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Some Aspects of Single-Particle and Two-Particle Inclusive Reactions in 12.7-GeV/c K^+p and 7-GeV/c π^+p Collisions*

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We present single-particle momentum spectra for π^-, K^0, $\overline{\Lambda}^0$, and Λ^0 particles produced in K^+p collisions at 12.7 GeV/c, and for π^- mesons produced in π^+p collisions at 7 GeV/c. We examine the correlations between the transverse and longitudinal momentum components of these particles and compare them to those expected as a result of momentum conservation alone. We also investigate the transverse-momentum correlations in two-particle inclusive reactions and discuss the underlying kinematic constraints on these correlations.

I. INTRODUCTION

Total cross sections for hadron-hadron collisions appear to remain relatively constant in the high-energy domain, and since the amount of quasi-two-body production decreases rapidly with increasing bombarding energy, the major part of high-energy total cross sections necessarily involves processes in which many final-state particles are produced. Consequently, the study of multiparticle reactions is an essential part of the investigation of high-energy processes. Although a detailed theoretical analysis of multiparticle reactions would be at present prohibitively complicated, Feynman, Yang, and others[1,2] have suggested that the characteristics of a single particle, summed over all final states, can yield important dynamical information concerning strong interactions. Such "inclusive" processes may be schematically represented by

$$a+b \to c+\text{anything}, \tag{1}$$

where c is the single particle being analyzed, and are the next level of sophistication above a total cross-section measurement which may be regarded as the analysis of the process

$$a+b \to \text{anything} \tag{2}$$

in which all final states are summed over. It has also been suggested[3] that more complicated processes in which two final-state particles are detected, and their momentum correlations analyzed, i.e., the processes

$$a+b \to c+d+\text{anything}, \tag{3}$$

may also be amenable to present theoretical analysis.

The theoretical application of Regge-pole ideas to inclusive reactions has implied that the momentum spectra of individual final-state particles approach limiting behavior differently in various kinematic regions. For example, in the region of target or beam fragmentation these limits are approached as $1/\sqrt{s}$, while in the central, or "pionization," region these limits are reached more slowly, approximately as $s^{-1/4}$. For two-particle inclusive reactions, the distribution in the correlation angle between the transverse momentum of the two particles is expected to approach limiting behavior as $1/\sqrt{s}$ in the appropriate single-Regge limit. Even at present accelerator energies, the $1/\sqrt{s}$ approach to asymptotic behavior may be rapid enough so that useful tests of the Regge ideas involved in these predictions may be carried out.

We present data from two[4,5] experiments utilizing the 80-in. BNL bubble chamber. The following reactions have been investigated:

$$K^+p \to \pi^- + \cdots, \quad 12.7 \text{ GeV}/c \tag{4}$$

$$K^+p \to K^0 + \cdots, \quad 12.7 \text{ GeV}/c \tag{5}$$

$$K^+p \to \overline{\Lambda}^0 + \cdots, \quad 12.7 \text{ GeV}/c \tag{6}$$

$$K^+p \to \Lambda^0 + \cdots, \quad 12.7 \text{ GeV}/c \tag{7}$$

$$\pi^+p \to \pi^- + \cdots, \quad 7 \text{ GeV}/c \tag{8}$$

$$K^+p \to K^0 + \pi^- + \cdots, \quad 12.7 \text{ GeV}/c \tag{9}$$

$$K^+p \to \overline{\Lambda}^0 + \pi^- + \cdots, \quad 12.7 \text{ GeV}/c \tag{10}$$

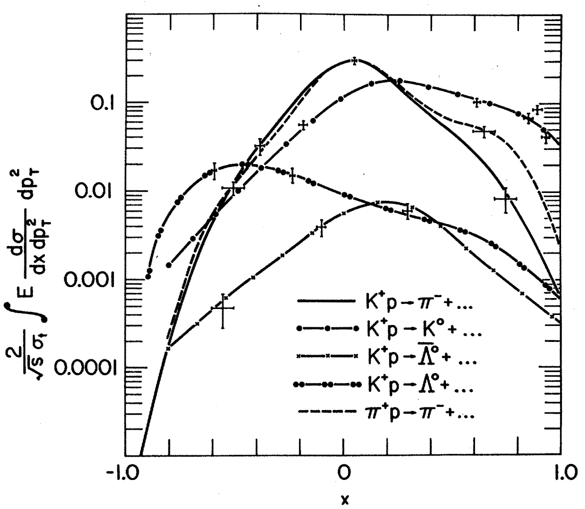

FIG. 1. $\frac{2}{\sqrt{s}\,\sigma_T}\int E\frac{d^2\sigma}{dx\,dp_T^2}dp_T^2$ vs x for reactions (4)–(8). Curves are polynomial fits to data; some typical error bars are shown.

