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Correlation Functions in High-Energy Multiple Production and an Extension of Feynman's Gas Analog*

Richard C. Arnold
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
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A picture of multiple-production cross sections suggested by Feynman is extended to include transverse-momentum distributions and phase-space boundary effects in the single-particle distributions by introducing an external confining potential on an axially symmetric plasma. Two-particle correlation functions are discussed in such an analog system, using closure approximations such as the Debye-Hückel equation for a nonuniform plasma, with the interparticle potentials obtained from two-particle elastic scattering properties. Effective correlation lengths are found to depend on the single-particle distribution in important ways. In particular, the central-region correlations are of shorter range than those at large transverse momentum or near phase-space boundaries in certain cases. A bootstrap mechanism for indefinitely rising Regge trajectories is also outlined.

I. ONE-DIMENSIONAL ANALOG FOR HIGH-ENERGY COLLISION ENSEMBLES

A concise picture of the distributions seen and expected in final-state particles from high-energy hadron collisions has been proposed by Feynman[1] and further utilized by Wilson[2] in discussing observables in high-energy collisions. This distribution is essentially one-dimensional; the transverse momentum of secondary particles is strongly damped, whereas the longitudinal momenta have a spread which increases as the collision energy increases. If the particle momenta are expressed in terms of rapidity (y), many current models suggest the distribution in y contains a flat plateau whose width is proportional to the maximum rapidity (Y), proportional to the logarithm of the incoming energy of the collision. Since the distribution in transverse momentum seems to be relatively independent of energy (and certainly is in these current models), and is sharply damped in that variable, one sees the development of a single-particle spectrum in the shape of a cigar at very high energies. Feynman and Wilson discuss this system, the ensemble of final states in collision processes of a given type, as if it were a gas or liquid confined in a tube. The effects of the walls have been discussed only very qualitatively, in terms of a finite correlation length at the edges.

In such a many-body ensemble, certain simple distribution functions usually characterize the basic properties of the constituent particles and their interactions. The one-particle distribution function, the density, is the single-particle inclusive spectrum. Two-particle joint density functions are the same as two-particle inclusive distributions. Probability distributions of gaps, or holes, correspond to "diffraction-dissociation" cross sections when the gaps are large, in the rapidity variable.[3] Since the probability of obtaining large rapidity gaps is exponentially small for most of the distribution, and the density of the central region (y plateau) is independent of volume (Y) asymptotically, the relevant familiar physical analog would seem to be a liquid,[1] which has essentially constant density and a relatively large free-energy requirement for making a cavity or hole.

Two-particle correlations in rapidity in such a system would be expected to exhibit a short correlation length, or at least a finite one independent of Y. This means that for large rapidity separation, the two-particle density factorizes into the product of the corresponding one-particle density functions. This is explicitly shown in some models,[4] when the rapidity of both particles is sufficiently separated from the ends of the distribution ($y=0$, $y=Y$).

Such a one-dimensional picture is not adequate to discuss correlations in transverse momenta, nor does it suggest any geometrical link between transverse-momentum distributions and structure of hadrons or elastic scattering. Some models[5] suggest the latter type of connections, but do not illuminate the role of correlations or give any clues about estimating their magnitude or their net effect in the system. In Sec. II we extend the analog described above to essentially three-dimensional analogs which allow discussions of a wide variety of phenomena.

II. THREE-DIMENSIONAL ANALOG: CONFINED PLASMA

Consider a plasma, i.e., a gas of strongly interacting particles, confined in an axially symmetric configuration to a region which can be of various lengths in the z direction. The confining potential fields may incorporate, for example, a wire along the origin in the transverse directions which attracts the plasma by a Coulomb-like interaction. We will denote the confining potential symbolically by $U(r)$. We do not intend to explore in this paper any detailed connection between U and the single-particle density; it will simply be assumed that the single-particle (i.e., average) density is a given function of analog coordinate space, which we identify with momentum space in the hadron collision ensemble.

