Lepton-Number and Chirality Nonconservation in Weak Processes*

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We propose a description of leptonic and semileptonic weak processes which involves the use of only three distinct lepton field operators $\psi_{e^-}(x)$, $\psi_{\mu^-}(x)$, and $\psi_\nu(x)$ and postulates such an expression for the leptonic weak current $l'_\alpha(\psi_{e^-}(x), \psi_{\mu^-}(x), \psi_\nu(x))$ that (1) neither the conservation of total lepton number nor the "conservation of neutrino chirality" holds exactly, and (2) the extent of lepton-number nonconservation and neutrino-chirality nonconservation becomes maximal at high interlepton momentum transfers. As a result, various weak processes forbidden in the conventional description and so far unobserved at relatively small momentum transfers are expected to proceed with non-negligible rates at higher momentum transfers. A discussion is given of certain of these processes and particular attention is focused on the production processes $\nu_\mu+p\to e^++n$, $\nu_\mu+p\to e^-+\Delta^{++}$, $\nu_\mu+p\to\mu^++n$, and $\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A)$, and on the decay processes $K^+\to\mu^++e^++\pi^-$ and $\mu^+\to e^++\gamma$.

I. INTRODUCTION

Some years ago, Konopinski and Mahmoud[1] proposed an assignment of lepton numbers to the electron, muon, and neutrino as follows:

$$L=1 \quad \text{for} \quad e^-,\ \mu^+,\ \nu,$$
$$L=-1 \quad \text{for} \quad e^+,\ \mu^-,\ \bar\nu, \qquad (1)$$
$$L_{\text{tot}}=\sum_j L_j$$

and assumed, in addition, that the total lepton number L_{tot} is conserved in all processes. This assignment of lepton numbers, together with the now available experimental information on neutrino and antineutrino helicities, corresponds to the identification

$$|\nu_e(\vec p)\rangle=|\nu(\vec p,-\tfrac12)\rangle : \ L=1,$$
$$|\bar\nu_\mu(\vec p)\rangle=|\nu(\vec p,\tfrac12)\rangle : \ L=1, \qquad (2)$$
$$|\bar\nu_e(\vec p)\rangle=|\bar\nu(\vec p,\tfrac12)\rangle : \ L=-1,$$
$$|\nu_\mu(\vec p)\rangle=|\bar\nu(\vec p,-\tfrac12)\rangle : \ L=-1.$$

In Eq. (2), $\vec p$ refers to the momentum, and $\frac12$ or $-\frac12$ to the helicity, of the neutrino or antineutrino state, while the "neutrinos" ν_e and ν_μ, and the "antineutrinos" $\bar\nu_e$ and $\bar\nu_\mu$, are those emitted in the decay of charged pions or in some equivalent semileptonic weak process:

$$\pi^+\to e^++\nu_e \quad \text{or, e.g.,} \quad e^-+p\to\nu_e+n,$$
$$\pi^+\to\mu^++\nu_\mu \quad \text{or, e.g.,} \quad \mu^-+p\to\nu_\mu+n, \qquad (3)$$
$$\pi^-\to e^-+\bar\nu_e \quad \text{or, e.g.,} \quad e^++n\to\bar\nu_e+p,$$
$$\pi^-\to\mu^-+\bar\nu_\mu \quad \text{or, e.g.,} \quad \mu^++n\to\bar\nu_\mu+p.$$

Equations (1)–(3) imply a leptonic weak current of the form

$$l_\alpha(x)=\psi^\dagger_{e^-}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_\nu(x)+\psi^\dagger_{\mu^-}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\bar\nu}(x). \qquad (4)$$

We also take $\gamma_\alpha=\gamma_\alpha^\dagger=\tilde\gamma_\alpha^\dagger(1-2\delta_{\alpha4})$ so that

$$\psi_{e^+}(x)=\tilde\psi^\dagger_{e^-}(x),\quad \psi_{\mu^+}(x)=\tilde\psi^\dagger_{\mu^-}(x),\quad \psi_{\bar\nu}(x)=\tilde\psi^\dagger_\nu(x). \qquad (5)$$

We note that the $l_\alpha(x)$ of Eqs. (4) and (5) employ a *single* neutrino field operator $\psi_\nu(x)$ so that in this case $m_{\nu_e}=m_{\nu_\mu}=m_\nu$; on the other hand, the conventional leptonic weak current

$$[l_\alpha(x)]_{\text{conv}}=\psi^\dagger_{e^-}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\nu_e}(x)+\psi^\dagger_{\mu^-}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\nu_\mu}(x)$$

employs *two* distinct neutrino field operators $\psi_{\nu_e}(x)=\tilde\psi^\dagger_{\bar\nu_e}(x)$ and $\psi_{\nu_\mu}(x)=\tilde\psi^\dagger_{\bar\nu_\mu}(x)$ so that here m_{ν_μ} *may* be different from m_{ν_e}.

Equations (1)–(5) forbid all processes which do not conserve L_{tot}. Thus, e.g., the ($\Delta L_{\text{tot}}=2$) processes

$$\nu_\mu+p\to e^-+\Delta^{++} \qquad (6)$$

and

$$\nu_\mu + p \to \mu^+ + n \qquad (7)$$

are characterized by transition amplitudes proportional to ($\lambda' = \frac{1}{2}$ or $-\frac{1}{2}$)

$$\langle e^-(\vec{p}', \lambda')|\psi^\dagger_{e^-}\gamma_4\gamma_\alpha(1+\gamma_5)\psi_\nu|\nu_\mu(\vec{p})\rangle = \langle e^-(\vec{p}', \lambda')|\psi^\dagger_{e^-}\gamma_4\gamma_\alpha(1+\gamma_5)\psi_\nu|\bar{\nu}(\vec{p}, -\tfrac{1}{2})\rangle \qquad (8)$$

and to

$$\langle \mu^+(\vec{p}', \lambda')|\psi^\dagger_{\bar{\nu}}\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\mu^-}|\nu_\mu(\vec{p})\rangle = \langle \mu^+(\vec{p}', \lambda')|\psi^\dagger_{\mu^+}\gamma_4\gamma_\alpha(1-\gamma_5)\psi_\nu|\bar{\nu}(\vec{p}, -\tfrac{1}{2})\rangle\,, \qquad (9)$$

respectively, and each of these matrix elements vanishes since ψ_ν annihilates a state without any ν. Further, Eqs. (1)–(5) imply a "neutrino-chirality-conservation" principle since the $l_\alpha(x)$ of Eqs. (4) and (5) is invariant under the transformation $\psi_\nu(x) \to \gamma_5\psi_\nu(x)$, $\psi_{e^-}(x) \to \psi_{e^-}(x)$, $\psi_{\mu^-}(x) \to -\psi_{\mu^-}(x)$ and since $(m_\nu/|\vec{p}|)$ can be considered as negligible in all cases of practical interest. As a result, processes which conserve $L_{\rm tot}$ but which violate neutrino-chirality conservation (Ch_ν conservation) are also forbidden. Thus, e.g., the $(\Delta L_{\rm tot} = 0)$ process

$$\nu_\mu + p \to e^+ + n \qquad (10)$$

is characterized by a transition amplitude proportional to

$$\begin{aligned}\langle e^+(\vec{p}', \lambda')|\psi^\dagger_\nu\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{e^-}|\nu_\mu(\vec{p})\rangle &= \langle e^+(\vec{p}', \lambda')|\psi^\dagger_{e^+}\gamma_4\gamma_\alpha(1-\gamma_5)\psi_{\bar{\nu}}|\bar{\nu}(\vec{p}, -\tfrac{1}{2})\rangle \\ &= u^\dagger_{e^+}(\vec{p}', \lambda')\gamma_4\gamma_\alpha\left(1 + \vec{\sigma}\cdot\hat{p}\frac{|\vec{p}|}{(|\vec{p}|^2+m_\nu^2)^{1/2}} - \gamma_5\gamma_4\frac{m_\nu}{(|\vec{p}|^2+m_\nu^2)^{1/2}}\right)u_{\bar{\nu}}(\vec{p}, -\tfrac{1}{2}) \\ &= u^\dagger_{e^+}(\vec{p}', \lambda')\gamma_4\gamma_\alpha\left(1 - \frac{|\vec{p}|}{(|\vec{p}|^2+m_\nu^2)^{1/2}} - \gamma_5\gamma_4\frac{m_\nu}{(|\vec{p}|^2+m_\nu^2)^{1/2}}\right)u_{\bar{\nu}}(\vec{p}, -\tfrac{1}{2}) \qquad (11)\end{aligned}$$