$$K^+p \to \Lambda^0 + \pi^- + \cdots, \quad 12.7\ \text{GeV}/c \tag{11}$$

$$\pi^+p \to \pi^- + \pi^- + \cdots, \quad 7\ \text{GeV}/c \tag{12}$$

We have discussed the data for the target and projectile fragmentation regions of reactions (4) and (8) in a previous publication.[6] Here we present the remaining single-particle momentum distributions, examine the interdependence observed between p_l and p_T,[7] and discuss the transverse-momentum correlations for the two-particle inclusive reactions.

II. SINGLE-PARTICLE DISTRIBUTIONS

For fixed s, and ignoring spin effects, it is customary to define a single-particle distribution function

$$f(p_T, p_l) \equiv E\frac{d^2\sigma}{dp_T^2 dp_l}.$$

The inclusive distributions

$$f(x) = \frac{2}{\sqrt{s}\,\sigma_T}\int E\frac{d^2\sigma}{dx dp_T^2}dp_T^2$$

(where $x = 2p_l/\sqrt{s}$ and E is the center-of-mass energy of the particle under consideration) for reactions (4)–(8) are shown in Fig. 1.[8] The rapidity (ω) distributions for these events (where $\omega = 0.5\ln[(E+p_l)/(E-p_l)]$) are shown in Fig. 2 ($E$ and p_l again refer to center-of-mass quantities). In general none of these distributions are centered on nor are symmetric about $x=0$. For the "beamlike" K^0 and $\bar{\Lambda}^0$ the distributions are strongly pitched forward[9] in the center-of-mass system while for the "targetlike" Λ^0 the integrated cross

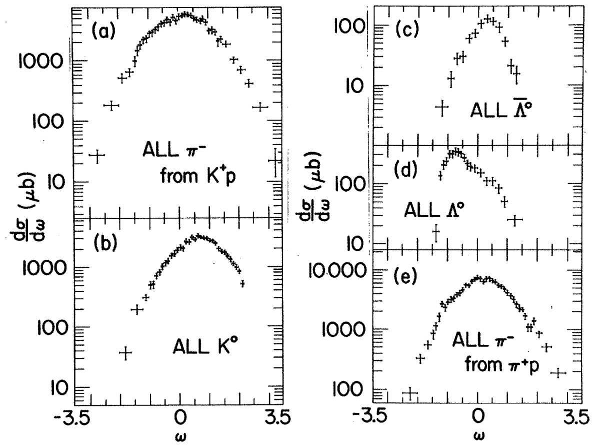

FIG. 2. Rapidity (ω) distributions for reactions (4)–(8) (see text for definition of ω).

section is considerably larger in the backward hemisphere.

The distributions for π^- mesons, which in K^+p and π^+p collisions cannot be regarded as either beamlike or targetlike, are symmetric and centered about $x=0$ in a Lorentz frame where the ratio of target to beam momentum is 1.6 ± 0.1.[10] We note that the normalized π^- production cross sections [i.e., $f(x)$] for both experiments are in approximate agreement (to better than 15%) for $x < 0.4$.[11]

If the K^0 and $\bar{\Lambda}^0$ can be considered as fragments of the K^+, and the Λ^0 as a fragment of the proton, then the energy dependences of the differential cross

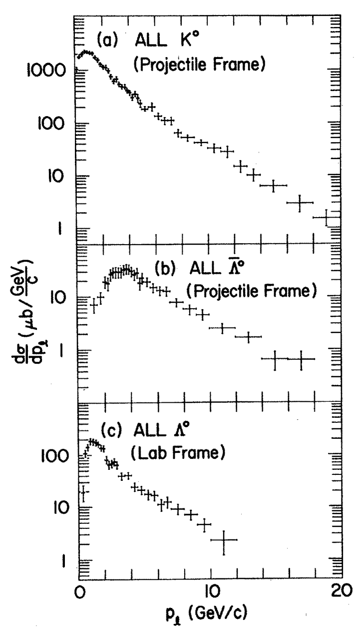

FIG. 3. $d\sigma/dp_l$ vs p_l for reactions (7) in the laboratory and $d\sigma/dp_l$ vs p_l for reactions (5)–(6) in the projectile frame.

