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Time-Reversal Invariance and the Phase of the ρ-ω Mixing*

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We use the unitarity sum to rederive in a simple way an explicit formula for the ρ-ω mixing phase in terms of $|\epsilon|$, Γ_ω, Γ_ρ, and $m_\omega^2 - m_\rho^2$. We then present a systematic study of the condition imposed by T (or CPT) invariance and unitarity on the relative phase and strength of the mixing. It is shown that the departure from T invariance in the ρ-ω system is, in principle, directly demonstrable through the measurement of the ρ-ω mixing phase in $e^+e^- \to \pi^+\pi^-$. The connection between the phase discrepancy and the breakdown of microscopic reversibility is also discussed. Finally, two possible graphical representations for the ρ-ω mixing parameters which exhibit directly the condition imposed by unitarity and time-reversal invariance on the relative phases and strengths of the ρ-ω mixing are given.

I. INTRODUCTION

The discovery of the G-parity-violating effect[1,2] $\omega \to 2\pi$ seen as an interference dip in the ρ-ω spectrum has reactivated the problem of ρ-ω mixing in recent years.[3-5] In this paper, we wish to report a systematic study of the condition imposed by time-reversal invariance and unitarity on the phase and strength of the ρ-ω mixing and present a simple method of calculating the mixing phase from the so-called unitarity sum.

Although the problem of ρ-ω mixing has often been claimed[4] to be equivalent to the neutral-kaon-decay problem and the phase of the ρ-ω mixing has been estimated previously,[3,5] nevertheless, we feel that the connection between the two problems has not yet been thoroughly explored and the role of the unitarity sum in the determination of the ρ-ω mixing phase has not yet been fully exploited. Here, we wish to reveal gradually through our discussion this close connection between the two mixing problems and then use the Bell-Steinberger unitarity sum to obtain an explicit formula for the ρ-ω mixing phase in terms of the ρ and ω masses and widths, assuming CPT invariance. The result obtained by us as well as those reported recently[3] seem to show that the theoretical estimated phase of the ρ-ω mixing is not in excellent agreement with its earlier experimental value.[6] There has been an attempt[5] to correct this discrepancy by considering (i) not only the mass mixing of ρ and ω vector mesons but also their vector mixing[7] and (ii) the energy dependence of the ρ width, but the situation is not improved much. On the contrary, the new estimate might even widen the discrepancy, taking into account the fact that in the latter estimate, the ρ-ω mixing phase may be smaller than 112° which is the value estimated in Ref. 3. We are, therefore, content with the mass mixing alone. In view of this difficulty, we believe that it would be interesting to consider a detailed investigation of the condition imposed by unitarity and time-reversal invariance on the mixing phase. This condition might serve as a guideline for future experiments measuring the phase concerned. The results of this investigation will be reported in a subsequent section of this paper. Besides, we show that the departure from T invariance in the electromagnetic interaction would, in principle, be directly demonstrable through the measured phase of the ρ-ω mixing in $e^+e^- \to \pi^+\pi^-$ alone, should the accuracy of the determination of that phase be improvable and that the time-reversal asymmetry estimated with the earlier measured phase of the Orsay group[1,6] seems to be significant. These arguments can be made much more easily by using the unitarity sum of the ρ-ω mixing. We are then tempted to connect the phase discrepancy in the ρ-ω mixing, should this discrepancy ever exist, with the breakdown of a microscopic reversibility.

Finally, a plot of all the results above in a graph to give an off-hand look at the situation would be very desirable.[8] We shall discuss the graphical representation of time-reversal invariance when only the relative phase and strength of the ρ-ω mixing in $e^+e^- \to \pi^+\pi^-$ is accessible to good measurements. Also given is the graphical representation for the case when the relative phases and strengths of the ρ-ω mixing in both $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to 3\pi$ are assumed to be determinable from experiments. The latter is somewhat less practical and more academically oriented because of the difficulty in the experiment of $e^+e^- \to 3\pi$ at least to our knowledge. Nevertheless, we still like to consider it as an alternative graphical representation of the ρ-ω mixing because it also yields equivalent information to the former about the conditions imposed by unitarity and T invariance in the ρ-ω mixing.

II. ALTERNATIVE METHOD OF DERIVATION OF THE ρ-ω MIXING PHASE

While in the problem of neutral kaon decay, the CP noninvariance effect mixes states of opposite CP eigenvalues, here, in the ρ-ω system, the G-parity-violation effect mixes states of opposite G parities. It is well known that in the K^0-$\bar{K}^0$ mixing, with CPT invariance, the K_L and K_S mixing states are given by[9]

$$|K^0_{L,S}\rangle = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}}\left[(1+\epsilon)|K^0\rangle \pm (1-\epsilon)|\bar{K}^0\rangle\right]. \tag{1}$$

When the interaction responsible for the CP noninvariance mixing is turned off, the mixing coefficient ϵ will be zero and $|K^0_{L,S}\rangle$ become states of definite CP eigenvalues

$$|K^0_{1,2}\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\bar{K}^0\rangle\right). \tag{2}$$

Now, in the ρ-ω mixing problem, it is more likely that the counterparts of $|K^0_1\rangle$ and $|K^0_2\rangle$ should be $|\rho^0\rangle$ and $|\omega^0\rangle$ which are states of definite G parity. We may therefore write

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle + |\bar{\alpha}\rangle\right), \tag{3}$$

$$|\omega^0\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle - |\bar{\alpha}\rangle\right), \tag{4}$$

where $|\alpha\rangle$ and $|\bar{\alpha}\rangle$ are two "fictive states" which are the counterparts of $|K^0\rangle$ and $|\bar{K}^0\rangle$ in the neutral-kaon-decay problem and introduced just for convenience. They must satisfy the following conditions:

$$\underline{G}|\alpha\rangle = |\bar{\alpha}\rangle, \tag{5}$$

$$\underline{G}|\bar{\alpha}\rangle = |\alpha\rangle. \tag{6}$$

$\underline{G}$ represents the G-parity operator. It should be stressed once that the introduction of these fictive states $|\alpha\rangle$ and $|\bar{\alpha}\rangle$ is actually not strictly necessary. We use them here however to make our argument more simple and the one-to-one comparison between the two mixing problems as lucid as possible. Since both $|\rho^0\rangle$ and $|\omega^0\rangle$ are states of positive CP eigenvalues, $|\alpha\rangle$ and $|\bar{\alpha}\rangle$ also satisfy the following conditions:

$$CP|\alpha\rangle = |\alpha\rangle, \tag{7}$$

$$CP|\bar{\alpha}\rangle = |\bar{\alpha}\rangle. \tag{8}$$

When the electromagnetic interaction responsible for the G-violating effect is turned on, there occurs a mixing of the G-parity eigenstates and with CPT invariance, we obtain the counterparts of Eq. (1) for the ρ-ω mixing problem as

$$|\rho\rangle = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}}\left[(1-\epsilon)|\alpha\rangle + (1+\epsilon)|\bar{\alpha}\rangle\right], \tag{9a}$$

$$|\omega\rangle = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}}\left[(1+\epsilon)|\alpha\rangle - (1-\epsilon)|\bar{\alpha}\rangle\right]. \tag{9b}$$

In fact, Eqs. (9a) and (9b) can be derived by the following argument. With CPT invariance, we have

$$\langle \Theta\alpha|\underline{W}|\Theta\bar{\alpha}\rangle = \langle\bar{\alpha}|\underline{W}|\alpha\rangle, \tag{10}$$

where $\Theta = CPT$ operator and $\underline{W}$ is the mass matrix of the ρ-ω system. Applying conditions (7) and (8) to the states $|\alpha\rangle$ and $|\bar{\alpha}\rangle$, we deduce that W must be symmetric in the subspace of $|\alpha\rangle$ and $|\bar{\alpha}\rangle$ states:

$$\langle\alpha|W|\bar{\alpha}\rangle = \langle\bar{\alpha}|W|\alpha\rangle. \tag{11}$$

