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Limiting Fragmentation and the Charge Ratio of Cosmic-Ray Muons*

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Combining the steeply falling primary cosmic-ray spectrum with the scaling hypothesis for the spectrum of produced secondary particles, we illustrate how to correlate data at machine energies with cosmic-ray phenomena. We find that the μ^+/μ^- ratio is expected to be energy-independent and greater than 1, regardless of the specific model chosen for multiparticle hadronic reactions. An analysis of accelerator data gives the value $\mu^+/\mu^- \approx 1.56$, to be compared with the experimental result ~ 1.25. Possible causes of this discrepancy are discussed.

The purpose of this paper is to examine some implications for cosmic-ray spectra of recent theoretical and experimental developments in hadronic multiparticle production; in particular, to examine some of the implications of the scaling[1] or limiting fragmentation hypothesis.[2] Given the nature of the primary cosmic-ray spectrum, one can use the scaling hypothesis to correlate data at machine energies with cosmic-ray phenomena.

We concentrate primarily on the cosmic-ray μ^+/μ^- ratio, in order to see (a) whether scaling and accelerator data can explain this ratio, and (b) whether the observed constancy of the ratio discriminates between models of high-energy reactions. Experimentally, the μ^+/μ^- ratio at sea level is known to be approximately 1.2–1.3 from energies of a few GeV up to 1 TeV (10^{12} eV), as shown in Fig. 1.[3] Although Chou and Yang[4] cite this constancy as evidence in favor of the class of high-energy theories that give rise to constant topological cross sections σ_n, we find that it is a much more general result implied by any model consistent with scaling.[5]

To understand the process qualitatively, let us distinguish between two mechanisms for production of hadrons (and consequently of muons), pionization and fragmentation. Fragmentation products of the projectile (or target) are those particles produced at low energy in the projectile (or target) rest frame. Pionization refers to the copious production of particles at low energy in the center-of-mass system. In models with a finite correlation length, such as the multiperipheral model, the charge of the incident particles is reflected only in the fragmentation products; the pionization products are independent of the nature of the incident particles. Since in such models the ratio of the number of pionization to fragmentation products increases logarithmically with the incident particle energy, one might expect the μ^+/μ^- ratio to decrease asymptotically to unity. The distinctive feature of cosmic-ray phenomena which invalidates this reasoning is the steep decline of the primary flux with increasing energy; experimentally, $dN/dE \propto E^{-2.7}$ for energies up to 1 TeV. Although pions of energy E_π can be produced by primaries with any energy $E \gtrsim E_\pi$, the steeply falling primary spectrum suppresses the contribution for $E \gg E_\pi$, and enhances the contribution for $E \approx E_\pi$. Since pions having $E_\pi \approx E$ are projectile fragments, we arrive at the conclusion that projectile fragmentation is the dominant process in producing cosmic-ray pions (and muons) of any given energy, despite the existence of pionization.[6,7]

In order to make these considerations quantitative in a way which elucidates the essential physics, let us first consider an oversimplified model, in which the primary spectrum consists exclusively of protons which interact once with the atmospheric nuclei, producing pions which all decay into muons. For simplicity, kaon production is ignored here. The problem is most conveniently formulated in terms of the single-particle distribution for pions[8] produced in a proton-proton collision,

$$f^{\pm}_{p\pi}(E_\pi, E_p) \equiv \frac{E_\pi}{\sigma^{\text{inel}}_{pp}} \frac{d\sigma^{\pm}_{p\to\pi}}{dE_\pi}, \tag{1}$$

where E_p and E_π are the laboratory energies of the

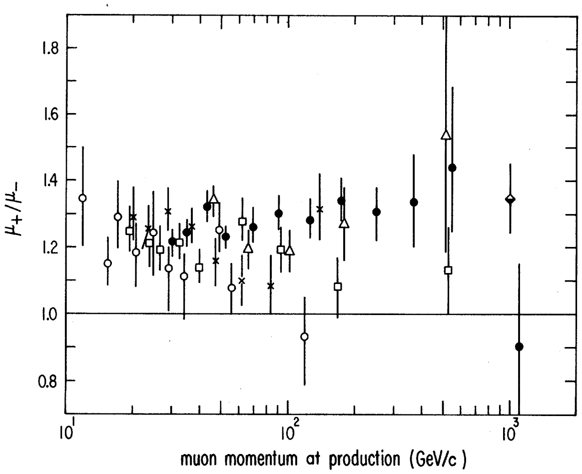

FIG. 1. Compilation of the muon charge ratio at sea level as a function of muon energy.