sections for these inclusive reactions, in the regions pertinent to fragmentation, are of paramount interest. In Fig. 3 we present $d\sigma/dp_l$ for reactions (5) – (7) in the rest frames most appropriate for fragmentation studies, namely, the projectile frame for reactions (5) and (6) and the laboratory frame for reaction (7), so as to allow a convenient comparison with future experiments.

We have investigated the properties of the single-particle distribution function to see if it can be separated into a product of functions of p_T and p_l,[12] i.e., whether we can assume $f(p_T, p_l) = g(p_T)h(p_l)$. In Fig. 4 we show the mean values of p_T ($\langle p_T \rangle$) for various regions of p_l in reactions (4) and (8), as well as the means $\langle p_l \rangle$ and variances ($\langle p_l^2 \rangle - \langle p_l \rangle^2$) of the p_l distributions for various regions of p_T. Data are presented both for the $d^2\sigma/dp_T^2 dp_l$ and the $Ed^2\sigma/dp_T^2 dp_l$ distributions as calculated using center-of-mass variables. We define for each p_l interval

$$\langle p_T \rangle = \int p_T \frac{d\sigma}{dp_T} dp_T \quad \text{and} \quad \langle p_T \rangle_E = \int E p_T \frac{d\sigma}{dp_T} dp_T$$

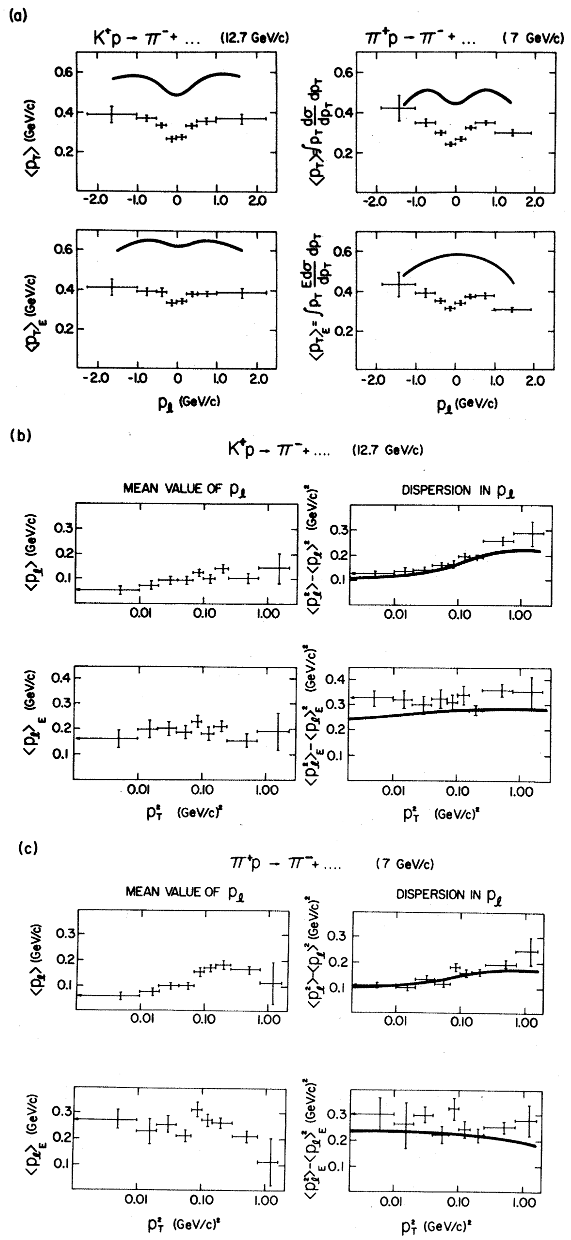

FIG. 4. (a) Mean p_T vs p_l for reactions (4) and (8) both unweighted and weighted by E. (b) Mean and variance of p_l vs p_T^2 for reaction (4) both unweighted and weighted by E. (c) Same as (b) for reaction (8).