In this connection, the condition imposed by CPT invariance is equivalent to the one by T invariance. It should be noted that the CPT invariance condition makes the diagonal elements of the mass matrix in the neutral-kaon-decay problem become equal instead. Thus, with CPT invariance, $|\rho\rangle$ and $|\omega\rangle$ can be given in the following form[10]:

$$|\rho\rangle = \frac{1}{(|p|^2+|q|^2)^{1/2}}\left(p|\alpha\rangle + q|\bar{\alpha}\rangle\right), \tag{12}$$

$$|\omega\rangle=\frac{1}{(|p|^2+|q|^2)^{1/2}}(q|\alpha\rangle+p|\bar{\alpha}\rangle). \tag{13}$$

By setting $p=1-\epsilon$ and $q=1+\epsilon$, (9a) and 9(b) are obtained. Making use of (3) and (4), we may deduce from (9a) and 9(b) that

$$|\rho\rangle=\frac{1}{(1+|\epsilon|^2)^{1/2}}(-\epsilon|\omega^0\rangle+|\rho^0\rangle), \tag{14}$$

$$|\omega\rangle=\frac{1}{(1+|\epsilon|^2)^{1/2}}(|\omega^0\rangle+\epsilon|\rho^0\rangle). \tag{15}$$

These are exact forms for the mixing states $|\rho\rangle$ and $|\omega\rangle$ in terms of the definite G-parity states $|\rho^0\rangle$ and $|\omega^0\rangle$ with its mixing parameter ϵ. In this connection, the mixing states $|\rho\rangle$ and $|\omega\rangle$ do not have a definite G parity, but, rather, are predominantly in the positive- and negative-G-parity states, respectively. Since $|\epsilon|^2$ is very small as confirmed by experiment ($|\epsilon|^2\simeq 0.36\%$), $|\rho\rangle$ and $|\omega\rangle$ may approximately be written as

$$|\rho\rangle\simeq-\epsilon|\omega^0\rangle+|\rho^0\rangle, \tag{16}$$

$$|\omega\rangle\simeq\epsilon|\rho^0\rangle+|\omega^0\rangle. \tag{17}$$

Through the one-to-one correspondence between the two problems of ρ-ω mixing and neutral-kaon decay, it becomes clear that there should also be a Bell-Steinberger sum[11] for the ρ-ω mixing problem written explicitly as follows:

$$[\Gamma_\rho m_\rho+\Gamma_\omega m_\omega+i(m_\omega{}^2-m_\rho{}^2)]\langle\rho|\omega\rangle =\sum_f\langle f|T|\omega\rangle\langle f|T|\rho\rangle^*. \tag{18}$$

Note that the counterparts of M_L, M_S, $\frac{1}{2}\gamma_L$, and $\frac{1}{2}\gamma_S$ are $m_\omega{}^2$, $m_\rho{}^2$, $\Gamma_\omega m_\omega$, and $\Gamma_\rho m_\rho$, respectively.

We are ready now to use the unitarity sum to derive the explicit formula for the ρ-ω mixing phase in terms of Γ_ρ, Γ_ω, m_ρ, and m_ω.[12] Let us assume that the G parity is conserved for all the matrix elements of $|\rho^0\rangle$ and $|\omega^0\rangle$ states. This assumption is similar to the requirement of CP invariance for all the matrix elements of $|K^0\rangle$ and $|\bar{K}^0\rangle$ in superweak theory.[13] In other words, the electromagnetic interaction is considered here as negligible compared to other strong interactions. The taking of zero values for the G-parity-violating transition amplitudes of $|\rho^0\rangle$ and $|\omega^0\rangle$ is equivalent to the neglect of the imaginary part of $W_{\omega^0\rho^0}$ compared to its real part. This can easily be verified. This neglect was indeed considered by Gourdin *et al.*[3] in their derivation of the ρ-ω mixing phase. It is, therefore, not surprising to see that we shall obtain exactly the same result as Gourdin *et al.*, although our method of derivation is certainly much simpler than theirs in the sense that we can avoid several complicated approximations made by these authors. Besides, within this assumption, our expression for the ρ-ω mixing phase is an exact one. It should also be noted that the unitarity sum has been used before to obtain the ρ-ω mixing phase[14] although there again, some approximations have already been made in the derivation. With the assumption of G-parity conservation for all the matrix elements of $|\rho^0\rangle$ and $|\omega^0\rangle$, the right-hand side of Eq. (18) has the simple form

$$\frac{1}{1+|\epsilon|^2}\left(\epsilon\sum_{g_n=1}|\langle n|T|\rho^0\rangle|^2-\epsilon^*\sum_{g_n=-1}|\langle n|T|\omega^0\rangle|^2\right), \tag{19}$$

where for convenience the set of decay states $|n\rangle$ are chosen to be of definite G parity, either with $g_n=+1$ or $g_n=-1$. The real part of Eq. (18) can then be written as

$$-2(m_\omega{}^2-m_\rho{}^2)\,\mathrm{Im}\epsilon =\mathrm{Re}\epsilon\left(\sum_{g_n=1}|\langle n|T|\rho^0\rangle|^2-\sum_{g_n=-1}|\langle n|T|\omega^0\rangle|^2\right). \tag{20}$$

Besides Eq. (18), unitarity also requires

$$2\Gamma_\rho m_\rho=\sum_n|\langle n|T|\rho\rangle|^2, \tag{21}$$

$$2\Gamma_\omega m_\omega=\sum_n|\langle n|T|\omega\rangle|^2. \tag{22}$$

If we again choose the complete set of states $|n\rangle$ as the set of definite G-parity states, we obtain

$$2(\Gamma_\omega m_\omega-\Gamma_\rho m_\rho) =-\frac{1-|\epsilon|^2}{1+|\epsilon|^2}\left(\sum_{g_n=1}|\langle n|T|\rho^0\rangle|^2-\sum_{g_n=-1}|\langle n|T|\omega^0\rangle|^2\right). \tag{23}$$

Equations (20) and (23) yield

$$\tan\phi_\epsilon=\frac{1+|\epsilon|^2}{1-|\epsilon|^2}\,\frac{\Gamma_\omega m_\omega-\Gamma_\rho m_\rho}{m_\omega{}^2-m_\rho{}^2}. \tag{24}$$

Thus, using the unitarity condition and the fact that the G-parity-violating transition amplitudes are small, we can obtain an explicit formula for the phase of the mixing. With $|\epsilon|^2$ very small, the phase of the ρ-ω mixing parameter is

$$\phi_\epsilon=\tan^{-1}\left(\frac{\Gamma_\omega m_\omega-\Gamma_\rho m_\rho}{m_\omega{}^2-m_\rho{}^2}\right) \tag{25}$$

which is exactly the same result of Gourdin *et al.*[3] For the purpose of direct comparison, we shall use the values given in Ref. 1:

$$m_\rho=773\text{ MeV},\quad \Gamma_\rho=111\text{ MeV},$$
$$m_\omega=783\text{ MeV},\quad \Gamma_\omega=12\text{ MeV}, \tag{26}$$

to find that

$$\phi_\epsilon = 101° \text{ (mod } 180°). \tag{27}$$

It is worth noting that the uncertainty in the value of the ρ width does not modify significantly the value of ϕ_ϵ. If the relative phase of the photon–vector-meson couplings $f_{\rho\gamma}$ and $f_{\omega\gamma}$ is taken to be of the order of 10°,[3] the relative phase of the ρ and ω production amplitudes in $e^+e^- \to \pi^+\pi^-$ is then equal to 111°(mod 180°) compared to the experimental value of 164° ± 28°.[1]

III. TIME-REVERSAL INVARIANCE IN THE ρ-ω SYSTEM

In this section, we shall present a systematic study of the condition imposed by time-reversal invariance and unitarity on the relative phase and strength of the ρ-ω mixing. We shall then point out that the experimental values of the relative phase and mixing strength obtained in an earlier experiment by the Orsay group[1] appear to violate the condition set by T invariance and unitarity.

