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Two-Channel Separable ΛN Potentials *†

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An attempt is made to use a two-channel YN potential with a single nonlocal separable (NLS) form in each of the four potential matrix elements to represent the low-energy S-wave ΛN interaction. For each spin state equations are developed for expressing the Λ-channel, Σ-channel, and cross-channel potential strengths in terms of the values of the ΛN scattering length and effective range both for full and for no suppression of the Σ channel with general forms for the NLS potential shapes. The condition that there be no resonance due to a bound state in the uncoupled Σ channel is developed. The combined results are proposed as a method for determining which combinations of Λ-channel and Σ-channel NLS potential shapes may be eliminated as not capable of representing a meson-theoretic potential model of the low-energy ΛN interaction. Application of the results is made to the meson-theoretic potential of Brown, Downs, and Iddings. With Yamaguchi forms for both NLS shapes a ${}^1S_0$ resonance below the Σ-channel threshold due to an uncoupled Σ-channel bound state is always obtained.

I. INTRODUCTION

Recently Satoh and Nogami[1] have constructed nonlocal separable (NLS) potentials for the low-energy, S-wave, two-channel interaction $(\Lambda N, \Sigma N) \rightarrow (\Lambda N, \Sigma N)$. The potential parameters in this model were chosen to give the Λ-nucleon (i.e., the Λ channel) scattering length and effective range, the position of the ${}^3S_1$ ΛN resonance, and, further, to give agreement with low-energy $\Sigma N \rightarrow \Sigma N$ (i.e., Σ channel) and $\Sigma N \rightarrow \Lambda N$ scattering data. The resulting potentials were then used in a calculation of the binding energy of the Λ in nuclear matter. In this paper we present an alternative method of determining the parameters of this kind of potential which we believe is more relevant for use in problems where the hyperon-nucleon interaction at energies near the ΛN threshold is required. In addition this method may provide a convenient tool for judging which NLS potential shapes may actually be used to represent the hyperon-nucleon interaction.

The motivation for the SN work was to estimate the effect of Σ-channel suppression in a many-nucleon hypernuclear system. In hypernuclear systems the $\Lambda N \rightarrow \Sigma N$ transition may be at least partially suppressed because (for example) the final states that conserve isospin are not as available as they are for the free hyperon-nucleon interaction.[2,3] In other words, if, following Brown, Downs, and Iddings,[4] we write the 2×2 coupled-channel potential V as

$$V=\begin{pmatrix} V_{\Lambda\Lambda} & \epsilon V_{\Lambda\Sigma} \\ \epsilon V_{\Sigma\Lambda} & V_{\Sigma\Sigma} \end{pmatrix}, \tag{1}$$

where V_{XY} represents the interaction $YN \rightarrow XN$, then for the free hyperon-nucleon interaction $\epsilon = 1$, for this same interaction with the Σ channel partially suppressed $0 < \epsilon < 1$, and $\epsilon = 0$ when the Σ channel is fully suppressed. If Σ-channel suppression in hypernuclei is important – and a number of works indicate that it is[1,2] – we should construct an NLS potential which not only reproduces the correct ΛN scattering parameters when $\epsilon = 1$, but which also reproduces at least some ΛN scattering parameters when $\epsilon < 1$. This $\epsilon < 1$ information cannot, of course, be obtained directly from two-body experiments. It can be obtained from a meson-theoretic potential (MTP) which has had its parameters adjusted to fit the two-body experimental data.

Our basic procedure in this paper then is to attempt to construct low-energy NLS ΛN potentials (a) whose parameters have been adjusted to fit the ΛN scattering lengths and effective ranges at $\epsilon = 0$ and $\epsilon = 1$, (b) which include the presence of the (closed) Σ channel, and (c) which incorporate the existence (or nonexistence) of resonances between the ΛN and ΣN thresholds. We take the input information implied in (a) and (c) from BDI who, to our knowledge, are the only authors to publish an MTP with ΛN scattering lengths (a) and effective ranges (r_0) given for values of ϵ other than $\epsilon = 1$.

The choice of fitting a and r_0 at $\epsilon = 0$ as a measure of the Σ-channel suppression was made partly on the grounds of simplicity and partly on the grounds of treating the ΛN amplitude with the Σ channel suppressed in the same manner as it is treated with the Σ channel unsuppressed. For problems such as that of calculating the binding energy of the Λ in nuclear matter it might make more sense to use some property of the $\epsilon < 1$ NLS ΛN amplitude at a large negative (≈ -30 MeV) energy as the property to be fit to the corresponding MTP value. This type of criticism applies much more strongly to NLS potentials that have been adjusted to fit hyperon-nucleon data at energies $\geqslant 80$ MeV above the ΛN threshold such as was done previously. It might also be argued that it would be more sensible to fit the ΛN amplitude with the Σ channel partially suppressed rather than with the Σ channel fully suppressed. Here we again plead simplicity and offer the additional rebuttal that in the cases of greatest concern detailed below our results show that our exact fit of a and r_0 at $\epsilon = 0, 1$ yields a fit to within 0.2% at $0 < \epsilon < 1$.

Condition (b) above is taken into account by the use of a two-channel formalism.