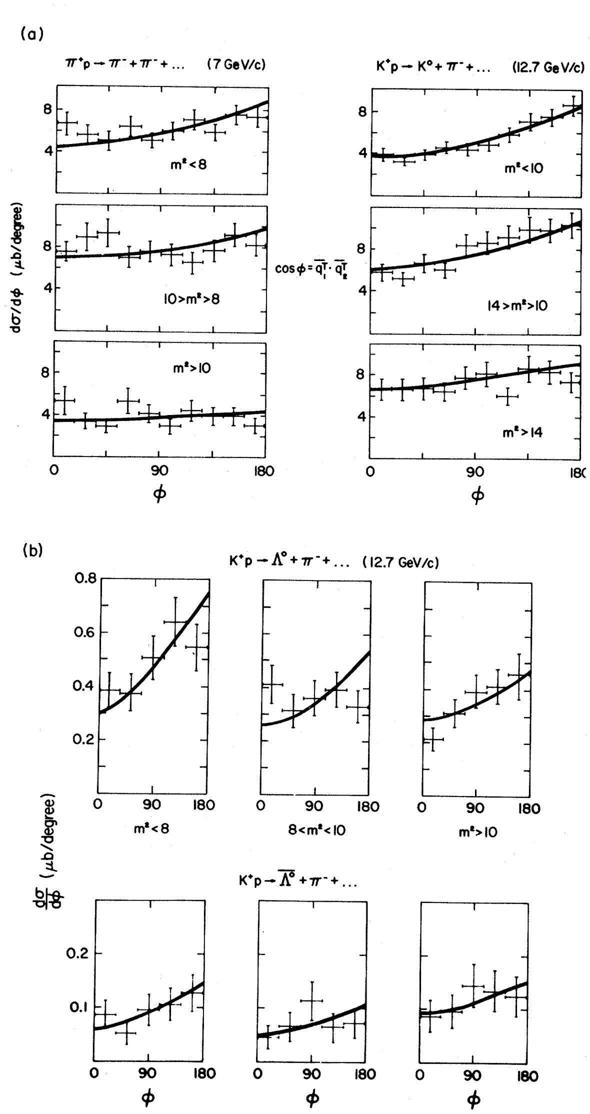

FIG. 5. (a) $d\sigma/d\phi$ for reactions (9) – (12) for different regions of missing mass squared (m^2) accompanying two final-state particles. (b) $d\sigma/d\phi$ for reactions (10) – (11) for different regions of missing mass squared (m^2) accompanying two final-state particles. Curves are phase space for average reaction.

and similarly $\langle p_l \rangle$ and $\langle p_l \rangle_E$. The curves drawn on the figures are the result of phase-space Monte Carlo calculations for the processes $K^+p \to K^+p + 4\pi$ and $\pi^+p \to p + 4\pi$. (These reactions correspond to the average multiplicity in the two experiments and thus represent typical kinematics for the data.) We find strong correlations between p_l and p_T when we consider the distributions in $d^2\sigma/dp_T^2dp_l$. For the invariant cross sections $Ed^2\sigma/dp_T^2dp_l$, on the other hand, we find that $\langle p_T \rangle_E$ is only somewhat correlated with p_l; neither the mean value of p_l, nor its variance, appears to be a function of p_T, however.[13] The distributions obtained from the phase-space calculation are also slowly varying and are in general agreement with the trends in the data, although, as expected, the average values of transverse momenta are much higher.[14]

III. TWO-PARTICLE CORRELATIONS

We now consider two-particle inclusive reactions. In particular, we wish to discuss the distribution in the angle ϕ between the transverse momenta of particles 1 and 2 in the general two-body inclusive reaction (3). The angle ϕ has been discussed by several authors,[3] and although no definite predictions concerning $d\sigma/d\phi$ have been made, we present here $d\sigma/d\phi$ distributions both as a function of the squared missing mass accompanying the two particles in question, and as a function of the absolute rapidity difference between these two particles, in hopes of stimulating further theoretical investigation of this angle.

