$$\vec{j}\cdot\frac{d\vec{r}}{dt}-\left(\vec{j}\cdot\frac{d\vec{r}}{dt}\right)_{t_0}=cK\int_{\tau_0}^{\tau}d\tau\,\frac{e^{\tau/\epsilon}}{\dot{t}^2}\,. \quad (6)$$

Since τ increases monotonically with t, $\dot{t}=E$ the energy of the particle, in units of its rest mass, instead of decreasing should increase more rapidly than $e^{\tau/2\epsilon}$ as $t\to\infty$, so that $d(\vec{j}\cdot\vec{r})/dt$ may remain bounded. Finally, since the longitudinal speed $v_\parallel$ is constant and the transverse component of velocity asymptotically tends to zero,

$$E(t\to\infty)\equiv E_A=(1-v_\parallel^2/c^2)^{-1/2}. \quad (7)$$

Thus the particle may remain relativistic for large t, if $(d\vec{j}\cdot\vec{r}/dt)/c$ is not very small (Shen[3]). For charged particles accelerated to relativistic energy and injected at random into a strong magnetic field so that

$$\langle v_\parallel^2\rangle=\frac{1}{3}\left\langle\frac{d\vec{r}}{dt}\cdot\frac{d\vec{r}}{dt}\right\rangle_0\,,$$

the ratio of the average loss of kinetic energy to that of the incident E_0 is given by

$$(E_0'-E_A')/E_0'=1-[3(2E_0^2+1)]^{-1/2}, \quad (8)$$

where $E'=\langle E^2\rangle^{1/2}$. Their ratio tends to $\frac{2}{3}$ only in the nonrelativistic case when E_0-1 is small, but for relativistic particles $(E_0\gg 1)$ it is

$$\frac{E_0'-E_A'}{E_0'}\simeq 1-\left(\frac{3}{2}\right)^{1/2}\frac{1}{E_0'}\left(1-\frac{1}{4E_0'^2}\right). \quad (9)$$

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Comment on the Exterior-Interior Separation in the Three-Body Problem*

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The exterior-interior separation of the three-body wave function is derived directly from the Faddeev equations by using the Kowalski representation for the two-body t matrices. The formalism clarifies the relationship between the quantum-mechanical observables in three-body breakup experiments and the two- and three-body wave functions in the interior regions. A complete parametrization of three-body breakup amplitudes suggested by this approach provides the generalization of the Watson-Migdal formula to include interference between resonances in different two-body channels.

It has been demonstrated[1] that if the three-particle wave function is known in the finite region where each of the three particles can interact with at least one other, the wave function in the exterior region is determined. In contrast to the two-body problem, where the exterior wave function can be determined from the interior wave function by a quadrature, determination of the exterior three-body wave function requires the solution of one-variable coupled integral equations driven by the interior part. Physically these equations result from the long-range "eternal triangle" effect[2-4]; outgoing waves from one pairwise interaction can perturb the interaction between a second pair at any distance, so long as the second pair are within the range of their own interaction. In this note, we show how a general property of the two-particle t-matrices allows this exterior-interior separation to be made directly in the Faddeev equations, without the *ad hoc* assumptions used in a previous[5] treatment of the problem. The resulting formalism clarifies the relationship between three-body observables and two-body phase shifts, two-body wave functions inside the range of forces, and the three-body wave function in the interior region where three-body forces could be present.

In order to bring out the essential simplicity of the dynamics, we restrict ourselves in this note to the breakup of a single bound pair in the state of zero total angular momentum, assuming only S-wave interactions between three spinless particles. Inclusion of spin, additional bound states leading to rearrangement collisions, and higher angular momenta are straightforward extensions which multiply the number of channels and indices without introducing qualitatively different dynamical phenomena. The radial wave function in configuration space is

$$U(x, y{:}z) = \tilde{\psi}_\gamma(x)\sin(z+\gamma^2)^{1/2}y + \frac{1}{\pi^2}\int_0^\infty dp^2 \int_0^\infty dp^2 \frac{K(p,q{:}z)}{p^2+q^2-z}\sin px \sin qy\,, \tag{1}$$

where

$$\vec{x} = \left(\frac{2m_2m_3}{m_2+m_3}\right)^{1/2}(\vec{r}_2-\vec{r}_3)\,, \tag{2a}$$

$$\vec{y} = \left(\frac{2m_1(m_2+m_3)}{m_1+m_2+m_3}\right)^{1/2}\left(\vec{r}_1 - \frac{m_2\vec{r}_2+m_3\vec{r}_3}{m_2+m_3}\right). \tag{2b}$$