Our inclusion of condition (c) is based on the fact that, if the low-energy ΛN amplitude is sensitive to the presence of the Σ-channel threshold at ≈ 78 MeV above the ΛN threshold, it should be sensitive to the presence (or absence) of other singularities near the physical region at lesser energies. Condition (c) does not, of course, guarantee that our NLS potential will yield the same ΛN phase shift as a given MTP for all energies below the Σ-channel threshold, but if our NLS potential predicts a resonance where the MTP does not, then such a phase-shift match would be impossible. This condition means that we demand our NLS potential yield a $^3S_1$ resonance at about 75 MeV above the ΛN threshold and no $^1S_0$ resonance between the ΛN and ΣN thresholds. However, the MTP used by BDI included a tensor force while we are going to investigate only central NLS forms. Therefore, we place more emphasis on the application of our results to the $^1S_0$ rather than the $^3S_1$ ΛN interaction.

So much for the buildup. The letdown is, our attempt failed. With the same general form of NLS potential as was used by SN, the application of conditions (a) and (b), and the use of the BDI values for a and r_0 at $\epsilon = 0, 1$, we always obtained a $^1S_0$ resonance. The analysis of the equations involved in this failure is what makes the attempt worthwhile.

In Sec. II we outline the derivation of, and give the results for obtaining the strengths of the various $XN \to YN$ potentials in terms of the shapes assumed for these potentials and the values of $a \equiv a_\epsilon$ and $r_0 \equiv r_{0\epsilon}$ for $\epsilon = 0, 1$. The equations for a_ϵ and $r_{0\epsilon}$ for other values of ϵ in terms of these end-point values are also given as is the condition for the existence of a resonance below the ΣN threshold. These relations are discussed in some detail. The general relation that must hold among the potential parameters if there is to be no resonance due to a bound state in the uncoupled Σ channel is also obtained.

In Sec. III we use Yamaguchi[5] shapes for the NLS potential in each channel. The BDI values of a_0, r_{00}, a_1, and r_{01} for both spin channels are used in our formalisms and the numerical results for the other parameters are obtained. The SN values for the potential parameters are used to calculate a_0 and r_{00} and comparison with the BDI results is made. We then vary some of the BDI and the SN parameters for the $^1S_0$ potential and note the effect on the $^1S_0$ resonance in the light of the discussion of Sec. II. Finally we suggest further lines of attack on this problem.

II. NLS YN POTENTIAL

The potential under investigation has the form given in Eq. (1) with the relative YN momentum-space representation

$$\langle \vec{k}_X | V_{XY} | \vec{k}_Y' \rangle = \lambda_{XY} v_X(k_X) v_Y(k_Y') , \qquad (2)$$

where k_X (k_Y') is the relative hyperon-nucleon wave vector for hyperon channel X (Y). If then E_X is the center-of-mass energy in the X channel and $\Delta = M_\Sigma - M_\Lambda$, where M_X is the mass of hyperon X, we have $k_\Lambda^2 = 2\mu_\Lambda E_\Lambda$, $k_\Sigma^2 = r^2(k_\Lambda^2 - k_0^2)$, with $k_0^2 = 2\mu_\Lambda \Delta$, $r^2 = (\mu_\Sigma / \mu_\Lambda)$, and μ_X is the reduced mass in channel X. We limit our discussion to physical energies below (or at) the Σ-channel threshold so that $k_\Sigma^2 \leqslant 0$ and $k_\Lambda \leqslant k_0$. We take the nucleon mass to be 938.9 MeV and use the values $M_\Lambda = 1115.4$ MeV and $M_\Sigma = 1193$ MeV. Then $k_0 = 281.28$ MeV/c. For convenience we define the strength parameters $\lambda_\Lambda = \mu_\Lambda \lambda_{\Lambda\Lambda}/2\pi$, $\lambda_\Sigma = \mu_\Sigma \lambda_{\Sigma\Sigma}/2\pi$, and $\lambda_X^2 = \mu_\Lambda \mu_\Sigma \lambda_{\Lambda\Sigma}/4\pi^2$, where $\lambda_{YX} = \lambda_{XY}$.

After using the potential given in Eq. (2) in the coupled-channel Lippmann-Schwinger equation for a given spin the resulting ΛN S-wave scattering amplitude $f_{\Lambda\Lambda}$ may be written

$$f_{\Lambda\Lambda} = -\mu_\Lambda \langle \vec{k}_\Lambda | t_{\Lambda\Lambda} | \vec{k}_\Lambda' \rangle / 2\pi = v_\Lambda(k_\Lambda) \tau_{\Lambda\Lambda} v_\Lambda(k_\Lambda) ,$$

where, as shown in Ref. 6,

$$\tau_{\Lambda\Lambda} = \gamma_\Lambda / (1 + \gamma_\Lambda g_\Lambda)$$

and

$$\gamma_\Lambda = \lambda_\Lambda + \epsilon^2 \lambda_x^{\,2} g_\Sigma (1 + \gamma_\Sigma g_\Sigma)^{-1},$$

with

$$g_\Lambda = \frac{2}{\pi} \int_0^\infty \frac{v_\Lambda^{\,2}(q) q^2 dq}{q^2 - k_\Lambda^{\,2} - i\eta}, \quad \eta \to 0^+$$

$$g_\Sigma = \frac{2}{\pi} \int_0^\infty \frac{v_\Sigma^{\,2}(q) q^2 dq}{q^2 + r^2(k_0^{\,2} - k_\Lambda^{\,2})}.$$

In this last relation we have taken advantage of our restriction $k_\Lambda \leq k_0$. The multiple-scattering interpretation of this form of $f_{\Lambda\Lambda}$ is manifest.