In Fig. 5 we display $d\sigma/d\phi$ for reactions (9)–(12) for different intervals in the square of the missing mass (m^2) accompanying the two final-state particles. In general, the ϕ distributions vary with m^2 with the specific dependence appearing to be a function of the reaction in question. In Fig. 6 we present the rapidity differences ($\Delta\omega$) for reactions

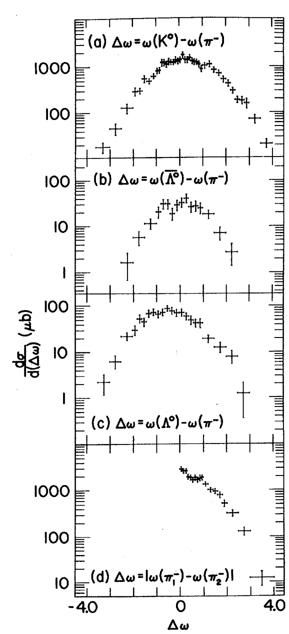

FIG. 6. Differences in rapidities ($\Delta\omega$) of the two detected outgoing particles in reactions (9)–(12).

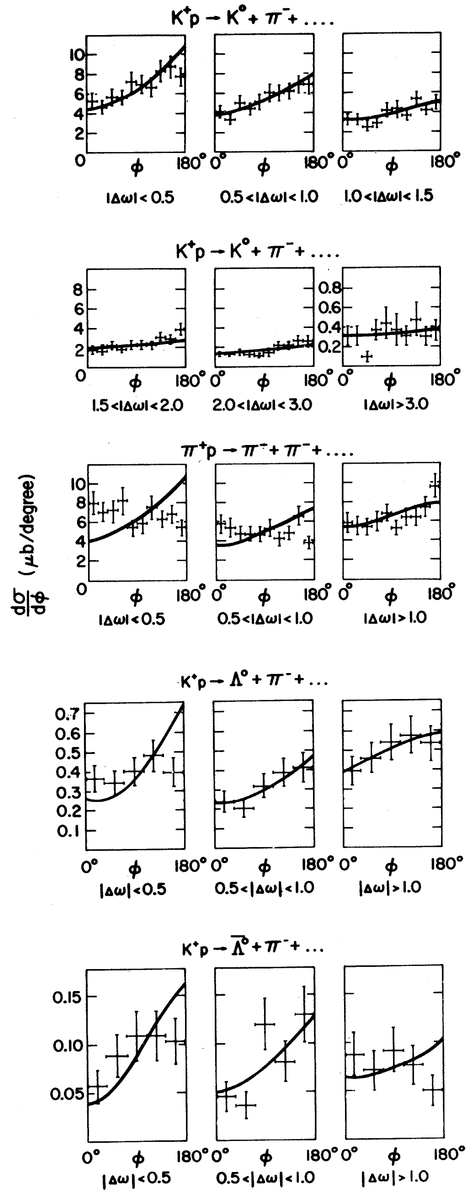

FIG. 7. (a) $d\sigma/d\phi$ for reactions (9), (12) for varying absolute rapidity differences ($|\Delta\omega|$). (b) $d\sigma/d\phi$ for reactions (10), (11) for varying absolute rapidity differences ($|\Delta\omega|$). Curves are phase space for average reaction.

TABLE I. Forward-backward ratios for reactions (9)–(12) for m^2 intervals.

Reaction	Interval (GeV^2)	$(F-B)/(F+B)$
$K^+p \to K^0 + \pi^- + \cdots$ (12.7 GeV/c)	$m^2 < 10$	-0.26 ± 0.04
	$10 < m^2 < 14$	-0.21 ± 0.04
	$14 < m^2$	-0.06 ± 0.05
$K^+p \to \bar{\Lambda}^0 + \pi^- + \cdots$ (12.7 GeV/c)	$m^2 < 8$	-0.28 ± 0.11
	$8 < m^2 < 10$	-0.05 ± 0.11
	$10 < m^2$	-0.22 ± 0.08
$K^+p \to \Lambda^0 + \pi^- + \cdots$ (12.7 GeV/c)	$m^2 < 8$	-0.17 ± 0.05
	$8 < m^2 < 10$	0.00 ± 0.06
	$10 < m^2$	-0.20 ± 0.06
$\pi^+p \to \pi^- + \pi^- + \cdots$ (7 GeV/c)	$m^2 < 8$	-0.09 ± 0.04
	$8 < m^2 < 10$	0.02 ± 0.04
	$10 < m^2$	0.07 ± 0.07