primary proton and secondary pion, respectively, $\sigma_{pp}^{\text{inel}}$ is the total inelastic proton-proton cross section, and $\pm$ refers to the charge of the observed pion. If we assume a primary spectrum of the form

$$\frac{dN}{dE}=N_0E^{-(1+\gamma)} \tag{2}$$

(where $\gamma\approx 1.7$), then the pion spectrum in this model is

$$\pi^{\pm}(E_\pi)\equiv\frac{dn_\pi^{\pm}(E_\pi)}{dE_\pi} = \frac{(\text{const})}{E_\pi}\int_{E_\pi}^{\infty} dE\,E^{-(1+\gamma)}f_{p\pi}^{\pm}(E_\pi,E)\,. \tag{3}$$

Applying the hypothesis of limiting fragmentation or scaling one has

$$\lim_{E\to\infty} f_{p\pi}^{\pm}(E_\pi,E)=\tilde{f}_{p\pi}^{\pm}(x)\,, \tag{4}$$

where

$$x=\sqrt{2}\,P_{\pi L}^{\text{c.m.}}/[M_N(E+M_N)]^{1/2} \tag{5}$$

is the usual Feynman scaled variable ($P_{\pi L}^{\text{c.m.}}$ is the pion longitudinal momentum in the center-of-mass frame, M_N is the nucleon mass); for $E, E_\pi\to\infty$, $x\approx E_\pi/E$. We can simplify Eq. (3) to the form

$$\pi^{\pm}(E_\pi)=(\text{const})E_\pi^{-(1+\gamma)}Z_{p\pi}^{\pm}\,, \tag{6}$$

where

$$Z_{p\pi}^{\pm}=\int_0^1 \tilde{f}_{p\pi}^{\pm}(x)x^{\gamma-1}dx\,. \tag{7}$$

Consequently, in this simple model, we have

$$\frac{\mu^+(E_\mu)}{\mu^-(E_\mu)}=\frac{\pi^+(E_\pi)}{\pi^-(E_\pi)}=\frac{Z_{p\pi}^{+}}{Z_{p\pi}^{-}}\,. \tag{8}$$

This simple model already embodies the relevant salient features: (1) From Eqs. (6)–(8), we see that the μ^+/μ^- ratio is explicitly independent of the muon energy. (2) $\mu^+/\mu^->1$ because the proton fragments more often into π^+ than π^-. (3) The ratio μ^+/μ^- depends on the power γ of the primary spectrum, and thus may have an implicit energy dependence through variations in the power law of the primary spectrum. (4) The nature of the target nuclei in the atmosphere is unimportant. This follows from the fact that only fast forward-moving fragments with $x>0$ contribute to the integral for $Z_{p\pi}^{\pm}$, and to the extent factorization is valid, $\tilde{f}_{p\pi}^{\pm}(x>0)$ depends only on the nature of the projectile particle, not the target. (The actual propagation of cosmic rays through the atmosphere does depend on the average atomic weight of the atmospheric nuclei, but, as we shall discuss later, such dependence for the muon charge ratio will be expected to be fairly mild.) (5) Pionization products have very little effect on the μ^+/μ^- ratio. The power-law behavior of the primary spectrum, appearing as the weighting factor $x^{\gamma-1}$ in the integrand of $Z_{p\pi}^{\pm}$, suppresses contributions from $x\approx 0$, the pionization region. For $x>0$, it is expected that

$$\tilde{f}_{p\pi}^{+}(x)>\tilde{f}_{p\pi}^{-}(x) \tag{9}$$

as a result of the initial proton charge. This is indeed the case in laboratory experiments performed up to 30 GeV/c,[9-12] and is expected to hold asymptotically as well. Since the integrand in $Z_{p\pi}^{\pm}$ never becomes negative, it follows that $Z_{p\pi}^{+}>Z_{p\pi}^{-}$, and that $\mu^+/\mu^->1$.