The scattering boundary condition is obtained by $z \to E + i0^+$, where E is related to the laboratory energy E_L of a beam of particles 1 incident on bound pairs at rest by

$$E = \frac{m_2+m_3}{m_1+m_2+m_3}E_L - \epsilon_\gamma\,. \tag{3}$$

K has a pole at $q^2 = E+\gamma^2$ which we write as $\phi_\gamma(p)T_\gamma(z)/(z+\gamma^2-q^2)$; here $T_\gamma = (\eta e^{2i\delta_\gamma}-1)/2i$ is the elastic scattering amplitude. On-shell unitarity is ensured, provided only δ_γ real, $0<\eta<1$, and

$$\eta^2 = 1 - \frac{4}{\pi(E+\gamma^2)^{1/2}}\int_0^E dp^2 p(E-p^2)^{1/2}|K(p,(E-p^2)^{1/2}{:}E)|^2\,. \tag{4}$$

The full K matrix is related to the Faddeev channels by

$$K(p, q{:}z) = K_1(p, q{:}z) + \sum_{s=2,3}\frac{1}{2\sin 2\mu_{1s}}\int_{|\xi-\mu_{1s}|}^{\min(\xi+\mu_{1s},\,\pi-\xi-\mu_{1s})} d\phi\, K_s(P\cos\phi, P\sin\phi{:}z)\,,$$
$$\xi = \tan^{-1}(q/p), \quad P^2 = p^2+q^2, \quad \cos^2\mu_{1s} = m_1m_s/(m_1+m_{s'})(m_s+m_{s'})\,, \tag{5}$$

where the K_s satisfy the Faddeev equations:

$$K_s = \frac{1}{2\pi q}\sum_{s'\neq s}\frac{1}{\sin 2\mu_{ss'}}\int_0^\infty dq'^2\int_{p_{ss'}^{-\,2}(q^2)}^{p_{ss'}^{+\,2}(q^2)} dp'^2\frac{t_s(p,\bar{p}{:}z-q^2)}{p'^2+q'^2-z}\left[\delta_{1s'}\psi_\gamma(p')\delta(q'-(z+\gamma^2)^{1/2}) + K_{s'}(p',q'{:}z)\right],$$
$$p_{ss'}^{\pm}(q^2) = p'\cot\mu_{ss'} \pm \sqrt{q^2}\csc\mu_{ss'}, \qquad \bar{p}^2 = p'^2+q'^2-q^2, \quad \psi_\gamma(p') = \phi_\gamma(p')/(p'^2+\gamma^2)\,. \tag{6}$$

The basic property of the kernel in these equations which allows a separation into interior and exterior parts is the Kowalski representation[6] for the two-particle t-matrices:

$$t_s(p,\bar{p}{:}z-q^2) = F^s_{z-q^2}(p)\tau_s(z-q^2)F^s_{z-q^2}(\bar{p}) + (p^2+q^2-z)\tfrac{1}{2}\pi r_s(p,\bar{p},z-q^2)$$
$$= F^s_{z-q^2}(p)\tau_s(z-q^2)F^s_{z-q^2}(\bar{p}) + (p'^2+q'^2-z)\tfrac{1}{2}\pi r_s(\bar{p},p,z-q^2)\,,$$
$$F^s_{p^2}(p) = 1, \quad \tau_s(p^2) = e^{i\delta_s(p)}\sin\delta_s(p)/p\,. \tag{7}$$

Here r_s is the resolvent kernel of the nonsingular integral equation[7,8] for the interior two-body wave function $F^s_{z-q^2}(p)$. This representation breaks down if the two-body phase shift $\delta_s(p)$ has zeros, but Osborn[9] has shown that the separable form can be restored for this case as well by adding one term for each zero of the phase, so we will assume this has been done when necessary. Inserting this representation into Eq. (6), we see that K_s is the sum of two terms:

$$K_s(p, q{:}z) = F^s_{z-q^2}(p)\tau_s(z-q^2)H_s(q{:}z) + (p^2+q^2-z)I_s(p,q{:}z)$$

