

with a zero (pole) situated outside the range  $\zeta_1$ ,  $\zeta_2$ . For a point between  $\zeta_1$  and  $\zeta_2$ , the distance between the line  $M_0 = M_{\text{true}}$  and the graph can be plausibly regarded as a probability density (normalized to the area below the line) for the zero (pole) to be located at that point.

(6)  $M_0$  is nothing but the  $L^\infty$  norm of the minimal possible amplitude on the unknown part of the cuts  $\Gamma_2$ . Another way of locating the singularities is the study of the variation of  $\Re\mathcal{K}_0(z_0)$ , the minimum of the  $L^2$  norm on  $\Gamma_2$ , of the analytic functions close to the histogram on  $\Gamma_1$  (Ref. 9).

(7) The more general case of variable error  $\epsilon(z)$  can be easily reduced to the one above (Sec. 4 of Ref. 5) by multiplying the histogram with a new Carleman function  $C_1$ , defined as having the modulus equal to  $\epsilon/\epsilon(z)$  on  $\Gamma_1$  and  $\epsilon/M$  on  $\Gamma_2$ , where  $\epsilon$  is an arbitrary number.

#### ACKNOWLEDGMENT

We acknowledge with pleasure a long discussion with Professor N. N. Bogoliubov, in which we were glad to see his interest and to hear his views on these kinds of questions.

<sup>1</sup>G. Callucci, L. Fonda, and G. C. Ghiraldi, *Phys. Rev.* **166**, 1719 (1968).

<sup>2</sup>S. Ciulli and G. Nenciu, *J. Math. Phys.* (to be published).

<sup>3</sup>An analogous phenomenon arose in Cutkosky and Deo's problem of extrapolation of the Chew-Low type [*Phys. Rev. Letters* **22**, 1272 (1968)]. The presence of a singularity required a much larger number of optimal polynomials to fit the histogram to the same desired accuracy.

<sup>4</sup>For explicit formulas, see Sec. 2 of Ref. 5.

<sup>5</sup>S. Ciulli and J. Fischer, *Nucl. Phys.* **B24**, 537 (1970).

<sup>6</sup>The negative-frequency part comes mainly from the fact that  $\hbar$  is defined as being zero on the boundary  $\Gamma_2$ , and not from the irregularity caused by the error, as one might believe.

<sup>7</sup>A possible way of doing this is to isolate the rapidly varying part of the Carleman function and to compute the integral via the moments of  $\exp(i/\pi) \ln(M/\epsilon) \ln \theta$ ; we stand by the interested reader with all necessary programs.

<sup>8</sup>This truncation acts as an additional small noise, which lowers the value of  $M_0$ .

<sup>9</sup>I. Sabba-Stefanescu, *Nucl. Phys.* (to be published).

## Symmetrization Effects in Spectator Momentum Distributions\*

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The traditional identification of the slow nucleon as the spectator in deuteron-breakup collisions is shown to be valid only when there is a significant difference in the speeds of the two nucleons. When both have similar speeds, symmetrization effects cannot be ignored. These effects will be particularly important (a) for low-momentum-transfer events and (b) for "high-momentum spectator" events. A new comparison technique free of these difficulties is suggested. By studying the transverse momentum spectrum for both final-state nucleons, one avoids ambiguous spectator identification. In addition, the effect of the interference in such a distribution reveals the spatial symmetry of the final two-nucleon state. From this information it is possible to determine the relative contributions of spin-flip and -nonflip processes, and thereby to reconstruct the free-nucleon scattering accurately.

In this paper we consider the impulse approximation for the general deuteron disintegration processes  $xd \rightarrow y'pp$ . A traditional element of the experimental data analysis of these reactions has involved comparing the momentum distribution of the slow nucleon, assumed to be the spectator, with the deuteron momentum-space density. We shall show that there is a non-negligible class of events in which that simple comparison is incor-

rect because of the necessity of symmetrizing the two-nucleon final state.