which vanishes as $m_\nu/|\vec{p}| \to 0$. Finally, the Eqs. (1)–(5) allow all processes which satisfy both $L_{\rm tot}$ conservation and Ch_ν conservation. Thus, e.g., the $(\Delta L_{\rm tot} = 0)$ process

$$\nu_\mu + p \to \mu^- + \Delta^{++} \qquad (12)$$

has a transition amplitude proportional to [neglecting henceforth all terms $\sim (m_\nu/|\vec{p}|)$]

$$\begin{aligned}\langle \mu^-(\vec{p}', \lambda')|\psi^\dagger_{\mu^-}\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\bar{\nu}}|\nu_\mu(\vec{p})\rangle &= \langle \mu^-(\vec{p}', \lambda')|\psi^\dagger_{\mu^-}\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\bar{\nu}}|\bar{\nu}(\vec{p}, -\tfrac{1}{2})\rangle \\ &= u^\dagger_{\mu^-}(\vec{p}', \lambda')\gamma_4\gamma_\alpha(1-\vec{\sigma}\cdot\hat{p})u_{\bar{\nu}}(\vec{p}, -\tfrac{1}{2}) \\ &= 2u^\dagger_{\mu^-}(\vec{p}', \lambda')\gamma_4\gamma_\alpha u_{\bar{\nu}}(\vec{p}, -\tfrac{1}{2}) \qquad (13)\end{aligned}$$

and this does not vanish. The discussion of Eqs. (6)–(13) can be extended to all other processes and to any order in the weak interaction and one arrives at the general conclusion that the theory based on Eqs. (1)–(5) with its implication of $L_{\rm tot}$ conservation and Ch_ν conservation is completely equivalent in its predictions to the conventional theory based on the above-quoted $[l_\alpha(x)]_{\rm conv}$ with its implication of $(L_e)_{\rm tot}$ conservation and $(L_\mu)_{\rm tot}$ conservation

$$[(L_e)_{\rm tot} = \sum_j (L_e)_j,\quad (L_\mu)_{\rm tot} = \sum_j (L_\mu)_j;\quad L_e = 1\ (-1)\ \text{for}\ e^-\ (e^+),\ \nu_e\ (\bar{\nu}_e);\ L_\mu = 1\ (-1)\ \text{for}\ \mu^-\ (\mu^+),\ \nu_\mu\ (\bar{\nu}_\mu)].^2$$

II. GENERALIZATION OF EXPRESSION FOR LEPTONIC WEAK CURRENT

We now proceed to generalize the theory of Eqs. (1)–(5) by replacing the leptonic weak current $l_\alpha(x)$ of Eqs. (4) and (5) by the leptonic weak current

$$\begin{aligned}l'_\alpha(x) &= \psi^\dagger_{e^-}(x)\gamma_4\gamma_\alpha[(1+\gamma_5)+\eta_e(1-\gamma_5)][\psi_\nu(x)+\xi\psi_{\bar{\nu}}(x)] + \psi^\dagger_{\mu^-}(x)\gamma_4\gamma_\alpha[(1+\gamma_5)+\eta_\mu(1-\gamma_5)][\psi_{\bar{\nu}}(x)+\xi^*\psi_\nu(x)]\,, \\ \psi_{e^+}(x) &= \tilde{\psi}^\dagger_{e^-}(x),\quad \psi_{\mu^+}(x) = \tilde{\psi}^\dagger_{\mu^-}(x),\quad \psi_{\bar{\nu}}(x) = \tilde{\psi}^\dagger_\nu(x),\end{aligned} \qquad (14)$$

where the parameters η_e, η_μ, and ξ may be considered as functions of $(-(\partial/\partial x_\beta)(\partial/\partial x_\beta))$ so that, e.g.,

$$\begin{aligned}\langle \mu^-(\vec{p}', \lambda')|\psi^\dagger_{\mu^-}\gamma_4\gamma_\alpha\eta_\mu(1-\gamma_5)\psi_{\bar{\nu}}|\bar{\nu}(\vec{p}, \tfrac{1}{2})\rangle &\to \langle \mu^-(\vec{p}', \lambda')|\eta_\mu[\psi^\dagger_{\mu^-}\gamma_4\gamma_\alpha(1-\gamma_5)\psi_{\bar{\nu}}]|\bar{\nu}(\vec{p}, \tfrac{1}{2})\rangle \\ &= \eta_\mu((p'-p)^2)\langle \mu^-(\vec{p}', \lambda')|\psi^\dagger_{\mu^-}\gamma_4\gamma_\alpha(1-\gamma_5)\psi_{\bar{\nu}}|\bar{\nu}(\vec{p}, \tfrac{1}{2})\rangle\,. \qquad (15)\end{aligned}$$

Because of Eqs. (3) and (14), we must replace the identification of the neutrino and antineutrino states given in Eq. (2) by

$$
\begin{aligned}
&|\nu_e(\vec{p})\rangle = N_e^{-1/2}[|\nu(\vec{p}, -\tfrac{1}{2})\rangle + \eta_e^*|\nu(\vec{p}, \tfrac{1}{2})\rangle + \xi^*|\overline{\nu}(\vec{p}, -\tfrac{1}{2})\rangle + \eta_e^*\xi^*|\overline{\nu}(\vec{p}, \tfrac{1}{2})\rangle],\\
&|\overline{\nu}_\mu(\vec{p})\rangle = N_\mu^{-1/2}[|\nu(\vec{p}, \tfrac{1}{2})\rangle + \eta_\mu|\nu(\vec{p}, -\tfrac{1}{2})\rangle + \xi^*|\overline{\nu}(\vec{p}, \tfrac{1}{2})\rangle + \eta_\mu\xi^*|\overline{\nu}(\vec{p}, -\tfrac{1}{2})\rangle],\\
&|\overline{\nu}_e(\vec{p})\rangle = N_e^{-1/2}[|\overline{\nu}(\vec{p}, \tfrac{1}{2})\rangle + \eta_e|\overline{\nu}(\vec{p}, -\tfrac{1}{2})\rangle + \xi|\nu(\vec{p}, \tfrac{1}{2})\rangle + \eta_e\xi|\nu(\vec{p}, -\tfrac{1}{2})\rangle],\\
&|\nu_\mu(\vec{p})\rangle = N_\mu^{-1/2}[|\overline{\nu}(\vec{p}, -\tfrac{1}{2})\rangle + \eta_\mu^*|\overline{\nu}(\vec{p}, \tfrac{1}{2})\rangle + \xi|\nu(\vec{p}, -\tfrac{1}{2})\rangle + \eta_\mu^*\xi|\nu(\vec{p}, \tfrac{1}{2})\rangle],\\
&N_e \equiv (1+|\eta_e|^2)(1+|\xi|^2), \quad N_\mu \equiv (1+|\eta_\mu|^2)(1+|\xi|^2),
\end{aligned} \tag{16}
$$

which shows that the "neutrinos" ν_e and ν_μ and the "antineutrinos" $\overline{\nu}_e$ and $\overline{\nu}_\mu$, emitted in the decay of charged pions are now neither in eigenstates of lepton number [see Eq. (1)] nor in eigenstates of helicity ($l'_\alpha(x)$ is not invariant under the transformation

$$
\begin{aligned}
&\psi_\nu(x) + \xi\psi_{\overline{\nu}}(x) \to \gamma_5[\psi_\nu(x) + \xi\psi_{\overline{\nu}}(x)],\\
&\psi_{e^-}(x) \to \psi_{e^-}(x),\\
&\psi_{\mu^-}(x) \to -\psi_{\mu^-}(x)
\end{aligned}
$$

so that Ch_ν conservation is violated). The "wrong" lepton number and helicity admixtures in $|\nu_e(\vec{p})\rangle$, $|\overline{\nu}_\mu(\vec{p})\rangle$, $|\overline{\nu}_e(\vec{p})\rangle$, and $|\nu_\mu(\vec{p})\rangle$ depend on the values of ξ and of η_e and η_μ at the interlepton momentum transfer appropriate to the neutrino or antineutrino process in question and they may become relatively large at large momentum transfer (see below). In particular, for $\xi=\eta_e=\eta_\mu=1$, these admixtures are of such a magnitude that

$$
\begin{aligned}
|\nu_e(\vec{p})\rangle &= |\nu_\mu(\vec{p})\rangle = |\overline{\nu}_e(\vec{p})\rangle = |\overline{\nu}_\mu(\vec{p})\rangle\\
&= \tfrac{1}{2}[\,|\nu(\vec{p}, -\tfrac{1}{2})\rangle + |\nu(\vec{p}, \tfrac{1}{2})\rangle + |\overline{\nu}(\vec{p}, -\tfrac{1}{2})\rangle + |\overline{\nu}(\vec{p}, \tfrac{1}{2})\rangle].
\end{aligned}
$$