As is well known, the electromagnetic interaction which does not conserve the G parity mixes states of opposite G parities with each other and in the most general case, without considering any kind of invariance for electromagnetic interaction, one can write the mixing states $|\rho\rangle$ and $|\omega\rangle$ as

$$|\rho\rangle = \frac{1}{[2(1+|\epsilon'|^2)]^{1/2}}\left[(1-\epsilon')|\alpha\rangle + (1+\epsilon')|\overline{\alpha}\rangle\right], \tag{28}$$

$$|\omega\rangle = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}}\left[(1+\epsilon)|\alpha\rangle - (1-\epsilon)|\overline{\alpha}\rangle\right], \tag{29}$$

where ϵ and ϵ' are mixing parameters. In terms of the states of definite G parity $|\rho^0\rangle$ and $|\omega^0\rangle$, we have

$$|\rho\rangle = \frac{1}{(1+|\epsilon'|^2)^{1/2}}\left(|\rho^0\rangle - \epsilon'|\omega^0\rangle\right), \tag{30}$$

$$|\omega\rangle = \frac{1}{(1+|\epsilon|^2)^{1/2}}\left(|\omega^0\rangle + \epsilon|\rho^0\rangle\right). \tag{31}$$

The difference between the off-diagonal elements of the mass-matrix $\underline{W}$ in the $|\alpha\rangle$ and $|\overline{\alpha}\rangle$ basis is equal to the difference of the off-diagonal elements of $\underline{W}$ in the $|\rho^0\rangle$ and $|\omega^0\rangle$ basis:

$$\langle\alpha|\underline{W}|\overline{\alpha}\rangle - \langle\overline{\alpha}|\underline{W}|\alpha\rangle = \langle\omega^0|\underline{W}|\rho^0\rangle - \langle\rho^0|\underline{W}|\omega^0\rangle. \tag{32}$$

The definitions (3) and (4) of $|\rho^0\rangle$ and $|\omega^0\rangle$ have been used to obtain (32). Since in $\langle\rho^0|\underline{W}|\omega^0\rangle$ and $\langle\omega^0|\underline{W}|\rho^0\rangle$, only the electromagnetic interaction can cause a nonzero connection between two states of opposite G parity, the time-reversal invariance of the electromagnetic interaction would make

$$\langle\rho^0|\underline{W}|\omega^0\rangle = \langle\omega^0|\underline{W}|\rho^0\rangle \tag{33}$$

and hence, from (32), the mass matrix $\underline{W}$ will be symmetric in the $|\alpha\rangle$ and $|\overline{\alpha}\rangle$ subspace. This would then imply

$$\epsilon' = \epsilon. \tag{34}$$

As was stressed before, this condition is also imposed by CPT invariance. Thus, the T (or CPT) noninvariance in electromagnetic interaction would imply a difference between ϵ and ϵ'. Conversely, the observation of a clear-cut nonzero value for one of the expressions $\text{Re}(\epsilon-\epsilon')$ and $\text{Im}(\epsilon-\epsilon')$ is sufficient to consider seriously a possible time-reversal noninvariance in the electromagnetic interaction. There also exist the following relations:

$$\text{Re}\langle\rho|\omega\rangle = \frac{\text{Re}(\epsilon-\epsilon')}{(1+|\epsilon|^2)^{1/2}(1+|\epsilon'|^2)^{1/2}}, \tag{35}$$

$$\text{Im}\langle\rho|\omega\rangle = \frac{\text{Im}(\epsilon+\epsilon')}{(1+|\epsilon|^2)^{1/2}(1+|\epsilon'|^2)^{1/2}}. \tag{36}$$

Since both $|\epsilon|^2$ and $|\epsilon'|^2$ are usually expected to be very small, to the order of a good approximation of 0.36%, one may have

$$\text{Re}(\epsilon-\epsilon') = \text{Re}\langle\rho|\omega\rangle, \tag{37}$$

$$\text{Im}(\epsilon+\epsilon') = \text{Im}\langle\rho|\omega\rangle. \tag{38}$$

Thus, the observation of a nonzero value for $\text{Im}\langle\rho|\omega\rangle$ would demonstrate a G-parity violation in electromagnetic interactions independently to any symmetry assumption, while the nonzero value for $\text{Re}\langle\rho|\omega\rangle$ would directly demonstrate a time-reversal noninvariance.

The Bell-Steinberger unitarity sum for the ρ-ω mixing[11] may now be used to estimate the upper and lower limits of $\text{Re}\langle\rho|\omega\rangle$. This unitarity sum may be written in the following form:

$$(1+\delta^2)^{1/2}\langle\rho|\omega\rangle = -i(m_\omega{}^2 - m_\rho{}^2)^{-1} \times e^{-i\phi_W}\sum_n \langle n|T|\omega\rangle\langle n|T|\rho\rangle^*, \tag{39}$$

where

$$\delta = \frac{\Gamma_\rho m_\rho + \Gamma_\omega m_\omega}{m_\omega{}^2 - m_\rho^2} \tag{40}$$

and

$$\tan\phi_W = -\delta, \tag{41}$$

and the angle ϕ_W must be such that $\cos\phi_W > 0$ and $\sin\phi_W < 0$. The real and imaginary parts of Eq. (39) yield

$$(1+\delta^2)^{1/2}\text{Re}\langle\rho|\omega\rangle = (m_\omega{}^2 - m_\rho^2)^{-1} \times \text{Im}\left(e^{-i\phi_W}\sum_n \langle n|T|\omega\rangle\langle n|T|\rho\rangle^*\right), \tag{42}$$

and

$$(1+\delta^2)^{1/2}\,\mathrm{Im}\langle\rho|\omega\rangle = -(m_\omega^{\,2}-m_\rho^{\,2})^{-1} \times \mathrm{Re}\Big(e^{-i\phi_W}\sum_n \langle n|T|\omega\rangle\langle n|T|\rho\rangle^*\Big), \tag{43}$$

respectively. It is not very difficult to find that the contribution to the sum on the right-hand side of Eqs. (42) and (43) comes predominantly from the 2π channel. A relatively small but finite contribution may also come from the 3π and $\pi\gamma$ channels. A contribution from all the remaining channels n is very small (probably much less than a few percent with the upper limits of available data[15]), because of the smallness of both $|\langle n|T|\omega\rangle|$ and $|\langle n|T|\rho\rangle|$. It is possible to write

$$\sum_n \langle n|T|\omega\rangle\langle n|T|\rho\rangle^* = \eta_{\pi\pi}\Gamma_{\rho\to 2\pi}m_\rho + \sum_{n\neq 2\pi}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to n}\Gamma_{\rho\to n})^{1/2}e^{i\phi_n}, \tag{44}$$

where $\eta_{\pi\pi}$ is the ratio of the 2π decay amplitudes of ρ and ω, respectively,

$$\eta_{\pi\pi} = \langle 2\pi|T|\omega\rangle/\langle 2\pi|T|\rho\rangle, \tag{45}$$