We begin from the impulse approximation in the form

$$F(xd \rightarrow yNN) = \left\langle f \left| \sum_n t_n \right| i \right\rangle, \quad (1)$$

where  $t_n$  is the transition matrix for the corresponding free-nucleon process and  $|f\rangle$  and  $|i\rangle$  are

the final and initial states. The initial state we write as

$$|i\rangle = |x\rangle|\varphi_d\rangle|\chi_d\rangle,$$

where  $|\chi_d\rangle$  is the spin-isospin wave function of the deuteron and

$$|\varphi\rangle = 2^{-1/2} \int d^3p \varphi(\vec{p})|\vec{p}\rangle|-\vec{p}\rangle$$

is its spatial state in the deuteron rest frame. The final state we write similarly as

$$|f\rangle = |y\rangle|\varphi_f\rangle|\chi_f\rangle,$$

with

$$|\varphi_f\rangle = \int d^3p \gamma_{\pm}(\vec{p})|\frac{1}{2}\vec{\Delta} - \vec{p}\rangle|\frac{1}{2}\vec{\Delta} + \vec{p}\rangle$$

describing the two-nucleon system recoiling with momentum transfer  $\vec{\Delta}$ . According to the antisymmetry or symmetry of the final spin-isospin state  $|\chi_f\rangle$ , over-all antisymmetry requires that  $\gamma_{\pm}(-\vec{p}) = \pm\gamma_{\pm}(\vec{p})$ .

The matrix elements of the single-nucleon operators may now be taken between the initial and final states. They will involve the free-nucleon scattering amplitudes, which we shall denote by

$$f_i(\vec{p}_x, \vec{P}; \vec{p}_y, \vec{P}') = \langle\chi_f|\langle y|\langle\vec{P}'|t_i|x\rangle|\vec{P}\rangle|\chi_i\rangle$$

to describe the free-nucleon process  $xN \rightarrow yN'$ , with the momenta of  $x$ ,  $N$ ,  $y$ , and  $N'$  given respectively by  $\vec{p}_x$ ,  $\vec{P}$ ,  $\vec{p}_y$ , and  $\vec{P}'$ . A straightforward evaluation of Eq. (1), making use of the symmetry assumed for  $\gamma_{\pm}(\vec{p})$ , then yields

$$F(xd \rightarrow yNN) = 2^{-1/2} \int d^3p \varphi(\vec{p})\gamma_{\pm}(\frac{1}{2}\vec{\Delta} - \vec{p}) \times f_{\pm}(\vec{p}_x, \vec{p}; \vec{p}_y, \vec{p} + \vec{\Delta}), \quad (2)$$

where

$$f_{\pm}(\vec{p}_x, \vec{P}; \vec{p}_y, \vec{P}') = f_1(\vec{p}_x, \vec{P}; \vec{p}_y, \vec{P}') \pm f_2(\vec{p}_x, \vec{P}; \vec{p}_y, \vec{P}').$$

The dependence of the matrix elements  $f_{\pm}$  on  $\vec{p}$  may be neglected in most situations,<sup>1</sup> allowing us to separate the free-nucleon amplitudes from the nuclear effects, yielding

$$F(xd \rightarrow yNN) = \Phi_{\pm}(\vec{\Delta})f_{\pm}(\vec{p}_x, \vec{0}; \vec{p}_y, \vec{\Delta}), \quad (3)$$

where

$$\Phi_{\pm}(\Delta) = 2^{-1/2} \int d^3p \varphi(\vec{p})\gamma_{\pm}(\frac{1}{2}\vec{\Delta} - \vec{p}).$$

A particularly simple result now follows if we neglect final-state interactions among the two nucleons and write  $|\varphi_f\rangle$  as plane-wave states by taking

$$\gamma_{\pm}(\vec{p}) = 2^{-1/2} [\delta^3(\vec{p} - \vec{q}) \pm \delta^3(\vec{p} + \vec{q})].$$

The amplitudes (3) are then given by

$$F_{\pm}(\frac{1}{2}\vec{\Delta} - \vec{q}, \frac{1}{2}\vec{\Delta} + \vec{q}) = \frac{1}{2} [\varphi(\frac{1}{2}\vec{\Delta} - \vec{q}) \pm \varphi(\frac{1}{2}\vec{\Delta} + \vec{q})] \times f_{\pm}(\vec{p}_x, \vec{0}; \vec{p}_y, \vec{\Delta}). \quad (4a)$$

