Bootstrap Calculation of the K^* Meson Using the Effective-Potential Method

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We use the effective-potential method of Balázs, with tensor-meson as well as vector-meson exchange included as suggested by an earlier calculation in the $\pi\pi$ case, to calculate πK scattering. Taking the strengths of the potentials due to the exchange of other particles from the experimental parameters of the particles, we "bootstrap" the K^*, and obtain self-consistent values for the K^* mass and width of 930 MeV and 50 MeV, in very good agreement with experiment. Some uncertainty in the predicted values, especially of the mass, results from the necessity of using SU(3) to obtain the $\rho K\bar{K}$ coupling, and the fact that the experimental values of the ρ and K^* widths suggest substantial SU(3) breaking. Results for other partial waves of the πK system are presented, and output K^* and K^{**} Regge trajectories, with slopes about $\frac{1}{2}$ and $\frac{1}{3}$ of the "canonical" value of 1 BeV^{-2}, respectively, are obtained.

In an earlier paper[1] (hereafter referred to as I) we have used the effective energy-dependent potential method developed by Balázs[2] to carry out a "double bootstrap" calculation of the ρ and f^0 resonance parameters in $\pi\pi$ scattering. The force due to the exchange of both ρ and f^0 is included in the input, and both are obtained, with self-consistent masses and widths, as output resonances. The self-consistent values of the parameters are in reasonable agreement with experiment. In particular, the agreement is far better than when only the ρ-exchange force is included in the input.[2,3]

It is our purpose here to inquire to what extent similar results hold for the πK problem. We have used the effective-potential method, with the effects of f^0 as well as ρ exchange in the t channel, and $K^{**}(1420)$ as well as $K^*(890)$ exchange in the u channel, included. The potentials are easily obtained by analogy with the $\pi\pi$ case. There the ρ-exchange potential is given by[2,3]

$$V(r,s) = -24(2\beta_{11})q_\rho{}^2\Gamma_\rho s^{-1/2}P_1(1+s/2q_\rho{}^2)\,e^{-m_\rho r}/r$$
$$= -12(2\beta_{11})\Gamma_\rho s^{-1/2}(2q_\rho{}^2+s)\,e^{-m_\rho r}/r\,. \qquad (1)$$

In Eq. (1), s, as usual, stands for the c.m. energy squared, q_ρ is the $\pi\pi$ c.m. momentum at $s=m_\rho{}^2$, $\beta_{11}=\frac{1}{2}$ is an element of the $\pi\pi$ isospin crossing matrix, and the factor of 2 is present because the t- and u-channel potentials are equal in this case. Γ_ρ, the reduced width, is related to $\Gamma_{\rho s}$, the resonance half-width in the energy-squared variable s, by

$$\Gamma_\rho = m_\rho\Gamma_{\rho s}/8q_\rho{}^3\,, \qquad (2)$$

and the factor of 24 in the first line of (1) is the product of 8 [from Eq. (2)] and $2l+1$, with $l=1$ for ρ exchange. To get the corresponding result for the ρ-exchange potential in the $I=\frac{1}{2}$ state in πK scattering, which will contribute to generating the K^* and K^{**}, we replace β_{11} by the element $\beta_{\frac{1}{2}1}=1$ of the πK crossing matrix (with no factor of 2 now, since there is no ρ in the u channel), we replace Γ_ρ by $(\Gamma_\rho\Gamma'_\rho)^{1/2}$, and $1+s/2q_\rho{}^2$ by $(q_\rho{}^2+q'_\rho{}^2+s)/2q_\rho q'_\rho$. Γ'_ρ and q'_ρ are the analogous quantities to Γ_ρ and q_ρ for the ρ in the $K\bar{K}$ channel, so that $q'_\rho{}^2 = m_\rho{}^2/4 - m_K{}^2$. Making these replacements in (1), one gets for V_ρ, the ρ-exchange potential in the πK isospin-$\frac{1}{2}$ state,

$$V_\rho(r,s) = -12(\Gamma_\rho\Gamma'_\rho)^{1/2}s^{-1/2}(q_\rho{}^2+q'_\rho{}^2+s)\,e^{-m_\rho r}/r\,. \qquad (3)$$

