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Light-Cone Analysis of Soft-Pion Production in Deep-Inelastic Electron-Proton Scattering*

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Partial conservation of axial-vector current and light-cone current commutators in the gluon-quark model are used to study the scaling behavior of one-soft-pion production in deep-inelastic e-p scattering. A very interesting sum rule which relates the inverse square of the pion decay constant to the integral of the scaling functions is obtained, namely, $f_\pi^{-2} = \int_0^1 (d\omega/\omega)(F_2^{\pi^+} - F_2^{\pi^-})$.

I. INTRODUCTION

In view of the success of describing the scaling behavior of deep-inelastic lepton-proton scattering by the analysis of the current commutator near the light cone, it is natural to think that by crossing symmetry it should also work for similar inclusive electromagnetic processes such as $e^+ + e^- \to$ hadron + anything or $e^- + N \to e^-$ + hadron + anything. But unfortunately, due to the complexity of the unitarity equation which contains several unwanted terms (the semidisconnected parts) in addition to what one wants,[1,2] the conventional light-cone analysis is in general not applicable unless these unwanted terms happen to cancel each other out. However, in the case that the produced hadron is a soft pion, we find that by incorporating PCAC (partially conserved axial-vector current) it enables us to bypass this difficulty and use the conventional light-cone analysis to obtain some interesting results. In fact, we obtain a sum rule about the scaling function for the deep-inelastic process $e^- + p \to e^-$ + soft pion + anything, which is discussed in Sec. II. In Sec. III, some relevant points are discussed.

II. THE LIGHT-CONE ANALYSIS AND THE SUM RULE

Consider the deep-inelastic scattering

$$e^- + p \to e^- + \pi + \text{anything } (X). \tag{1}$$

The structure tensor $W_{\mu\nu}$ describing the process is defined by

$$W_{\mu\nu} = \frac{1}{2\pi} \int e^{iq\cdot x} d^4x \sum_X \langle p | J_\mu^{\text{em}}(x) | \pi, X\rangle \langle \pi, X | J_\nu^{\text{em}}(0) | p\rangle , \tag{2}$$

where an average over the proton spin is understood, q is the momentum of the virtual photon, and J_μ^{em} is the electromagnetic current. The corresponding momentum of the proton and pion are denoted, respectively, by p and q'. From Lorentz invariance, it is easy to see that the tensor $W_{\mu\nu}$ contains four structure functions. However, for our purpose, we are only interested in discussing two structure functions defined conventionally,

$$W_{\mu\nu} = \delta_{\mu\nu} W_1^\pi + \frac{p_\mu p_\nu}{m^2} W_2^\pi + \cdots . \tag{3}$$

The functions W_1^π and W_2^π are functions of q^2, $p\cdot q$, $q'\cdot q$, and $p\cdot q'$.

The general scaling behaviors of these functions have been studied by Ellis,[2] assuming that multiple products of operators have light-cone singularities similar to those for the product of two electromagnetic currents. We are interested in learning more about the scaling functions in the soft-pion limit by incorporating PCAC and light-cone current commutators recently derived by Gross and Treiman in the gluon-quark model.[3]

As is pointed out in Sec. I, we cannot simply cross the pion over from the final state to the initial state in order to use the light-cone current commutators. To see how to bypass these difficulties, let us first consider the scattering amplitude S_ν,

$$S_\nu \equiv \langle X, \pi | J_\nu^{\text{em}}(0) | p\rangle . \tag{4}$$

We use the Lehmann-Symanzik-Zimmermann reduction formula to reduce in the pion, then use PCAC,

$$\partial_\mu A_\mu^a = f_\pi m_\pi{}^2 \pi^a, \quad f_\pi = 0.95 m_\pi , \tag{5}$$

to obtain

$$S_\nu = f_\pi{}^{-1} \langle X | A_\nu^a(0) | p\rangle + i f_\pi{}^{-1} q'_\mu \int e^{iq'\cdot y} d^4y \langle X | T(A_\mu^a(y) J_\nu^{\text{em}}(0)) | p\rangle , \tag{6}$$

where a is the SU(3) index for the pion. For convenience, the produced pion is considered to be π^+, so that $a = 1 - i2$. In the soft-pion limit, only the external-line insertions on the initial and final proton and on the hyperons or baryon resonance in the final state X would contribute to the second term in (6). The contributions of these pole terms were studied recently by Ioffe[4] and found to be small – of the same order of magnitude as the PCAC corrections. This point is further discussed in Sec. III. If we neglect these small contributions, then $W_{\mu\nu}$ in (2) would take the form

$$W_{\mu\nu}=\frac{f_\pi^{-2}}{2\pi}\int e^{iq\cdot x}d^4x\sum_X\langle p|A_\mu^{a\dagger}(x)|X\rangle\langle X|A_\nu^a(0)|p\rangle$$
$$=\frac{f_\pi^{-2}}{2\pi}\int e^{iq\cdot x}d^4x\langle p|A_\mu^{a\dagger}(x)A_\nu^a(0)|p\rangle$$
$$=\frac{f_\pi^{-2}}{2\pi}\int e^{iq\cdot x}d^4x\langle p|[A_\mu^{a\dagger}(x),A_\nu^a(0)]|p\rangle\,. \qquad (7)$$

We see immediately that the conventional light-cone analysis can then be applied to (7) to obtain some interesting results about the scaling functions.