Since we are interested in the momenta of the two nucleons, which are given by the arguments of  $\varphi$  in Eq. (4a), it is appropriate to rewrite them as  $\vec{p}_{1,2} = \frac{1}{2}\vec{\Delta} \pm \vec{q}$ . Writing  $f_{\pm}$  simply as a function of momentum transfer, we then obtain

$$F_{\pm}(\vec{p}_1, \vec{p}_2) = \frac{1}{2} [\varphi(\vec{p}_1) \pm \varphi(\vec{p}_2)] f_{\pm}(\vec{p}_1 + \vec{p}_2). \quad (4b)$$

In a standard normalization,<sup>1</sup> the differential cross section for (4) is

$$d\sigma = \frac{M}{p_x} \frac{d^3p_y d^3p_1 d^3p_2}{E_y E_1 E_2} \delta^4(p_i - p_f) \sum_{\text{spins}} (|F_+|^2 + |F_-|^2), \quad (5)$$

where  $M$  is the nucleon mass and  $E_{1,2}$  are the energies of the nucleons. The  $\delta$  function constrains the initial and final energies and requires that  $\vec{p}_x - \vec{p}_y = \vec{\Delta}$ . For peripheral processes, the component of  $\vec{\Delta}$  parallel to  $\vec{p}_x$  is fixed by energy conservation; for simplicity we shall assume here that it vanishes, as for elastic scattering, although our conclusions do not require that assumption. Then (5) becomes

$$d\sigma = \frac{M}{p_x} \frac{d^3p_1 d^3p_2}{E_1 E_2} \delta(\vec{p}_x \cdot (\vec{p}_1 + \vec{p}_2)) \sum_{\text{spins}} (|F_+|^2 + |F_-|^2). \quad (6)$$

Now let us examine

$$|F_{\pm}|^2 = \frac{1}{4} |\varphi(\vec{p}_1) \pm \varphi(\vec{p}_2)|^2 |f_{\pm}(\vec{\Delta})|^2, \quad (7)$$

where we have written  $f_{\pm}$  as a function of momentum transfer  $\vec{\Delta} = \vec{p}_1 + \vec{p}_2$  only. The contribution of  $\varphi(\vec{p}_{1,2})$  describes the process in which  $\vec{p}_{1,2}$  is the momentum of the spectator (i.e., of the particle which does not interact). If, for example,  $\vec{p}_1$  is slow and  $\vec{p}_2$  is fast, then because  $\varphi(\vec{p})$  decreases rapidly with increasing  $p$  we will have  $\varphi(\vec{p}_1) \gg \varphi(\vec{p}_2)$  and

$$|F_{\pm}|^2 \approx \frac{1}{4} |\varphi(\vec{p}_1)|^2 |f_{\pm}(\vec{\Delta})|^2. \quad (8)$$

This equation represents the usual experimental assumption,<sup>2</sup> with  $p_1$  identified as the *slow* nucleon;  $\varphi(\vec{p}_2)$  is customarily completely neglected. If  $p_1 \approx p_2$ , however, the two terms will be comparable, and  $\varphi(\vec{p}_2)$  cannot be neglected. In other words, the slow nucleon can be identified as the spectator only if the other nucleon is much faster; otherwise Eq. (8) will not be valid. (When  $\vec{p}_1 \approx \vec{p}_2$ , of course, the neglect of final-state interactions is not justified; but since events of this type will presumably be a small fraction even of those for which the magnitudes of  $p_1$  and  $p_2$  are equal, it is probably valid to neglect them.) Quantitatively, we estimate

$\varphi(p_1) \gg \varphi(p_2)$  when  $p_2^2 - p_1^2 \gtrsim 0.1$  (GeV/c)<sup>2</sup>, a condition violated by about 10–15% of the events in a typical sample.<sup>3</sup>