After this simplified discussion, we proceed now to consider a more realistic model which takes into account the propagation of cosmic rays through the atmosphere, the neutron component of the primaries, and the production and decay of charged kaons in addition to pions. We have followed the general approach described by Barrett *et al.*[13] and obtained the muon spectra in terms of the single-particle spectra f_{pp}, f_{pn}, $f_{p\pi}$, f_{pk}, f_{nk}, and $f_{\pi\pi}$, all defined analogously to the previous $f_{p\pi}^{\pm}$. To illustrate this approach, we present here a brief discussion of the propagation of nucleons through the atmosphere.

Denoting by $\mathcal{P}(E,y)$ ($\mathfrak{N}(E,y)$) the proton (neutron) flux at energy E and atmospheric depth y (g/cm^2), we describe the propagation of these particles through the atmosphere by the following diffusion equation:

$$\frac{\partial\mathcal{P}(E,y)}{\partial y}=-\frac{\mathcal{P}(E,y)}{\lambda_N}+\frac{1}{\lambda_N E}\int_E^{\infty}dE'[\mathcal{P}(E',y)\tilde{f}_{pp}(E/E') +\mathfrak{N}(E',y)\tilde{f}_{np}(E/E')]\,, \tag{10a}$$

$$\frac{\partial \mathfrak{N}(E,y)}{\partial y} = -\frac{\mathfrak{N}(E,y)}{\lambda_N} + \frac{1}{\lambda_N E}\int_E^\infty dE'[\mathfrak{N}(E',y)\tilde{f}_{nn}(E/E') + \mathcal{P}(E',y)\tilde{f}_{pn}(E/E')], \tag{10b}$$

where λ_N is the nucleon-interaction mean free path in the atmosphere and the $\tilde{f}_{ab}(x)$ are defined analogously to $\tilde{f}^{\pm}_{p\pi}(x)$ in Eqs. (1) and (4). In writing down Eqs. (10), we have ignored the regeneration of nucleons in the atmosphere by secondary-pion fragmentation. These equations are simplified by the following charge-symmetry relations[14]:

$$\tilde{f}_{pp}(x) = \tilde{f}_{nn}(x), \tag{11a}$$

$$\tilde{f}_{pn}(x) = \tilde{f}_{np}(x). \tag{11b}$$

Inserting these relations into Eqs. (10) and assuming a separable dependence[15] on y and E, we obtain the solutions:

$$\mathcal{P}(E,y) = (\tfrac{1}{2}N_0 e^{-y/\Lambda_N} + \tfrac{1}{2}\Delta_0 e^{-y/\Lambda'_N})E^{-(1+\gamma)}, \tag{12a}$$

$$\mathfrak{N}(E,y) = (\tfrac{1}{2}N_0 e^{-y/\Lambda_N} - \tfrac{1}{2}\Delta_0 e^{-y/\Lambda'_N})E^{-(1+\gamma)}, \tag{12b}$$

where $\Lambda_N \equiv \lambda_N[1-(Z_{pp}+Z_{pn})]^{-1}$ is the attenuation length of nucleons in the atmosphere and $\Lambda'_N \equiv \lambda_N[1-(Z_{pp}-Z_{pn})]^{-1}$. Note that the initial primary spectrum is given by

$$\mathcal{P}(E,0) + \mathfrak{N}(E,0) = N_0 E^{-(1+\gamma)},$$

as expected. It is also worth mentioning that both Λ_N/λ_N and Λ'_N/λ_N are explicitly energy-independent as a consequence of the power-law behavior of the primary spectrum. From Eqs. (12), the charge composition of nucleons at atmospheric depth y is given by

$$\frac{\Delta}{N}(y) = \frac{\mathcal{P}(E,y)-\mathfrak{N}(E,y)}{\mathcal{P}(E,y)+\mathfrak{N}(E,y)} = \frac{\Delta_0}{N_0}\exp\left[-y\left(\frac{1}{\Lambda'_N}-\frac{1}{\Lambda_N}\right)\right]. \tag{13}$$

It is known experimentally that $\Delta_0/N_0 \approx 0.74 \pm 0.01$.[16] If f_{pp} and f_{pn} are experimentally determined, Δ/N is then a known function of y from the above equations. It should be interesting to measure Δ/N as a function of y directly and compare with the results obtained from laboratory data on f_{pp} and f_{pn} through Eq. (13).