If we assume the validity of SU(3) (as we shall see below, there are some difficulties with this) and make the standard[4] assumption that the coupling constants to which SU(3) applies are proportional to the reduced widths, we can express both Γ_ρ and Γ'_ρ in terms of Γ^*, the reduced width for $K^*\to\pi K$, by the relations[5]

$$\Gamma_\rho = \tfrac{8}{3}\Gamma_{K^*}\,, \qquad (4a)$$

$$\Gamma'_\rho = \tfrac{2}{3}\Gamma_{K^*}\,, \qquad (4b)$$

whereupon the expression for V becomes

$$V_\rho(r,s) = -16\Gamma_{K^*}s^{-1/2}(q_\rho{}^2+q'_\rho{}^2-s)\,e^{-m_\rho r}/r\,. \qquad (3')$$

Similarly, one can write down expressions for V_f, V^*, V^{**}, the potentials in the πK isospin-$\frac{1}{2}$ state due to f^0, K^*, and K^{**} exchange.

One obtains

$$V_f(r,s) = -(20/\sqrt{6})(\Gamma_f\Gamma'_f)^{1/2}4q_f{}^2q'_f{}^2s^{-1/2}P_2(1+s/2q_fq'_f)\,e^{-m_f r}/r\,, \qquad (5)$$

$$V^*(r,s) = -(-1)^I 4\Gamma^* s^{-1/2}(m^{*2} - \Sigma - q^{*2} + s)\, e^{-m^* r}/r\,, \tag{6}$$

$$V^{**}(r,s) = (-1)^I 6.67\Gamma^{**}_{el} 4 q^{**4} s^{-1/2} P_2(1 + (\Sigma - m^{**2} - s)/2q^{**2}) e^{-m^{**} r}/r\,. \tag{7}$$

In these equations, q_f and q_f' are the $\pi\pi$ and $K\bar{K}$ c.m. momenta at the energy of the f, q^* and q^{**} are πK c.m. momenta at the energies of the K^* and K^{**}, whose masses are denoted by m^* and m^{**}, respectively, and $\Sigma = 2m_K^2 + 2m_\pi^2$. Γ_f and Γ_f' are the reduced partial $\pi\pi$ and $K\bar{K}$ widths of the f, so that

$$\Gamma_f = m_f \Gamma_{f\pi\pi s}/8q_f^5\,, \tag{8}$$

with $\Gamma_{f\pi\pi s}$ being the partial half-width in s for the decay $f^0 \to \pi\pi$, with an analogous relation holding between Γ_f' and the partial half-width for $f^0 \to K\bar{K}$. Γ^{**}_{el} is the reduced width of the K^{**} in the πK channel, defined by analogy with Eq. (8).

In principle, one could now carry out the exact analog of the calculation in I. One could take m_ρ, m_f, Γ_ρ, and Γ_f from experiment, or, almost equivalently, from the theoretical values obtained in (1). Γ_ρ' could be expressed in terms of Γ^* by SU(3). For Γ_f', one could either used the approximate experimental value[6] or attempt to relate it to Γ^{**} or Γ_f by SU(3). With these parameters fixed, one could then proceed to seek values of the masses and widths of the K^* and K^{**} which, when used in the input potentials, would give rise to self-consistent output resonances in the isospin-$\frac{1}{2}$ P and D waves. There are two difficulties with this approach. The first is that SU(3) does not seem to work well for the vector-meson widths. SU(3) predicts $\gamma_{\rho\pi\pi}{}^2/\gamma_{K^*K\pi}{}^2 = \frac{8}{3}$, where the γ's are the indicated coupling constants, while experimentally Γ_ρ and Γ^* have a ratio of 1 to within about 5% for a ρ width of 125 MeV. Put another way, using the ρ width and SU(3) leads to a value of Γ_ρ' only about $\frac{3}{8}$ as large as, and hence a value of V_ρ about 60% of, what is obtained if one used the K^* width to obtain Γ_ρ'. Secondly, the K^{**} is quite inelastic compared with the f^0, with a πK branching ratio of only about 50%. Hence it seems unrealistic to attempt to obtain the K^{**} in a calculation of the present type, in which inelasticity is neglected.