Thus the ν_e, ν_μ, $\overline{\nu}_e$, and $\overline{\nu}_\mu$ produced in processes for which $\xi=\eta_e=\eta_\mu=1$ behave identically in all subsequent processes. We also note that, according to Eq. (14), $\eta_e=\eta_\mu=1$ corresponds to a parity-conserving $l'_\alpha(x)$ and that more generally, the extent of parity nonconservation in any leptonic or semileptonic weak process (as measured for example by the expectation values of the helicities of the emitted charged leptons) depends on the values of η_e and η_μ appropriate to the momentum transfer of the process [see, e.g., Eqs. (17) and (18), below]. Finally, we explicitly admit the possibility that $\xi \neq \xi^*$, $\eta_e \neq \eta_e^*$, $\eta_\mu \neq \eta_\mu^*$, which corresponds to a violation of CP invariance by the weak Hamiltonian containing $l'_\alpha(x)$,[3] and we take $\eta_e=\eta_\mu$, which is required for the validity of μ-e universality.

III. SOME CONSEQUENCES OF THE GENERALIZED LEPTONIC WEAK CURRENT

We proceed to give a more systematic discussion of the predictions of the theory based on Eqs. (1), (3), and (14)–(16). In this theory, the longitudinal spin polarization (=twice the expectation value of the helicity) of the e^+, ν_e and e^-, $\overline{\nu}_e$ emitted in nuclear β decay is given by

$$
\begin{aligned}
&S_{\text{long}}(e^+) = -S_{\text{long}}(e^-) = \left(\frac{v_e}{c}\right)\left(\frac{1-|\eta|^2}{1+|\eta|^2}\right),\\
&S_{\text{long}}(\overline{\nu}_e) = -S_{\text{long}}(\nu_e) = \left(\frac{1-|\eta|^2}{1+|\eta|^2}\right),
\end{aligned} \tag{17}
$$

whence, using the available experimental limit on[4]

$$1 - \frac{|S_{\text{long}}(e^\pm)|}{(v_e/c)},$$

$$|\eta| < 0.1 \text{ for } |(p_{\nu_e}+p_e)^2|^{1/2} < 5 \text{ MeV}/c. \tag{18}$$

Further, in muon decay

$$\mu^+ \to \overline{\nu}_\mu + e^+ + \nu_e, \tag{19}$$

the Michel parameter ρ, which determines the shape of the electron momentum spectrum, is given by

$$\rho = \tfrac{3}{4}[1 - \tfrac{1}{2}P(\nu_e, \overline{\nu}_\mu)], \tag{20}$$

where $P(\nu_e, \overline{\nu}_\mu)$ is the probability that a ν_e and a $\overline{\nu}_\mu$ of the same momentum have the same lepton number and the same helicity, i.e., $P(\nu_e, \overline{\nu}_\mu)$ is the probability of overlap between the $|\nu_e(\vec{p})\rangle$ and the $|\overline{\nu}_\mu(\vec{p})\rangle$ states. Equation (16) yields

$$
\begin{aligned}
P(\nu_e, \overline{\nu}_\mu) &= |\langle\overline{\nu}_\mu(\vec{p})|\nu_e(\vec{p})\rangle|^2\\
&= 4|\eta|^2/(1+|\eta|^2)^2
\end{aligned} \tag{21}
$$

so that, using the available experimental limit on $(1-\frac{4}{3}\rho)$,[5]

$$|\eta| < 5\times10^{-2} \text{ for } |(p_{\overline{\nu}_e} - p_{\mu^+})^2|^{1/2} < 100 \text{ MeV}/c. \tag{22}$$

Let us now consider again the various processes that may be initiated by a high-energy neutrino emitted in pion decay and incident on a proton [Eq. (3) and Eqs. (6), (7), (10), (12)]. As noted above, the process [Eq. (10)][6]

$$\nu_\mu + p \to e^+ + n \tag{23}$$

is forbidden by the theory of Eqs. (1)–(4) because of its violation of Ch_ν conservation [Eq. (11)], and it is also forbidden by the conventional theory because $\Delta(L_e)_{\text{tot}} = -1$ and $\Delta(L_\mu)_{\text{tot}} = -1$. On the other hand, in the theory of Eqs. (1), (3), and (14)–(16), this process is characterized by a cross section $d\sigma(\nu_\mu + p \to e^+ + n)/d\omega$ proportional to the probability of overlap $P(\nu_\mu, \overline{\nu}_e)$ between the incident $|\nu_\mu(\vec{p})\rangle$ state and the $|\overline{\nu}_e(\vec{p})\rangle$ state which would be produced if the e^+ were absorbed in $e^+ + n \to \overline{\nu}_e + p$ [see Eq. (3)]. In fact, we have, using Eq. (16),

$$\frac{d\sigma(\nu_\mu + p \to e^+ + n)/d\omega}{d\sigma(\overline{\nu}_e + p \to e^+ + n)/d\omega} = P(\nu_\mu, \overline{\nu}_e)$$

$$= |\langle \overline{\nu}_e(\vec{p}) | \nu_\mu(\vec{p})\rangle|^2$$

$$= \frac{|[\eta(\mathrm{II}) + \eta(\mathrm{I})][1 + \xi(\mathrm{II})\xi^*(\mathrm{I})]|^2}{[1 + |\eta(\mathrm{II})|^2][1 + |\eta(\mathrm{I})|^2][1 + |\xi(\mathrm{II})|^2][1 + |\xi(\mathrm{I})|^2]}, \tag{24}$$

where the I and II in $\eta(\mathrm{I})$, $\eta(\mathrm{II})$ and $\xi(\mathrm{I})$, $\xi(\mathrm{II})$ refer to the squares of the momentum transfers in Eq. (3) and Eq. (23), i.e.,

$$\mathrm{I} \equiv (p_{\mu^+} + p_{\nu_\mu})^2 = p_\pi{}^2 = -m_\pi{}^2,$$

$$\mathrm{II} \equiv (p_{e^+} - p_{\nu_\mu})^2 = (\vec{p}_{e^+} - \vec{p}_{\nu_\mu})^2 - (E_{e^+} - E_{\nu_\mu})^2 \cong 2E_{\nu_\mu}{}^2(1 - \hat{p}_{\nu_\mu} \cdot \hat{p}_{e^+}).$$

Since $|\mathrm{II}| \gg |\mathrm{I}|$ for large E_{ν_μ} and not too small $\cos^{-1}(\hat{p}_{\nu_\mu} \cdot \hat{p}_{e^+})$ we may expect that

$$\eta(\mathrm{II}) \gg \eta(\mathrm{I}) \cong 0, \quad \xi(\mathrm{II}) \gg \xi(\mathrm{I}) \cong 0 \tag{25}$$

whence, combining Eqs. (25) and (24),

$$\frac{d\sigma(\nu_\mu + p \to e^+ + n)/d\omega}{d\sigma(\overline{\nu}_e + p \to e^+ + n)/d\omega} \cong \frac{|\eta(\mathrm{II})|^2}{[1 + |\eta(\mathrm{II})|^2][1 + |\xi(\mathrm{II})|^2]}. \tag{26}$$

Equation (26) and the experimental limit on $d\sigma(\nu_\mu + p \to e^+ + n)/d\omega$ (Ref. 7) provide a limit on the Ch_ν-nonconservation parameter $\eta(\mathrm{II})$, viz.,

$$|\eta(\mathrm{II})| < 0.1 \quad \text{for} \quad |(p_{e^+} - p_{\nu_\mu})^2|^{1/2} < 2\ \mathrm{GeV}/c. \tag{27}$$