which satisfy the coupled equations

$$H_s(q{:}z) = \frac{1}{2\pi q}\sum_{s'\neq s}\frac{1}{\sin 2\mu_{ss'}}\int_0^\infty dq'^2\int_{p_{ss'}^{-\,2}(q^2)}^{p_{ss'}^{+\,2}(q^2)} dp'^2\frac{F^s_{z-q^2}(\bar{p})}{p'^2+q'^2-z}\left[\delta_{1s'}\psi_\gamma(p')\delta(q'-(z+\gamma^2)^{1/2}) + K_{s'}(p',q'{:}z)\right],$$
$$I_s(p,q{:}z) = \frac{1}{4q}\sum_{s'\neq s}\frac{1}{\sin 2\mu_{ss'}}\int_0^\infty dq'^2\int_{p_{ss'}^{-\,2}(q^2)}^{p_{ss'}^{+\,2}(q^2)} dp'^2\frac{r_s(p,\bar{p}{:}z-q^2)}{p'^2+q'^2-z}\left[\delta_{1s'}\psi_\gamma(p')\delta(q'-(z+\gamma^2)^{1/2}) + K_{s'}(p',q'{:}z)\right]. \tag{8}$$

In order to demonstrate the unitarity of the K_s obtained by solving these equations, the simplest route is to first derive the equations for $\tilde{K}_s$ which, in the operator sense, contain $\tilde{K}_{s'}G_0(z)t_s$ rather than $t_sG_0(z)K_{s'}$ as above; this is trivial if we start from the configuration-space equation for $U(x, y{:}z)$. Given the equations in both orders, Freedman, Lovelace, and Namyslowski[10] have proved that the unitarity of K_s follows immediately from the unitarity of t_s. But Kowalski[6] has shown that the full off-shell unitarity of t_s is maintained for *arbitrary* real values of r_s, including $r_s = 0$. Thus, we can exploit this arbitrariness in r_s to obtain an arbitrary function $I_s(p, q:z)$ to insert in the one-variable equation for $H_s(q:z)$ without destroying the unitarity of K_s. This proves rigorously that the on-shell phenomenology previously presented[4] can be made unitary, and that we can even extend the phenomenology to three-body off-shell amplitudes by exploiting the arbitrariness of r_s in the second half of Eq. (8). On-shell three-body unitarity is trivial; we need merely require that the integrals converge, that the I_s have no singularities which clash with the vanishing of the $(p^2 + q^2 - z)$ coefficient, and that the I_s contain constants adjusted to ensure that Eq. (4) (or its generalization when more channels are included) is satisfied.

The separation of K into functions of one and two (vector) variables already occurs in Faddeev's treatment[11] [cf. Eq. (5.23) of Ref. 11], but he uses $\phi_\gamma^s(p)/(z+\gamma^2-q^2)$ rather than our choice of $F^s_{z-q^2}(p)\tau_s(z-q^2)$ for the coefficient of $H_s(q:s)$. These coincide at the pole; all we have done is to include the continuum singularities of the two-body on-shell amplitude in addition to the bound-state pole. Faddeev did not choose to do this; as noted above, the difficulty discussed by Osborn[9] would then have restricted the class of two-body interactions he could consider. Now that this difficulty has been solved, our approach can be understood as a logical extension of Faddeev's treatment, like most valid work on the three-body problem during the last decade.

The separation we propose here is also not new in another sense. For separable interactions, $r_s = 0$ and the Faddeev equations reduce to the one-variable equation for $H_s(q:z)$ given above [Eq. (8) with $I_s = 0$]. Thus all calculations using separable interactions necessarily confine their dynamics to the long-range ("eternal triangle") region. It is, therefore, not surprising that they are reasonably satisfactory for elastic scattering and breakup,[12] but give quite different binding energies for the triton[13] than local potentials fitted to the same two-nucleon low-energy parameters,[14] unless they are supplemented by three-body forces[15] which reintroduce I_s phenomenologically.

The full observable breakup amplitudes are known if we know $H_s(q:z)$ for $0 < q^2 < E$, while the elastic scattering amplitude $T_\gamma(z) = H_s((E+\gamma^2)^{1/2}:z))$ completes the description of the on-shell three-body T matrix. Viewed in this light, Eq. (8) for $H_s(q:z)$ restricted to these values appears to be a formula for computing the physical scattering amplitude if the wave function $K_s(p,q:z)$ is known. If this were true, it would be strictly analogous to the formula[8] for the two-body amplitude $\tau_s(k^2)$ in terms of the wave function $F^s_{k^2}(p)$:

$$\tau_s(k^2) = \tau_s^B(k,k) \Big/ \left(1 + \frac{2}{\pi}\int_0^\infty dp\, \frac{p^2 \tau_s^B(k,p) F^s_{k^2}(p)}{p^2 - k^2 - i\epsilon}\right), \tag{9}$$

$$\tau_s^B(k,p) = -\frac{m}{2\hbar^2}\int_0^\infty dx^2 \int_0^\infty dx'\, j_0(kx)\, V(x,x')\, j_0(px').$$

