the absorptive part, one would expect scaling according to the total cross section). We, however, do not wish to insist on factorization in diffraction excitation. Furthermore, as discussed in Sec. III, these values include some double excitation. This approach is possible because double excitation does not change appreciably the distribution outside the leading particle effect.

[57a]After this manuscript was submitted, we learned of new data from the ISR showing that the ratio $pp(\pi^+)$ to $pp(\pi^-)$ continues to be about 2.0 ± 0.5 for $0.2 < x < 0.3$ and

500 and 1500 GeV/c [A. Bertin *et al.*, Phys. Letters 38B, 260 (1972)]. This strongly supports the fragmentation model.

[58]P. Franzini, in *High Energy Collisions* (Ref. 20a); M. Miesowicz, in *Proceedings of the Sixth Interamerican Seminar on Cosmic Rays, La Paz, Bolivia, 1970* (Universidad Mayor de San Andrés, La Paz, 1970), Vol. III, p. 564.

[59]Chan Hong-Mo *et al.* Phys. Rev. Letters 26, 672 (1971); J. Ellis *et al.*, Phys. Letters 35B, 227 (1971); S.-H. H. Tye and G. Veneziano, *ibid.* 38B, 30 (1972).

[60]This holds at least when the ρ meson is produced with zero helicity.

[61]We are grateful to Dr. J. Harte for pointing out this effect.

[62]It is a pleasure to thank Dr. C. DeTar for an enlightening discussion of this point.

[63]J. Finkelstein and K. Kajantie, Nuovo Cimento 56, 659 (1968).

[64]J. Erwin *et al.*, Phys. Rev. Letters 27, 1534 (1971).

[65]A. Pignotti and P. Ripa, Phys. Rev. Letters 27, 1538 (1971).

[66]We are grateful to Dr. Ludlam for many stimulating discussions and, in particular, for access to his data on kaon-induced reactions.