Phenomenological Study of Single-Particle Distributions near the Kinematical Limits*

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(Received 5 May 1971)

We present a phenomenological analysis of the Regge-pole predictions for single-particle distributions from inclusive reactions of the type $a+b\to c+$anything near the kinematical limits of the outgoing particle c. A prototype comparison with experimental data is made via a detailed analysis of the p, $\bar{p}$, π^+, π^-, K^+, and K^- distributions from inelastic pp scattering at 19.2 GeV/c and also some π^- distributions from pp scattering at 30 GeV/c. Our finding is that the distributions are not yet dominated by the known leading Regge poles. Some explanations are given. The prospects of this kind of study at higher machine energies are also discussed.

I. INTRODUCTION

The single-particle distributions from inclusive reactions of the type

$$a+b\to c+\text{anything } (X) \qquad (1)$$

have recently become the subject of various investigations.[1,2] Near the kinematical limits for particle c, when $s\gg s'\gg 1$ GeV^2, and for reasonably small values of t [where s and s' are, respectively, the center-of-mass (c.m.) energy squared and the missing mass squared and t is the square of the invariant momentum transfer between particles a and c], the Regge-pole model gives a simple but quantitative prediction for the invariant cross section for the particle c,[3]

$$E\frac{d^2\sigma}{dp_l dp_t{}^2}=\frac{d^2\sigma}{dt\,d(s'/s)}\simeq|\beta_\alpha^{ac}(t)|^2\left(\frac{s}{s'}\right)^{2\alpha(t)-1}\eta_P^{\alpha\alpha}(t)\beta_P^{bb}(0)\,. \qquad (2)$$

In Eq. (2) E, p_l, and p_t are, respectively, the energy, longitudinal momentum, and transverse momentum of particle c, $\alpha(t)$ is the dominant Regge trajectory exchanged between particles a and c, $\beta_\alpha^{ac}(t)$ is the coupling of the Regge pole $\alpha(t)$ to particles a and c, and $\eta_P^{\alpha\alpha}(t)$ and $\beta_P^{bb}(0)$ are the triple-Reggeon coupling and the Pomeranchukon-particle-b coupling, respectively. Their precise meaning will be explained in Sec. II. In this paper, we present an analysis of the presently available data parametrized according to Eq. (2). The finding is that this parametrization is not valid at the present machine energies in the sense that none of the $\alpha(t)$'s obtained correspond to the known leading-Regge-pole exchange between particles a and c.

In Sec. II, for completeness, we give a brief derivation of Eq. (2) and discuss the kinematical conditions for its validity and possible corrections to it. In Sec. III, the content and applications of Eq. (2) are discussed and a procedure of data analysis in this form is presented. Section IV contains the results of our data analysis of inelastic pp reactions at 19.2 GeV/c (Ref. 4) and 30 GeV/c (Ref. 5). The results are summarized in Figs. 3–8 and Table I. In Sec. V, we discuss the results of this analysis and the prospects for this kind of study at higher machine energies.

II. DERIVATION

When the c.m. energy squared s is much greater than the missing mass squared $s'\equiv(p_a+p_b-p_c)^2$ and for reasonably small momentum transfer squared $t=(p_a-p_c)^2$, the amplitude for the reaction $a+b\to c+X$ is like that of a quasi-two-particle reaction. The leading Regge pole exchanged in the ac channel gives

$$\frac{d^2\sigma}{dt\,ds'}\simeq\frac{1}{s^2}|\beta_\alpha^{ac}(t)|^2\left(\frac{s}{s'}\right)^{2\alpha(t)}A(s',t)\,. \qquad (3)$$

Here $A(s',t)$ can be considered as the absorptive part of the forward scattering amplitude for Reggeon-particle scattering: $\alpha(t)+b\to\alpha(t)+b$ (see Fig. 1). As usual, the criterion for the asymptotic behavior represented by Eq. (3) is that the absolute value of

$$\cos\theta_t=\frac{2st+t^2-t(m_a{}^2+m_b{}^2+m_c{}^2+s')+(m_a{}^2-m_c{}^2)(m_b{}^2-s')}{\{[t-(m_a-m_c)^2][t-(m_a+m_c)^2][t-(s'^{1/2}+m_b)^2][t-(s'^{1/2}-m_b)^2]\}^{1/2}} \qquad (4)$$