In view of these problems, we have carried through a less ambitious program for the πK case than for the $\pi\pi$ problem in I. We use the experimental values of the K^{**} parameters to determine V^{**}, and do not attempt to determine these self-consistently, although we do give the resulting output D-wave results. We also use the experimental values[6,7] for the f^0 parameters in determining V_f. The $K\bar{K}$ branching ratio of the f^0 is quite uncertain experimentally; the present experimental value of Γ_f'/Γ_f is consistent with SU(3) and the assumption that the f^0 is primarily an SU(3) singlet, as suggested by the Gell-Mann–Okubo formula for the squared masses of the tensor octet. Our results are not extremely sensitive to reasonable changes in the value of Γ_f'. Even for the K^* we can carry out only a partial bootstrap because, having chosen an input value of Γ^* and hence of V^*, the experimental violations of SU(3) do not leave us with a well-defined rule for obtaining Γ_ρ', and hence V_ρ. It seems reasonable to restrict Γ_ρ' to values ranging roughly from those obtained by using SU(3) together with the input value of Γ^*, or the physical ρ width, respectively. The latter value of Γ_ρ' leads to a V_ρ about 60% as large as if Γ_ρ' is obtained from the input value of Γ^*. Hence we must expect that, rather than unique values for the K^* mass and width, we will get a range of such values, depending on the value we take for Γ_ρ'.

Carrying out the calculation, we find that we obtain self-consistent values of 930 MeV and 50 MeV for the K^* mass and width, respectively. These results are, clearly, in very good agreement with the physical values of 890 MeV and 50 MeV. These values correspond to using Eq. (3′) for V_ρ, i.e., to using the value of Γ_ρ' obtained from SU(3) and the K^*, rather than the ρ, width. Choosing Γ_ρ' and hence V_ρ at the other end of our acceptable range yields about the same value for the K^* width, but a self-consistent mass about 100 MeV larger, i.e., about 1030 MeV. Thus, the results for the K^* are, if anything, better than those obtained in I for the ρ. We find it of particular interest that it is possible to obtain a bootstrap value in agreement with the rather narrow K^* width of 50 MeV, in view of the well-known difficulty with many bootstrap calculations of obtaining widths much larger than the experimental values.

As in the $\pi\pi$ case, the results for the K^* bootstrap are much better when the exchange of the tensor mesons as well as of the vector mesons is included in the input force than if one assumes, as in the original bootstrap calculations, that vector-meson exchange only is the dominant force in producing the output vector-meson resonances. We have repeated the calculation described above with the f^0 and K^{**} exchange potentials set equal to zero. Using the physical value for the K^* width, and the corresponding maximum strength within our permissible range for V_ρ, yields far too weak an attractive force. There was no resonance pro-

duced up to 1700 MeV, the highest energy for which we did the calculation; the P-wave phase shift had reached only about 65° at that energy. It was possible to obtain a self-consistent K^* at 910 MeV, about the physical value, using only K^* exchange and V_ρ as given by Eq. (3′), but the self-consistent value of the width is 120 MeV, 2.4 times too large. Finkelstein[3] also makes a bootstrap calculation of the K^* using the effective-potential method with only ρ and K^* exchange. However, his expressions for the ρ and K^* exchange potentials [Eqs. (5) of Ref. 3] are too large by a factor of 2 so that his solution is not, in fact, self-consistent. Also, using Finkelstein's values for the potentials yields not only the output resonance which he identified as the K^*, but, in fact, also a bound state. This can be seen from the results obtained when we calculated the output P-wave amplitude for several potentials of strength less than the large values which come from using Finkelstein's values for the widths and his Eqs. (5). We found that, as the potential was reduced in strength from the value used in Ref. 3 towards magnitudes comparable with the potential strengths in our solution, the resonance initially at 890 MeV moved to higher and higher energies, as one would expect. However, as the potential strength was reduced, a new resonance appeared at *lower* energies (near threshold), indicating that as the potential strength was reduced from the value in Ref. 3 a bound state emerged from below threshold.