The expressions for the Λ-channel scattering length and effective range for a given spin as functions of the suppression parameter may be found from the expansion in powers of $k_\Lambda^{\,2}$,

$$ik_\Lambda + 1/f_\Lambda = -\alpha_\epsilon + 0.5 r_{0\epsilon} k_\Lambda^{\,2} + \cdots, \tag{3}$$

where $\alpha_\epsilon \equiv 1/a_\epsilon$. We now assume that a_0 and r_{00} as well as a_1 and r_{01} are known.

For $\epsilon = 0$ the equations obtained from Eq. (3) for a_0 and r_{00} as functions of the Λ-channel potential parameters may be manipulated so as to yield the relations

$$u_0(A_2 - 0.5 r_{00}) + u_2(\alpha_0 + A_0) = 0 \tag{4}$$

and

$$\lambda_\Lambda = 1/[u_0(\alpha_0 + A_0)]. \tag{5}$$

Here

$$-g^P/v_\Lambda^{\,2}(k_\Lambda) \equiv A_0 + A_2 k_\Lambda^{\,2} + \cdots, \tag{6}$$

$$v_\Lambda^{\,2}(k_\Lambda) \equiv u_0 + u_2 k_\Lambda^{\,2} + \cdots, \tag{7}$$

and g^P is the principal-value part of g_Λ defined above. Equation (4) is a relation among the Λ-channel potential shape parameters and the values of a_0 and r_{00}, whereas Eq. (5) yields the Λ-channel strength in terms of a_0 and the parameters in v_Λ. For a one-parameter shape (e.g., a Yamaguchi shape) Eqs. (4) and (5) completely determine the parameters in $V_{\Lambda\Lambda}$.

For $\epsilon = 1$ we may now obtain equations for a_1 and r_{01} from Eq. (3) and, with the help of Eq. (5), invert them to yield the following expressions for λ_Σ and $\lambda_x^{\,2}$:

$$\lambda_\Sigma = -[1 - \xi(g_{\Sigma 2} k_0^{\,2}/g_{\Sigma 0})]/g_{\Sigma 0} \tag{8}$$

and

$$\lambda_x^{\,2} = \frac{\xi(\alpha_1 - \alpha_0)[g_{\Sigma 2} k_0^{\,2}/(g_{\Sigma 0})^2]}{u_0(\alpha_0 + A_0)(\alpha_1 + A_0)}, \tag{9}$$

where the dimensionless parameter ξ is given by

$$\xi = \frac{(\alpha_1 + A_0)(\alpha_1 - \alpha_0)}{k_0^{\,2}(\alpha_0 + A_0)[\frac{1}{2}(r_{00} - r_{01}) + u_r(\alpha_1 - \alpha_0)]}, \tag{10}$$

with $u_r \equiv u_2/u_0$. The quantities $g_{\Sigma 0}$ and $g_{\Sigma 2}$ are defined by the expansion

$$g_\Sigma \equiv g_{\Sigma 0} + g_{\Sigma 2} k_\Lambda^{\,2} + \cdots; \tag{11}$$

they depend only on the shape $v_\Sigma(k_\Sigma)$ and the difference in threshold energies of the two channels in the form rk_0.

The first use we make of Eqs. (5), (8), and (9) is to eliminate λ_Λ, λ_Σ, and $\lambda_x^{\,2}$ from the general equations for a_ϵ and $r_{0\epsilon}$ in terms of the potential parameters that are obtained from Eq. (3). With the additional use of Eq. (4) to eliminate A_2 we obtain

$$a_\epsilon = N_\epsilon / D_\epsilon, \tag{12}$$

$$r_{0\epsilon} = r_{00} + 2 M_\epsilon / D_\epsilon^{\,2}, \tag{13}$$

where

$$N_\epsilon = (\alpha_1 + A_0) - \epsilon^2(\alpha_1 - \alpha_0),$$

$$M_\epsilon = \epsilon^2(\alpha_0 + A_0)[\tfrac{1}{2}(r_{01} - r_{00})(\alpha_0 + A_0) + u_r(\alpha_1 - \alpha_0)(1 - \epsilon^2)],$$

$$D_\epsilon = \alpha_0(\alpha_1 + A_0) + \epsilon^2 A_0(\alpha_1 - \alpha_0).$$

The point to be brought out here is that no Σ-*channel potential parameter appears explicitly* in Eqs. (12) and (13). The Λ-channel scattering parameters a_ϵ and $r_{0\epsilon}$ for all $0 < \epsilon < 1$ are determined by a_0, r_{00}, a_1, r_{01}, and the parameters in $v_\Lambda(k_\Lambda)$. In particular, if $v_\Lambda(k_\Lambda)$ is a one-parameter shape, that parameter is determined by Eq. (4), and Eqs. (12) and (13) then yield a_ϵ and $r_{0\epsilon}$. Conversely, it is fruitless to try to determine the parameters of $v_\Sigma(k_\Sigma)$ by further fitting of a_ϵ or $r_{0\epsilon}$ at values of ϵ other than those used already.

Finally, to check whether or not a resonance exists at an energy between the Λ- and Σ-channel thresholds we look for values of $k_\Lambda = k_a + ik_b$ that cause $\tau_{\Lambda\Lambda}$, and hence $f_{\Lambda\Lambda}$, to become infinite. We find k_Λ such that

$$1 + \gamma_\Lambda g_\Lambda = 0. \tag{14}$$

For a resonance in the region of interest, k_Λ not only satisfies Eq. (14) but k_b is negative and close to the real k_Λ axis and the ΛN phase shift goes through $\frac{1}{2}\pi$ at a value of $k_\Lambda < k_0$.