We proceed to treat the process [Eq. (6)]

$$\nu_\mu + p \to e^- + \Delta^{++} \tag{28}$$

which, as noted above, is forbidden by the theory of Eqs. (1)–(5) because $\Delta L_{\mathrm{tot}} = 2$ [Eq. (8)], and is also forbidden by the conventional theory because $\Delta(L_e)_{\mathrm{tot}} = 1$ and $\Delta(L_\mu)_{\mathrm{tot}} = -1$. In the theory of Eqs. (1), (3), and (14)–(16) this process is characterized by a cross section $d\sigma(\nu_\mu + p \to e^- + \Delta^{++})/d\omega$ proportional to $P(\nu_\mu \nu_e)$ and we have, using Eq. (16),

$$\frac{d\sigma(\nu_\mu + p \to e^- + \Delta^{++})/d\omega}{d\sigma(\nu_e + p \to e^- + \Delta^{++})/d\omega} = P(\nu_\mu, \nu_e)$$

$$= |\langle \nu_e(\vec{p}) | \nu_\mu(\vec{p})\rangle|^2$$

$$= \frac{|[\xi(\mathrm{II}) + \xi(\mathrm{I})][1 + \eta(\mathrm{II})\eta^*(\mathrm{I})]|^2}{[1 + |\xi(\mathrm{II})|^2][1 + |\xi(\mathrm{I})|^2][1 + |\eta(\mathrm{II})|^2][1 + |\eta(\mathrm{I})|^2]}$$

$$\cong \frac{|\xi(\mathrm{II})|^2}{[1 + |\xi(\mathrm{II})|^2][1 + |\eta(\mathrm{II})|^2]}. \tag{29}$$

Equation (29) and the experimental limit on $d\sigma(\nu_\mu + p \to e^- + \Delta^{++})/d\omega$ (Ref. 7) provide a limit on the L_{tot} -nonconservation parameter $\xi(\mathrm{II})$, viz.,

$$|\xi(\mathrm{II})| < 0.1 \quad \text{for} \quad |(p_{e^-} - p_{\nu_\mu})^2|^{1/2} < 2\ \mathrm{GeV}/c. \tag{30}$$

Continuing, we treat the process [Eq. (7)]

$$\nu_\mu + p \to \mu^+ + n, \tag{31}$$

which, as noted above, is forbidden by the theory of Eqs. (1)–(5) because $\Delta L_{\mathrm{tot}} = 2$ [Eq. (9)], and is also forbidden by the conventional theory because $\Delta(L_\mu)_{\mathrm{tot}} = -2$. In the theory of Eqs. (1), (3), and (14)–(16) this process is characterized by a cross section $d\sigma(\nu_\mu + p \to \mu^+ + n)/d\omega$ proportional to $P(\nu_\mu, \overline{\nu}_\mu)$ and we have, using Eq. (16),

$$\frac{d\sigma(\nu_\mu+p\to\mu^++n)/d\omega}{d\sigma(\bar\nu_\mu+p\to\mu^++n)/d\omega}=P(\nu_\mu,\bar\nu_\mu)$$
$$=|\langle\bar\nu_\mu(\vec{p})|\nu_\mu(\vec{p})\rangle|^2$$
$$=\frac{|[\xi(\mathrm{II})+\xi(\mathrm{I})][\eta^*(\mathrm{II})+\eta^*(\mathrm{I})]|^2}{[1+|\xi(\mathrm{II})|^2][1+|\xi(\mathrm{I})|^2][1+|\eta(\mathrm{II})|^2][1+|\eta(\mathrm{I})|^2]}$$
$$\cong\frac{|\xi(\mathrm{II})|^2|\eta(\mathrm{II})|^2}{[1+|\xi(\mathrm{II})|^2][1+|\eta(\mathrm{II})|^2]}\,. \quad (32)$$

Equations (32) and the experimental limit on $d\sigma(\nu_\mu+p\to\mu^++n)/d\omega$ (Ref. 7) provide a limit on the product of the L_{tot}-nonconservation parameter and the Ch_ν-nonconservation parameter, viz.,

$$|\xi(\mathrm{II})||\eta(\mathrm{II})|<0.1 \text{ for } |(p_{\mu^+}-p_{\nu_\mu})^2|^{1/2}<2 \text{ GeV}/c. \quad (33)$$

A comparison of Eq. (33) with Eqs. (30) and (27) shows that combination of the latter two equations provides a far more stringent limit on $|\xi(\mathrm{II})||\eta(\mathrm{II})|$ than the former equation.

It is to be noted that the $d\sigma(\nu_\mu+p\to\mu^++n)/d\omega$ of Eq. (32) is different from zero only if *both* the L_{tot}-nonconservation parameter ξ *and* the Ch_ν-nonconservation parameter η are different from zero. Thus even if $\xi=1$, which according to Eq. (14) corresponds to a Majorana neutrino and hence to a generally maximal nonconservation of L_{tot}, i.e., even with

$$l'_\alpha(x)=\psi^\dagger_{e^-}(x)\gamma_4\gamma_\alpha[(1+\gamma_5)+\eta(1-\gamma_5)]\Phi_\nu(x)$$
$$+\psi^\dagger_{\mu^-}(x)\gamma_4\gamma_\alpha[(1+\gamma_5)+\eta(1-\gamma_5)]\Phi_\nu(x), \quad (34)$$
$$\Phi_\nu(x)\equiv\psi_\nu(x)+\psi_{\bar\nu}(x)=\psi_\nu(x)+\tilde\psi^\dagger_\nu(x)=\tilde\Phi^\dagger_\nu(x),$$

$\nu_\mu+p\to\mu^++n$ is forbidden as long as Ch_ν conservation holds so that $\eta=0$. This "double forbiddenness" of $\nu_\mu+p\to\mu^++n$ is a reflection of the fact that the $(\Delta L_{\mathrm{tot}}=2)$ process

$$\pi^++p\to\mu^++\mu^++n$$
$$(\pi^+\to\mu^++\nu_\mu \text{ followed by } \nu_\mu+p\to\mu^++n) \quad (35)$$

is a process of the "double β-decay" type since the two produced leptons are identical (both are muons and both have the same charge). On the other hand, the "singly forbidden" $(\Delta L_{\mathrm{tot}}=2)$ process

$$\pi^++p\to\mu^++e^-+\Delta^{++}$$
$$(\pi^+\to\mu^++\nu_\mu \text{ followed by } \nu_\mu+p\to e^-+\Delta^{++}) \quad (36)$$

and the "singly forbidden" $(\Delta L_{\mathrm{tot}}=0)$ process

$$\pi^++p\to\mu^++e^++n$$
$$(\pi^+\to\mu^++\nu_\mu \text{ followed by } \nu_\mu+p\to e^++n) \quad (37)$$

proceed at a finite rate if, respectively, $\xi\neq0$, $\eta=0$ [Eq. (29)] and $\eta\neq0$, $\xi=0$ [Eq. (24)] – this last circumstance is a reflection of the fact that in Eq. (36) and in Eq. (37) the two produced leptons are not identical.

We also note that the calculated rates of $(\Delta L_{\mathrm{tot}}=2)$ nuclear double β decays such as $\mathrm{Te}^{130}\to e^-+e^-+\mathrm{Xe}^{130}$ ($\mathrm{Te}^{130}\to e^-+\bar\nu_e+I_0^{130},I_1^{130},\ldots$ followed by $\bar\nu_e+I_0^{130},I_1^{130},\ldots\to e^-+\mathrm{Xe}^{130}$, the intermediate $\bar\nu_e$ and $I_0^{130},I_1^{130},\ldots$ being virtual) are proportional to

$$P(\bar\nu_e,\nu_e)=|\langle\nu_e(\vec{p})|\bar\nu_e(\vec{p})\rangle|^2$$
$$=\frac{|4\xi\eta|^2}{[(1+|\xi|^2)(1+|\eta|^2)]^2} \quad (38)$$

[see Eq. (16) and compare with Eq. (35) and Eq. (32)] and when taken together with the corresponding measured lifetimes (e.g., the measured value for the *sum* of the $\mathrm{Te}^{130}\to e^-+e^-+\mathrm{Xe}^{130}$ rate *and* the $\mathrm{Te}^{130}\to e^-+e^-+\bar\nu_e+\bar\nu_e+\mathrm{Xe}^{130}$ rate) yield the limit[8]

$$|\xi||\eta|\lesssim10^{-4} \text{ for } |(p_{\bar\nu_e}+p_{e^-})^2|^{1/2}<60 \text{ MeV}/c. \quad (39)$$

Equations (18), (22), (27), (30), (33), and (39) show that the L_{tot}-nonconservation parameter ξ and the Ch_ν-nonconservation parameter η are no larger than a few percent at momentum transfers $\lesssim m_p$.