This external field may not be connected with interparticle interactions in any essential way. For example, it is possible in a plasma to neglect external electric fields compared with magnetic field forces; whereas, in considering correlation effects, the interparticle electric field (Coulomb potential) is the most important contribution.

In hadron collisions, we can proceed conceptually by building a model for the single-particle density with parameters depending essentially only on the initial two-particle colliding system. This will enable a relation to be maintained phenomenologically between the hadron-hadron high-energy scattering and the transverse and longitudinal scaling properties and other characteristics of the single-particle spectrum.[5] However, when correlations are to be examined, it is necessary to introduce additional parameters into the description which characterize the relatively low-energy interactions between the produced particles in the analog system.

A convenient way of separating these effects is provided by the integral-equation formalism appropriate to a classical gas or liquid in equilibrium with specified external field and internal potentials.[6] Transverse coordinates will be the transverse momenta of produced particles; longitudinal coordinates will be the rapidities of such particles. A model for the single-particle density ρ_1 is first used; then the parameters of interparticle scattering are introduced in a way analogous to the plasma analog.

Qualitative properties of the single-particle density are assumed as follows: (1) In the central region, where both y and $Y-y$ are large compared with some fixed correlation length, the density is constant in y and depends only on $k_\perp$, the transverse momentum. (2) The dependence of the density on $k_\perp^2$ is very sharply damped, with exponential decrease at large $k_\perp$. (3) Near the phase-space boundaries, which are the extreme ends of the system in the z direction (y near 0 or Y), the behavior is given by Regge phenomenology with an appropriate exponent characteristic of the quantum numbers involved in the reaction. The possibilities here are usually assumed to be of the form[7]

$$\rho_1(x, k_\perp) \sim (1-x)^{1-2\alpha(k_\perp^2)} \text{ as } x \to 1,$$

where

$$x = \frac{2(k_\perp^2 + m^2)^{1/2}}{M} \sinh(y - Y)$$
$$= 2k_\parallel/\sqrt{s}$$

and $\alpha(t)$ depends on the quantum-number differences involved; $\alpha(0)$ may be 1, $\frac{1}{2}$, or zero or negative.

III. INTEGRAL EQUATIONS FOR CORRELATION FUNCTIONS

In a nonuniform system, it is convenient in the classical theory of liquids to consider the two-particle correlation function $G(r_1, r_2)$ defined[6] by

$$G(r_1, r_2) = \frac{\rho_2(r_1, r_2)}{\rho_1(r_1)\rho_1(r_2)} - 1, \tag{1}$$

where ρ_2 is the two-particle (joint) density function. In our class of systems, we notice immediately that if G is slowly varying in transverse momentum, the sharp cutoff in ρ_1 also appears in the observable ρ_2.

In such classical systems, the interparticle interaction is most conveniently introduced through an auxiliary function, called the "direct correlation function," $c(r_1, r_2)$, which is assumed to describe a "direct" or "unshielded" correlation between two particles, while the function G contains all the accumulated indirect effects induced by the medium. In the simplest theory, c is taken as directly proportional to the interparticle potential; this leads to the Debye-Hückel equation.[6] In more sophisticated theories, c is assumed to be a more "regular" function of the potential, such as in the Percus-Yevick equation.[6]

The general connection between c and G is expressed in the integral relation

$$G(r_1, r_2) = c(r_1, r_2) + \int dr\, c(r_1, r)\rho_1(r)G(r, r_2). \tag{2}$$

Specific closure approximations for c then lead to integral equations for G. In a system with relatively weak long-range potentials, the linear Debye-Hückel approximation $c = v(r_1 - r_2)$ is a reasonable assumption. In systems with hard-core potentials, the Percus-Yevick approximation,

$$c(r_1, r_2) = (1 - e^{-v(r_1, r_2)})[1 + G(r_1, r_2)],$$

which leads to a quadratic integral equation for G, is more realistic.