Several remarks about Eqs. (3)–(14) are appropriate at this point. First, we look at the signs of the terms in Eqs. (9) and (10). From Eq. (6) and the definition of g^P it follows that $A_0 < 0$. From a number of experimental and theoretical sources[7-9] $\alpha_1 < 0$. From BDI or SN and the results given in Sec. III $\alpha_0 < 0$, $(\alpha_1 - \alpha_0) > 0$, and $(r_{00} - r_{01}) > 0$. We are not claiming these last three inequalities hold for all the MTP that fit the on-shell YN data, but we are restricting our discussion to cases where they do hold. In such cases the sign of ξ is the same as the sign of the quantity in square brackets in the denominator of Eq. (10). From its definition, for $t_{\Lambda\Lambda}$ to have the correct Λ-channel threshold behavior,[5] $u_0 > 0$. Thus, if $v_\Lambda{}^2(k_\Lambda)$ increases as k_Λ increases from zero, $u_2 > 0$, $u_r > 0$, this square-bracketed quantity > 0, and finally $\xi > 0$. For $u_2 < 0$ – as is the case with the most commonly used shapes[10] – both signs of ξ are possible, with $\xi < 0$ if $(r_{00} - r_{01})$ is too small relative to $|a_0 - a_1|$. In addition it follows from the above definitions of g_Σ, $g_{\Sigma 0}$, and $g_{\Sigma 2}$ that $g_{\Sigma 0} > 0$ and $g_{\Sigma 2} > 0$. Equation (10) now exhibits the possibility that for $u_2 < 0$, $\lambda_X{}^2 < 0$. *It may be the case that we cannot fit the model used here to the given values of a_0, r_{00}, a_1, and r_{01} for any shapes $v_\Lambda(k_\Lambda)$ and $v_\Sigma(k_\Sigma)$.*

Second, rather than deal with the general condition for resonance in Eq. (14), we shall limit our discussion here – although not the calculations in Sec. III – to the existence, or rather nonexistence, of a ΛN resonance below the Σ-channel threshold due to a bound state in the uncoupled Σ channel. If the Σ channel were uncoupled from the Λ channel (i.e., if we set $\epsilon = 0$ *after* we have determined all the potential parameters) then by an interchange of the Λ and Σ subscripts in the above definitions of $f_{\Lambda\Lambda}$ and $\tau_{\Lambda\Lambda}$, we see that a bound state in the Σ channel exists at

$$k_\Sigma{}^2 = -r^2(k_0{}^2 - k_B{}^2) \leq 0,$$

that is, at $k_\Lambda = k_B$, such that

$$1 + \lambda_\Sigma g_{\Sigma B} = 0,$$

where $g_{\Sigma B}$ is the value of g_Σ at $k_\Lambda = k_B$. For the $^1S_0$ state no ΛN resonance below the Σ threshold of any kind is either predicted by meson theory or discovered experimentally. In particular a $^1S_0$ resonance due to a bound state in the uncoupled Σ channel does not exist. For the $^1S_0$ state we must have

$$1 + \lambda_\Sigma g_\Sigma > 0, \quad 0 \leq k_\Lambda \leq k_0 . \tag{15}$$

From the definition of g_Σ given above it follows that for $0 \leq k_\Lambda \leq k_0$ g_Σ is a positive monotonic increasing function of k_Λ. Even if it turns out that $\lambda_\Sigma < 0$, the inequality (15) will be satisfied for all $0 \leq k_\Lambda \leq k_0$ if it is satisfied at $k_\Lambda = k_0$. Therefore, our no-resonance condition becomes

$$1 + \lambda_\Sigma g_{\Sigma T} > 0, \tag{16}$$

where $g_{\Sigma T}$ is the value of g_Σ at the Σ-channel threshold, i.e., at $k_\Lambda = k_0$. With λ_Σ given by Eq. (8) and with some algebra, inequality (16) may be written

$$\xi > \eta = g_{\Sigma 0} g_{\Sigma 1} / g_{\Sigma T} g_{\Sigma 2} k_0{}^2 , \tag{17}$$

where

$$g_{\Sigma 1} = \frac{2}{\pi} \int_0^\infty \frac{v_\Sigma{}^2(q) x^2 dq}{q^2 + x^2} ,$$

and for completeness we exhibit

$$g_{\Sigma 0} = \frac{2}{\pi} \int_0^\infty \frac{v_\Sigma{}^2(q) q^2 dq}{q^2 + x^2} ,$$

$$k_0{}^2 g_{\Sigma 2} = \frac{2}{\pi} \int_0^\infty \frac{v_\Sigma{}^2(q) q^2 x^2 dq}{(q^2 + x^2)^2} ,$$

and

$$g_{\Sigma T} = \frac{2}{\pi} \int_0^\infty v_\Sigma{}^2(q) dq ,$$

with $x = rk_0$.