$\Gamma_{\omega\to n}$ is the partial decay rate of ω into the channel n, and ϕ_n is the relative phase of $\langle n|T|\omega\rangle$ and $\langle n|T|\rho\rangle$. The partial decay rates $\Gamma_{\omega\to n}$ and $\Gamma_{\rho\to n}$ provide the magnitude of the product $\langle n|T|\omega\rangle \times\langle n|T|\rho\rangle^*$ and thereby a limit on the absolute value of the sum on the right-hand side of Eq. (44). Therefore, Eq. (42) can be put in the following form:

$$\mathrm{Re}\langle\rho|\omega\rangle = (1+\delta^2)^{-1/2}(m_\omega^{\,2}-m_\rho^{\,2})^{-1}\Big(\mathrm{Im}(e^{-i\phi_W}\eta_{\pi\pi}\Gamma_{\rho\to 2\pi}m_\rho) + \sum_{n\neq 2\pi}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to n}\Gamma_{\rho\to n})^{1/2}\sin(\phi_n-\phi_W)\Big). \tag{46}$$

For the purpose of direct comparison, we choose the values of Ref. 1 for Γ_ω, Γ_ρ, m_ω, and m_ρ which yield $\phi_W \simeq -81°$. It is worth noting that the uncertainty in the value of the ρ width does not modify significantly the value of ϕ_W. If time-reversal invariance is conserved, then $\mathrm{Re}\langle\rho|\omega\rangle = 0$ and Eq. (46) implies that

$$\mathrm{Im}(e^{-i\phi_W}\eta_{\pi\pi}) = -\frac{1}{m_\rho\Gamma_{\rho\to 2\pi}}\sum_{n\neq 2\pi}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to n}\Gamma_{\rho\to n})^{1/2}\sin(\phi_n-\phi_W), \tag{47}$$

which leads to the following inequality:

$$|\eta_{\pi\pi}|\,|\sin(\phi_{\pi\pi}-\phi_W)| \leq M_T, \tag{48}$$

$$M_T = \frac{1}{m_\rho\Gamma_{\rho\to 2\pi}}\sum_{n\neq 2\pi}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to n})^{1/2}(\Gamma_{\rho\to n})^{1/2}, \tag{49}$$

$\phi_{\pi\pi}$ is the relative phase of the ρ-ω mixing and $|\eta_{\pi\pi}|$, its mixing strength. If some partial decay rates are not known, the experimental upper limits furnish a corresponding upper bound. The greatest contributions to M_T likely come from the 3π and $\pi\gamma$ decay channels. With $\Gamma_{\omega\to\pi\gamma}$ equal to 9.4% of the total decay rate Γ_ω and $\Gamma_{\rho\to\pi\gamma} \lesssim 0.2\%$ of Γ_ρ, we obtain

$$\frac{1}{m_\rho\Gamma_\rho}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to\pi\gamma})^{1/2}(\Gamma_{\rho\to\pi\gamma})^{1/2} \lesssim 0.45\times 10^{-2}, \tag{50}$$

while for the 3π channel, using the experimental result $\Gamma_{\omega\to 3\pi}\sim 0.87\Gamma_\omega$ together with the fact that the value of $|\eta_{3\pi}|$ is very small and presumably of the same order of magnitude as $|\eta_{\pi\pi}|$, i.e., $|\eta_{3\pi}| \lesssim 0.06$, we derive

$$\frac{1}{m_\rho\Gamma_\rho}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to 3\pi})^{1/2}(\Gamma_{\rho\to 3\pi})^{1/2} \lesssim 0.56\times 10^{-2}. \tag{51}$$

All the contributions from other channels are very small because of the smallness of the partial decay rates of both ρ and ω. A conservative estimate based on poorly known upper limits of the corresponding partial decay rates yields an upper limit for such contributions probably not more than 0.19×10^{-2}:

$$\frac{1}{m_\rho\Gamma_\rho}\sum_{n\neq 2\pi,\,3\pi,\,\pi\gamma}(\Gamma_{\omega\to n}\Gamma_{\rho\to n}m_\omega m_\rho)^{1/2} \lesssim 0.19\times 10^{-2}. \tag{52}$$

Consequently, we may obtain an upper limit for M_T as

$$M_T \lesssim 1.2\times 10^{-2}. \tag{53}$$

This upper limit for M_T may be lowered if more precise data for the partial decay rates of ρ and ω are available. The restriction on $\eta_{\pi\pi}$ will then be strengthened accordingly. In order to satisfy the T-invariance condition, the value for $|\sin(\phi_{\pi\pi}-\phi_W)|$ must be such that

$$|\sin(\phi_{\pi\pi}-\phi_W)| \lesssim 1.2\times 10^{-2}/|\eta_{\pi\pi}|. \tag{54}$$

Therefore, the greatest value of the upper limit for $|\sin(\phi_{\pi\pi}-\phi_W)|$ must correspond to the smallest value of $|\eta_{\pi\pi}|$, i.e., $|\eta_{\pi\pi}| = 0.04$ and hence

$$|\sin(\phi_{\pi\pi}-\phi_W)| \lesssim 0.333 \tag{55}$$

or

$$|\phi_{\pi\pi} - \phi_W| \lesssim 20^\circ. \tag{56}$$

Consequently, unitarity and time-reversal invariance require $\phi_{\pi\pi}$ to stay in the following range of angles (mod 180°):

$$79^\circ \leq \phi_{\pi\pi} \leq 119^\circ. \tag{57}$$

Conversely, one can, in principle, use Eq. (46) to estimate the value of $\mathrm{Re}\langle\rho|\omega\rangle$ with the measured values of $|\eta_{\pi\pi}|$ and $\phi_{\pi\pi}$ and may thereby conclude that time-reversal (or *CPT*) invariance in the ρ-ω system is conserved or not. As an illustrative example, we compute $\mathrm{Re}\langle\rho|\omega\rangle$ with the earlier results of the Orsay experiment.[1] The earlier Orsay experiment[1] provides the following values for $|\eta_{\pi\pi}|$:

$$|\eta_{\pi\pi}| = 0.06 \pm 0.02 \tag{58}$$

and, for the ρ-ω mixing phase in $e^+e^- \to \pi^+\pi^-$,

$$\alpha_{\pi\pi} = 164^\circ \pm 28^\circ. \tag{59}$$

If the estimated relative phase of the photon–vector-meson couplings $f_{\omega\gamma}$ and $f_{\rho\gamma}$ is accepted to be of the following order,[16]

$$\phi_\omega - \phi_\rho \simeq 8^\circ \pm 2^\circ, \tag{60}$$

the Orsay experiment then yields an experimental value for $\phi_{\pi\pi}$ as

$$\begin{aligned}\phi_{\pi\pi} &= \alpha_{\pi\pi} - (\phi_\omega - \phi_\rho)\\ &= 156^\circ \pm 30^\circ.\end{aligned} \tag{61}$$

Equation (46) implies that the value of $\mathrm{Re}\langle\rho|\omega\rangle$ lies within the following upper and lower limits:

$$\mathrm{Re}\langle\rho|\omega\rangle = (1+\delta^2)^{-1/2}(m_\omega^2 - m_\rho^2)^{-1}\Gamma_\rho m_\rho\left(|\eta_{\pi\pi}|\sin(\phi_{\pi\pi}-\phi_W) \pm \frac{1}{\Gamma_\rho m_\rho}\sum_{n\neq 2\pi}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to n}\Gamma_{\rho\to n})^{1/2}\right) \tag{62}$$

or, numerically,

$$\mathrm{Re}\langle\rho|\omega\rangle = (1+\delta^2)^{-1/2}(m_\omega^2 - m_\rho^2)^{-1}\Gamma_\rho m_\rho[|\eta_{\pi\pi}|\sin(\phi_{\pi\pi}-\phi_W) \pm 1.2\times 10^{-2}]. \tag{63}$$