There are two physical situations in which both nucleons are expected to have similar speeds. The first, and more important, is when the momentum transfer  $\vec{\Delta}$  is small, for  $\vec{p}_1 + \vec{p}_2 = \vec{\Delta}$  then implies that  $p_1 \approx p_2$ . Since it is in this region that deuteron effects are most significant anyway, a correct analysis here is essential for extracting the free-nucleon data. The second case in which  $\varphi(p_2)$  cannot be neglected is when the slower proton has a relatively high momentum, because the other nucleon is then ordinarily not much faster, so  $p_1 \approx p_2$  here also. In both of these situations the effect of symmetrization is to enhance the spatially symmetric final states, by up to a factor 4, and to suppress correspondingly the antisymmetric ones. This result is also reflected in the net differential cross section calculated by the closure approximation, i.e., by integrating (6) over  $\vec{q} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$ , yielding

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} [1 + S(\vec{\Delta})] \sum_{\text{spins}} |f_+(\vec{\Delta})|^2 + \frac{1}{2} [1 - S(\vec{\Delta})] \sum_{\text{spins}} |f_-(\vec{\Delta})|^2, \quad (9)$$

where  $S(\vec{\Delta})$  is the deuteron form factor.

In a separate paper<sup>4</sup> we have shown that for an arbitrary charge-exchange process one can write

$$\frac{1}{2} \sum_{\text{spins}} |f_+(\vec{\Delta})|^2 = \frac{1}{3} |D|^2, \quad (10)$$

$$\frac{1}{2} \sum_{\text{spins}} |f_-(\vec{\Delta})|^2 = |C|^2 + \frac{2}{3} |D|^2,$$

where  $|C|^2$  and  $|D|^2$  correspond to spin-nonflip and -flip terms in the free-nucleon reaction. Inserting (10) in (6) we find

$$d\sigma = \frac{M}{p_x} \frac{d^3 p_1 d^3 p_2}{E_1 E_2} \delta(\vec{p}_x \cdot \vec{\Delta}) \times \left[ \frac{1}{3} |D|^2 |\varphi(p_1) + \varphi(p_2)|^2 + \left( \frac{2}{3} |D|^2 + |C|^2 \right) |\varphi(p_1) - \varphi(p_2)|^2 \right], \quad (11)$$

implying that at least two-thirds of the charge-exchange reaction leads to the antisymmetric final state. It follows that symmetrization effects will always suppress high-momentum spectators in these processes. Consequently any excess of high-momentum spectators must be due to multiple-scattering effects.<sup>5</sup> In non-charge-exchange processes, however, no such simple result follows. If one assumes the process is dominated by isoscalar exchange, then the scattering will lead entirely to the symmetric final state for spin nonflip, and two-thirds to the symmetric state for spin flip.

In either case an enhancement of high-momentum spectators, in addition to that resulting from double scattering, would follow.

It is possible, of course, to give an explicit expression for the distribution of the slower nucleons, namely,

$$\frac{d\sigma}{d^3 p_{\text{slow}}} = \int \frac{d\sigma}{d^3 p_1 d^3 p_2} \theta(p_1 - p_2) d^3 p_1 + \int \frac{d\sigma}{d^3 p_1 d^3 p_2} \theta(p_2 - p_1) d^3 p_2,$$

where  $\theta$  is a step function. Analytic evaluation of this form seems intractable, however, even using the crudest wave functions, and in any case it requires assuming a parametrization of  $f_{\pm}$ . In order to avoid the necessity of discarding events with  $p_1 \approx p_2$  in data analysis, therefore, we need to develop techniques which do not rely on identifying the slow nucleon as the spectator.

These are, in fact, fairly easily obtained by forgetting about spectators and looking at distributions with respect to *all* nucleons. The momentum distribution of individual nucleons can be calculated by integrating over the momentum of the second nucleon,

$$\frac{d\sigma}{d^3 p} = \frac{d\sigma_+}{d^3 p} + \frac{d\sigma_-}{d^3 p},$$

$$\frac{d\sigma_{\pm}}{d^3 p} = \int d^3 p' \delta(p_1 + p'_1) |\varphi(\vec{p}) \pm \varphi(\vec{p}')|^2 \times \frac{M}{2 p_x^2 E E'} \sum_{\text{spins}} |f_{\pm}(\vec{p} + \vec{p}')|^2. \quad (12)$$