Once we have a self-consistent solution, we can proceed to calculate the scattering in other angular momentum and isospin states of the πK system, using the self-consistent values of the potential. [For the remainder of the paper, we choose the bootstrap solution with $m^* = 930$ MeV and V_ρ given by (3′), since that is the solution closest to the experimental values.] To begin with, we attempt to determine the properties of the K^* Regge trajectory by obtaining the scattering amplitudes for the isospin-$\frac{1}{2}$ odd-signature partial waves with $j = 2$ and 3; of course the odd-signature $j = 2$ amplitude is unphysical, being calculated with a potential with the "wrong" factors of $(-1)^I$ in the u-channel (K^* and K^{**}) potentials. We find that the theoretical K^* trajectory rises at least to $j = 3$. The $j = 2$ and $j = 3$ intercepts of the trajectory are at $s = 2.45$ BeV^2 and $s = 4.43$ BeV^2. This yields average values for the trajectory slope, α', of $\alpha' = 0.63$ BeV^{-2} in the interval between $j = 1$ and $j = 2$, and $\alpha' = 0.52$ BeV^{-2} in the interval from $j = 2$ to $j = 3$. Hence the predicted trajectory is approximately linear, although turning over slightly in the interval in question. However, the predicted slope is only about half of the "canonical" value of 1 BeV^{-2}. This is very similar to what was found for ρ trajectory in I. In the case of the ρ, fits to high-energy scattering data, as well as the masses of the ρ and g mesons, presumed to be the $j = 3$ recurrence of the ρ, yield a value for the slope close to the canonical value. Presumably the same is true for the K^*, belonging as it does to the same SU(3) multiplet, but there is little direct evidence on the K^* trajectory either from fits to high-energy data involving the exchange of a unit of strangeness or from the identification of the mass of the $j = 3$ recurrence.

Next we consider the physical (even-signature) $j = 2$ partial-wave amplitude. In a completely self-consistent solution, one would obtain an output K^{**} resonance in this partial wave, as one obtained an output f^0 in I. In fact, we find only that the phase shift rises monotonically, but slowly, reaching only 12° at the energy of the K^{**}, and about 35° at 2 BeV, which was as far as we extended the calculation. The model does in fact predict an even-signature trajectory, as we find a resonance in the (unphysical) $j = 1$ partial wave of the even-signature amplitude at 1750 MeV, to be compared with a value of about 1 BeV for the $j = 1$ intercept of the physical trajectory if it has the canonical slope. The $j = 0$ intercept of the K^{**} trajectory, which would be at about $s = 0$ physically if the slope is canonical, does lie below threshold in our model, occurring as a bound state in the $I = \frac{1}{2}$, $j = 0$ partial wave. (However, as discussed below, one cannot place great reliance on the results of the model for $j = 0$.) Although we cannot detect the bound state directly, we can infer its presence. In the first place, we find no resonance in the S wave, corresponding to a $j = 0$ trajectory intercept, above threshold. Secondly, we find a negative scattering length with our actual potential, although it is attractive in the S wave, while when the strength of the attractive potential is somewhat reduced, the bound state emerges through threshold as a resonance. Assuming the $j = 0$ intercept occurs between threshold ($s = 0.41$ BeV^2) and $s = 0$, and using the value of the $j = 1$ intercept, we get a value of approximately 0.35 BeV^{-2} for the slope of the K^{**} trajectory, so that the theoretical value of the slope of this trajectory is too small. As we have already mentioned, inelastic channels are clearly important in the case of this trajectory, at least at energies corresponding to those of the K^{**}, so that we would not expect to be able to reproduce the K^{**} with any quantitative accuracy. One of the principal reasons why the K^{**} slope turns out to be smaller than that of other trajectories calculated in the model (the K^* or the trajectories obtained in I) is the K^{**} exchange force. This force is strong but of short range, fairly strongly energy-dependent since it comes from