Hence, with the central values of $\phi_{\pi\pi}$ and $|\eta_{\pi\pi}|$ obtained by the Orsay experiment, i.e., $\phi_{\pi\pi} = 156^\circ$ and $|\eta_{\pi\pi}| = 0.06$, we obtain

$$\mathrm{Re}(\epsilon' - \epsilon) = (1+\delta^2)^{-1/2}(m_\omega^2 - m_\rho^2)^{-1}\Gamma_\rho m_\rho(5.03\times 10^{-2} \pm 1.2\times 10^{-2}) \simeq (4.46 \pm 1.07)\times 10^{-2}. \tag{64}$$

The deviation of $\mathrm{Re}(\epsilon' - \epsilon)$ from the zero value is at least greater than 3.4×10^{-2} and can be considered as rather great compared to $|\epsilon| \sim 0.06$. Even if the smallest value allowable by the Orsay experiment is taken for $\sin(\phi_{\pi\pi} - \phi_W)$, the value found for $\mathrm{Re}(\epsilon' - \epsilon)$ still is of the order $(2 \pm 1.07)\times 10^{-2}$. Thus, Eq. (64) shows that it is possible to detect a time-reversal noninvariance in electromagnetic interaction with a careful measurement of the relative phase and strength of the ρ-ω mixing in $e^+e^- \to \pi^+\pi^-$, provided that some improvement can be made in the accuracy of such a phase determination. With the statistically poor data available, T (or *CPT*) invariance seems to be violated. Whether the fault should be blamed on the time-reversal noninvariance in the ρ-ω system or on the breakdown of vector-meson-dominance theory is a matter of taste. To our prejudiced view, it would be safer to leave the vector-meson-dominance theory intact and explore a possible T noninvariance in electromagnetic interaction to accommodate the situation. It should also be remarked that if the G-parity conservation is not violated, $\mathrm{Re}(\epsilon' - \epsilon)$ will identically be equal to zero regardless of the time-reversal noninvariance. Thus, the observation of a small value for $\mathrm{Re}(\epsilon' - \epsilon)$ might also trigger a possibility of G-parity conservation. This situation, however, can occur only if ϵ and ϵ' are both equal to zero and there were experimental evidences showing that this is not the case [it was possible to estimate a value of about 0.06 for $|\epsilon|$ (Ref. 1)].

IV. BREAKDOWN OF MICROSCOPIC REVERSIBILITY

In this section, we shall investigate the relationship between the time-reversal noninvariance and the breakdown of microscopic reversibility of the ρ-ω mixing system[17] and then show that the nonzero value for $\mathrm{Re}(\epsilon' - \epsilon)$ may be considered as equivalent to a kind of microscopic nonreversibility.

Inversion of Eqs. (30) and (31) yields

$$|\rho^0\rangle = (1+\epsilon\epsilon')^{-1}(1+|\epsilon'|^2)^{1/2}(|\rho\rangle + \epsilon'|\omega\rangle), \tag{65}$$

and

$$|\omega^0\rangle = (1+\epsilon\epsilon')^{-1}(1+|\epsilon|^2)^{1/2}(|\omega\rangle - \epsilon|\rho\rangle). \tag{66}$$

After a time t has elapsed, the amplitude of a ρ state changes by a factor

$$\Theta_\rho = \exp[-(\Gamma_\rho m_\rho + i m_\rho^2)t],$$

while that of an ω state is multiplied by

$$\Theta_\omega = \exp[-(\Gamma_\omega m_\omega + i m_\omega^2)t].$$

Therefore, according to Eqs. (65) and (66), a state created initially as $|\rho^0\rangle$ will be found after a lapse of time t to have transformed into

$$|\rho^0, t\rangle = (1+\epsilon\epsilon')^{-1}[(\Theta_\rho + \epsilon\epsilon'\Theta_\omega)|\rho^0\rangle + \epsilon'(\Theta_\omega - \Theta_\rho)|\omega^0\rangle], \quad (67)$$

while the initial state $|\omega^0\rangle$ will transform during the same time into

$$|\omega^0, t\rangle = (1+\epsilon\epsilon')^{-1}[(\Theta_\omega + \epsilon\epsilon'\Theta_\rho)|\omega^0\rangle + \epsilon(\Theta_\omega - \Theta_\rho)|\rho^0\rangle]. \quad (68)$$

Consequently, the probability to find, after a time t, that a ρ^0 state has changed into an ω^0 state is $|\epsilon'/\epsilon|^2$ times the probability to find that an ω^0 state has transformed into a ρ^0 state. Thus, the probability for a vector meson produced with a negative G parity to transform spontaneously into one with a positive G parity should be different from that for a $G=1$ vector meson to reappear spontaneously into one with $G=-1$. Observation of this effect manifested through $|\epsilon| \neq |\epsilon'|$ would directly demonstrate the failure of reciprocity, i.e., T invariance.

In a similar way, we can show that the nonzero value for $\mathrm{Re}(\epsilon' - \epsilon)$ may be regarded as equivalent to the breakdown of microscopic reversibility in the transformation of the fictive states $|\alpha\rangle$ and $|\bar{\alpha}\rangle$. In fact, inversion of Eqs. (28) and (29) yields

$$|\alpha\rangle = [2(1+\epsilon\epsilon')]^{-1}\{[2(1+|\epsilon'|^2)]^{1/2}(1-\epsilon)|\rho\rangle + [2(1+|\epsilon|^2)]^{1/2}(1+\epsilon')|\omega\rangle\}, \quad (69)$$

$$|\bar{\alpha}\rangle = [2(1+\epsilon\epsilon')]^{-1}\{[2(1+|\epsilon'|^2)]^{1/2}(1+\epsilon)|\rho\rangle - [2(1+|\epsilon|^2)]^{1/2}(1-\epsilon')|\omega\rangle\}. \quad (70)$$

After a lapse of time t, the states created initially as $|\alpha\rangle$ and $|\bar{\alpha}\rangle$ have transformed into $|\alpha, t\rangle$ and $|\bar{\alpha}, t\rangle$, respectively, with

$$|\alpha, t\rangle = [2(1+\epsilon\epsilon')]^{-1}\{[(1-\epsilon)(1-\epsilon')\Theta_\rho + (1+\epsilon')(1+\epsilon)\Theta_\omega]|\alpha\rangle + (1+\epsilon')(1-\epsilon)(\Theta_\rho - \Theta_\omega)|\bar{\alpha}\rangle\} \quad (71)$$

and

$$|\bar{\alpha}, t\rangle = [2(1+\epsilon\epsilon')]^{-1}\{(1+\epsilon)(1-\epsilon')(\Theta_\rho - \Theta_\omega)|\alpha\rangle + [(1+\epsilon)(1+\epsilon')\Theta_\rho + (1-\epsilon')(1-\epsilon)\Theta_\omega]|\bar{\alpha}\rangle\}. \quad (72)$$

The probability for an α state to transform into $\bar{\alpha}$ after time t is

$$P_{\bar{\alpha}\alpha}(t) = \frac{|(1+\epsilon')(1-\epsilon)|^2|\Theta_\rho - \Theta_\omega|^2}{4|1+\epsilon\epsilon'|^2}, \quad (73)$$

while the probability for an $\bar{\alpha}$ state to change into α in the same time is

$$P_{\alpha\bar{\alpha}}(t) = \frac{|(1-\epsilon')(1+\epsilon)|^2|\Theta_\rho - \Theta_\omega|^2}{4|1+\epsilon\epsilon'|^2}. \quad (74)$$

Thus

$$\frac{P_{\alpha\bar{\alpha}}(t) - P_{\bar{\alpha}\alpha}(t)}{P_{\alpha\bar{\alpha}}(t) + P_{\bar{\alpha}\alpha}(t)} = \frac{2[\mathrm{Re}\epsilon(1+|\epsilon'|^2) - \mathrm{Re}\epsilon'(1+|\epsilon|^2)]}{(1+|\epsilon|^2)(1+|\epsilon'|^2) - 4\,\mathrm{Re}\epsilon\,\mathrm{Re}\epsilon'} \simeq 2\,\mathrm{Re}\langle\rho|\omega\rangle, \quad (75)$$

to the lowest order of $|\epsilon|$ and $|\epsilon'|$. Consequently, the breakdown of reversibility in the transformation of $|\alpha\rangle$ and $|\bar{\alpha}\rangle$ states is equivalent to the nonzero value for $2(\mathrm{Re}\epsilon - \mathrm{Re}\epsilon')$, i.e., the discrepancy in the ρ-ω mixing phase.