The energy  $\delta$  function naturally introduces cylindrical coordinates; writing  $p_l$  and  $p_t$  for longitudinal and transverse components, we may integrate (12) over  $p_l$  and obtain the net distribution of transverse nucleon momenta as

$$\frac{d\sigma_{\pm}}{dp_t} = \pi p_t \int d^3 p' |\varphi(\vec{p}'') \pm \varphi(\vec{p}')|^2 \times \frac{M}{p_x^2 E E'} \sum_{\text{spins}} |f_{\pm}(\vec{p}'' + \vec{p}')|^2, \quad (13)$$

where  $\vec{p}''$  has components  $p_t$  and  $-p'_t$ .

We can evaluate (13) by assuming that the free scattering has a simple parametrization, e.g.,

$$\sum_{\text{spins}} \frac{\pi M}{p_x^2 E E'} |f_{\pm}(\vec{p}'' + \vec{p}')|^2 \propto \exp[-a(\vec{p}' + \vec{p}'')^2]. \quad (14)$$

If we also assume a Gaussian wave function

$$\varphi(\vec{p}) = (c/\pi)^{3/4} \exp(-\frac{1}{2} c p^2),$$

it follows that

$$\frac{d\sigma_{\pm}}{dp_t} \propto cp_t \left\{ \frac{1}{a} \exp[-cp_t^2] + \frac{1}{a+c} \exp[-acp_t^2/(a+c)] \right. \\ \left. \pm \frac{4}{2a+c} \exp[-\frac{1}{2}c(4a+c)p_t^2/(2a+c)] \right\}. \quad (15)$$

The first two terms of (15) correspond to the distributions for the spectator and struck nucleons, respectively; the third is the interference between them, and would be absent without symmetrization in (4).

Typical curves corresponding to (4) are shown, along with the no-interference result, in Fig. 1. The structure of the curve around  $p_t = 0.2$  GeV/c is sensitive to the sign of the interference. This additional piece of experimental information can be very helpful in learning about the free-nucleon process, since knowing the relative contributions of  $d\sigma_{\pm}/dp_t$  reveals the relative magnitudes of  $|f_{\pm}|^2$ . For charge-exchange processes, in particular, this determines  $|C|^2$  and  $|D|^2$ , from which the free-nucleon scattering can be reconstructed.

To use this analysis, one would assume simple parametrizations of  $|f_{\pm}|^2$ , as above, and fit them simultaneously to  $d\sigma/d\Omega$ , as given by Eq. (9), and to  $d\sigma/dp_t = d\sigma_+/dp_t + d\sigma_-/dp_t$ . In a peripheral process, the slope  $a$  in (14) will be fixed quite strongly by  $d\sigma/d\Omega$  in the region  $\Delta^2 \gtrsim 0.3$  (GeV/c)<sup>2</sup>, where  $S(\Delta) \approx 0$ . The behavior of each term as  $\Delta^2 \rightarrow 0$  can be assumed kinematically known, and the magnitude of its contribution will then be revealed by the data for  $p_t \approx 0.2$  GeV/c.

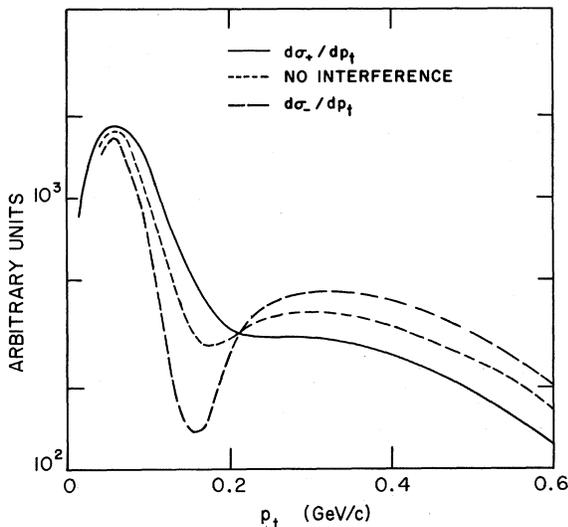


FIG. 1. Distributions in nucleon transverse momenta obtained using a Gaussian deuteron form factor [Eq. (15) in text] with constructive, destructive, and no interference between the final-state nucleons. The curves are normalized to have the same area.