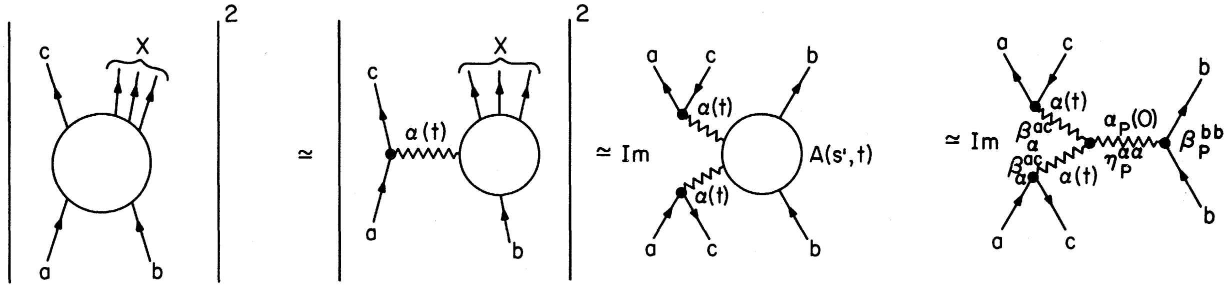

FIG. 1. Graphical illustration of the dominant contribution to the single-particle distribution from $a+b \to c+X$. The straight lines represent the particles and the wavy lines represent the Reggeons.

should be large. Furthermore, if s' is reasonably large, we assume that $A(s', t)$ can be Reggeized in the same way as the usual two-body reactions, then

$$A(s', t) = \eta_P^{\alpha\alpha}(t)\beta_P^{bb}(0)(s')^{\alpha_P(0)}, \tag{5}$$

where $\alpha_P(0)$ is the intercept of the Pomeranchuk trajectory, $\beta_P^{bb}(0)$ is the usual Regge coupling between particle b and the Pomeranchukon, and $\eta_P^{\alpha\alpha}(t)$ is the triple-Reggeon coupling among $\alpha(t)$, $\alpha(t)$, and $\alpha_P(0)$. Combining Eqs. (3) and (5), we have

$$\frac{d^2\sigma}{dt\,d(s'/s)} \simeq |\beta_\alpha^{ac}(t)|^2\left(\frac{s}{s'}\right)^{2\alpha(t)-1}\eta_P^{\alpha\alpha}(t)\beta_P^{bb}(0)(s')^{\alpha_P(0)-1} \tag{6a}$$

$$= [\,|\beta_\alpha^{ac}(t)|^2 s^{\alpha(t)-1}]s^{\alpha(t)}[(s')^{\alpha_P(0)-2\alpha(t)}\eta_P^{\alpha\alpha}(t)\beta_P^{bb}(0)]\,. \tag{6b}$$

If the reaction $a+b\to c+X$ is dominated by the same trajectory $\alpha(t)$ as that in the two-body reaction $a+c\to c+a$, the first factor in Eq. (6b) is simply

$$\left(\frac{d\sigma}{dt}(a+c\to c+a)\right)^{1/2}.$$

It is important to notice that

$$\frac{d^2\sigma}{dt\,d(s'/s)}\Big/\left(\frac{d\sigma}{dt}\right)^{1/2}$$

still depends explicitly on s, s', and t.[6]

As mentioned above, Eq. (6) is valid provided that

$$s \gg s' \text{ and } t \text{ is small such that } |\cos\theta_t| \gg 1 \tag{7a}$$

and

$$s' \gg 1\,. \tag{7b}$$

At finite energies, there are two kinds of corrections to Eq. (6). If condition (7a) is not well satisfied, there are additional terms in Eq. (6) due to the secondary trajectory $\alpha_2(t)$ exchanged between particles a and c. The energy dependences of such corrections are of the form

$$s^{\alpha(t)+\alpha_2(t)-1}s'^{\alpha_P(0)-\alpha(t)-\alpha_2(t)}, \tag{8}$$

where $\alpha_2(t) < \alpha(t)$. Due to the complexity of the inclusive reactions, there are other corrections which do not exist for two-body reactions. Examples are shown in Figs. 2(a) and 2(b). For these