Although it was possible to relate the T noninvariance manifested through $\mathrm{Re}(\epsilon' - \epsilon) \neq 0$ to the breakdown of microscopic reprocity in the transformation of the states $|\alpha\rangle$ and $|\bar{\alpha}\rangle$, admittedly, the physical concept of these states has not yet become clear to us. Highly speculatively, we may tentatively create a new kind of quantum number called "superstrangeness" and then consider $|\alpha\rangle$ and $|\bar{\alpha}\rangle$ as states of positive and negative "superstrangeness," respectively. The superstrangeness quantum number will perhaps be conserved in the absence of all kinds of interaction. When other interactions are turned on, the superstrangeness is no longer conserved and the vector meson states $|\rho^0\rangle$ and $|\omega^0\rangle$ of definite G parity ought to be defined as incoherent mixtures of these opposite superstrangeness states:

$$|\omega^0\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle - |\bar{\alpha}\rangle), \quad (76)$$

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |\bar{\alpha}\rangle). \quad (77)$$

Within this assumption, the breakdown of reversibility discussed above may be seen as the inequality between the probability for a positive superstrangeness state $|\alpha\rangle$ to change into a negative superstrangeness state $|\bar{\alpha}\rangle$ and that for $|\bar{\alpha}\rangle$ to undergo the inverse transformation.

V. GRAPHICAL REPRESENTATION OF TIME-REVERSAL INVARIANCE IN THE ρ-ω MIXING

The condition imposed by T (or CPT) invariance in the ρ-ω system on its mixing phase and strength can best be seen by plotting it on a graph.

Two types of graphical representations of time-reversal invariance will be shown below, one with $\eta_{\pi\pi}$ as variable and the other with both $\eta_{\pi\pi}$ and $\eta_{3\pi}$ as variables.

A. Graphical Representation of Time-Reversal Invariance with only $\eta_{\pi\pi}$ Accessible to Good Measurement

In Sec. IV we have shown that if time-reversal invariance is conserved, then one must have the following condition imposed on $\eta_{\pi\pi}$:

$$|\eta_{\pi\pi}|\,|\sin(\phi_{\pi\pi}-\phi_W)|\leq M_T, \tag{48}$$

with

$$M_T=\frac{1}{m_\rho\Gamma_{\rho\to 2\pi}}\sum_{n\neq 2\pi}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to n})^{1/2}(\Gamma_{\rho\to n})^{1/2}. \tag{49}$$

The upper limit of M_T has been estimated to be

$$M_T\lesssim 1.2\times 10^{-2}, \tag{53}$$

and this upper limit may be lowered if more precise data for the partial decay rates of ρ and ω

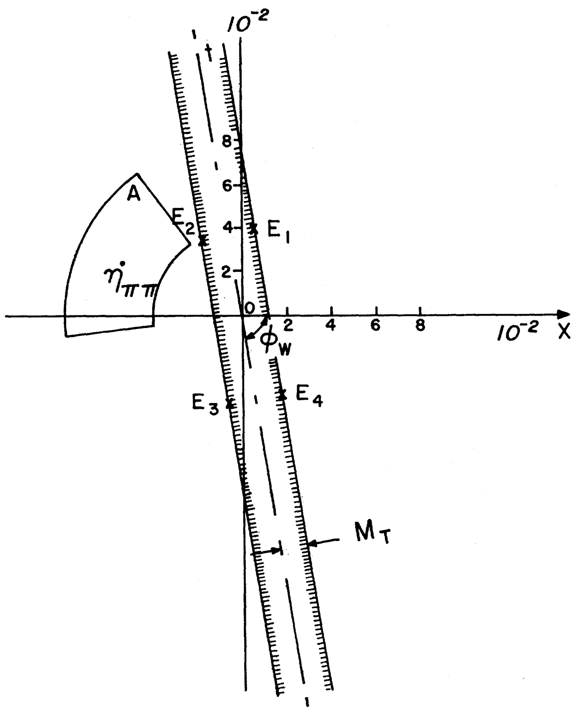

FIG. 1. The curved rectangular box A indicates the experimental value $\eta_{\pi\pi}$ given in Ref. 1 with its central dot corresponding to the central value of $\eta_{\pi\pi}$. Unitarity and time-reversal (or CPT) invariance require $\eta_{\pi\pi}$ to fall within the shaded strip whose width is $2M_T$. The greatest "allowed" deviations of the mixing phase $\phi_{\pi\pi}$ from $\phi_W\simeq -81°$ (mod 180°) correspond to the points E_1, E_2, E_3, and E_4.

are available. If M_T could be neglected, Eq. (48) would require $\phi_{\pi\pi}=\phi_W$, i.e., the complex vector $\eta_{\pi\pi}$ to lie along the straight line t through the origin, making an angle $\phi_W\simeq -81°$ with the positive x axis (see Fig. 1). For a finite value of M_T, condition (48) means that $\eta_{\pi\pi}$ is required to lie, if conditions imposed by both unitarity and T invariance are to be observed, within a distance M_T off the t axis, i.e., inside a band of width $2M_T$ indicated in Fig. 1 by shading. The distance of $\eta_{\pi\pi}$ from the t axis is a measure of the absolute value of the sum on the right-hand side of Eq. (44), i.e., of the magnitude of G-parity-violation amplitudes in channels other than 2π. In the graphical representation, the vector $\eta_{\pi\pi}$ corresponding to the experimental value obtained by the earlier Orsay experiment[1] lies within a curved rectangular box A as shown in Fig. 1. With the Orsay experiment results, we see that $\eta_{\pi\pi}$ definitely lies outside the allowed unitarity-T-invariance strip. Thus, the experimental value for $\eta_{\pi\pi}$ gives an indication that time reversal might not be conserved in the ρ-ω system. The greatest value allowed for $|\phi_{\pi\pi}-\phi_W|$ must correspond to the vector $\eta_{\pi\pi}$ located at the points E_1, E_2, E_3, and E_4 indicated in Fig. 1.