The light-cone current commutators in the gluon-quark model recently derived by Gross and Treiman[3] read

$$[A_\mu^a(x),A_\nu^b(0)]\underset{x^2\approx 0}{\approx}[V_\mu^a(x),V_\nu^b(0)]$$
$$\underset{x^2\approx 0}{\approx}\frac{1}{4\pi}\partial_\alpha[\epsilon(x_0)\delta(x^2)]\{if_{abc}[S_{\mu\nu\alpha\beta}V_\beta^c(S;x)+i\epsilon_{\mu\nu\alpha\beta}A_\beta^c(A;x)]+d_{abc}[S_{\mu\nu\alpha\beta}V_\beta^c(A;x)-i\epsilon_{\mu\nu\alpha\beta}A_\beta^c(S;x)]\}\,, \qquad (8)$$

where $S_{\mu\nu\alpha\beta}=\delta_{\mu\alpha}\delta_{\nu\beta}+\delta_{\mu\beta}\delta_{\nu\alpha}-\delta_{\mu\nu}\delta_{\alpha\beta}$.

The symmetric and antisymmetric bilocal currents are defined by

$$V_\beta^c(S;x)=\overline{\psi}(x)\gamma_\beta\tfrac{1}{2}\lambda^c\exp\left(-ig\int_0^x dz_\mu B_\mu(z)\right)\psi(0)+(x\longleftrightarrow 0)\,,$$
$$V_\beta^c(A;x)=\overline{\psi}(x)\gamma_\beta\tfrac{1}{2}\lambda^c\exp\left(-ig\int_0^x dz_\mu B_\mu(z)\right)\psi(0)-(x\longleftrightarrow 0)\,, \qquad (9)$$

$$A_\beta^c(S;x)=\overline{\psi}(x)\gamma_\beta\gamma_5\tfrac{1}{2}\lambda^c\exp\left(-ig\int_0^x dz_\mu B_\mu(z)\right)\psi(0)+(x\longleftrightarrow 0)\,,$$
$$A_\beta^c(A;x)=\overline{\psi}(x)\gamma_\beta\gamma_5\tfrac{1}{2}\lambda^c\exp\left(-ig\int_0^x dz_\mu B_\mu(z)\right)\psi(0)-(x\longleftrightarrow 0)\,, \qquad (10)$$

where ψ is the quark field, B is the neutral gluon field, and g is the gluon-quark coupling constant. From Lorentz invariance, the matrix elements of these bilocal operators can be written

$$\langle p|V_\beta^c(S;x)|p\rangle=p_\beta G_S^c(x\cdot p)+\text{trace term}\,, \qquad (11)$$
$$\langle p|V_\beta^c(A;x)|p\rangle=p_\beta G_A^c(x\cdot p)+\text{trace term}\,. \qquad (12)$$

Expressing the bilocal currents $V^c(S;x)$ and $V^c(A;x)$ in terms of the sum of local operators by doing the short-distance expansion, we can explicitly calculate the functions G_S^c and G_A^c. In fact, from (9) we have

$$V_\beta^c(S;x)=\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)+\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)-x_\mu[(\partial_\mu+igB_\mu)\overline{\psi}(x)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)+\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c(\partial_\mu-igB_\mu)\psi(x)]_{x=0}$$
$$+\tfrac{1}{2}x_\mu x_\nu[(\partial_\mu+igB_\mu)(\partial_\nu+igB_\nu)\overline{\psi}(x)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)+\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c(\partial_\mu-igB_\mu)(\partial_\nu-igB_\nu)\psi(x)]_{x=0}$$
$$+\cdots\,,$$
$$V_\beta^c(A;x)=\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)-\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)-x_\mu[(\partial_\mu+igB_\mu)\overline{\psi}(x)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)-\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c(\partial_\mu-igB_\mu)\psi(x)]_{x=0}$$
$$+\tfrac{1}{2}x_\mu x_\nu[(\partial_\mu+igB_\mu)(\partial_\nu+igB_\nu)\overline{\psi}(x)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)-\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c(\partial_\mu-igB_\mu)(\partial_\nu-igB_\nu)\psi(x)]_{x=0}$$
$$+\cdots\,. \qquad (13)$$