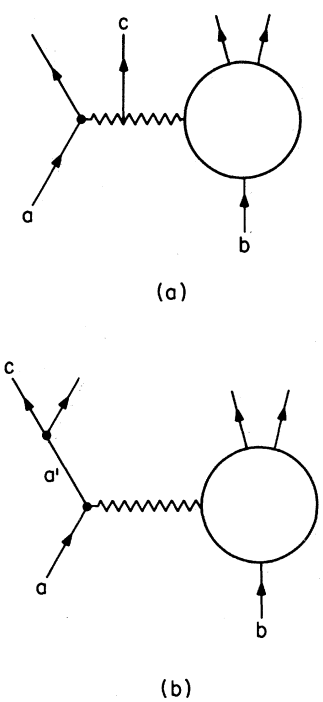

FIG. 2. Graphical illustration of some correction terms in which the outgoing particle c is not produced from the graph shown in Fig. 1.

types of corrections to be important, it can be shown that s' will usually be comparable to s. Hopefully these corrections will be negligible for large s/s' corresponding to large $|\cos\theta_t|$. Without a detailed model, the question of how large $|\cos\theta_t|$ should be can be answered only from a phenomenological analysis.

When the condition (7b) is not well satisfied, we have secondary contributions to $A(s', t)$. The energy dependence of their contributions to Eq. (6) will have the form

$$s^{2\alpha(t)-1}s'^{\alpha_{P'}(0)-2\alpha(t)}, \tag{9}$$

where $\alpha_{P'}(0)$ is the intercept of the P' trajectory.

However, it might be interesting to point out that in all the distributions we have studied here except the proton distribution, the missing-mass channels are exotic ones (i.e., no resonances can contribute); therefore we expect this correction term possibly to be small.

If $\alpha_P(0)=1$, Eq. (6) reduces to Eq. (2). Notice that the invariant cross section then becomes a function of t and the scaling variable s'/s. It can be shown that these two variables are related to the limiting variables of Feynman,[1] $x=2p_l/s^{1/2}$ and p_t in the c.m. frame, by

$$s'/s \simeq 1-x, \quad t\simeq m_a^2+m_c^2-(p_t^2+m_c^2)/x \tag{10}$$

and also to the limiting variables of Benecke *et al.*,[2] p_l and p_t in the laboratory frame, by

$$\begin{aligned} s'/s &\simeq (p_l^{\text{boundary}}-p_l)/2m_a, \\ t &= m_a^2+m_c^2-2m_a(p_l^2+p_t^2+m_c^2)^{1/2}. \end{aligned} \tag{11}$$

Therefore, as $s \gg s'$, Eq. (2) gives the behavior of the limiting distribution near the kinematical boundary.

III. PHENOMENOLOGICAL ANALYSIS

In this section we discuss the comparison of Eq. (6) with experimental data. For a given set of data at various values of s, s', and t, one can first find out whether Eq. (6) is indeed correct. More modestly, we can always assume that the behaviors in s and s' in Eq. (6b) are dominated by effective trajectories $\alpha_{\text{eff}}(t)$ and $\alpha_{\text{eff}'}(t)$, respectively. For fixed s', $\alpha_{\text{eff}}(t)$ can be determined by plotting $\ln[d^2\sigma/dt\,d(s'/s)]$ vs $\ln s$ for various fixed values of t. These fixed-t curves should be straight lines if Eq. (6) is valid. The slopes of these plots then give the powers, $n(t)$, in $s^{n(t)}$. Similarly, the power, $n'(t)$, in $(1/s')^{n'(t)}$, can be determined by plotting $\ln[d^2\sigma/dt\,d(s'/s)]$ vs $\ln(1/s')$ for various values of t and fixed s. The effective trajectories can then be obtained from the relations

$$n(t)=2\alpha_{\text{eff}}(t)-1, \tag{12a}$$

$$n'(t)=2\alpha_{\text{eff}'}(t)-1. \tag{12b}$$

If $n(t)=n'(t)$, the cross section possesses the scaling

$$\frac{d^2\sigma}{dt\,d(s'/s)} \propto \left(\frac{s}{s'}\right)^{n(t)}.$$

Furthermore, if $\alpha_{\text{eff}}(t)$ thus obtained turns out to be indeed the dominant $\alpha(t)$ for the reaction $a+c \to c+a$, the first factor in Eq. (6b) simply becomes $[d\sigma/dt(a+c\to c+a)]^{1/2}$. With $\alpha_P(0)$ and $\beta_P^{bb}(0)$ given from the total cross-section analysis, we can determine the triple-Reggeon coupling[7] $\eta_P^{\alpha\alpha}(t)$ by

$$\eta_P^{\alpha\alpha}(t)=\left(\frac{(s')^{2\alpha(t)-\alpha_P(0)}}{\beta_P^{bb}(0)}\right)\frac{\dfrac{d^2\sigma}{dt\,d(s'/s)}(a+b\to c+X)}{s^{\alpha(t)}\left(\dfrac{d\sigma}{dt}(a+c\to c+a)\right)^{1/2}}. \tag{13}$$