$j=2$ exchange, and repulsive for the even-signature amplitude. The growing importance of this repulsive force at higher energies, both because of its short range and its energy dependence, tends to slow the rise of the K^{**} trajectory. It may be that, in the physical case, this effect is compensated for by the effects of inelastic channels which are neglected here, or it may be that one should not take this strongly repulsive force completely literally, at least at higher energies.

We observe in passing that, in general, the trajectories found in the effective-potential model, both here and in I, tend, at least where it is possible to tell, to be roughly linear, in contrast with those found from energy-independent Yukawa potentials,[8] which tend to turn over at low values of j. Trivedi[9] has emphasized the energy dependence of the potential as a possible mechanism for generating infinitely rising trajectories. On the other hand, the slopes of the trajectories obtained in the present model tend to be too small. This suggests that the linearity and observed value of the slope of physical trajectories may result from a combination of the energy dependence of the effective potential plus the alternative mechanism[10] in which rising trajectories result from coupling to inelastic channels containing spinning particles, in which a large j is possible without large values of l and the corresponding large centripetal barriers.

We next investigate the $I=\frac{3}{2}$ P-wave amplitude, altering the potentials of Eqs. (3) and (5)–(7) by inserting the appropriate crossing matrix elements for the case of isospin $\frac{3}{2}$ in the direct channel. The longest-range force, ρ exchange, is now repulsive, though only half as strong as for $I=\frac{1}{2}$, while K^* and K^{**} are four times as strong and, in states of odd angular momentum, attractive and repulsive, respectively. The net result is a weak repulsive force in the low-energy range. The phase shift decreases slowly with energy, reaching about $-8.5°$ at a πK mass of 1.2 BeV, which was as far as the calculation was carried. Experimentally, not much is known directly about this phase shift, except for the absence of observed resonances. Measurements do indicate that the total isospin-$\frac{3}{2}$ cross section is small, of the order of 2 mb or a little more in the energy range under consideration.[11] Our P-wave phase shift is consistent with this; at 1.2 BeV it would contribute a partial cross section of about 1.2 mb, about 50% of the observed value, and at lower energies much less.

For the sake of completeness, one can also attempt to calculate the scattering in the S waves in the two isospin states. The reliability of the model is highly questionable here, due to the probable sensitivity of the S waves, with their lack of centripetal barrier, to the unknown details of the potential at short range. Some very limited success was attained in the calculation of the $\pi\pi$ S waves.[12] However, in the present case, we find not even qualitative agreement between the πK S waves and either the experimental information available[13] or the prediction of current algebra.[14] The sign of the scattering lengths is wrong in both isospin states, our theory yielding a negative value for $a_{1/2}$, as already mentioned, and a positive value for $a_{3/2}$. The fact that $a_{1/2}<0$, even though the force is attractive, presumably reflects the $j=0$ intercept of the K^{**} trajectory occurring below threshold. The theoretical $I=\frac{1}{2}$ S-wave phase shift remains negative throughout the energy region under consideration; there is no sign of an S-wave resonance such as was found in the $\pi\pi$ calculation[12] and suggested by experiment in the πK case.[13] The $I=\frac{3}{2}$ S-wave phase shift remains positive throughout the energy range; this results from the fact that the potential in our model is actually attractive, the attractive f^0 and K^{**} exchange forces actually dominate the long-range but weak repulsive K^* and ρ exchange forces at all energies. (The latter forces are weak because of the narrow K^* width and the small value of $\beta_{3/2,1}$, respectively.) Because of the tendency of the vector-meson and tensor-meson exchange forces to cancel, however, the net potential in this state is small, and we would expect it therefore to be unusually sensitive to the presence of additional short-range components of the potential, so that the results for the isospin-$\frac{3}{2}$ S wave are likely to be especially unreliable. A similar problem is encountered in the $I=2$ $\pi\pi$ state.[8] We are not inclined to believe that the poor results for the S-wave amplitudes should be taken as a serious argument against the validity of the model for the higher partial waves.

