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Restrictions on Two-Channel Separable ΛN Potentials*

P. S. Damle and L. H. Schick
University of Wyoming, Laramie, Wyoming
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A previously suggested method for representing a meson-theoretic ΛN potential by a two-channel separable potential is shown to limit the shape of the separable Λ-channel potential for arbitrary shape of the separable Σ-channel potential.

In the preceding paper[1] it was proposed that, in view of the apparent importance of suppression of Λ-Σ conversion when a Λ is bound in a multinucleon system,[2,3] for such problems a simple physical way to represent a given ΛN meson-theoretic potential (MTP) by a two-channel nonlocal separable (NLS) potential was to choose the NLS potential to (a) have the same ΛN scattering length and effective range as those of the given MTP, (b) to have the same ΛN scattering length and effective range as those of the given MTP when in both potentials the Σ channel is fully suppressed, and (c) to have the same resonances below the Σ-channel threshold as predicted by the given MTP; e.g., for the $^1S_0$ ΛN potential there should be no such resonance, and in particular there should be no bound state in the uncoupled Σ channel to cause such a resonance.

In Ref. 1, as here, attention was focused on the $^1S_0$ interaction so that the complications of a tensor force were avoided. The very common form of NLS two-channel ΛN potential

$$V=\begin{pmatrix} \lambda_\Lambda U_{\Lambda\Lambda} & \epsilon\lambda_x U_{\Lambda\Sigma} \\ \epsilon\lambda_x U_{\Sigma\Lambda} & \lambda_\Sigma U_{\Sigma\Sigma}\end{pmatrix}, \tag{1a}$$

where in momentum space ($c=\hbar=1$)

$$\langle \vec{k}_x | U_{xy} | \vec{k}_y\rangle = v_x(k_x)v_y(k_y), \tag{1b}$$

$x, y=\Lambda$ or Σ, and ϵ is the suppression parameter (i.e., $0\leqslant\epsilon\leqslant 1$, $\epsilon=1$ yields the completely unsuppressed, free-particle, hyperon-nucleon interaction, and $\epsilon=0$ yields this interaction with the Σ channel fully suppressed), was chosen for investigation. It was then shown that the conditions (a), (b), and (c) imposed a limitation on the possible shapes for v_Λ and v_Σ. In particular it was shown that with Yamaguchi[4] shapes for v_Λ and v_Σ, the use of conditions (a) and (b), and the use of the MTP of Brown, Downs, and Iddings (BDI),[5] condition (c) was violated; i.e., such an NLS representation of the BDI potential as given by Eqs. (1) and conditions (a), (b), and (c) was impossible with Yamaguchi shapes for v_Λ and v_Σ. In this note we wish to present results of a much more general nature.

The limitation on the shapes v_Λ and v_Σ obtained in Ref. 1 could be written as an inequality, $\xi>\eta$. Here ξ is a dimensionless parameter which depends only on k_0 (the value of k_Λ at the Σ-channel threshold), on the ΛN scattering length a_ϵ and effective range $r_{0\epsilon}$ at the values $\epsilon=0$ and $\epsilon=1$, and on the potential shape v_Λ, while η depends only on the shape v_Σ and on k_0. Explicitly

$$\xi=\frac{(\alpha_1+A_0)(\alpha_1-\alpha_0)}{k_0^2(\alpha_0+A_0)[\frac{1}{2}(r_{00}-r_{01})+u_r(\alpha_1-\alpha_0)]}, \tag{2}$$

where $\alpha_j\equiv 1/a_j$, u_r is defined by

$$v_\Lambda^2(k_\Lambda)\equiv u_0[1+u_r k_\Lambda^2+O(k_\Lambda^4)+\cdots],$$

and A_0 is defined by

$$-g^p/v_\Lambda^2(k_\Lambda)\equiv A_0+A_2k_\Lambda^2+O(k_\Lambda^4)+\cdots,$$

with

$$g^p=\frac{1}{\pi}P\int_{-\infty}^{\infty}(q^2-k_\Lambda^2)^{-1}v_\Lambda^2(q)q^2dq.$$