We conclude our discussion of the various processes initiated by a neutrino incident on a proton with a comment on the process [Eq. (12)]

$$\nu_\mu+p\to\mu^-+\Delta^{++}. \quad (40)$$

This process is allowed by the theory of Eqs. (1)–(5) because $\Delta L_{\mathrm{tot}}=0$ and Ch_ν conservation is satisfied and it is also allowed by the conventional theory because $\Delta(L_e)_{\mathrm{tot}}=0$ and $\Delta(L_\mu)_{\mathrm{tot}}=0$. In the theory of Eqs. (1), (3), and (14)–(16), this process is again allowed, and, in fact, is characterized by a cross section independent of ξ and η since $P(\nu_\mu,\nu_\mu)=|\langle\nu_\mu(\vec{p})|\nu_\mu(\vec{p})\rangle|^2=1$.

IV. FURTHER CONSEQUENCES OF THE GENERALIZED WEAK CURRENT

We proceed to treat several additional second-order weak processes from the point of view of the theory of Eqs. (1), (3), and (14)–(16). These second-order weak processes, just as the processes of Eqs. (35)–(37), may be viewed as consisting of a sequence of two first-order weak processes in the first of which an intermediate neutrino is emitted and in the second of which it is reabsorbed. In these cases, however, in contradistinction to the cases of Eqs. (35)–(37) and as in the case of ($\Delta L_{\rm tot}=2$) nuclear double β decay, the intermediate neutrino is, in general, virtual rather than real.

The additional second-order weak processes in question are the "doubly forbidden" ($\Delta L_{\rm tot}=\pm 2$) processes [compare with Eq. (35) and Eq. (32)]

$$K^+\to\begin{Bmatrix}\mu^++\mu^++\pi^-\\ e^++e^++\pi^-\end{Bmatrix}\quad\left(K^+\to\begin{Bmatrix}\mu^++\nu_\mu+\pi^0,\rho^0,\dots\\ e^++\nu_e+\pi^0,\rho^0,\dots\end{Bmatrix}\text{ followed by }\begin{Bmatrix}\nu_\mu+\pi^0,\rho^0,\dots\to\mu^++\pi^-\\ \nu_e+\pi^0,\rho^0,\dots\to e^++\pi^-\end{Bmatrix}\right),\tag{41}$$

$$\Sigma^-\to\begin{Bmatrix}\mu^-+\mu^-+p\\ e^-+e^-+p\end{Bmatrix}\quad\left(\Sigma^-\to\begin{Bmatrix}\mu^-+\bar\nu_\mu+n,\Delta^0,\dots\\ e^-+\bar\nu_e+n,\Delta^0,\dots\end{Bmatrix}\text{ followed by }\begin{Bmatrix}\bar\nu_\mu+n,\Delta^0,\dots\to\mu^-+p\\ \bar\nu_e+n,\Delta^0,\dots\to e^-+p\end{Bmatrix}\right),\tag{42}$$

the "singly forbidden" ($\Delta L_{\rm tot}=\pm 2$) processes [compare with Eq. (36) and Eq. (29)]

$$K^+\to\begin{Bmatrix}\mu^++e^-+\pi^+\\ e^++\mu^-+\pi^+\end{Bmatrix}\quad\left(K^+\to\begin{Bmatrix}\mu^++\nu_\mu+\pi^0,\rho^0,\dots\\ e^++\nu_e+\pi^0,\rho^0,\dots\end{Bmatrix}\text{ followed by }\begin{Bmatrix}\nu_\mu+\pi^0,\rho^0,\dots\to e^-+\pi^+\\ \nu_e+\pi^0,\rho^0,\dots\to\mu^-+\pi^+\end{Bmatrix}\right),\tag{43}$$

$$\Sigma^+\to\begin{Bmatrix}\mu^-+e^++p\\ e^-+\mu^++p\end{Bmatrix}\quad\left(\Sigma^+\to\begin{Bmatrix}\mu^-+\bar\nu_\mu+\Delta^{++},\{\pi^++p\},\dots\\ e^-+\bar\nu_e+\Delta^{++},\{\pi^++p\},\dots\end{Bmatrix}\text{ followed by }\begin{Bmatrix}\bar\nu_\mu+\Delta^{++},\{\pi^++p\},\dots\to e^++p\\ \bar\nu_e+\Delta^{++},\{\pi^++p\},\dots\to\mu^++p\end{Bmatrix}\right),\tag{44}$$

$$\mu^+\to e^++\gamma\quad(\mu^+\to\bar\nu_\mu+\pi^+,\rho^+,\dots\text{ followed by }\bar\nu_\mu+\pi^+,\rho^+,\dots\to e^++\gamma),\tag{45}$$

and the "singly forbidden" ($\Delta L_{\rm tot}=0$) processes [compare with Eq. (37) and Eq. (24)]

$$K^+\to\mu^++e^++\pi^-\quad(K^+\to\mu^++\nu_\mu+\pi^0,\rho^0,\dots\text{ followed by }\nu_\mu+\pi^0,\rho^0,\dots\to e^++\pi^-),\tag{46}$$

$$\Sigma^-\to\mu^-+e^-+p\quad(\Sigma^-\to\mu^-+\bar\nu_\mu+n,\Delta^0,\dots\text{ followed by }\bar\nu_\mu+n,\Delta^0,\dots\to e^-+p),\tag{47}$$

$$\mu^-+(A,Z)\to e^++(A,Z-2)$$
$$[\mu^-+(A,Z)\to\nu_\mu+(A,Z-1)_0,(A,Z-1)_1,\dots\text{ followed by }\nu_\mu+(A,Z-1)_0,(A,Z-1)_1,\dots\to e^++(A,Z-2)].\tag{48}$$

The predicted rates of these second-order weak processes, in comparison to the rates of first-order weak processes with approximately the same phase space, are [compare Eq. (32)]

$$\frac{\Gamma\left(K^+\to\begin{Bmatrix}\mu^++\mu^++\pi^-\\ e^++e^++\pi^-\end{Bmatrix}\right)}{\Gamma\left(K^+\to\begin{Bmatrix}\mu^++\nu_\mu+\pi^0\\ e^++\nu_e+\pi^0\end{Bmatrix}\right)}\approx\frac{\Gamma\left(\Sigma^-\to\begin{Bmatrix}\mu^-+\mu^-+p\\ e^-+e^-+p\end{Bmatrix}\right)}{\Gamma\left(\Sigma^-\to\begin{Bmatrix}\mu^-+\bar\nu_\mu+n\\ e^-+\bar\nu_e+n\end{Bmatrix}\right)}\approx\left(\frac{G}{\sqrt2}\frac{m_p^2}{4\pi^2}\right)^2\frac{|4\xi\eta|^2}{(1+|\xi|^2)^2(1+|\eta|^2)^2}<6\times10^{-17},\tag{49}$$

where $G=10^{-5}/m_p^2$ is the universal weak coupling constant and $|\xi|<0.1$, $|\eta|<0.1$, for $|(p_{\nu_{\mu,e}}+p_{\mu,e})^2|^{1/2}\lesssim m_p$ [see Eqs. (30) and (27)]. Similarly [compare Eq. (24)],

$$\frac{\Gamma(K^+\to\mu^++e^++\pi^-)}{\Gamma(K^+\to\mu^++\nu_\mu+\pi^0)}\approx\frac{\Gamma(\Sigma^-\to\mu^-+e^-+p)}{\Gamma(\Sigma^-\to\mu^-+\bar\nu_\mu+n)}\approx\frac{\sigma(\mu^-+(A,Z)\to e^++(A,Z-2))}{\sigma(\mu^-+(A,Z)\to\nu_\mu+(A,Z-1))}\approx\left(\frac{G}{\sqrt2}\frac{m_p^2}{4\pi^2}\right)^2\left(\frac{4|\eta|^2}{(1+|\eta|^2)^2}\right)<2\times10^{-15}.\tag{50}$$