If $\alpha_{\text{eff}}(t)$ turns out to be lower than the dominant $\alpha(t)$, it shows that, for some dynamical reason, $\eta_P^{\alpha\alpha}(t)$ is small. Then one has to determine the coupling $\beta_{\alpha_2}^{ac}(t)$ for the secondary trajectory in $d\sigma/dt(a+c\to c+a)$ in order to determine $\eta_P^{\alpha\alpha_2}(t)$. However, if the $\alpha_{\text{eff}}(t)$ turns out to be very low, it is questionable whether condition (7a) is satisfied. The correction terms given by Eq. (8) must be important.

If $n(t)\neq n'(t)$, then $d^2\sigma/dt\,d(s'/s)$ does not scale in s'/s. However, if the difference between $n'(t)$ and $n(t)$ is a constant, we may still have $\alpha_{\text{eff}}(t)=\alpha_{\text{eff}'}(t)$ and the difference is attributed to the secondary contribution, $\alpha_{P'}(0)$, in $A(s', t)$ [see Eq. (9)]. If the difference is t-dependent, then Eq. (6) is totally unsatisfied and, as discussed in Sec. II, there must be more complicated corrections.

IV. RESULTS FROM THE DATA ANALYSIS

In this section, we analyze the single-particle distribution from inelastic pp scattering experiments along the lines presented in Sec. III. In principle, Eq. (2) should be valid only when the conditions (7a) and (7b) are well satisfied. For the presently available data, the kinematical region in which all these conditions are satisfied is too limited to allow detailed Regge behavior to be checked. Nevertheless, we wish to make a prototype comparison to illustrate the ideas in Sec. III and also to provide a background for working with future higher-energy experiments which can be carried out at, say, the CERN colliding-beam machine and the NAL 200-GeV accelerator.

The data we analyze below are high-precision measurements of the single-particle distributions from 19.2-GeV/c inelastic pp scattering done by

Allaby *et al.*[4] The observed outgoing particles are p, $\bar{p}$, π^+, π^-, K^+, and K^-. These data were taken in terms of the natural variables p and θ in the laboratory frame. To study the s' dependence at fixed values of t, we use an interpolating program to obtain the invariant cross section as a function of s' at fixed t. The experimental errors are typically 3–6%. Together with our estimated interpolation errors the over-all errors are typically 6–10%.

As discussed in Sec. III, we plot $\ln[d^2\sigma/dt\,d(s'/s)]$ vs $\ln(1/s')$ at various values of t and fixed s (= 37.8 GeV^2). The slopes $n'(t)$, and therefore the values of $\alpha_{\text{eff}'}(t)$, are obtained from a rough linear fit. (This rough fit is meant only to show the qualitative features.) In Figs. 3–8 we present the plots for the cross section $d^2\sigma/dt\,d(s'/s)$, on a logarithmic scale, vs $\ln(1/s')$ and for the $\alpha_{\text{eff}'}(t)$ thus obtained. In Table I, we give all the $\alpha_{\text{eff}'}(t)$ and the corresponding known leading Regge trajectories.