A Gaussian wave function for the deuteron is not realistic, of course, and (15) can consequently have only qualitative validity. Unfortunately, the integrals involved cannot be performed analytically using any of the standard wave functions. We have therefore carried out calculations analogous to (15) using a multi-Gaussian wave function (with  $p$  in GeV/c)

$$\varphi(\vec{p}) = 0.9824 \exp[-(9.228p)^2] \\ + 0.01715 \exp[-(2.990p)^2] \\ + 0.0004136 \exp[-(0.6551p)^2], \quad (16)$$

which resembles the Hulthén wave function quite closely out to  $p \approx 0.9$  GeV/c. For larger values of  $p$  the integrand of (13) is small because of  $|f_{\pm}|^2$ , so the resulting integrals should be fairly accurate. The results obtained for  $d\sigma_{\pm}/dp_t$ , using  $a = 6$  (GeV/c)<sup>-2</sup> and  $a = 9$  (GeV/c)<sup>-2</sup>, are shown in Fig. 2. The peak at small  $p_t$  corresponds to the first term of (15), and its shape is essentially independent of  $a$ ; at large  $p_t$ , the dominant contribution corresponds to the second term of (15), and shrinks rapidly as  $a$  increases.

The parametrization (14) may be inappropriate, however, for amplitudes describing spin-flip processes, requiring instead

$$\frac{\pi M}{p_x^2 E E'} |f_{\pm}(\vec{\Delta})|^2 \propto \Delta^2 e^{-a\Delta^2}. \quad (17)$$

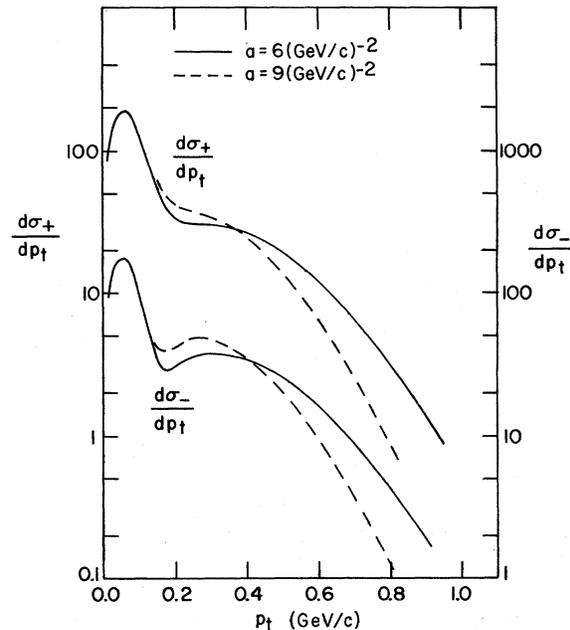


FIG. 2. Predicted distributions of nucleon transverse momenta corresponding to diffraction peak slopes of  $a = 6.0$  (GeV/c)<sup>-2</sup> and  $a = 9.0$  (GeV/c)<sup>-2</sup> and dominance of symmetric or antisymmetric final states. The curves have been normalized to the same area of 32.0 (GeV/c)<sup>2</sup>.

Integrating (13) with this parametrization yields a rather lengthy expression most simply written as

$$\frac{d\sigma'_{\pm}}{dp_t} = -\frac{\partial}{\partial a} \left( \frac{d\sigma_{\pm}}{dp_t} \right), \quad (18)$$

where  $d\sigma_{\pm}/dp_t$  refers to (15), or to the curves of Fig. 2 if the wave function (16) is used. In Fig. 3 are shown the curves corresponding to the latter case.

The effects of double scattering on all of these results are negligible. The correction to the first term of (15), for example, is significant compared with that term around  $p_t = 0.4$  GeV/c; but both are buried under the much larger contribution of the *second* term of (15) in that region. Thus the dominant contributions to  $d\sigma_{\pm}/dp_t$  arise entirely from the impulse approximation.