In the hadron collision ensemble, we will not necessarily be able to use experience borrowed from classical fluids. However, as a working framework, we may tentatively adopt some hypotheses concerning c which can be tested by experiments. We should, at a minimum, provide a concise language for expressing empirical data on multiple production; this is easier if we have a specific model for purposes of illustration and whose features can be completely explored.

We will assume the most convenient formulation is in terms of covariant 4-momentum vectors restricted to the mass shell. Thus, we begin with the ansatz

$$G(k_1, k_2) = c(k_1, k_2) + \int d^4k\, c(k_1, k)\rho(k)G(k, k_2)\,, \tag{3}$$

where $\rho(k) = \rho_1(\vec{k})\,\delta(k^2 - m^2)$ is interpreted as a density of particle states.

We will take this density to be determined independently of c by experimental data or a model which may not provide an associated G.

To obtain a definite model for c, we will make two assumptions, the first being much weaker than the second.

(1) c is a function only of $(k_1 + k_2)^2 = s_{12}$. This is motivated by our desire to reflect the two-particle resonance structure in the correlation function, while keeping as simple as possible the functional dependences involved.

(2) c is directly related to the total interaction cross section of the two outgoing particles at the given subenergy; and this relation can be obtained by formally setting the single-particle density to zero in Eq. (3), while identifying G in this case as the ratio of nonconstant to constant terms in the total cross section[8] for the two particles at energy $(s_{12})^{1/2}$.

Thus, explicitly,

$$c(s') = [\sigma_T(s') - \sigma_T(\infty)]/\sigma_T(\infty)\,. \tag{4}$$

In most current models of total cross sections, there is a small, long-range (weakly energy-dependent) contribution to this expression from Regge cuts of various types,[9] together with a secondary contribution which is strong at low energies but dies out rapidly with s'. The latter contributions are associated with resonance structure in the two-particle system. The former contribution is present in all cases.

Such assumptions on c do not guarantee 4-momentum conservation, so they cannot be universally applicable. We will assume that *in the central region* these constraints are not important at high energies. We will not quantitatively investigate these model assumptions near the "walls," the fragmentation regions, in this paper.

The secondary Regge-pole contributions in (4) will yield (in c) a rapidity correlation length of order $[1 - \alpha(0)]^{-1}$, which is of order unity, in the central region. This will be qualitatively unchanged when the solution of (3) is obtained, and G will also exhibit such a correlation length, as discussed by Feynman[1] and verified in dual models.[8] In addition to secondary Regge poles, one expects high-lying singularities near the Pomeranchukon, such as shielding cuts or multiple-scattering cuts, to contribute a very long correlation length in the ensemble.

Since we wish to illustrate certain important consequences of our approach, which are not necessarily contained in other models, we will consider quantitatively (in Sec. V) only the long-range contribution (cuts) to c in this paper. We will show how these long-range correlations become suppressed in the central region, leaving only short-range correlations, while in the regions where ρ_1 is small, at large transverse momentum or at certain phase-space boundaries, it is possible to detect long-range effects.

The cut contributions for cuts near $J = 1$ in the J plane will have the form

$$c(s') \sim a(\ln s')^{-n} \tag{5}$$

[if we consider the variation of $\ln(s)$ to be more important than $s^{\alpha_c - 1}$], where n is unity in some models, $\frac{1}{2}$ in others. We will take $n = 1$ for simplicity.

It is convenient now to reexpress Eq. (3) in terms of rapidity and transverse momentum coordinates. The integration d^4k becomes simply $dy\, d^2q$ (where q is the two-dimensional transverse momentum); and, in the central region, the single-particle density depends only on q^2 and not on y. With Y of order $\ln s$, the maximum rapidity in the laboratory frame for a conventional experimental configuration, we have

$$G(y_1, q_1; y_2, q_2) = c(y_1, q_1; y_2, q_2) + \int_0^Y dy \int_{-\infty}^{\infty} d^2q\, c(y_1, q_1; y, q) \times \rho_1(y, q)G(y, q; y_2, q_2)\,. \tag{6}$$

Now s_{12} can easily be expressed in terms of these variables:

$$s_{12} = 2[m^2 + (\kappa_1\kappa_2)^{1/2}\cosh(y_1 - y_2) - q_1 \cdot q_2]\,, \tag{7}$$

where $\kappa_i = q_i^{\,2} + m^2$.