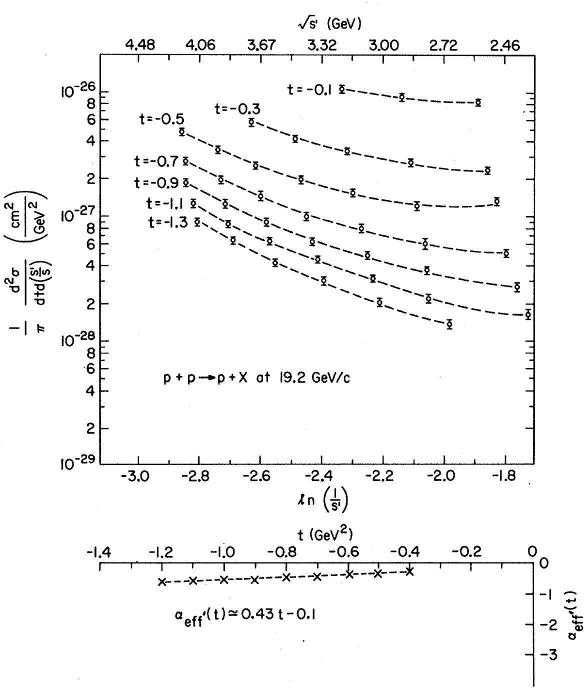

FIG. 3. The plots of $(1/\pi)[d^2\sigma/dt\,d(s'/s)]$, on a logarithmic scale, vs $\ln(1/s')$; and $\alpha_{\text{eff}'}(t)$ vs t for the reaction $p+p \to p+X$ at 19.2 GeV/c (s = 37.8 GeV^2) (Ref. 4). For reference, a scale for the missing mass $(s')^{1/2}$ is also presented in parallel with the $\ln(1/s')$ axis in the first plot. The dashed lines are hand drawn to guide the eyes. The error bars of the cross section represent the combination of the experimental errors and our estimated interpolation error. For easier recognition, not all the interpolated cross sections used to determine $\alpha_{\text{eff}'}(t)$ are presented. Furthermore, $\alpha_{\text{eff}'}(t)$ are determined only from those fixed-t curves which are approximately straight lines.

Unfortunately, there are no similar data for us to do the same interpolation at different incident energies. Thus we cannot do a systematic analysis for the s dependence to obtain $\alpha_{\text{eff}}(t)$. Therefore the scaling behavior of $d^2\sigma/dt\,d(s'/s)$ in s'/s, as given by Eq. (2), cannot be verified. However, we do calculate the $\alpha_{\text{eff}}(t)$ at $t \simeq 0$ by comparing this 19.2-GeV/c data with the 30-GeV/c $pp \to \pi^- X$ data by Anderson *et al.*[5] The result is that $\alpha_{\text{eff}}(t \simeq 0) \simeq -2$, which is consistent with $\alpha_{\text{eff}'}(t) \simeq -2$ as shown in Table I.

V. CONCLUSIONS AND DISCUSSIONS

We want to point out the following features of the analysis presented in the previous sections.

(1) The distributions of $\ln[d^2\sigma/dt\,d(s'/s)]$ as a function of $\ln s'$ all lie approximately on straight lines and rapidly decrease with s', except that the proton distribution from $p+p \to p+X$ shows a clear flattening as $\ln s'$ decreases.

(2) All the $\alpha_{\text{eff}'}(t)$ obtained are strikingly lower than the known leading Regge trajectories $\alpha_{\text{leading}}(t)$. From the discussion in Sec. III, this fact indicates that there is no evidence of appreciable coupling among $\alpha_{\text{leading}}(t)$, $\alpha_{\text{leading}}(t)$, and $\alpha_P(0)$, in particular

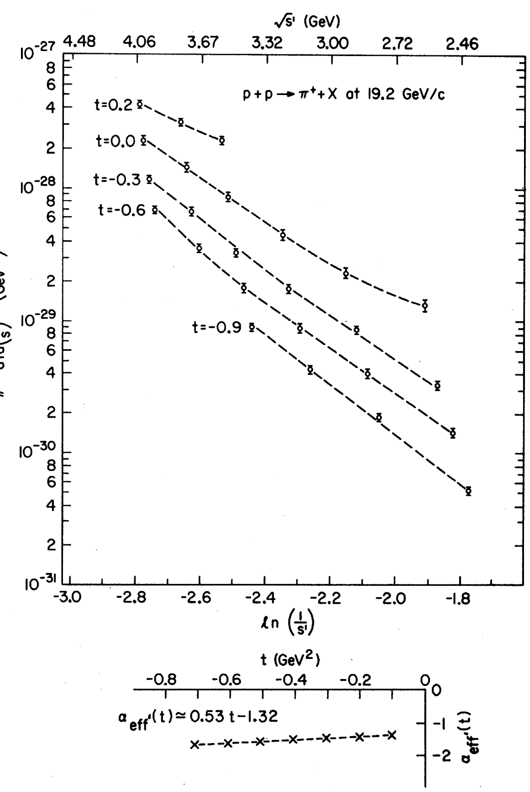

FIG. 4. The plots, similar to those in Fig. 3, of the cross section and $\alpha_{\text{eff}'}(t)$ for $p+p \to \pi^+ + X$ at 19.2 GeV/c (Ref. 4).

for the triple-Pomeranchukon coupling in the $p+p \to p+X$ case.