Further [compare Eq. (29)],

$$\frac{\Gamma\left(K^+\to\begin{Bmatrix}\mu^++e^-+\pi^+\\ e^++\mu^-+\pi^+\end{Bmatrix}\right)}{\Gamma\left(K^+\to\begin{Bmatrix}\mu^++\nu_\mu+\pi^0\\ e^++\nu_e+\pi^0\end{Bmatrix}\right)}\approx\frac{\Gamma\left(\Sigma^+\to\begin{Bmatrix}\mu^-+e^++p\\ e^-+\mu^++p\end{Bmatrix}\right)}{\Gamma\left(\Sigma^-\to\begin{Bmatrix}\mu^-+\bar\nu_\mu+p\\ e^-+\bar\nu_e+p\end{Bmatrix}\right)}\approx\left(\frac{G}{\sqrt2}\frac{\Lambda^2}{4\pi^2}\right)^2\left(\frac{4|\xi|^2}{(1+|\xi|^2)^2}\right)\cong\left(\frac{G}{\sqrt2}\frac{\Lambda^2}{4\pi^2}\right)^2=8\times10^{-11}\tag{51}$$

and

$$\frac{\Gamma(\mu^+\to e^+ +\gamma)}{\Gamma(\mu^+\to e^+ +\nu_e+\bar{\nu}_\mu)} \approx \frac{\alpha\left(\frac{G}{\sqrt{2}}\frac{\Lambda^2}{4\pi^2}\right)^2(4\pi)^{-1}}{(96\pi^3)^{-1}}\,\frac{4|\xi|^2}{(1+|\xi|^2)^2}$$

$$\cong \frac{\alpha\left(\frac{G}{\sqrt{2}}\frac{\Lambda^2}{4\pi^2}\right)^2(4\pi)^{-1}}{(96\pi^3)^{-1}} = 2\times 10^{-10}, \quad (52)$$

where Λ, the cutoff momentum of the semileptonic weak interactions, has been taken as $7m_p$ on the basis of the experimental limit on $\Gamma(K_L^0\to\mu^+ +\mu^-)$ (Ref. 9) and where, in accord with the idea that a radical change in the structure of the weak interactions is likely to occur at momentum transfers (and energies) $\gtrsim$ the cutoff momentum, ξ is assumed to be close to unity at

$$|(p_{\nu_{\mu,e}}\pm p_{\mu,e})^2|^{1/2}\approx\Lambda\approx 7m_p\,.$$

We note that the amplitudes for the second-order weak processes of Eqs. (41), (42), and (46)–(48), in which the two emitted leptons have the *same* charge, converge, and are characterized by a "natural cutoff" [$\approx$(hadron diameter)$^{-1}\approx\frac{1}{2}m_p - m_p$] in the summation over the momenta of the intermediate virtual-neutrino states.[8] On the other hand, the second-order weak processes in Eqs. (43)–(45), where the two emitted leptons have *opposite* charge ($\mu^+\to e^+ +\gamma$ is equivalent to $\gamma\to\mu^+ + e^-$), are associated with a potentially divergent amplitude arising from the summation over the momenta of the intermediate virtual-neutrino states, and this divergence is avoided only by explicit introduction of the cutoff Λ.[8] The branching ratios in Eqs. (49)–(52) are as small as they are essentially because the processes in Eqs. (41)–(48) are supposed to be second-order weak; in spite of this, the theoretical estimate of the branching ratio of

$$\frac{\Gamma(\mu^+\to e^+ +\gamma)}{\Gamma(\mu^+\to e^+ +\nu_e+\bar{\nu}_\mu)}$$

in Eq. (52) is only about 25 times smaller than the corresponding experimental limit.[10] Thus, a further search for $\mu^+\to e^+ +\gamma$ would be worthwhile.[11]

Finally, a special comment may be made with regard to the ($\Delta L_{\rm tot}=0$) processes of Eqs. (46)–(48). Kisslinger[12] has proposed that the weak-interaction Hamiltonian contains not only the usual (singly charged current)$\times$(singly charged current) terms

$$\frac{G}{\sqrt{2}}\{l_\alpha^\dagger(x)l_\alpha(x)+[l_\alpha^\dagger(x)h_\alpha(x)+h_\alpha^\dagger(x)l_\alpha(x)]+\cdots\}$$

[$l_\alpha(x)$ as in Eqs. (4) and (5) or, from our point of view, $l_\alpha(x)\to l'_\alpha(x)$ of Eq. (14), $\langle 0|h_\alpha(x)|\pi^+\rangle\neq 0$, $\langle n|h_\alpha(x)|p\rangle\neq 0$, etc.] but also a (doubly charged current)$\times$(doubly charged current) term

$$\frac{G_{\rm doub}}{\sqrt{2}}[\lambda_\alpha^\dagger(x)k_\alpha(x)+k_\alpha^\dagger(x)\lambda_\alpha(x)], \quad (53)$$

with

$$\lambda_\alpha(x)=\left[\left(\frac{1+\gamma_5}{\sqrt{2}}\right)\psi_{e^-}(x)\right]^\dagger\gamma_4\gamma_\alpha\left[\left(\frac{1+\gamma_5}{\sqrt{2}}\right)\psi_{\mu^+}(x)\right]$$
$$=\psi_{e^-}^\dagger(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\mu^+}(x)$$
$$=\psi_{e^-}^\dagger(x)\gamma_4\gamma_\alpha(1+\gamma_5)\tilde{\psi}_{\mu^-}^\dagger(x), \quad (54)$$
$$\langle\pi^-|k_\alpha(x)|K^+\rangle\neq 0,\quad \langle\Delta^-|k_\alpha(x)|p\rangle\neq 0,$$
$$\langle\Sigma^-|k_\alpha(x)|p\rangle\neq 0,\quad \text{etc.}$$

From Eqs. (53) and (54) one has, instead of Eq. (50),

$$\frac{\Gamma(K^+\to\mu^+ + e^+ +\pi^-)}{\Gamma(K^+\to\mu^+ +\nu_\mu+\pi^0)}\approx\frac{\Gamma(\Sigma^-\to\mu^- + e^- + p)}{\Gamma(\Sigma^-\to\mu^- +\bar{\nu}_\mu+n)}$$
$$\approx\frac{\sigma(\mu^- + p\to e^+ +\Delta^-)}{\sigma(\mu^- + p\to\nu_\mu+n)}$$
$$\approx\left(\frac{G_{\rm doub}}{G}\right)^2 \quad (55)$$

if it is assumed that, at similar momentum transfer,

$$\left|\frac{\langle\pi^-|k_\alpha|K^+\rangle}{\langle\pi^0|h_\alpha|K^+\rangle}\right|\approx\left|\frac{\langle p|k_\alpha^\dagger|\Sigma^-\rangle}{\langle n|h_\alpha^\dagger|\Sigma^-\rangle}\right|$$
$$\approx\left|\frac{\langle\Delta^-|k_\alpha|p\rangle}{\langle n|h_\alpha|p\rangle}\right|\approx 1. \quad (56)$$

Further,

$$\frac{\sigma(\mu^- +(A,Z)\to e^+ +(A,Z-2))}{\sigma(\mu^- +(A,Z)\to\nu_\mu+(A,Z-1))}$$
$$\approx\frac{\sigma(\mu^- + p\to e^+ +\Delta^-)}{\sigma(\mu^- + p\to\nu_\mu+n)}\,\mathcal{P}(\Delta^-)$$
$$+\frac{\sigma(\mu^- +\Delta^{++}\to e^+ + n)}{\sigma(\mu^- + p\to\nu_\mu+n)}\,\mathcal{P}(\Delta^{++})$$
$$\approx\frac{\sigma(\mu^- + p\to e^+ +\Delta^-)}{\sigma(\mu^- + p\to\nu_\mu+n)}[\mathcal{P}(\Delta^-)+\mathcal{P}(\Delta^{++})], \quad (57)$$

where $\mathcal{P}(\Delta^-)$ is the probability that a Δ^- is present in the nucleus $(A,Z-2)$ and $\mathcal{P}(\Delta^{++})$ is the probability that a Δ^{++} is present in the nucleus (A,Z). $\mathcal{P}(\Delta^-)$ and $\mathcal{P}(\Delta^{++})$ can be estimated as[12]

$$\mathcal{P}(\Delta^-)\approx\mathcal{P}(\Delta^{++})\approx 10^{-2} \quad (58)$$

so that, combining Eqs. (54), (55), (57), and (58),

$$\frac{\sigma(\mu^- +(A,Z)\to e^+ +(A,Z-2))}{\sigma(\mu^- +(A,Z)\to\nu_\mu+(A,Z-1))}\approx\left(\frac{G_{\rm doub}}{G}\right)^2\times 10^{-2}. \quad (59)$$