As an example of how this analysis may be used, let us consider the process  $K^+d \rightarrow K^0pp$ . Here we know kinematically that the spin-flip amplitude must vanish in the forward direction. Therefore we write

$$\begin{aligned} |C|^2 &= c_0 e^{-a\Delta^2}, \\ |D|^2 &= d_0 \Delta^2 e^{-a\Delta^2}. \end{aligned} \quad (19)$$

The net differential cross section (9) is then

$$\frac{d\sigma}{d\Omega}(K^+d \rightarrow K^0pp) = \{c_0[1 - S(\vec{\Delta})] + d_0\Delta^2(1 - \frac{1}{3}S(\vec{\Delta}))\} e^{-a\Delta^2}. \quad (20)$$

Experimental data can be fitted directly to (20),

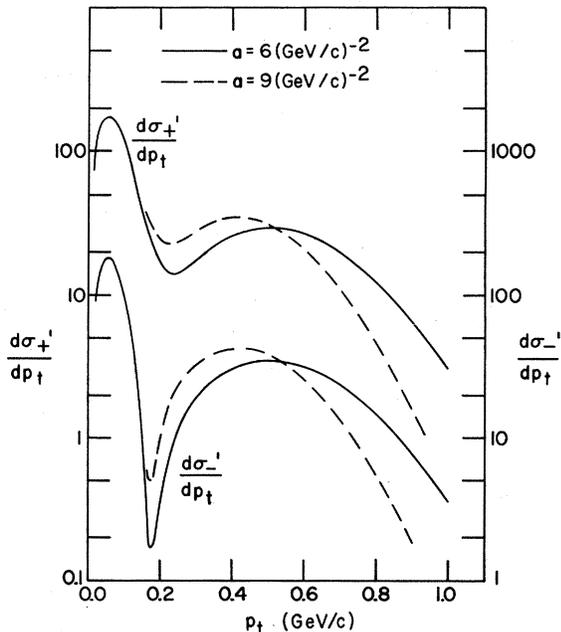


FIG. 3. Curves analogous to those of Fig. 2 for processes dominated by nucleon spin-flip terms.

but unless the data are extremely precise the result will be relatively insensitive to the ratio  $c_0/d_0$  because both multiply terms which are vanishing. Perhaps for this reason, Firestone, Alexander, and Goldhaber<sup>6</sup> assumed  $d_0 \equiv 0$  in analyzing their data, while Cline, Penn, and Reeder<sup>7</sup> took  $c_0/d_0$  to have the same value as the ratio of corresponding quantities in  $\pi^-p \rightarrow \pi^0n$ . Choosing the wrong value here affects only those data points in the region where  $S(\vec{\Delta})$  is significant, of course; but those are some of the most interesting and important ones.

The *correct* value of  $c_0/d_0$  can be obtained by fitting these parametrizations simultaneously to  $d\sigma/dp_t$ . Using the above notation, one finds that

$$\frac{d\sigma}{dp_t}(K^+d \rightarrow K^0pp) = c_0 \frac{d\sigma_-}{dp_t} + d_0 \left( \frac{1}{3} \frac{d\sigma_+}{dp_t} + \frac{2}{3} \frac{d\sigma_-}{dp_t} \right). \quad (21)$$

Figures 2 and 3 show that the term multiplying  $d_0$  will dip much more deeply than that which multiplies  $c_0$ . In other words, one can determine  $c_0/d_0$  from the depth of the structure of the  $p_t$  distribution. It is hoped that future experiments on this reaction will not ignore this extra bit of experimental information.