The remarkable thing about inequality (17) is the simplicity with which it exhibits the dependence of the "no-resonance" condition on the two channels. The left-hand side of the inequality depends only on the input information a_1, a_0, r_{00}, and r_{01} (available from a meson-theoretic potential) and the shape $v_\Lambda(k_\Lambda)$, whereas the right-hand side depends only on the shape $v_\Sigma(k_\Sigma)$ and the existence of a mass difference between the two channels. For a given set of input parameters (i.e., a given $^1S_0$ MTP) and a given v_Λ (v_Σ) there may be none, one, or only a few v_Σ (v_Λ) for which this inequality is satisfied. In other words we have a method for eliminating certain combinations of shapes $v_\Lambda(k_\Lambda)$ and $v_\Sigma(k_\Sigma)$ as being incapable of representing the given $^1S_0$ MTP. Whether this method is a useful tool depends on how discriminating it is, which may to a large extent depend on the actual values taken on by a_0, a_1, r_{00}, and r_{01}. Along this line we point out that the simplest way to avoid a Σ bound-state induced resonance is to have $\lambda_\Sigma \geq 0$. From Eq. (8) this would be the case when

$$\xi \geq (g_{\Sigma 0}/g_{\Sigma 2} k_0{}^2) > 1,$$

where the rightmost inequality follows from the explicit expressions for $g_{\Sigma 0}$ and $g_{\Sigma 2}$ given above. In such a case, as it must for consistency, $\xi > \eta$ for any shape $v_\Sigma(k_\Sigma)$.

III. APPLICATIONS

For our first application of the method discussed above we used Yamaguchi shapes for both $v_\Lambda(k_\Lambda)$ and $v_\Sigma(k_\Sigma)$, so that $v_X(k_X) = 1/(k_X{}^2 + \beta_X{}^2)$, for $X = \Lambda$ or Σ. For a given spin we have five potential parameters λ_Λ, β_Λ, λ_Σ, β_Σ, and λ_X. The first two of these we fixed from the values of a_0 and r_{00}; i.e., from Eqs. (4) and (5). The variable β_Σ was chosen as the free parameter. For a range of values of β_Σ Eqs. (8) and (9) were used to determine λ_Σ and λ_X. For each value of β_Σ we used Eqs. (12) and (13) to find a_ϵ and $r_{0\epsilon}$ at $\epsilon = 0.25$, 0.50, and 0.75, and we solved Eq. (14) to find the k_Λ-

TABLE I. NLS potential parameters for the BDI values of a and r_0 at $\epsilon=0$ and $\epsilon=1$. For all spin-zero cases $1/\beta_\Lambda=0.801\,45$ F and $\lambda_{\Lambda\Lambda}=-2.0557\times10^5$ MeV^2. For all spin-one cases $1/\beta_\Lambda=0.918\,36$ F and $\lambda_{\Lambda\Lambda}=-7.4829\times10^4$ MeV^2.

$(\beta_\Lambda/\beta_\Sigma)$	Spin	$-\lambda_{\Sigma\Sigma}/10^5$ (MeV^2)	$\lambda_{\Lambda\Sigma}/10^5$ (MeV^2)
0.5	0	55.714	1.2011
	1	43.897	2.2091
1.0	0	11.332	0.702 17
	1	9.8301	1.3065
2.0	0	2.8730	0.435 26
	1	2.7751	0.818 72
3.0	0	1.4269	0.337 07
	1	1.4624	0.637 30
6.0	0	0.502 28	0.224 65

plane poles of $\tau_{\Lambda\Lambda}$ that would yield an ΛN resonance below the Σ threshold. For the input parameters we chose the values of a_0, r_{00}, a_1, and r_{01} given by BDI.

In Table I we give for both the $^1S_0$ and $^3S_1$ states the values of the potential parameters we obtained using the BDI input. The first column of this table gives the ratio of the Σ-channel range parameter $1/\beta_\Sigma$ to the Λ-channel range parameter $1/\beta_\Lambda$. For a simple one- or two-pion-exchange model this ratio would be 2.0. The sign of $\lambda_{\Lambda\Sigma}$ is arbitrary.

The values of the BDI input parameters for both spin states are given in Table II as are the BDI values of a_ϵ and $r_{0\epsilon}$ for $0<\epsilon<1$. The values of a_ϵ and $r_{0\epsilon}$ we obtained for $0<\epsilon<1$ after matching *exactly* the BDI values at $\epsilon=0$ and $\epsilon=1$ are also given in this table. For the spin-zero case we obtained an excellent fit (within 0.2%) at all ϵ. The spin-one parameters are more rapidly varying functions of ϵ than are the spin-zero parameters, so that the over-all agreement with the BDI values is not as good, the discrepancy being as large as 5%. Again we point out that our results in Table II are obtained for all values of β_Σ and indeed these same values would be obtained no matter what the shape of $v_\Sigma(k_\Sigma)$.

TABLE II. ΛN scattering lengths and effective ranges as functions of the Σ-channel suppression.

ϵ	a_ϵ (F)		$r_{0\epsilon}$ (F)	
	BDI	Ours	BDI	Ours
		Spin 0		
1.00	−2.25	−2.25	3.47	3.47
0.75	−2.15	−2.151	3.56	3.554
0.50	−2.08	−2.083	3.62	3.618
0.25	−2.04	−2.043	3.66	3.657
0.00	−2.03	−2.03	3.67	3.67
		Spin 1		
1.00	−2.12	−2.12	3.31	3.31
0.75	−1.35	−1.416	4.49	4.255
0.50	−1.00	−1.049	5.66	5.409
0.25	−0.86	−0.866	6.56	6.470
0.00	−0.81	−0.81	6.92	6.92

When we went to test for ΛN resonances below the Σ-channel threshold, we found that *all the potentials* whose parameters are given in Table I as well as all those for much larger values of β_Σ *produced such a resonance*. The $^3S_1$ resonance actually occurred at a slightly lower energy (i.e., $k_\Lambda\approx180$ MeV/c) than the $^1S_0$ resonance. According to the BDI MTP that we are fitting, no $^1S_0$ resonance should exist below the Σ-channel threshold, while the $^3S_1$ resonance should appear much closer to this threshold (i.e., at $k_\Lambda\approx270$ MeV/c) and it should be due to a cooperative effect between the channels, not to a bound state in the uncoupled Σ channel. To eliminate the resonance we allowed r_{00} to vary away from the value given by BDI and repeated our calculations. Because we did not treat the tensor force used by BDI correctly we do not press any conclusions based on results obtained for the $^3S_1$ channel, but merely present these results for completeness.