We now argue that the ($\Delta L_{\rm tot}=0$) processes of Eqs. (46)–(48), which we are here considering as first order in $G_{\rm doub}$, should be inhibited, relative to the allowed first-order processes in G such as

$K^+\to\mu^++\nu_\mu+\pi^0$, $\Sigma^-\to\mu^-+\bar{\nu}_\mu+n$, and $\mu^-+(A,Z)\to\nu_\mu+(A,Z-1)$, by a chirality conservation principle similar to the neutrino-chirality conservation principle that inhibits the process $\nu_\mu+p\to e^++n$ relative to the allowed process $\nu_e+p\to e^++n$ [compare the discussion associated with Eqs. (23)–(27) with that associated with Eq. (40)]. The inhibition in question can be effected by replacing Eqs. (53) and (54) by

$$\frac{G}{\sqrt{2}}\{[\lambda'_\alpha(x)]^\dagger k_\alpha(x)+k^\dagger_\alpha(x)\lambda'_\alpha(x)\}, \tag{60}$$

with

$$\begin{aligned}\lambda'_\alpha(x)&=\{\tfrac{1}{2}[(1+\gamma_5)+\delta^*(1-\gamma_5)]\psi_{e^-}(x)\}^\dagger\\&\quad\times\gamma_4\gamma_\alpha\{\tfrac{1}{2}[(1-\gamma_5)+\delta(1+\gamma_5)]\psi_{\mu^+}(x)\}\\&=\delta[\psi^\dagger_{e^-}(x)\gamma_4\gamma_\alpha\psi_{\mu^+}(x)]\\&=\delta[\psi^\dagger_{e^-}(x)\gamma_4\gamma_\alpha\bar{\psi}^\dagger_{\mu^-}(x)]\end{aligned} \tag{61}$$

so that $G_{\rm doub}/G$ corresponds to δ. Equations (60), (61) and (57), (58) yield

$$\begin{aligned}\frac{\Gamma(K^+\to\mu^++e^++\pi^-)}{\Gamma(K^+\to\mu^++\nu_\mu+\pi^0)}&\approx\frac{\Gamma(\Sigma^-\to\mu^-+e^-+p)}{\Gamma(\Sigma^-\to\mu^-+\bar{\nu}_\mu+n)}\\&\approx\frac{\sigma(\mu^-+p\to e^++\Delta^-)}{\sigma(\mu^-+p\to\nu_\mu+n)}\\&\approx10^2\frac{\sigma(\mu^-+(A,Z)\to e^++(A,Z-2))}{\sigma(\mu^-+(A,Z)\to\nu_\mu+(A,Z-1))}\\&\approx|\delta|^2\end{aligned} \tag{62}$$

which, with the experimental limit on (see Ref. 13)

$$\frac{\sigma(\mu^-+(A,Z)\to e^++(A,Z-2))}{\sigma(\mu^-+(A,Z)\to\nu_\mu+(A,Z-1))},$$

gives

$$|\delta|<10^{-3}\ \text{for}\ |(p_{e^+}-p_{\mu^-})^2|^{1/2}<100\ \text{MeV}/c, \tag{63}$$

whence

$$\begin{aligned}\frac{\Gamma(K^+\to\mu^++e^++\pi^-)}{\Gamma(K^+\to\mu^++\nu_\mu+\pi^0)}&\approx\frac{\Gamma(\Sigma^-\to\mu^-+e^-+p)}{\Gamma(\Sigma^-\to\mu^-+\bar{\nu}_\mu+n)}\\&\approx\frac{\sigma(\mu^-+p\to e^++\Delta^-)}{\sigma(\mu^-+p\to\nu_\mu+n)}\\&\approx10^2\frac{\sigma(\mu^-+(A,Z)\to e^++(A,Z-2))}{\sigma(\mu^-+(A,Z)\to\nu_\mu+(A,Z-1))}\\&<10^{-6}.\end{aligned} \tag{64}$$

The limit in Eq. (64) is to be compared with the limit in Eq. (50).

V. FINAL COMMENTS AND CONCLUSIONS

In the theory of Eqs. (1)–(5), and also in the conventional theory, the trileptonic (hadronically inclusive) production process

$$\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A) \tag{65}$$

is allowed [$\Delta L_{\rm tot}=0$, $\Delta(L_e)_{\rm tot}=0$, $\Delta(L_\mu)_{\rm tot}=0$], while, for example, the trileptonic (hadronically inclusive) production process

$$\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A) \tag{66}$$

is forbidden [$\Delta L_{\rm tot}=2$, $\Delta(L_e)_{\rm tot}=1$, $\Delta(L_\mu)_{\rm tot}=-1$]. However in the theory of Eqs. (1), (3), and (14)–(16), the process of Eq. (66) is characterized by a cross section $d\sigma(\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A))/d\omega$ proportional to $P(\nu_\mu,\nu_e)$ so that [compare Eqs. (29) and (30)]

$$\begin{aligned}\frac{d\sigma(\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega}&=P(\nu_\mu,\nu_e)=|\langle\nu_e(\vec{p})|\nu_\mu(\vec{p})\rangle|^2\\&=\frac{|[\xi(\text{II})+\xi(\text{I})][1+\eta(\text{II})\eta^*(\text{I})]|^2}{[1+|\xi(\text{II})|^2][1+|\xi(\text{I})|^2][1+|\eta(\text{II})|^2][1+|\eta(\text{I})|^2]}\\&\cong\frac{|\xi(\text{II})|^2}{[1+|\xi(\text{II})|^2][1+|\eta(\text{II})|^2]}.\end{aligned} \tag{67}$$

Thus, with [see discussion after Eq. (52)]

$$\xi(\text{II})\cong\eta(\text{II})\cong1\ \text{for}\ |(p_{e^-}-p_{\nu_\mu})^2|^{1/2}\gtrsim7m_p, \tag{68}$$

we have

$$\frac{d\sigma(\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega}\cong\frac{1}{4},\quad|(p_{e^-}-p_{\nu_\mu})^2|^{1/2}\gtrsim7\ \text{GeV}/c, \tag{69}$$

while with [see Eqs. (30) and (27)]

$$|\xi(\text{II})|<0.1\ \text{and}\ |\eta(\text{II})|<0.1\ \text{for}\ |(p_{e^-}-p_{\nu_\mu})^2|^{1/2}<2m_p \tag{70}$$

we have

$$\frac{d\sigma(\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega}<10^{-2},\quad |(p_{e^-}-p_{\nu_\mu})^2|^{1/2}<2\text{ GeV}/c. \tag{71}$$

These predictions are to be compared with the prediction of the "multiplicative" lepton-number theory[14] based on

$$H_{\text{leptonic weak}}(x)=\frac{G}{2\sqrt{2}}\{[\psi^\dagger_{\mu^-}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\nu_\mu}(x)][\psi^\dagger_{\nu_e}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{e^-}(x)]$$
$$+[\psi^\dagger_{\overline{\nu}_\mu}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\mu^-}(x)][\psi^\dagger_{e^-}(x)\gamma_4\gamma_\alpha(1+\gamma_5)\psi_{\overline{\nu}_e}(x)]+\text{H.c.}\}$$

with its implication of the conservation of $(L_e)_{\text{tot}}+(L_\mu)_{\text{tot}}$ and of $(+1)^{(L_e)_{\text{tot}}}(-1)^{(L_\mu)_{\text{tot}}}$ in all processes, namely,

$$\frac{d\sigma(\nu_\mu+(Z,A)\to\mu^++e^-+\nu_e+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega}=1 \tag{72}$$

for all momentum transfers. We also wish to emphasize that nonobservation of the process $\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A)$ [Eqs. (66)–(71)] or, equivalently, nonobservation of the process

$$\nu_\mu+p\to e^-+\Delta^{++}\quad\text{[Eqs. (28)–(30)]}$$

or of the processes

$$K^+\to\mu^\pm+e^\mp+\pi^+,$$
$$\Sigma^+\to\mu^\mp+e^\pm+p,$$
$$\mu^+\to e^++\gamma\qquad\text{[Eqs. (43)–(45) and (51)–(52)]}$$

constitutes unequivocal proof of the vanishing of the L_{tot}-nonconservation parameter ξ and so of the conservation of total lepton number, independent of any restrictions which arise from the conservation of neutrino chirality (i.e., which arise from $\eta=0$).