As a second example, we consider the process  $K^+d \rightarrow K^{*0}(890)pp$ . In this case it is possible for

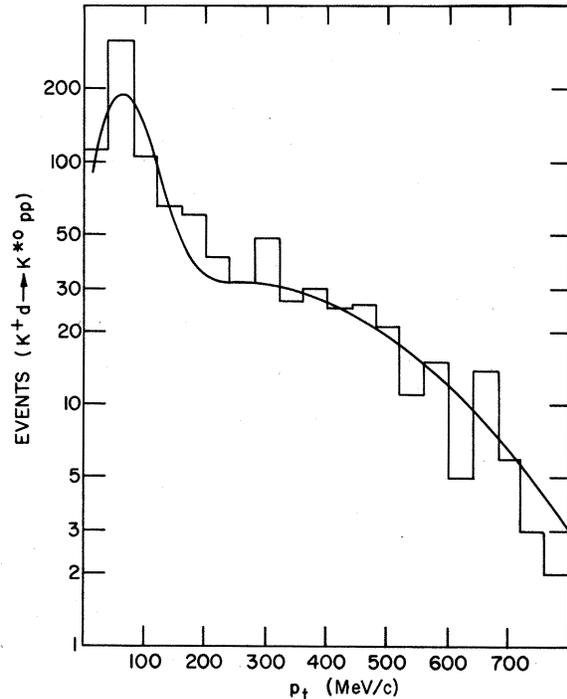


FIG. 4. Comparison of preliminary data (Ref. 3) for the proton transverse-momentum spectrum in  $K^+d \rightarrow K^{*0}pp$  at 4.2 GeV/c with  $d\sigma_{\pm}/dp_t$  for  $a = 6.0$  (GeV/c)<sup>-2</sup>.

the amplitude  $D$  to have components which do not vanish in the forward direction. Therefore we replace (19) by

$$\begin{aligned} |C|^2 &= c_0 e^{-a\Delta^2}, \\ |D|^2 &= d_0 \Delta^2 e^{-a\Delta^2} + d_1 e^{-a\Delta^2}. \end{aligned} \quad (22)$$

It follows then that

$$\begin{aligned} \frac{d\sigma}{d\Omega}(K^+d \rightarrow K^{*0}(890)pp) \\ = \{c_0[1 - S(\vec{\Delta})] + (d_1 + d_0\Delta^2)[1 - \frac{1}{3}S(\vec{\Delta})]\} e^{-a\Delta^2}, \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{d\sigma}{dp_t}(K^+d \rightarrow K^{*0}(890)pp) \\ = \frac{1}{3}d_1 \frac{d\sigma_+}{dp_t} + c_0 \frac{d\sigma_-}{dp_t} + d_0 \left( \frac{1}{3} \frac{d\sigma'_+}{dp_t} + \frac{2}{3} \frac{d\sigma'_-}{dp_t} \right). \end{aligned} \quad (24)$$

In Fig. 4 we show preliminary data<sup>3</sup> for this pro-

cess at 4.2 GeV/c, compared with the curve obtained with  $c_0 = d_0 = 0$  in Eq. (24). Including the latter two terms will always tend to deepen the dip. Thus the good agreement shown indicates that the scattering proceeds predominantly via the  $d_1$  term. The importance of  $d_1$  is also shown by the fact that  $d\sigma/d\Omega$  does not vanish in the forward direction; but the complete negligibility of  $c_0$  and  $d_0$  is indicated *only* by the  $p_t$  distribution. Unless one knows these quantities, it is not possible to reconstruct  $d\sigma/d\Omega(K^+n \rightarrow K^{*0}p)$  accurately in the small- $\Delta^2$  region.

Similar comparisons can be made for any experiment, provided one can guess an appropriate parametrization of  $|f_{\pm}|^2$ . In addition to removing the difficulty introduced by symmetrization in traditional "spectator distribution" analysis, our transverse-momentum technique makes available an important piece of previously neglected experimental information. We therefore hope that it will be utilized in future deuteron-breakup experiments.

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is greatly appreciated.

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<sup>5</sup>In a separate paper [Phys. Rev. Letters **27**, 276 (1971)] we have shown that multiple-scattering effects dominate the high-momentum spectator spectrum, and also yield an enhancement of that region, even when the spectator is unambiguously identifiable.

<sup>6</sup>A. Firestone, G. Alexander, and G. Goldhaber, Phys. Rev. Letters **25**, 958 (1970).

<sup>7</sup>D. Cline, J. Penn, and D. D. Reeder, Nucl. Phys. **B22**, 247 (1970).