On the other hand, $\eta=I_1I_2/I_3I_4$, where

$$I_j=\frac{2}{\pi}\int_0^\infty v_\Sigma^2(q)R_j dq,\quad j=1,\ldots,4$$

with $R_1 = x^2/(q^2+x^2)$, $R_2 = q^2/(q^2+x^2)$, $R_3 = 1$, $R_4 = q^2x^2/(q^2+x^2)^2$, $x = rk_0$, and r is the square root of the ratio of the Σ-channel reduced mass to the Λ-channel reduced mass. In addition, condition (b) requires

$$(A_2 - \tfrac{1}{2}r_{00}) + u_r(\alpha_0 + A_0) = 0\,. \tag{3}$$

We wish to report here that we have been able to show that $\eta > 1$ *for all* v_Σ such that the integrals I_j exist. Thus, the ability of the form of NLS potential given in Eqs. (1) to represent an MTP in the sense of conditions (a), (b), and (c) may be tested directly once a shape is chosen for v_Λ, independent of the shape of v_Σ; i.e., any shape $v_\Lambda(k_\Lambda)$ that yields $0 < \xi < 1$ is unacceptable. On the other hand, for a given a_1 and r_{01} as well as a given v_Λ, Eqs. (2) and (3) may be used to plot out the region $0 < \xi < 1$ in the a_0-r_{00} plane. This is the region for which an MTP with the given values of a_1 and r_{01} cannot be represented with this particular shape of NLS potential.

To prove $\eta > 1$ we wrote each of the products I_1I_2 and I_3I_4 as a double integral over the first quadrant in the qq' plane. We than changed integration variable to polar coordinates r and θ ($q = r\cos\theta$, $q' = r\sin\theta$) so that each double integration ran over $0 \le r \le \infty$ and $0 \le \theta \le \frac{1}{2}\pi$. The θ interval was broken into the range $0 \to \frac{1}{4}\pi$ and $\frac{1}{4}\pi \to \frac{1}{2}\pi$. For the higher range the transformation $\theta \to \frac{1}{2}\pi - \theta$ was made and the integrals over θ were recombined. At this point the result $\eta > 1$ became manifest.

In Fig. 1 we give an example of the use of the restriction $0 < \xi < 1$. We have chosen from the recent literature three different pairs of values for the ${}^1S_0$ ΛN scattering length and effective range ($a_1 = -1.70$ F, $r_{01} = 2.50$ F),[6] ($a_1 = -1.6523$ F, $r_{01} = 3.1717$ F),[2] and ($a_1 = -2.25$ F, $r_{01} = 3.47$ F).[5] For each of these we have assumed a Yamaguchi shape for v_Λ. Using Eqs. (2) and (3) we have found in the a_0-r_{00} plane the curves $\xi = 1$ for values of $a_0 > a_1$.[7] These have been plotted in Fig. 1. If a ${}^1S_0$ ΛN potential adjusted to fit one of the sets (a_1, r_{01}) given above predicts values of a_0 and r_{00} that lie *below* the corresponding $\xi = 1$ curve (i.e., in the region $0 < \xi < 1$) when the Σ channel is uncoupled (i.e., when $\epsilon = 0$), the NLS form given in Eq. (1) with a Yamaguchi shape for v_Λ cannot be used to represent this MTP in the sense of conditions (a), (b), and (c).

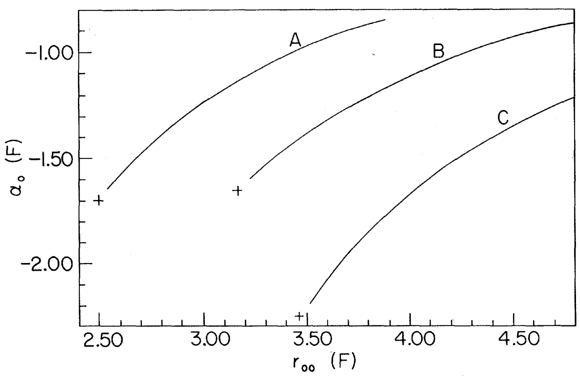

FIG. 1. The $\xi = 1$ curve for each of three sets of values of a_1 and r_{01} (each of which is denoted by +) plotted for $a_0 > a_1$. For curve A, $a_1 = -1.70$ F, $r_{01} = 2.50$ F. For curve B, $a_1 = -1.6523$ F, $r_{01} = 3.1717$ F. For Curve C, $a_1 = -2.25$ F, $r_{01} = 3.47$ F. The region $0 < \xi < 1$ lies below the corresponding $\xi = 1$ curve.

In those cases where an NLS representation of a given MTP fails, as it did in Ref. 1, we suspect the difficulty lies in the fact that all MTP's are cut off at very short range by some sort of strong repulsive core. The form given by Eq. (1) for the ΛN potential when $\epsilon = 0$, as is well known, cannot represent such a local potential whose phase shift changes sign.

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