In Table III we present the spin-zero results and in Table IV the spin-one results for the determina-

TABLE III. Existence of an uncoupled Σ-channel bound state and a ΛN resonance for spin 0 as a function of r_{00}. The parameters a_0, a_1, and r_{01} are kept at the BDI values.

r_{00} (F)	$1/\beta_\Lambda$ (F)	$\beta_\Lambda/\beta_\Sigma$	ξ	ξ/η	k_R (MeV/c)
3.67	0.801 45	0.5	0.595	0.364	205.3−i1.312
		1.0		0.407	208.0−i1.346
		3.0		0.486	211.7−i1.399
3.64	0.796 57	3.0	0.950	0.776	261.5−i1.125
		6.0		0.844	266.2−i1.179
		30.0		0.924	272.4−i1.299
3.63	0.794 94	3.0	1.186	0.969	280.7−i0.040
		6.0		1.053	···
		30.0		1.154	···
3.62	0.793 31	3.0	1.578	1.288	···
		6.0		1.401	···
		30.0		1.535	···
3.61	0.791 68	0.5	2.376	1.441	···
		1.0		1.608	···
		3.0		1.924	···
3.58	0.786 77	0.5	$\lambda_X{}^2<0$		
		1.0			
		3.0			

TABLE IV. Existence of an uncoupled Σ-channel bound state and a ΛN resonance for spin 1 as a function of r_{00}. The parameters a_0, a_1, and r_{01} are kept at the BDI values.

r_{00} (F)	$1/\beta_\Lambda$ (F)	$\beta_\Lambda/\beta_\Sigma$	ξ	ξ/η	k_R (MeV/c)
6.92	0.918 36	0.5	0.414	0.259	175.8–i11.81
		1.0		0.290	177.6–i12.04
		3.0		0.345	180.1–i12.40
6.32	0.867 60	0.5	0.609	0.377	209.9–i11.37
		1.0		0.422	212.9–i11.67
		3.0		0.504	217.5–i12.24
5.72	0.814 54	0.5	1.144	0.702	266.6–i7.537
		1.0		0.784	272.3–i7.259
		3.0		0.937	283.3–i5.630
		6.0		1.018	···
5.42	0.787 04	0.5	2.022	1.235	···
		1.0		1.379	···
		3.0		1.649	···

tion of ΛN resonances below the Σ-channel threshold. For both tables we have kept a_0, a_1, and r_{01} at the BDI values and varied r_{00} as shown in the leftmost column of each table. The first value r_{00} in each table is the BDI value. In both Tables III and IV the column on the far right gives the value k_R of k_Λ for which $f_{\Lambda\Lambda}$ becomes infinite. By looking at the ΛN phase shift for values of $k_\Lambda < k_0$, we checked that the values of k_R shown did yield a ΛN resonance below the Σ-channel threshold. In this column the spaces with three dots are those for which no resonance exists below the Σ threshold. Even in such cases a resonance may exist above this threshold, but the investigation of this point was outside the limits of this paper.[11] The second and third columns from the right in these tables contain the values of the parameters ξ and ξ/η discussed in Sec. II. In both Tables III and IV for each value of r_{00} we have tested for a resonance at a number of values of the ratio of the range parameters $(1/\beta_\Sigma)(1/\beta_\Lambda)$. The results shown in these tables exemplify the discussion given in the Sec. II.

From Table III we see that not only does our fit of the potential parameters to the BDI parameters predict a $^1S_0$ resonance, but also, from the values of ξ/η, we see that this resonance is due to a bound state in the uncoupled Σ channel. As we decrease the value of r_{00} we finally come to a value (3.63 F) for which, if the Σ-channel range parameter is large enough, the resonance disappears. For even smaller values of r_{00}, all values of $1/\beta_\Sigma$ give a resonance-free result. Finally, r_{00} gets so close to r_{01} that as discussed in Sec. II λ_x^2 becomes negative. We note that the presence or absence of the resonance is extremely sensitive to the value of r_{00}. A decrease in r_{00} of less than 2% is sufficient to make the resonance disappear.

We have shown that a *two-channel* NLS potential with *Yamaguchi shapes cannot reproduce the BDI MTP low-energy* ΛN *scattering parameters* with the Σ channel both unsuppressed and fully suppressed, *and at the same time reproduce the resonance structure of the BDI potential* for energies below the Σ-channel threshold. Because of the extreme sensitivity of this resonance to the value of the input parameters, which themselves are not all well determined by experiment,[9] it is possible that this form of NLS potential still could give a proper fit [in the sense of conditions (a), (b), and (c) given above] to a MTP that is adjusted to yield slightly different values of a_ϵ and $r_{0\epsilon}$ at $\epsilon = 0$ and $\epsilon = 1$. In any case sticking to the BDI values we would like to see if the result just stated may be extended to other than Yamaguchi shapes. From the values of ξ and ξ/η given in Tables III and IV we see that in all cases $\eta > 1$. If we consider the $^1S_0$ state we have in addition from Table III $\xi = 0.595$ with a Yamaguchi shape for v_Λ. Thus if we could show $\eta > 1$ for all shapes $v_\Sigma(k_\Sigma)$, the Yamaguchi shape for $v_\Lambda(k_\Lambda)$ would be ruled out. Even being able to prove $\eta < 1$ for a restricted class of shapes $v_\Sigma(k_\Sigma)$ would be a very useful result. We have not been able to prove $\eta > 1$ for all $v_\Sigma(k_\Sigma)$, nor have we been able to come up with a counterexample. We have only been able to show by direct calculation for a few common one-parameter shapes that $\eta > 1$.