B. Graphical Representation of Time-Reversal Invariance with both $\eta_{\pi\pi}$ and $\eta_{3\pi}$ Determinable from Experiment

In this case, Eq. (47) may be written as

$$\operatorname{Im}(e^{-i\phi_W}Z)=-\frac{1}{m_\rho\Gamma_\rho}\times\operatorname{Im}\left(e^{-i\phi_W}\sum_{n\neq 2\pi,3\pi}\langle n|T|\omega\rangle\langle n|T|\rho\rangle^*\right), \tag{78}$$

with

$$Z=\eta_{\pi\pi}+C\eta_{3\pi}^*, \tag{79}$$

where

$$\eta_{3\pi}=\langle 3\pi|T|\rho\rangle/\langle 3\pi|T|\omega\rangle \tag{80}$$

and

$$C=m_\omega\Gamma_{\omega\to 3\pi}/m_\rho\Gamma_{\rho\to 2\pi}. \tag{81}$$

Equation (78) implies the inequality

$$|\operatorname{Im}(e^{-i\phi_W}Z)|\leq M_T', \tag{82}$$

with

$$M_T'=\text{Maximum value of }\frac{1}{m_\rho\Gamma_\rho}\sum_{n\neq 2\pi,3\pi}(m_\omega m_\rho)^{1/2}(\Gamma_{\omega\to n}\Gamma_{\rho\to n})^{1/2}. \tag{83}$$

With the available data,[15] the upper limit of the sum on the right-hand side of Eq. (83) is estimated to be 0.77×10^{-2}, i.e.,

$$M'_T \lesssim 0.77\times10^{-2}, \qquad (84)$$

and the value of C is given by

$$C \simeq 0.87\Gamma_\omega/\Gamma_\rho \simeq 9.4\times10^{-2}. \qquad (85)$$

Consequently, unitarity and time-reversal invariance require

$$|Z|\,|\sin(\phi_Z-\phi_W)| \lesssim 0.75\times10^{-2}, \qquad (86)$$

where ϕ_Z is the phase of Z. In Fig. 2, the complex vector Z has to lie in a band of width $2M'_T=1.50\times10^{-2}$ whose median line is the t axis. One deduces that for a given $\eta_{\pi\pi}$, the limits between which $\eta^*_{3\pi}$ must lie, if Z is to be in the unitarity T-invariance band, are straight lines parallel to the t axis and displaced by

$$a_\pm = \frac{1}{C}\left[\,|\eta_{\pi\pi}|\sin(\phi_{\pi\pi}-\pi-\phi_W)\pm M'_T\right], \qquad (87)$$

respectively. These are shown with dashed lines in Fig. 2 with the central value of $\eta_{\pi\pi}$, whereby $a_+=0.617$ and $a_-=0.453$. The complex vector $\eta_{3\pi}$ must, therefore, be within a band reflected through the x axis from the one just mentioned. For $\eta_{\pi\pi}$ being within the curved rectangular box A, the allowed strip for $\eta_{3\pi}$ is widened with its boundary lines now displaced from t by

$$b_- = \frac{1}{C}\left[\,|\eta_{\pi\pi}|_{\min}\sin(\phi^{\min}_{\pi\pi}-\pi-\phi_W)-M'_T\right] \qquad (88)$$

and

$$b_+ = \frac{1}{C}\left[\,|\eta_{\pi\pi}|_{\max}\sin(\phi^{\max}_{\pi\pi}-\pi-\phi_W)+M'_T\right], \qquad (89)$$

respectively. These are shown by dot-dashed lines in Fig. 2. With

$$|\eta_{\pi\pi}|_{\min}=0.04, \quad |\eta_{\pi\pi}|_{\max}=0.08,$$

$$\phi^{\min}_{\pi\pi}=126°, \quad \phi^{\max}_{\pi\pi}=186°,$$

the numerical values of $b_\pm$ are

$$b_+=0.933 \qquad (90)$$

and

$$b_-=0.111. \qquad (91)$$

The corresponding strip is shaded and indicated by B in Fig. 2. Since the experimental value of $|\eta_{3\pi}|$ which might be obtained from the experiment with $e^+e^-\to3\pi$ is unlikely to be very much greater than $|\eta_{\pi\pi}|$, the condition set above on $\eta_{3\pi}$ seems to demonstrate that time-reversal invariance is

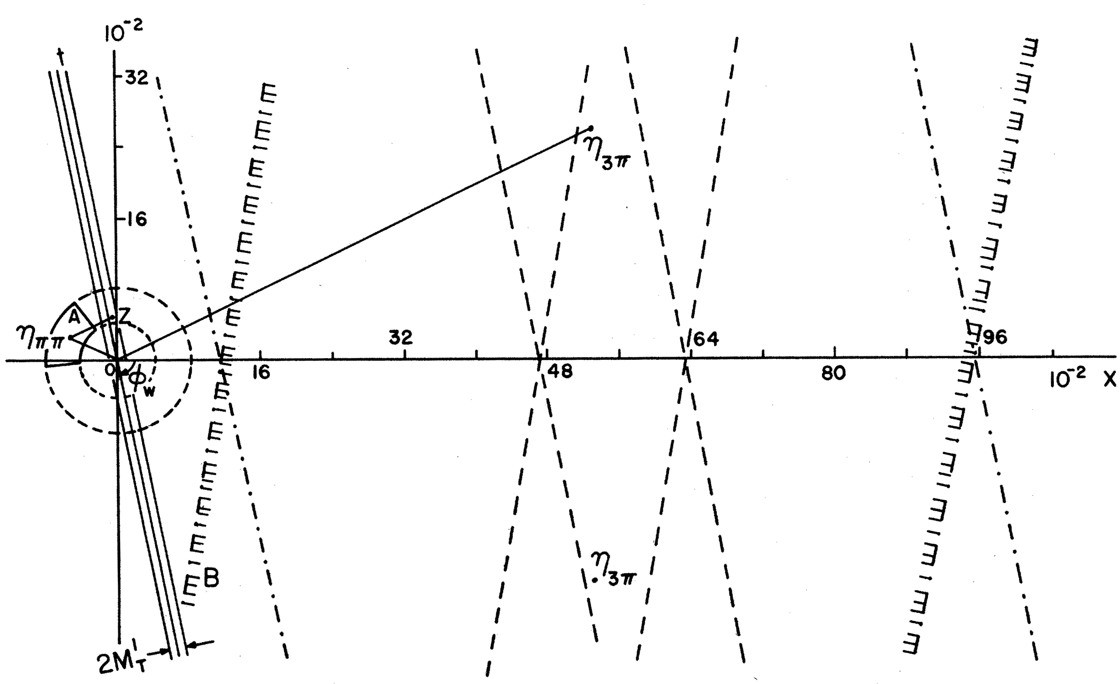

FIG. 2. The curved rectangular box A indicates the experimental value $\eta_{\pi\pi}$ given in Ref. 1 with its central dot corresponding to the central value of $\eta_{\pi\pi}$. Unitarity and time-reversal (or CPT) invariance require Z to fall within a narrow strip whose median line is t and whose width is $2M'_T$. With the central value of $\eta_{\pi\pi}$, $\eta^*_{3\pi}$ is required to be within a band limited by dashed lines, while with other experimental values of $\eta_{\pi\pi}$ given in Ref. 1, $\eta_{3\pi}$ must stay within a widened strip limited by dot-dashed lines, shaded and indicated by B, if the conditions imposed by unitarity and T invariance are to be observed.

not conserved. It is worth noting that even if $|\eta_{3\pi}|$ exceeds b_-, but is smaller than b_+, the phase $\phi_{3\pi}$ is still limited to be in the following range only:

$$90° + \phi_W - \theta \leq \phi_{3\pi} \leq 90° + \phi_W + \theta, \tag{92}$$

with

$$\theta = \cos^{-1}\left[\frac{1}{|\eta_{3\pi}|}\left(\frac{1}{C}|\eta_{\pi\pi}|_{\min}\sin(\phi_{\pi\pi}^{\min} - \pi - \phi_W) - M'_T\right)\right]. \tag{93}$$