(9) – (12), and in Fig. 7 we display $d\sigma/d\phi$ for various intervals in $|\Delta\omega|$. The angle ϕ is also seen to be a function of the $|\Delta\omega|$ band chosen, with this variation likewise depending on the specific reaction under consideration. In particular, for reaction (12), in which the particles under consideration are indistinguishable, there is an increased tendency for small ϕ values to occur relative to the corresponding behavior for the distinguishable particles involved in reactions (9) – (11). This effect is reminiscent of earlier observations made in $\bar{p}p$ annihilations.[15]

To investigate the effects of kinematics on $d\sigma/d\phi$ we consider the subreactions $K^+p \to Kp4\pi$, $K^+p \to \bar{\Lambda}pp3\pi$, $K^+p \to \Lambda KK3\pi$, and $\pi^+p \to \pi p3\pi$ for reactions (9) – (12), respectively. The curves in Figs. 5 and 7 show phase-space Monte Carlo calculations for these reactions. (The calculations are not very

TABLE II. Forward-backward ratios for reactions (9)–(12) for $|\Delta\omega|$ intervals.

Reaction	Interval	$(F-B)/(F+B)$
$K^+p \to K^0 + \pi^- + \cdots$ (12.7 GeV/c)	$|\Delta\omega| < 0.5$	-0.13 ± 0.04
	$0.5 < |\Delta\omega| < 1.0$	-0.17 ± 0.04
	$1.0 < |\Delta\omega| < 1.5$	-0.16 ± 0.05
	$1.5 < |\Delta\omega| < 2.0$	-0.15 ± 0.06
	$2.0 < |\Delta\omega| < 3.0$	-0.29 ± 0.06
	$3.0 < |\Delta\omega|$	-0.07 ± 0.13
$K^+p \to \bar{\Lambda}^0 + \pi^- + \cdots$ (12.7 GeV/c)	$|\Delta\omega| < 0.5$	-0.22 ± 0.10
	$0.5 < |\Delta\omega| < 1.0$	-0.31 ± 0.10
	$1.0 < |\Delta\omega|$	0.00 ± 0.11
$K^+p \to \Lambda^0 + \pi^- + \cdots$ (12.7 GeV/c)	$|\Delta\omega| < 0.5$	-0.12 ± 0.06
	$0.5 < |\Delta\omega| < 1.0$	-0.18 ± 0.07
	$1.0 < |\Delta\omega|$	-0.10 ± 0.05
$\pi^+p \to \pi^- + \pi^- + \cdots$ (7 GeV/c)	$|\Delta\omega| < 0.5$	0.06 ± 0.05
	$0.5 < |\Delta\omega| < 1.0$	0.01 ± 0.05
	$1.0 < |\Delta\omega|$	-0.09 ± 0.05

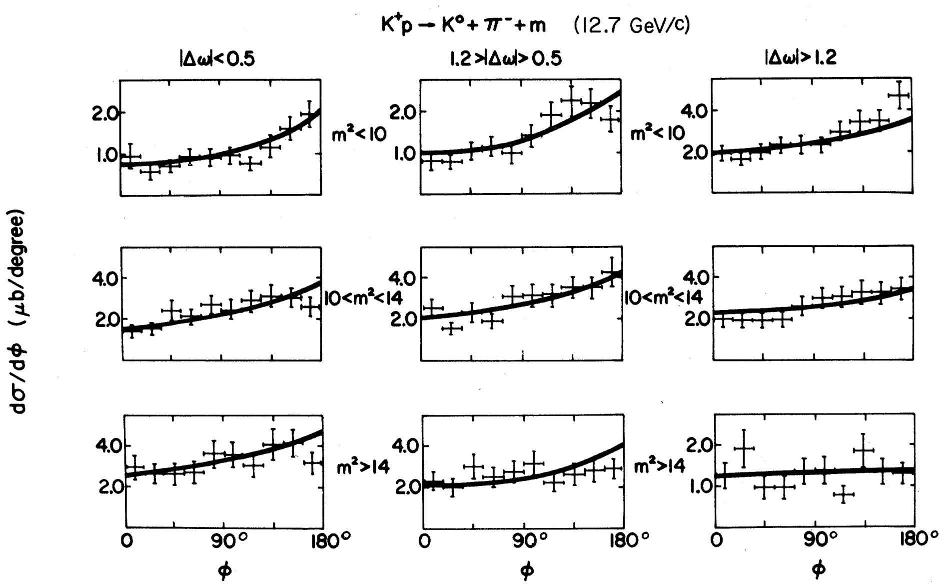

FIG. 8. $d\sigma/d\phi$ for reaction (9) for variation of both m^2 and $|\Delta\omega|$.