In conclusion, then, the effective-potential model with tensor-meson and vector-meson exchange yields a self-consistent K^* in very good agreement with experiment. Some uncertainty in the predicted values of the K^* parameters, particularly the mass, results from the necessity of using SU(3) to obtain the $\rho K\overline{K}$ coupling, and the fact that the experimental values of the ρ and K^* widths suggest substantial SU(3) breaking. The results for other properties of the πK system at low energies and for $j>0$ are qualitatively reasonable, though the Regge-trajectory slopes predicted, especially for the K^{**} trajectory, are too small. The latter effect may well be due, at least partially, to the neglect of inelasticity in the model. The results for the S-wave amplitude are very poor; however, it is not clear that the model should

reasonably be expected to produce reliable results for the S waves.

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Quantitative Regge Expressions with Scaling for Experimentally Measured Fast $\pi^{\pm}, K^{+}$ Inclusive Spectra and Relation to Backward-Elastic-Scattering Data*

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We represent the spectra for fast π^+, π^-, K^+ secondaries in pp collisions in Reggeized forms that describe the experimental data measured at several energies and that scale at high energies. We relate the inclusive amplitudes obtained to backward elastic $\pi^{\pm}p, K^+p$ scattering and predict both their relative magnitudes and their energy and angle dependence.

I. INTRODUCTION

The inclusive reaction $p+p \to X+$anything, where X is a specified secondary, offers the simplest probe of the dynamics of particle production processes, and has recently been the subject of experimental and theoretical study. Particle spectra have been measured in pp collisions at momenta of 12.4 GeV/c,[1] 19.2 GeV/c,[2] and 30 GeV/c.[3] General considerations about the features of the spectra have been proposed by Feynman[4] and Benecke, Chou, Yang, and Yen[5]; Vander Velde[6] has demonstrated the equivalence of their approaches and shown that the fast secondaries of Refs. 1–3 satisfy their predictions. Calculations of the spectra within specific models – the multi-Regge model[7–9] and the Hagedorn thermodynamic model[10] – have been compared with the measured data.

In this paper we show that the spectra of fast produced $\pi^{\pm}$, K^+ in pp collisions can be represented by Reggeized expressions that scale at large energy, and that such expressions are intrinsically related to the experimental backward $\pi^{\pm}p$ and K^+p elastic two-body differential cross sections. This approach will allow us not only to predict the inclusive fast spectra at high energies, but also to extract the properties of backward elastic cross sections at lower energies.

Consider first the process $p+p \to m+$anything of Fig. 1(a), with "m" a produced meson and M^2 representing the remaining secondaries. We will describe those produced $\pi^{\pm}, K^+$ that emerge with large momentum in the center-of-mass (c.m.) system, and hence with large positive momentum or small negative momentum in the lab. In a multi-Regge model, these mesons emerge at the end of the multiperipheral chain, with the incident proton traveling a link down the chain and peripherally scattering with the other incident proton [Fig. 1(b)]. We represent this process with the diagram of Fig. 1(c), the cross section of which is

$$\frac{d^2\sigma}{dt\,dM^2} = \frac{1}{s^2}\left(\frac{s}{M^2}\right)^{2\alpha(t)} \beta^2(t) M^2 \sigma_T(M^2) . \qquad (1)$$