We note that many of the characteristic features of our results are similar to those of the isobar model of Ref. 7. However, the explicit use of the scaling property enables us to sharpen the derivations so that the results are less model-dependent, and to relate known or measurable laboratory quantities to cosmic-ray phenomena. For instance, if we take the experimental values[17] $\Lambda_N = 120$ g/cm^2 and[16] $\lambda_N = 77.6$ g/cm^2, we find

$$Z_{pp} + Z_{pn} = 1 - \lambda_N/\Lambda_N = 0.353. \tag{14}$$

From 30-GeV/c accelerator data,[9] we obtain $Z_{pp} \approx 0.26$. Equation (14) then gives $Z_{pn} = 0.093$, and

$$\Lambda'_N = \frac{\lambda_N}{1 - Z_{pp} + Z_{pn}} = 93.2 \text{ g/cm}^2. \tag{15}$$

The charge composition of nucleons in air is then

$$\frac{\Delta}{N}(y) \approx \frac{\Delta_0}{N_0} e^{-0.0024y}. \tag{16}$$

This is in reasonable agreement with a cosmic-ray measurement of Δ/N at mountain altitude, referred to in Ref. 7. [Our Eq. (13) corresponds to their Eq. (III.2), for which they gave the numerical expression

$$\frac{\Delta}{N}(y) = \frac{\Delta_0}{N_0} e^{-0.0030y}, \tag{17}$$

based on the experiment of Lal *et al.*, as quoted in their Ref. 14.] We take this to be an encouraging sign for the validity of the general approach we have adopted.

We have treated the pion, kaon, and muon fluxes by similar diffusion equations, and obtained the resulting muon spectrum. The results are particularly simple in the limit of large muon energies ($E_\mu \cos\theta \gg 100$ GeV; θ is the zenith angle of the muon). Ignoring the generation of nucleons by π or K fragmentation and the conversion of π to K, or vice versa, we obtain:

$$\frac{\mu^+}{\mu^-} = \frac{A_\pi^+ R_\pi^+ + A_\pi^- R_\pi^- + k(A_K^+ R_K^+ + A_K^- R_K^-)}{A_\pi^+ R_\pi^- + A_\pi^- R_\pi^+ + k(A_K^+ R_K^- + A_K^- R_K^+)}, \tag{18}$$

where

$$A_\pi^\pm = Z_{p\pi}^\pm(1+\Delta_0/N_0) + Z_{n\pi}^\pm(1-\Delta_0/N_0), \tag{19}$$

$$R_\pi^\pm = \left(\frac{\ln(\Lambda_\pi/\Lambda_N)}{1/\Lambda_N - 1/\Lambda_\pi}\right) \pm \left(\frac{\ln(\Lambda'_\pi/\Lambda'_N)}{1/\Lambda'_N - 1/\Lambda'_\pi}\right), \tag{20}$$

with similar definitions for $A_K^\pm$ and $R_K^\pm$. Here, $Z_{p\pi}^\pm$, Λ_N, and Λ'_N are the same as before, $Z_{n\pi}^\pm$ is the corresponding quantity for neutron fragmenting into $\pi^\pm$,

$$\Lambda_\pi = \lambda_\pi(1 - Z_{\pi^+\pi^+} - Z_{\pi^+\pi^-})^{-1},$$

$$\Lambda'_\pi = \lambda_\pi(1 - Z_{\pi^+\pi^+} + Z_{\pi^+\pi^-})^{-1},$$

and λ_π is the pion-interaction mean free path in air. Similar quantities are introduced for the kaons and appear in $A_K^\pm$ and $R_K^\pm$ in exactly the same way. Finally,

$$k = \frac{\tau_\pi}{\tau_K} b_{K\mu}\left(\frac{M_K}{M_\pi}\right)^3\left(\frac{M_\pi^2 - M_\mu^2}{M_K^2 - M_\mu^2}\right) \times \left[1 - \left(\frac{M_\mu^2}{M_K^2}\right)^{2+\gamma}\right]\left[1 - \left(\frac{M_\mu^2}{M_\pi^2}\right)^{2+\gamma}\right]^{-1} = 2.5. \tag{21}$$

(τ_π is the pion lifetime; τ_K is the kaon lifetime; $b_{K\mu}^-$ is the branching ratio of K-μ decay; and $[-(1+\gamma)]$ is the power of the primary energy spectrum.)