Next we turn to the NLS potential used by SN. These authors, using a Yamaguchi shape in each channel with $\beta_\Lambda = \beta_\Sigma$, fit a_ϵ and $r_{0\epsilon}$ at $\epsilon = 1$ and the ΛN amplitude below the Σ-channel threshold. The values they used for their potential parameters are of no particular interest here, but the values they used for a_1 and r_{01} were (in both the $^1S_0$ and the $^3S_1$ states)

$$a_1 = -1.6523 \text{ F}, \quad r_{01} = 3.1717 \text{ F}. \tag{18}$$

Using the SN case-A spin-zero potential parameters, we calculated

$$a_0 = -1.1589 \text{ F}, \quad r_{00} = 3.7877 \text{ F} \tag{19}$$

and

$$\xi = 1.903, \quad \xi/\eta = 1.271. \tag{20}$$

The value for ξ/η in Eq. (20) indicates that the SN values of a_1, a_0, and r_{00} are not that far away from values that would give a $^1S_0$ resonance. To check this out we first varied a_0 away from the value given in Eq. (19) and then varied r_{00} away from the value given in Eq. (19). The results are shown in Table V.

For the first four rows of Table V r_{00}, a_1, and

TABLE V. Existence of an uncoupled Σ-channel bound state and a ΛN resonance for the SN case-A spin-zero values of a_1 and r_{01} as functions of a_0 and r_{00}.

a_0 (F)	r_{00} (F)	$1/\beta_\Lambda = 1/\beta_\Sigma$ (F)	ξ	ξ/η	k_R (MeV/c)
−1.458 91	3.7877	0.749 36	0.171	0.104	114.3−i1.811
−1.358 91	3.7877	0.733 98	0.349	0.211	162.0−i3.115
−1.258 91	3.7877	0.717 43	0.718	0.433	225.3−i3.940
−1.158 91	3.7877	0.699 55	1.903	1.271	⋯
−1.158 91	3.6877	0.686 70	7.096	4.725	⋯
−1.158 91	3.5877	0.673 71	$\lambda_X{}^2 < 0$		

r_{01} were kept at the SN values, while for the last three rows a_0, a_1, and r_{01} were kept at the SN values; i.e., the fourth row from the top contains all SN values. As indicated in the third column from the left the SN condition $\beta_\Sigma = \beta_\Lambda$ was used throughout the table. The three columns on the right contain values of the same variables given in the corresponding columns of Tables III and IV. Again the sensitivity of the existence of a resonance to the values of the parameters is evident. An increase of less than 10% in a_0 (or of less than 3% in $1/\beta_\Lambda$) is enough to cause a resonance to appear. A decrease in r_{00} of less than 5% is enough to make the fit of the NLS potential to the input data impossible.

Comparing our $^1S_0$ BDI results of Table III with our SN results of Table V we can say that the former yield a resonance because $(r_{00} - r_{01})$ is too large for the accompanying value of $|a_0 - a_1|$ while the latter is resonance-free because this is not the case. Because of the sensitivity to these scattering parameters, however, one could not tell this was so merely by looking at the values of the scattering parameters in the two cases. Turning our SN results around we would expect, on the basis of of the BDI results, an MTP adjusted to give the SN values of a_1 and r_{01} to give either a value of r_{00} larger than that in Eq. (19) and/or a value of a_0 more negative than that given in Eq. (19).

Further investigations in this whole area are clearly called for. It would be highly desirable for such work to have a number of different MTP's whose values of a_ϵ and $r_{0\epsilon}$ for $0 \leq \epsilon < 1$ are known and all of which have the same values of a_1, r_{01}, and the ΛN phase shift at energies up to the Σ-channel threshold. It would then be possible to investigate how the form of the low-energy half-off-shell ΛN amplitude is correlated (if at all) with the values of a_0 and r_{00}.

The main question of interest as far as the work described here is concerned which still has not been answered is, can an NLS potential of the form given in Eq. (2) be found which fits the $^1S_0$ BDI values for a_1, r_{01}, a_0, r_{00}, and the ΛN resonance structure below the Σ-channel threshold? Other questions of interest are: How dependent are our results on the particular shapes used for $v_\Lambda(k_\Lambda)$ and $v_\Sigma(k_\Sigma)$? Can the problem of the nonexistent resonances be avoided by choosing an MTP of the BDI form adjusted to fit slightly different values of a_1 and r_{01} or by using a different form of MTP (e.g., one that incorporates a two-pion-exchange term or one with soft-core potentials) so that a_0 and r_{00} are changed from the BDI values even though a_1 and r_{01} are not? Can the resonance problem be avoided by using sums of NLS potentials in some of the matrix elements of Eq. (1)? A number of these questions are at present under investigation. We expect to report further results in this area at a later date.

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†A preliminary report of this work was given in Bull. Am. Phys. Soc. 15, 570 (1970).

[1]E. Satoh and Y. Nogami, Phys. Letters 32B, 243 (1970), referred to hereafter as SN.

[2]A. R. Bodmer, Phys. Rev. 141, 1387 (1966).