Note, therefore, that if c is a function only of s_{12}, it is translationally invariant in terms of rapidity. Since ρ_1 is independent of y in the central region, which contains most of the region of integration in y for large Y, we may, in the central region, utilize the Fourier transform of this equation with respect to y to simplify the problem considerably.

Equation (6), together with assumptions (1) and (2), $\sigma_T(s)$ for the outgoing particles, and knowledge for ρ_1 (e.g., from a model[5] which ignores correlations), provide in principle a detailed prediction for one- and two-particle inclusive spectra which can be tested experimentally. This analog model appears somewhat simpler than dual resonance[4] or multiperipheral dynamics (Pignotti, Ref. 7).

IV. INDEFINITELY RISING REGGE TRAJECTORIES

Certain qualitative results obtained with a simple short-range ansatz for c are sufficiently interesting to be explicitly noted here. If we assume a smoothly behaved exchange mechanism, e.g., Regge-pole or elementary-pion exchange, which is damped exponentially in rapidity, the situation is very similar to a gas or liquid with specified short-range molecular potentials. Thus, solutions of (6) will exhibit qualitative behavior similar to that of realistic liquids.

One of these characteristic behaviors is the oscillatory radial behavior of the correlation function. All normal fluids have such a damped oscillatory component; the oscillations are strong at one or two molecular radii, but become very weak at large spacings. Our solutions of (6) will automatically exhibit such behavior; this is explicitly seen in the Fourier-transform solutions of Sec. V, when the nearest pole in the complex plane of the transform variable dominates.

There is a direct interpretation of such damped oscillations. In terms of s_{12}, it appears as an infinite series of decreasing-amplitude enhancements of the two-particle inclusive cross section as a function of the invariant (mass)^2 of the two-particle system. But the obvious identification of such a series is the coupling of particles 1 and 2 to an indefinitely rising Regge trajectory $\alpha(s_{12})$, whose couplings decrease rapidly with increasing s_{12}.

What may be concluded from this depends on the point of view adopted. One possible point of view is this: that indefinitely rising trajectories can be generated, from many-body dynamics as we have outlined, in a relatively quantitative way, given simple, smooth (nonoscillating) exchange mechanism. This provides still another "bootstrap" method, apparently independent of the duality approach or the elastic two-body N/D approaches. The consistency and utility of such a method remains to be investigated.

V. SOLUTIONS OF THE "LONG-RANGE" MODEL SYSTEM

With the specific assumption (5) for c, we can obtain interesting qualitative properties of the correlation function G with very little effort.

Consider first the solution of (6) in the central region, for large Y. The most important contribution to the integral, we suspect, will come from a large interval in y, and such that q^2 is small, since strong damping is present in the single-particle density. Thus, in the integral we should be able to replace $c(s')$ by an asymptotic approximation for large s' but small q^2. This would yield

$$c(y_1, q_1; y_2, q_2) \to a[\,|y_1 - y_2| + \ln(\kappa_1\kappa_2)^{1/2}]^{-1} \equiv c_A\,. \tag{8}$$

Next, a Fourier transform with respect to y may be performed on (6); if z is the variable conjugate to y, we obtain

$$\hat{G}(z;\kappa_1,\kappa_2) = \hat{c}(z;\kappa_1,\kappa_2) + \int d\kappa\, \hat{c}(z;\kappa_1,\kappa)\rho_1(\kappa)\hat{G}(z;\kappa,\kappa_2)\,, \tag{9}$$

where the caret indicates the Fourier-transformed function, e.g.,

$$\hat{c}(z;\kappa_1,\kappa_2) = \int_{-\infty}^{+\infty} dy\, e^{iyz} c(y;\kappa_1,\kappa_2)\,.$$