If we ignore the kaons, and assume that the pion charge-exchange fragmentation is small (i.e., $Z_{\pi^+\pi^-}\approx 0$), then Eq. (18) simplifies to

$$\frac{\mu^+}{\mu^-}=\left(\frac{Z^+_{p\pi}}{Z^-_{p\pi}}+\frac{S^-_\pi}{S^+_\pi}\right)\bigg/\left(1+\frac{S^-_\pi}{S^+_\pi}\frac{Z^+_{p\pi}}{Z^-_{p\pi}}\right), \tag{22}$$

where

$$S^\pm_\pi=\left(\frac{\ln(\Lambda_\pi/\Lambda_N)}{\Lambda_\pi/\Lambda_N-1}\right)\pm\frac{\Delta_0}{N_0}\left(\frac{\ln(\Lambda_\pi/\Lambda'_N)}{\Lambda_\pi/\Lambda'_N-1}\right). \tag{23}$$

To obtain Eq. (22), we have used the charge-symmetry relations[14]:

$$\tilde{f}_{p\pi^+}(x)=\tilde{f}_{n\pi^-}(x), \tag{24a}$$

$$\tilde{f}_{p\pi^-}(x)=\tilde{f}_{n\pi^+}(x). \tag{24b}$$

It is important to note that Eq. (22) depends only on the ratio λ_π/λ_N, and not on λ_π and λ_N separately. While λ_π or λ_N depends significantly on the average atomic weight A of the nuclei targets in the atmosphere, λ_π/λ_N has only a mild dependence (see Ref. 18), and the laboratory results at 40 GeV/c on carbon should be close to those for the atmosphere. This is also true for the full relation (18), which depends also on λ_K/λ_N; consequently, the muon charge ratio μ^+/μ^- can also be regarded as independent of the nature of the atmospheric nuclei, as we have mentioned earlier.

It is interesting to note that in Eq. (22) $Z^+_{p\pi}/Z^-_{p\pi}>1$ and $S^+_\pi/S^-_\pi>1$ always imply $\mu^+/\mu^->1$. One can show that $S^+_\pi/S^-_\pi>1$ as long as $\Lambda_N/\Lambda'_N>1$, which is always the case.

We have attempted to give a numerical estimate of μ^+/μ^- as given by Eqs. (22) and (18), using presently available accelerator data for the various single-particle distributions.[9-11,19,20] We find $Z_{\pi^+\pi^+}=Z_{\pi^-\pi^-}\approx 0.242$, $\lambda_\pi\approx 111$ g/cm^2 (from Ref. 18) and the ratio $Z^+_{p\pi}/Z^-_{p\pi}$ ranges from ~1.8 to ~2.25, depending on the data used and the method of estimation. While the suppression of contributions to the integrals $Z^\pm_{p\pi}$ from the $x\approx 0$ region is sufficiently effective to minimize the effects of pionization, the actual behavior of the single-particle distributions $f^\pm_{p\pi}$ at $x\approx 0$ may still be important for accurate determination of the ratio $Z^+_{p\pi}/Z^-_{p\pi}$. In this connection, we note that accelerator data at $E\lesssim 30$ GeV are probably not yet scaling for $x\lesssim 0.1$, as is evidenced by the inequality $f^+_{p\pi}(0)>f^-_{p\pi}(0)$ from these data.[11,21] Taking the (unweighted) average value $Z^+_{p\pi}/Z^-_{p\pi}=2.02$, we obtain the muon ratio from Eq. (22) as

$$\mu^+/\mu^-=1.56. \tag{25}$$

This is somewhat higher than the experimental values (see Fig. 1), although the latter fluctuate between 1.2 and 1.45 for $E>100$ GeV.[22]

We can readily identify two possible contributions to this discrepancy: the nonscaling behavior of present accelerator data at $x\approx 0$, and the possible inaccuracy of our factorization assumption. Finally, our estimate using existing accelerator data in Eq. (18) shows that contributions from kaons and from the pion charge-exchange fragmentation tend to cancel each other, so that the value of μ^+/μ^- is only slightly modified from Eq. (25).

In conclusion, we wish to emphasize the role which the scaling behavior of the single-particle distributions can play in the analysis of cosmic-ray phenomena. Although we have concentrated our attention on the muon charge ratio, the formulation can have much wider applications. As a general result, it allows one to correlate information obtained from particle production distributions at accelerator energies with the spectra of secondary cosmic rays at much higher energies.

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