(3) The $\alpha_{\text{eff}}'(t)$ for exotic exchanges, like $p+p \to K^- + X$ and $p+p \to \bar{p}+X$, are indeed much lower than those for nonexotic exchanges.

The result that $\alpha_{\text{eff}}'(t) \ll \alpha_{\text{leading}}(t)$ could be attributed to the fact that the conditions (7) are not yet satisfied. It cannot be solely due to the fact that condition (7b) is not satisfied. For a given $n'(t)$, as seen from Eq. (12b), replacing $\alpha_P(0)$ by anything less than $\alpha_P(0)=1$ would make $\alpha_{\text{eff}}'(t)$ even lower. Hence the condition (7a) must not yet be satisfied. In Fig. 9, we plot $|\cos\theta_t|$ as a function of t for various given s' and s. The curve with $s'=16$ and $s=40$ corresponds to approximately the largest missing-mass points in our analysis. Therefore, it has the smallest $|\cos\theta_t|$, which is about 2. Notice that $|\cos\theta_t|$ increases by a large amount as s' decreases from 16 to 4. In the range of t we have considered, $|\cos\theta_t|$ varies approximately from 2 to 8. This range of $|\cos\theta_t|$ is quite reasonable for the leading Regge pole to dominate in the usual two-body reactions $a+c \to c+a$, especially for the elastic $\pi^\pm p$ scatterings and $\pi^- p$ charge-exchange reaction.[8] With this qualification, the drastic decrease of $\ln[d^2\sigma/dt\,d(s'/s)]$ with $\ln s'$ and the absence of evidence for any change of this trend even for small $\ln s'$ is surprising. This fact seems to suggest that, in order for Eq. (2) to be valid, the values of $|\cos\theta_t|$ required for inclusive reactions must be considerably greater than those for two-body reactions.

As discussed in Sec. II, the reason for the requirement of larger values of $|\cos\theta_t|$ is the complexity of the inclusive reaction for large s'. The particle c from the reaction $a+b \to c+X$ can be produced in many ways other than the one shown in Fig. 1; for example, like the ones shown in Fig. 2. All these terms can be eliminated only by requiring large enough $|\cos\theta_t|$. Our conclusion is that this requirement is not yet met at the present energy.

For a reasonable variation of s' from 4 to 16 GeV^2 to determine $\alpha_{\text{eff}}'(t)$, from our $|\cos\theta_t|$ plot, we feel that the future 200-GeV/c protons from NAL may have barely enough energy for condition (7a) to be satisfied in order to provide a possible determination of the triple-Reggeon coupling.

ACKNOWLEDGMENTS

The authors thank Dr. A. H. Mueller, Dr. Frank Paige, Dr. Ronald F. Peierls, and Dr. Jiunn-Ming Wang for various helpful discussions.

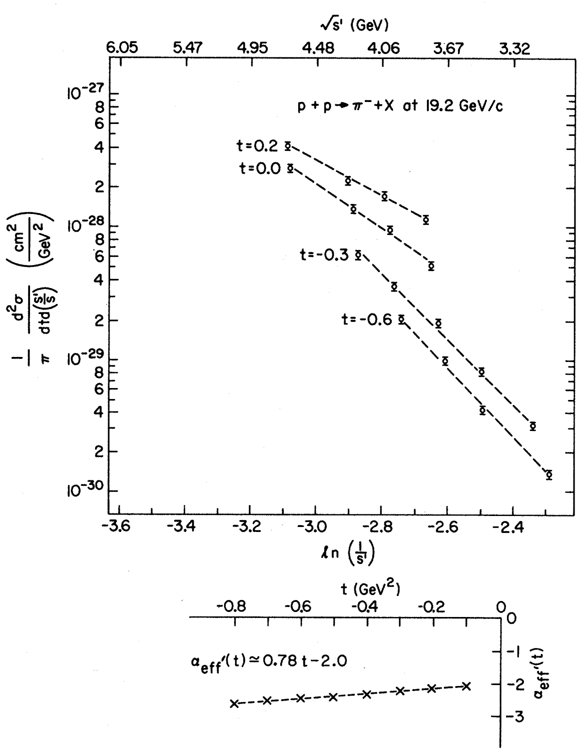

FIG. 5. The plots of the cross section and $\alpha_{\text{eff}}'(t)$ for $p+p \to \pi^- + X$ at 19.2 GeV/c (Ref. 4).