Actually, time-reversal invariance requires not only $\eta_{3\pi}$ to lie within the strip B mentioned above (this is the consequence of $\mathrm{Re}\epsilon = \mathrm{Re}\epsilon'$), but also imposes a second condition which is either $|\epsilon| = |\epsilon'|$ or $\mathrm{Im}\epsilon = \mathrm{Im}\epsilon'$. If the transition amplitudes via e.m. interaction from the states of definite G parity $|\rho^0\rangle$ and $|\omega^0\rangle$ to 3π and 2π states, respectively, are negligible, $|\epsilon| = |\epsilon'|$ implies

$$|\eta_{3\pi}| = |\eta_{\pi\pi}|. \tag{94}$$

In fact, the ρ and ω states can be written in terms of $|\rho^0\rangle$ and $|\omega^0\rangle$ as

$$|\rho\rangle \simeq |\rho^0\rangle - \epsilon'|\omega^0\rangle, \tag{95}$$

$$|\omega\rangle \simeq |\omega^0\rangle + \epsilon|\rho^0\rangle. \tag{96}$$

From Eqs. (95) and (96), one obtains

$$\eta_{\pi\pi} = \frac{\langle 2\pi|T|\omega^0\rangle + \epsilon\langle 2\pi|T|\rho^0\rangle}{\langle 2\pi|T|\rho^0\rangle - \epsilon'\langle 2\pi|T|\omega^0\rangle} \tag{97}$$

and

$$\eta_{3\pi} = \frac{\langle 3\pi|T|\rho^0\rangle - \epsilon'\langle 3\pi|T|\omega^0\rangle}{\langle 3\pi|T|\omega^0\rangle + \epsilon\langle 3\pi|T|\rho^0\rangle}. \tag{98}$$

Consequently, if $\langle 3\pi|T|\rho^0\rangle$ and $\langle 2\pi|T|\omega^0\rangle$ are negligible, Eqs. (97) and (98) become

$$\eta_{\pi\pi} \simeq \epsilon \tag{99}$$

and

$$\eta_{3\pi} \simeq -\epsilon', \tag{100}$$

respectively. Hence, the time-reversal invariance condition $|\epsilon| = |\epsilon'|$ may be expressed equivalently by $|\eta_{2\pi}| = |\eta_{3\pi}|$. Thus, Eq. (94) says that $\eta_{3\pi}$, besides being confined into the allowed shaded strip B, must also lie within a circular strip of external and internal radii equal to 0.08 and 0.04, respectively. With the existing experimental value for $\eta_{\pi\pi}$, indications are that time-reversal invariance may be violated, because this kind of symmetry imposes, with such a value for $\eta_{\pi\pi}$, a value for $|\eta_{3\pi}|$ very much greater than $|\eta_{\pi\pi}|$, and this of course cannot be accommodated by condition (94). For a given value of $|\eta_{\pi\pi}|$, the conditions imposed by T invariance are to be observed if

$$|\eta_{\pi\pi}| > \frac{1}{C}[|\eta_{\pi\pi}|\sin(\phi_{\pi\pi} - 180° - \phi_W) - M'_T], \tag{101}$$

when $\eta_{\pi\pi}$ is in the left-half of a plane divided by the t axis and if

$$|\eta_{\pi\pi}| > \frac{1}{C}[|\eta_{\pi\pi}|\sin(180° + \phi_W - \phi_{\pi\pi}) - M'_T], \tag{102}$$

when $\eta_{\pi\pi}$ is in the right-half of the plane. Conditions (101) and (102) imply

$$|\sin(\phi_{\pi\pi} - \phi_W - 180°)| \leq C + \frac{M'_T}{|\eta_{\pi\pi}|} \tag{103}$$

which, with the available experimental data, yields similar conditions imposed by unitarity and T invariance on $\phi_{\pi\pi}$ as obtained previously:

$$79° \leq \phi_{\pi\pi} \leq 119° \ (\mathrm{mod}\ 180°). \tag{104}$$

Rigorously speaking, usually $\langle 3\pi|T|\rho^0\rangle$ and $\langle 2\pi|T|\omega^0\rangle$ though small, are not, however, identically equal to zero. Therefore, without excluding these terms, Eqs. (97) and (98) yield

$$\eta_{\pi\pi} = \frac{\alpha_1 + \epsilon}{1 - \epsilon'\alpha_1} \tag{105}$$

and

$$\eta_{3\pi} = \frac{\alpha_2 - \epsilon'}{1 + \epsilon\alpha_2}, \tag{106}$$

respectively, with

$$\alpha_1 = \langle 2\pi|T|\omega^0\rangle / \langle 2\pi|T|\rho^0\rangle \tag{107}$$

and

$$\alpha_2 = \langle 3\pi|T|\rho^0\rangle / \langle 3\pi|T|\omega^0\rangle. \tag{108}$$

In current practice, one considers the G-violating transitions of $|\omega^0\rangle$ and $|\rho^0\rangle$ to 2π and 3π channels, respectively, as possible only through an emission and absorption of at least a virtual photon[3]; consequently, it is expected that $\langle 2\pi|T|\omega^0\rangle$ is one order of α smaller than $\langle 2\pi|T|\rho^0\rangle$. So is $\langle 3\pi|T|\rho^0\rangle$ relative to $\langle 3\pi|T|\omega^0\rangle$. α is the e.m. structure constant with its numerical value $\frac{1}{137} \simeq 0.72 \times 10^{-2}$. Thus, from Eqs. (105) and (106), one can have approximately

$$\epsilon \simeq \eta_{\pi\pi} - \alpha \tag{109}$$

and

$$-\epsilon' \simeq -\alpha + \eta_{3\pi}. \tag{110}$$

The T-invariance condition $|\epsilon| = |\epsilon'|$ is now equivalent to

$$|\eta_{\pi\pi} - \alpha| = |\eta_{3\pi} - \alpha|. \tag{111}$$

Equation (109) says that the complex vector ϵ must lie within a curved rectangular box A' obtained by displacing the curved rectangular box A by $-\alpha$, while Eq. (110) means that $-\epsilon'$ has to stay within a strip B' obtained by the same displacement of the shaded band B mentioned previously, if $\mathrm{Re}\epsilon$

$=\mathrm{Re}\,\epsilon'$ and unitarity are to be satisfied simultaneously. Now, to fulfill the second condition imposed by T invariance, $|\epsilon|$ can be equal to $|\epsilon'|$ only if the allowed area B' of $-\epsilon'$ overlaps the circular strip of external and internal radii equal to 0.04 and 0.08, respectively, and centered at O' which is obtained by displacing O by $-\alpha$. It is quite obvious that these requirements yield the same conditions (103) for the phase $\phi_{\pi\pi}$, because the whole new figure, referred to the figure of the previous case when α is neglected, just appears to be displaced altogether by a very small length $-\alpha$.

In brief, in the graphical representation I (Fig. 1), unitarity and time-reversal invariance require $\eta_{\pi\pi}$ to lie near the t axis at an inclination of

$$\phi_W = -\tan^{-1}\left(\frac{\Gamma_\rho m_\rho + \Gamma_\omega m_\omega}{m_\omega^2 - m_\rho^2}\right) \simeq -81°$$

to the positive real axis. The distance of $\eta_{\pi\pi}$ from the t axis is the measure of the magnitude of G-parity-violation amplitudes in decay channels other than 2π and can be bounded by a quantity M_T determined by the partial decay rates (or their upper limits) of those modes. In the graphical representation II (Fig. 2), unitarity and time-reversal invariance require

$$Z = \eta_{\pi\pi} + \frac{m_\omega \Gamma_{\omega\to 3\pi}}{m_\rho \Gamma_{\rho\to 2\pi}}\, \eta_{3\pi}^*$$

to lie near the polar axis t at an inclination $\phi_W \simeq -81°$ to the positive real axis. The distance of Z from t is a measure of G-violation amplitudes in channels other than 2π and 3π and can be bounded by a quantity M'_T determined by the partial decay rates (or their upper limits) of those modes. This restriction on Z implies that corresponding to an experimental value for $\eta_{\pi\pi}$ determined by the earlier Orsay experiment,[1] $\eta_{3\pi}$ must lie within a shaded band limited by the dot-dashed lines and indicated by B in Fig. 2. Besides, time-reversal invariance also requires $\eta_{3\pi}$ to have its magnitude of the same order as $|\eta_{\pi\pi}|$. It is unlikely that these two conditions can be satisfied simultaneously with the available experimental value[1] for $\eta_{\pi\pi}$.

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