The replacement of c by c_A inside the integral then leads to a particularly simple form for the Fourier transform, which factorizes in the q^2 variables:

$$\hat{c}_A(z;\kappa_1,\kappa_2) = a(\kappa_1\kappa_2)^{-iz/2}\,. \tag{10}$$

This factorization of the kernel immediately leads to an explicit solution of (9). Define the trace of the kernel as

$$h(z) = \int d\kappa\, \rho_1(\kappa)\hat{c}(z;\kappa,\kappa)\,, \tag{11}$$

where the replacement of c by the asymptotic expression c_A is implied (but an approximate solution is obtained even if c does not factorize exactly). Then the solution of (9) is

$$\hat{G}(z;\kappa_1,\kappa_2) = \frac{\hat{c}(z;\kappa_1,\kappa_2)}{1 - h(z)}\,. \tag{12}$$

From this expression, we can read off the qualitative properties of the correlation function in the central region.

The correlation length in rapidity is determined by the asymptotic behavior of G as a function of y. A contour integration of expression (12) shows that this asymptotic behavior is determined by the imaginary part of the location of the pole in the z plane in (12), which has the smallest imaginary part. Since the numerator of (12) in our model has singularities only at infinity, the correlation length is determined by the first zero of the denominator. With reasonable single-particle densities, in the central region, $h(z)$ attains the value unity for Imz of order unity, which means the correlation length in rapidity will be of order unity.

This estimate can be made more precise by assuming the q^2 dependence of G (or c) can be neglected compared with the rapid q^2 dependence of the single-particle density in the integral. Then we can approximate $h(z)$ by

$$h(z) \cong \hat{c}(z; m^2, m^2)\left[\int d\kappa\, \rho_1(\kappa)\right] \equiv b\hat{c}(z; m^2, m^2). \tag{13}$$

The coefficient b can be related to the asymptotic behavior of the mean production multiplicity for large Y, since it is the coefficient of Y in the integral over the single-particle spectrum:

$$\bar{n} = b \ln s + \text{const} \quad \text{as } s \to \infty .$$

Data, or models which scale, indicate b is of order unity.

The strength of the correlation, a, then determines the correlation length; if a is of order unity, as suggested by total cross-section data fits,[9] we obtain a denominator zero in (12) with imaginary part near unity, which gives us the correlation length in the central region.

This phenomenon of screening a long-range potential through self-consistent-field cooperative readjustment of the density is a general property of dynamical systems expressible in equations such as (2), and was the original idea exploited by Debye and Hückel in their theory of electrolytes, where the basic interaction is the Coulomb attraction or repulsion.

Qualitatively different behavior of G may be obtained if the produced particles are near phase-space boundaries, in cases where the single-particle density vanishes at the boundary. In those cases, for example, where baryon exchange determines the appropriate Regge pole $\alpha(q^2)$, the screening effect, as exhibited in (6), may be drastically reduced, and one may see the long-range correlation of c more directly. The most characteristic test of such long-range correlation would be observation of a transverse-transverse correlation term $\vec{q}_1 \cdot \vec{q}_2$ which decreases only like $P_{\text{lab}}^{-1/2}$ (as suggested by the form taken by s_{12} when one particle is near $y=0$, the other near $y=Y$) instead of a much more rapid decrease for particles in the central region. A similar effect should appear when $|\vec{q}_1|$ and $|\vec{q}_2|$ are both large compared with longitudinal momenta in the center-of-mass system.

Of course, assumption (1) may, in fact, be wrong in the regions near boundaries, since it is possible to construct models wherein the correlations do not vanish as P_{lab} increases, e.g., diffraction-dissociation models. It will be necessary to incorporate additional realistic model features as experience increases and data are obtained on two-particle inclusive cross sections.

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