sensitive to the center-of-mass energy nor to the value assumed for the final-state multiplicity.) The agreement between the Monte Carlo curves and the data is fairly good, particularly in the region of large m^2 or large $|\Delta\omega|$. We also investigated a "peripheral" phase-space model where each outgoing final-state particle was given a p_T dependence of $e^{-\alpha p_T^2}$ (for several α values) and found qualitatively similar predictions in $d\sigma/d\phi$ as for pure phase space.

For completeness, in Tables I and II we present the ratio $(F-B)/(F+B)$ for each missing mass squared and absolute rapidity difference interval previously discussed. Here F refers to $\int_0^{\pi/2}(d\sigma/d\phi)\,d\phi$ and B refers to $\int_{\pi/2}^{\pi}(d\sigma/d\phi)d\phi$. The availability of these ratios will permit quantitative comparisons to be made between our data and any forthcoming theoretical analyses.

Finally, in Fig. 8 we present $d\sigma/d\phi$ distributions for reaction (9) for simultaneous binning in both m^2 and in $|\Delta\omega|$; in Table III we present the corresponding values of $(F-B)/(F+B)$. The curves in Fig. 8 were calculated on the basis of simple phase space. As can be seen, the agreement between the data and the phase-space curves is rather good, and, in particular, there is a tendency for the distribution in ϕ to approach isotropy when both m^2 and $|\Delta\omega|$ are simultaneously made large. This behavior is consistent with what might be expected on the basis of current analyses of the angle ϕ.[3]

IV. SUMMARY

Single-particle inclusive distributions have been presented for comparison with other experiments. We find a correlation between low values of p_l and p_T which may be partially due to peripheral resonance production in these data. We have also investigated the correlations between the transverse momenta of pairs of outgoing particles in two-body inclusive reactions, and have compared these to the predictions of phase space for various m^2 and $|\Delta\omega|$ intervals.

TABLE III. Forward-backward ratios, $(F-B)/(F+B)$, for reaction (9) for simultaneous binning in m^2 and $|\Delta\omega|$.

m^2 interval (GeV^2) \ $|\Delta\omega|$ interval	$|\Delta\omega|<0.5$	$0.5<|\Delta\omega|<1.2$	$1.2<|\Delta\omega|$
$m^2<10$	-0.22 ± 0.08	-0.33 ± 0.05	-0.24 ± 0.05
$10<m^2<14$	-0.19 ± 0.06	-0.21 ± 0.06	-0.23 ± 0.06
$14<m^2$	-0.08 ± 0.06	-0.06 ± 0.06	0.00 ± 0.09

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[2]See, for example, A. H. Mueller, Phys. Rev. D 2, 2963 (1970); N. F. Bali *et al.*, Phys. Rev. Letters 25, 557 (1970); M. S. Chen *et al.*, Phys. Rev. Letters 26, 280 (1971); C. E. DeTar, Phys. Rev. D 3, 128 (1971); Chan Hong-Mo *et al.*, Phys. Rev. Letters 26, 672 (1971); C. Quigg, invited paper, Conference on the Phenomenology of Particle Physics, California Institute of Technology, Pasadena, Calif., 1971 (unpublished); R. D. Peccei and A. Pignotti, Phys. Rev. Letters 26, 1076 (1971); H. Abarbanel, Phys. Rev. D 3, 2227 (1971); D. Z. Freedman *et al.*, Phys. Rev. Letters 26, 1197 (1971); D. Horn, CERN Report No. CERN-Th-1387, 1971 (unpublished).

[3]See Abarbanel (Ref. 2), Quigg (Ref. 2), and Freedman *et al.* (Ref. 2).

[4]M. S. Farber *et al.*, Phys. Rev. D 1, 781 (1970); S. Stone *et al.*, Phys. Letters 32B, 515 (1970). We wish to thank P. L. Jain, W. Labudda, and G. Pappas of SUNY at Buffalo for providing a sample of six-pronged data from the K^+p experiment.