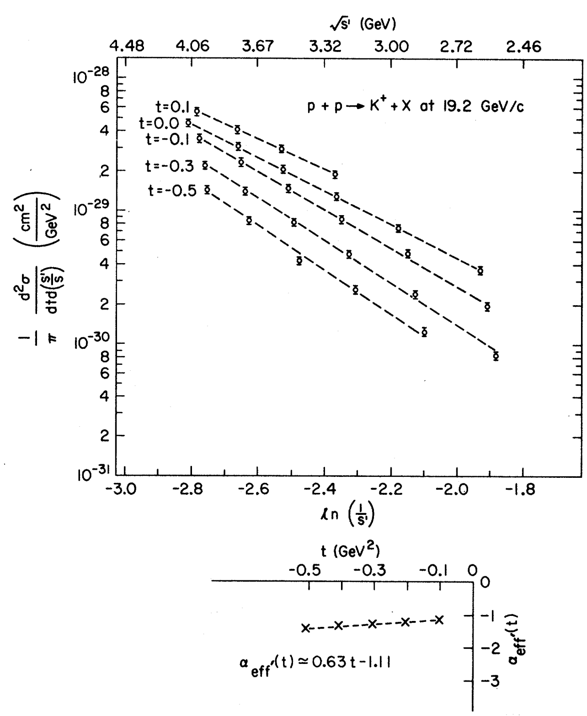

FIG. 6. The plots of the cross section and $\alpha_{\text{eff}}'(t)$ for $p+p \to K^+ + X$ at 19.2 GeV/c (Ref. 4).

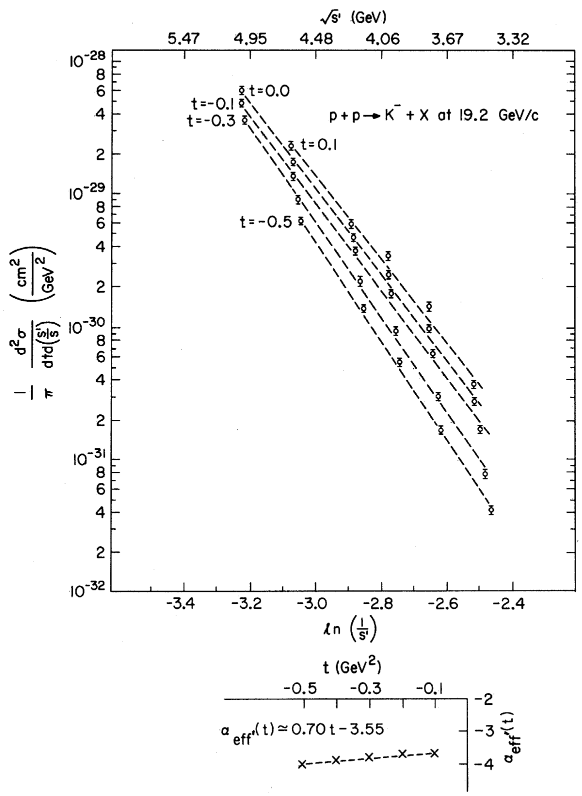

FIG. 7. The plots of the cross section and $\alpha_{\text{eff}'}(t)$ for $p+p \to K^- + X$ at 19.2 GeV/c (Ref. 4).