The local operator $\overline{\psi}(0)\gamma_\beta\frac{1}{2}\lambda^c\psi(0)$, which is explicitly independent of the gluon field, can easily be identified as the vector current $V^c(0)$; i.e.,

$$V_\beta^c(0)=\overline{\psi}(0)\gamma_\beta\tfrac{1}{2}\lambda^c\psi(0)\,.$$

So we have

$$\langle p|\bar{\psi}(0)\gamma_\beta \tfrac{1}{2}\lambda^c\psi(0)|p\rangle = \langle p|V_\beta^c(0)|p\rangle = N_c p_\beta, \quad (14)$$

where N_c is the form factor of the vector current at zero momentum transfer, for instance $N_3=\frac{1}{2}$. From (11)–(14), we find that

$$G_S^c(x\cdot p) = 2N_c + C_{S,2}^c(x\cdot p)^2/2! + \cdots, \quad (15)$$

$$G_A^c(x\cdot p) = C_{A,1}^c(x\cdot p) + C_{A,3}^c(x\cdot p)^3/3! + \cdots, \quad (16)$$

where the constants $C_{S,n}^c$ and $C_{A,n}^c$ are related to proton expectation values of the local operators appearing in (13). As far as our discussion is concerned, there is no need to identify them.

Sandwiching (8) between the proton states and taking the Fourier transform yields

$$\frac{1}{2\pi}\int e^{iq\cdot x}d^4x\langle p|[A_\mu^a(x), A_\nu^b(0)]|p\rangle = \frac{p_\mu p_\nu}{2p\cdot q}[if_{abc}\tilde{G}_S^c(\omega) + d_{abc}\tilde{G}_A^c(\omega)], \quad (17)$$

where $\omega = -q^2/2p\cdot q$ and $\tilde{G}_S^c(\omega)$ and $\tilde{G}_A^c(\omega)$ are the Fourier transforms of G_S^c and G_A^c, respectively.

On the other hand, in the Bjorken scaling limit it is pointed out that[2]

$$\frac{-p\cdot q}{m}W_2^\pi \xrightarrow[\omega,\,\rho',\text{ and } p\cdot q \text{ fixed}]{q^2\to\infty;} F_2^\pi(\omega,\rho',p\cdot q'), \quad (18)$$

where $\rho' = -2q\cdot q'/q^2$ and F_2^π is the scaling function. In the soft-pion limit we have $\rho'=0$ and $p\cdot q'=0$. It is pointed out by Brandt and Preparata[5] and by Ellis[2] that in the Bjorken scaling limit the dominant region of integration in (2) is that near the light cone $x^2\approx 0$. Thus by comparing the coefficient of $p_\mu p_\nu$ in (17) and (3), we find

$$F_2^{\pi^+}(\omega,\rho'=0,p\cdot q'=0) = f_\pi^{-2}\omega[\tilde{G}_S^3(\omega) + (\tfrac{2}{3})^{1/2}\tilde{G}_A^0(\omega) + (\tfrac{1}{3})^{1/2}\tilde{G}_A^8(\omega)], \quad (19)$$

However, from (15) and (16) we have

$$\tilde{G}_S^3(\omega) = \frac{1}{2\pi}\int e^{i\omega x\cdot p}G_S^3(x\cdot p)d(x\cdot p) = 2N_3\delta(\omega) - C_{S,2}^3\delta''(\omega)/2! + \cdots = \delta(\omega) - C_{S,2}^3\delta''(\omega)/2! + \cdots, \quad (20)$$

$$\tilde{G}_A^c(\omega) = \frac{1}{2\pi}\int e^{i\omega x\cdot p}G_A^c(x\cdot p)d(x\cdot p) = -iC_{A,1}^c\delta'(\omega) + iC_{A,3}^c\delta'''(\omega)/3! + \cdots. \quad (21)$$

Using (19)–(21), we find that

$$\int_{-1}^{1}\frac{d\omega}{\omega}F_2^{\pi^+}(\omega,\rho'=0,p\cdot q'=0) = f_\pi^{-2}.$$

Applying the crossing property of $W_{\mu\nu}$, we finally arrive at the sum rule

$$f_\pi^{-2} = \int_0^1\frac{d\omega}{\omega}[F_2^{\pi^+}(\omega,\rho'=0,p\cdot q'=0) - F_2^{\pi^-}(\omega,\rho'=0,p\cdot q'=0)],$$

where $F_2^{\pi^-}$ is the scaling function for the electroproduction of π^-.

III. DISCUSSION

In deriving our sum rule, we have neglected the contributions of pole terms corresponding to the emission of soft pions by baryons or baryonic resonances. These contributions are small, as is pointed out by Ioffe,[4] if the lab momenta of the baryons or baryonic resonances in the final state are small compared to their masses. However, for the time being, it cannot be justified either theoretically or experimentally that their lab momenta should be small. Since the validity of our sum rule can be tested experimentally, it can be used to clarify this point.

Also in deriving our sum rule, we have used PCAC. Since PCAC does not seem to be applicable at high energies, one tends to think that it cannot be applied here either. This presents no problem, because in the Bjorken scaling limit, $q\cdot q'$ only enters as the ratios $q\cdot q'/q^2$, $q\cdot q'/q\cdot p$, so that the accuracy of using PCAC here will be of the same order as that at low energies.

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