In conclusion, we may summarize the essential features of our theory [as specified in Eqs. (1), (3), and (14)–(16)] in the following way:

(1) Only three distinct lepton field operators, namely, $\psi_{e^-}(x)$, $\psi_{\mu^-}(x)$, $\psi_\nu(x)$, with each effecting the destruction (creation) of a particle (antiparticle) of lepton number 1 (−1), enter into the leptonic weak current $l'_\alpha(x)$.

(2) $l'_\alpha(x)$ is such that neither L_{tot} conservation nor Ch_ν conservation hold exactly – as a result, various leptonic and semileptonic weak processes forbidden in the conventional theory, e.g., $\nu_\mu+p\to e^++n$ and $\mu^+\to e^++\gamma$, proceed at a finite rate determined by the values of the L_{tot}-nonconservation parameter ξ and the Ch_ν-nonconservation parameter η. Moreover, the noninvariance of $l'_\alpha(x)$ under

$$\psi_\nu(x)+\xi\psi_{\overline{\nu}}(x)\to\gamma_5[\psi_\nu(x)+\xi\psi_{\overline{\nu}}(x)],$$
$$\psi_{e^-}(x)\to\psi_{e^-}(x),$$
$$\psi_{\mu^-}(x)\to-\psi_{\mu^-}(x),$$

i.e., Ch_ν nonconservation, implies that $m_\nu=m_{\nu_e}=m_{\nu_\mu}\neq0$ unless $(m_\nu)_{\text{bare}}$ just cancels the $(m_\nu)_{\text{self}}$ induced by H_{weak}.[15]

(3) ξ and η are assumed to increase with increasing interlepton momentum transfer and to reach unity at momentum transfers ≈ cutoff momentum of semileptonic weak interactions ≈7 GeV/c.

(4) The $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\overline{\nu}_e\rangle$, and $|\overline{\nu}_\mu\rangle$ states produced in such large-momentum-transfer processes are identical and are characterized by vanishing lepton number and helicity expectation values. Similarly, charged lepton states produced in conjunction with these neutrino or antineutrino states, e.g., the $|e^+\rangle$ state in

$$\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A)\quad\text{[Eq. (65)]}$$

and the $|\mu^+\rangle$ state in

$$\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A)\quad\text{[Eq. (66)]},$$

are characterized by vanishing helicity expectation values.

(5) The pairs of successive processes,

$$\nu_\mu\to e^++(n+\overline{p}),\pi^-,\ldots\to\overline{\nu}_e,$$

or

$$\nu_\mu\to e^-+(\Delta^{++}+\overline{p}),\pi^+,\ldots\to\nu_e,$$

or

$$\nu_\mu\to\mu^++(n+\overline{p}),\pi^-,\ldots\to\overline{\nu}_\mu$$

[Eqs. (23), (28), (31), and Eq. (3)],

where the intermediate lepton-hadron states are virtual, provide a mechanism for Pontecorvo's "neutrino oscillations."[16] The frequency of these oscillations (in the neutrino rest frame) should be $\approx(m_\nu)_{\text{self}}$ so that the oscillation length (in the laboratory frame) is

$$\approx\left(\frac{1}{(m_\nu)_{\text{self}}}\right)\left(\frac{E_\nu}{(m_\nu)_{\text{self}}}\right)\approx10^{20}\frac{1}{m_\mu}\frac{E_\nu}{m_\mu}\quad\text{(Ref. 15)}$$
$$=2\times10^3\text{ km}$$

for $E_\nu=1$ GeV.

(6) The cross section of the leptonic scattering process $\nu_\mu+e^-\to e^-+\nu_e$ is given by

$$\frac{d\sigma(\nu_\mu+e^-\to e^-+\nu_e)/d\omega}{d\sigma(\nu_\mu+e^-\to\mu^-+\nu_e)/d\omega} \cong \frac{d\sigma(\nu_\mu+(Z,A)\to e^-+e^++\nu_e+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega} \cong \frac{d\sigma(\nu_\mu+(Z,A)\to e^-+\mu^++\nu_\mu+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega}.$$

Thus our theory predicts that

$$\frac{d\sigma(\nu_\mu+e^-\to e^-+\nu_e)/d\omega}{d\sigma(\nu_\mu+e^-\to\mu^-+\nu_e)/d\omega} \cong \frac{d\sigma(\nu_\mu+(Z,A)\to e^-+e^++\nu_e+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega}$$

increases with increasing momentum transfer and becomes as large as $\cong\frac{1}{4}$ for

$$|(p_{e^-}-p_{\nu_\mu})^2|^{1/2}\gtrsim 7\text{ GeV}/c \text{ [Eqs. (69) and (71)]}.$$

In sharp contrast, the conventional theory forbids

$$\nu_\mu+e^-\to e^-+\nu_e \quad [\text{and } \nu_\mu+(Z,A)\to e^-+e^++\nu_e+(Z,A)]$$

and allows

$$\nu_\mu+e^-\to e^-+\nu_\mu \quad [\text{and } \nu_\mu+(Z,A)\to e^-+e^++\nu_\mu+(Z,A)]$$

only in the second-order weak approximation with a predicted cross section

$$\frac{d\sigma(\nu_\mu+e^-\to e^-+\nu_\mu)/d\omega}{d\sigma(\nu_\mu+e^-\to\mu^-+\nu_e)/d\omega} \cong \frac{d\sigma(\nu_\mu+(Z,A)\to e^-+e^++\nu_\mu+(Z,A))/d\omega}{d\sigma(\nu_\mu+(Z,A)\to\mu^-+e^++\nu_e+(Z,A))/d\omega} \approx\left(\frac{G}{\sqrt{2}}\frac{\Lambda^2}{4\pi^2}\right)^2\approx\left(\frac{G}{\sqrt{2}}\frac{(7m_p)^2}{4\pi^2}\right)^2\cong 10^{-10}$$

for all momentum transfers.[17]

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[2]See especially the papers of Bludman (Ref. 1) and the paper of Kabir (Ref. 1).

[3]H. Primakoff and S. P. Rosen, Phys. Rev. 184, 1925 (1969); H. Primakoff and D. Sharp, Phys. Rev. Letters 23, 501 (1969).

[4]See, e.g., W. J. Willis and J. Thompson, *Advances in Physics* (Wiley, New York, 1968).

[5]See, e.g., R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Interscience, New York, 1969).

[6]See also the discussion of this process in Pilkuhn (Ref. 1), and the comments of Bludman (Ref. 1), and of L. B. Okun, in Proceedings of the International Conference on High Energy Physics, CERN, Geneva, Switzerland, 1962 (unpublished).

[7]See, e.g., D. H. Perkins, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN, Geneva, 1969).

[8]See the discussion by Primakoff and Rosen (Ref. 3).

[9]$[\Gamma(K_L^0\to\mu^++\mu^-)]_{\rm exp}<0.04\text{ sec}^{-1}$ as given in A. R. Clark, T. Elioff, R. C. Field, H. J. Frisch, R. P. Johnson, L. T. Kerth, and W. A. Wenzel, Phys. Rev. Letters 26, 1667 (1971). This corresponds to $\Lambda\lesssim 7m_p$; see, e.g., H. Primakoff, in *Lectures on Elementary Particles and Quantum Field Theory, 1970,* Brandeis University Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1971), Vol. II.

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[11]P. Minkowski (SIN report, 1971) presents an estimate of $\Gamma(\mu^+\to e^++\gamma)$ which is much smaller than that of Eq. (52) and which is essentially obtainable from that of Eq. (52) by replacing $\Lambda=7m_p$ by m_μ and $4|\xi|^2/(1+|\xi|^2)^2\cong 1$ by $(10^{-3})^2$. Minkowski's calculation employs the Bogoliubov renormalization technique and considers that μ^+ goes to $e^++\gamma$ via intermediate virtual trileptonic states only (in our formalism via $\mu^+\to\bar\nu_\mu+e^++\nu_e\to e^+\to e^++\gamma$, etc.); on the other hand, in the calculation of Eqs. (45) and (52), μ^+ goes to $e^++\gamma$ via the (presumed dominant) path of intermediate virtual $\bar\nu_\mu$-hadron states [see Eq. (45)] and an explicit cutoff momentum is introduced.

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$$\approx\left(\frac{G}{\sqrt{2}}\frac{\Lambda^2}{4\pi^2}\right)^2(\mathrm{Re}\,\eta)\approx\left(\frac{G}{\sqrt{2}}\frac{(7m_p)^2}{4\pi^2}\right)^2\cong 10^{-10}.$$

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[17]See, e.g., Primakoff (Ref. 9).