[3]A. R. Bodmer and D. M. Rote, report (Univ. of Illinois Chicago Circle), 1970 (unpublished).

[4]J. T. Brown, B. W. Downs, and C. K. Iddings, Ann. Phys. (N.Y.) 60, 148 (1970); B. W. Downs, in *Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, 1969,* edited by A. R. Bodmer and L. G. Hyman (Argonne National Laboratory, Argonne, Ill., 1969), p. 51. Hereafter referred to as BDI.

[5]Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).

[6]L. H. Schick and Alan J. Toepfer, Phys. Rev. 170, 946 (1968).

[7]R. C. Herndon and Y. C. Tang, Phys. Rev. 159, 853 (1967); Y. C. Tang, in *Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, 1969,* edited by A. R. Bodmer and L. G. Hyman (Ref. 4), p. 276.

[8]B. Sechi-Zorn, B. Kehoe, J. Twitty, and R. A. Burnstein, Phys. Rev. 175, 1735 (1968).

[9]G. Alexander, in *Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, 1969*, edited by A. R. Bodmer and L. G. Hyman (Ref. 4), p. 5.

[10]For examples of commonly used NLS shapes see T. R. Mongan, Phys. Rev. 180, 1514 (1969).

[11]For resonances above the Σ-channel threshold see A. N. Kamal and H. J. Kreuzer, Phys. Rev. D 2, 2033 (1970).

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Restrictions on Two-Channel Separable ΛN Potentials*

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A previously suggested method for representing a meson-theoretic ΛN potential by a two-channel separable potential is shown to limit the shape of the separable Λ-channel potential for arbitrary shape of the separable Σ-channel potential.

In the preceding paper[1] it was proposed that, in view of the apparent importance of suppression of Λ-Σ conversion when a Λ is bound in a multinucleon system,[2,3] for such problems a simple physical way to represent a given ΛN meson-theoretic potential (MTP) by a two-channel nonlocal separable (NLS) potential was to choose the NLS potential to (a) have the same ΛN scattering length and effective range as those of the given MTP, (b) to have the same ΛN scattering length and effective range as those of the given MTP when in both potentials the Σ channel is fully suppressed, and (c) to have the same resonances below the Σ-channel threshold as predicted by the given MTP; e.g., for the ${}^1S_0$ ΛN potential there should be no such resonance, and in particular there should be no bound state in the uncoupled Σ channel to cause such a resonance.

In Ref. 1, as here, attention was focused on the ${}^1S_0$ interaction so that the complications of a tensor force were avoided. The very common form of NLS two-channel ΛN potential

$$V=\begin{pmatrix} \lambda_\Lambda U_{\Lambda\Lambda} & \epsilon\lambda_x U_{\Lambda\Sigma} \\ \epsilon\lambda_x U_{\Sigma\Lambda} & \lambda_\Sigma U_{\Sigma\Sigma} \end{pmatrix}, \tag{1a}$$

where in momentum space ($c=\hbar=1$)

$$\langle \vec{k}_x | U_{xy} | \vec{k}_y \rangle = v_x(k_x)v_y(k_y), \tag{1b}$$

$x, y=\Lambda$ or Σ, and ϵ is the suppression parameter (i.e., $0\leq\epsilon\leq 1$, $\epsilon=1$ yields the completely unsuppressed, free-particle, hyperon-nucleon interaction, and $\epsilon=0$ yields this interaction with the Σ channel fully suppressed), was chosen for investigation. It was then shown that the conditions (a), (b), and (c) imposed a limitation on the possible shapes for v_Λ and v_Σ. In particular it was shown that with Yamaguchi[4] shapes for v_Λ and v_Σ, the use of conditions (a) and (b), and the use of the MTP of Brown, Downs, and Iddings (BDI),[5] condition (c) was violated; i.e., such an NLS representation of the BDI potential as given by Eqs. (1) and conditions (a), (b), and (c) was impossible with Yamaguchi shapes for v_Λ and v_Σ. In this note we wish to present results of a much more general nature.

The limitation on the shapes v_Λ and v_Σ obtained in Ref. 1 could be written as an inequality, $\xi>\eta$. Here ξ is a dimensionless parameter which depends only on k_0 (the value of k_Λ at the Σ-channel threshold), on the ΛN scattering length a_ϵ and effective range $r_{0\epsilon}$ at the values $\epsilon=0$ and $\epsilon=1$, and on the potential shape v_Λ, while η depends only on the shape v_Σ and on k_0. Explicitly

$$\xi=\frac{(\alpha_1+A_0)(\alpha_1-\alpha_0)}{k_0^2(\alpha_0+A_0)[\frac{1}{2}(r_{00}-r_{01})+u_r(\alpha_1-\alpha_0)]}, \tag{2}$$

where $\alpha_j\equiv 1/a_j$, u_r is defined by

$$v_\Lambda^2(k_\Lambda)\equiv u_0[1+u_r k_\Lambda^2+O(k_\Lambda^4)+\cdots],$$

and A_0 is defined by

$$-g^p/v_\Lambda^2(k_\Lambda)\equiv A_0+A_2k_\Lambda^2+O(k_\Lambda^4)+\cdots,$$

with

$$g^p=\frac{1}{\pi}P\int_{-\infty}^{\infty}(q^2-k_\Lambda^2)^{-1}v_\Lambda^2(q)q^2dq.$$

On the other hand, $\eta=I_1I_2/I_3I_4$, where

$$I_j=\frac{2}{\pi}\int_0^\infty v_\Sigma^2(q)R_j dq, \quad j=1,\ldots,4$$