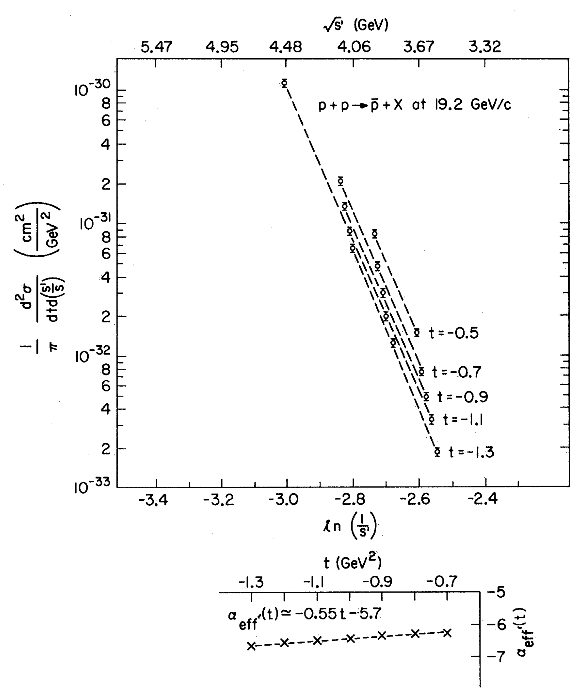

FIG. 8. The plots of the cross section and $\alpha_{\text{eff}'}(t)$ for $p+p \to \bar{p} + X$ at 19.2 GeV/c (Ref. 4).

TABLE I. Here we summarize our results of $\alpha_{\text{eff}'}(t)$ from Figs. 3–8. The leading $\alpha(t)$ is the dominant leading Regge exchange in the corresponding two-body $a+c \to a+c$ reaction. See Ref. 8.

Reactions	$\alpha_{\text{eff}'}(t)$	Leading $\alpha(t)$
$p+p \to p+X$	$-0.1 + 0.43t$	$\alpha_P(t) \approx 1 + 0.3t$
$p+p \to \pi^+ + X$	$-1.32 + 0.53t$	$\alpha_N(t) \approx -0.38 + 0.88t$
$p+p \to \pi^- + X$	$-2.0 + 0.78t$	$\alpha_\Delta(t) \approx 0.20 + 0.85t$
$p+p \to K^+ + X$	$-1.11 + 0.63t$	$\alpha(t) \approx -1$ [a]
$p+p \to K^- + X$	$-3.55 + 0.7t$	exotic
$p+p \to \bar{p} + X$	$-5.7 + 0.7t$	exotic

[a] This value is obtained from the energy dependence of the backward $K^+ + p \to p + K^+$ reaction. The exact trajectory is not clear. See again Ref. 8.

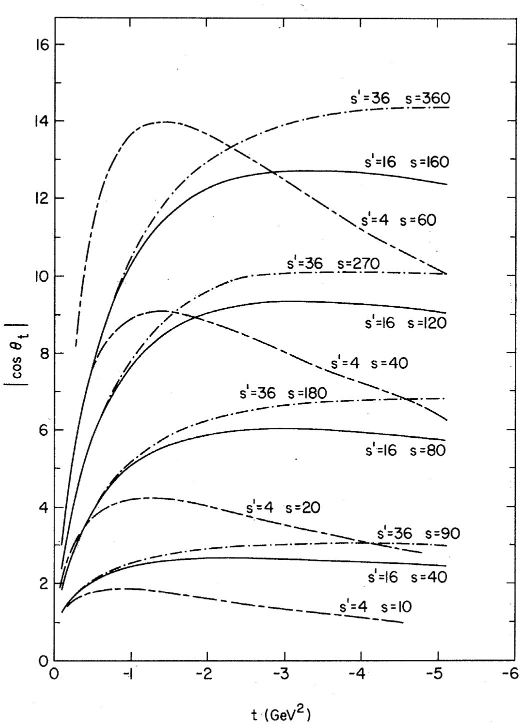

FIG. 9. Plot of $|\cos\theta_t|$, given by Eq. (4), vs t for some illustrative values of s and s'. Notice that $\cos\theta_t$ scales in s/s' near small $|t|$.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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[5]E. W. Anderson *et al.*, Phys. Rev. Letters 19, 198 (1967).

[6]Therefore it is not surprising that Allaby *et al.* [Phys. Letters 33B, 429 (1970)] observed *a* very strong s' dependence for different fixed t values in

$$\frac{d\sigma}{d\Omega\, dE}\bigg/\left(\frac{d\sigma}{dt}\right)^{1/2},$$

which is proportional to

$$\frac{d\sigma}{dt\, d(s'/s)}\bigg/\left(\frac{d\sigma}{dt}\right)^{1/2},$$

for the $p+p\to p+X$ distributions. In inelastic ep scattering, $\alpha(t)$ is the photon with its spin fixed at unity. Therefore, the s' dependence can be easily factored out by the Mott cross section if W_2 is indeed dominated by the Pomeranchukon.

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