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[6]M. S. Chen *et al.*, Phys. Rev. Letters 26, 1585 (1971).

[7]For previous studies in this area see the review papers by O. Czyzewski, in *Proceedings of the Fourteenth International Conference on High Energy Physics,* Vienna, 1968, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968); A. Wroblewski, in *Proceedings of the Fifteenth International Conference on High Energy Physics, Kiev, 1970* (Atomizdat, Moscow, 1971) and references therein.

[8]Details of the analysis of reactions (4)–(8) have been presented in S. L. Stone *et al.*, Nucl. Phys. B32, 19 (1971). For σ_T we use 17.4 mb for K^+p collisions and 23.4 mb for π^+p collisions.

[9]In all our distributions neutral-strange-particle events have been corrected for detection losses and neutral decays. The error bars displayed on the graphs contain systematic as well as statistical uncertainties. The K^0 distributions contain a small amount ($\lesssim 10\%$) of $\bar{K}^0$ background. The K^0 signal observed in the backward hemisphere may be a product of nuclear dissociation, e.g., $p \to \Sigma^+ + K^0$. The background in the Λ^0 and $\bar{\Lambda}^0$ distributions are approximately 6% and 20%, respectively.

[10]This effect was first reported in π^-p collisions by J. W. Elbert *et al.*, Phys. Rev. D 3, 2043 (1971). It was subsequently shown to hold as well in our K^+p and π^+p data [S. Stone *et al.*, Bull. Am. Phys. Soc. 16, 136 (1971) and Ref. 8] and in the K^+p data of the Davis group [W. Ko and R. L. Lander, Phys. Rev. Letters 26, 1290 (1971)]. See also N. N. Biswas *et al.*, Phys. Rev. Letters 26, 1589 (1971).

[11]When limiting behavior in x is reached for a given secondary particle, then the average multiplicity of this secondary particle increases logarithmically with the incident energy $\bar{n} = c\ln(E^{\text{inc}}/E_0)$ + const, where c is given by the intercept of

$$\frac{1}{\sigma_T}\int E\frac{d^2\sigma}{dx\,dp_T^{\,2}}\,dp_T^{\,2}$$

at $x=0$ and E_0 is some arbitrary constant. The value of c for reactions (4) and (8) is the same, namely, $c = 0.29 \pm 0.03$. We thank Dr. M. S. Chen, Dr. L. L. Wang, and Dr. T. F. Wong for suggesting this calculation.

[12]For other studies see references given in Ref. 7, Elbert (Ref. 10), and Biswas *et al.* (Ref. 10); J. W. Elbert *et al.*, Phys. Rev. Letters 20, 124 (1968); W. Ko and R. L. Lander, Phys. Rev. Letters 26, 1064 (1971).

[13]We have considered whether or not the apparent lack of orthogonality between p_l and p_T may be partially due to quasi-two-body resonance production. When a peripherally produced low-Q-value resonance decays into mesons, the decay π mesons have lower mean values of transverse momentum than freely produced π mesons. In the reaction $\pi^+p \to \pi^+p\pi^+\pi^-$ at 7 GeV/c, for example, the $\langle p_T\rangle$ for all π^- mesons is 0.334±0.004 GeV/c, but when the π^- meson results from the decay of a ρ^0 which is produced opposite a Δ^{++}, then $\langle p_T\rangle = 0.281 \pm 0.007$ GeV/c. Since in the latter reaction the ρ^0 decays (in its rest frame) approximately as $\cos^2\theta_{\pi\pi}$ ($\theta_{\pi\pi}$ is the angle of the π^- with respect to the beam direction), emitted π^- mesons tend to populate either small p_T and small p_l values or small p_T and large p_l values. It is interesting to note, however, that the six-pronged events, in which quasi-two-body production is suppressed, do not show the dip observed at $p_l = 1.5$ GeV/c in Fig. 4(a), but do show the dip at $p_l = 0$, suggesting that the correlation near $p_l = 0$ is not solely due to quasi-two-body events. We thank Dr. E. L. Berger for a discussion of this point.

[14]When we require all particles in our average multiplicity reaction to have small transverse momenta we find that this restriction has no substantial effect on our conclusions.

[15]G. Goldhaber *et al.*, Phys. Rev. 120, 300 (1960).