The analogy fails to be complete because even if we know the unobservable pieces of the wave function, that is, $I_s(p, q:z)$ and $H_s(q:z)$, $q^2 > E$, $q^2 \neq E + \gamma^2$, Eq. (8) is an integral equation for the functions

$$A_s(q:z) = [\theta(q^2) - \theta(q^2 - E)] H_s(q:z)$$

and the constant $T_\gamma(z)$. This integral equation is caused physically by the long-range ("eternal triangle") effect, as already noted. A dramatic example of this long-range interaction was discovered by Efimov[16] and rigorously proved by Amado and Noble[17]: The number of bound states of three identical bosons goes to infinity like $(1/\pi)\ln(|a|/R)$, where R is the range of forces and the S-wave scattering length a is large.

The separation of the three-body T matrix into observable and unobservable pieces allows us to construct an on-shell phenomenology for three-body breakup. The integral equations for the observables are

$$\begin{aligned} T_\gamma(z) &= \chi_\gamma(z) + \sum_{s=2,3}\int_0^E dq'^2 Q_{\gamma s}(q') A^s(q':z), \\ A_s(q:z) &= \chi_s(q:z) + (1-\delta_{1s}) Q_{s\gamma}(q) T_\gamma(z) + \sum_{s'\neq s}\int_0^E dq'^2 Q_{ss'}(q,q') A^{s'}(q':z), \end{aligned} \tag{10}$$

where

$$Q_{\gamma s}(q') = \frac{1}{2\pi \sin\mu_{1s}(E+\gamma^2)^{1/2}} \int_{p^-_{1s}{}^2(E+\gamma^2)}^{p^+_{1s}{}^2(E+\gamma^2)} dp'^2 \frac{\phi_\gamma(\bar{p})}{p'^2 + q'^2 + z} F^s_{z-q'^2}(p')\tau_s(z-q'^2),$$

$$Q_{s\gamma}(q) = \frac{1}{2\pi \sin\mu_{s1} q}\int_E^\infty dq'^2 \int_{p^-_{s1}{}^2(q^2)}^{p^+_{s1}{}^2(q^2)} dp'^2 \frac{F^s_{z-q^2}(\bar{p})}{p'^2 + q'^2 - z} F^1_{z-q'^2}(p')\tau_1(z-q'^2),$$

$$Q_{ss'}(q,q')=\frac{1}{2\pi\sin\mu_{ss'}q}\int_{p_{ss'}^{-}{}^{2}(q^2)}^{p_{ss'}^{+}{}^{2}(q^2)}dp'^2\frac{F^{s}_{z-q^2}(\bar{p})}{p'^2+q'^2-z}\,F^{s'}_{z-q'^2}(p')\tau_{s'}(z-q'^2),$$

which have been derived from Eq. (8) by the decomposition

$$\begin{aligned}&H_s(q{:}z)=[\theta(q^2)-\theta(q^2-E)]A^s(q{:}z)+\delta_{1s}\,\theta(q^2-E)T_\gamma(z)+\theta(q^2-E)\hat{H}_s(q{:}z),\\&\hat{H}_s(q{:}z)=H_s(q{:}z)-\delta_{1s}\,T_\gamma(z)\,.\end{aligned}\tag{11}$$

Therefore, the driving terms are determined from the unobservable pieces through

$$\begin{aligned}\chi_\gamma(z)=&\frac{1}{2\pi(E+\gamma^2)^{1/2}}\sum_{s'=2,3}\frac{1}{\sin2\mu_{1s'}}\int_0^\infty dq'^2\int_{p_{1s'}^{-}{}^{2}(E+\gamma^2)}^{p_{1s'}^{+}{}^{2}(E+\gamma^2)}dp'^2\frac{\phi_\gamma(\bar{p})}{p'^2+q'^2-z}\,[F^{s'}_{z-q'^2}(p')\tau_{s'}(z-q'^2)\theta(q'^2-E)\hat{H}_{s'}(q',E)\\&+(p'^2+q'^2-z)I_{s'}(p',q'{:}z)],\\\chi_s(q{:}z)=&\frac{1}{2\pi q}\sum_{s'\neq s}\frac{1}{\sin2\mu_{ss'}}\int_0^\infty dq'^2\int_{p_{ss'}^{-}{}^{2}(q^2)}^{p_{ss'}^{+}{}^{2}(q^2)}dp'^2\frac{F^{s}_{z-q^2}(\bar{p})}{p'^2+q'^2-z}\,[F^{s'}_{z-q'^2}(p')\tau_{s'}(z-q'^2)\theta(q'^2-E)\hat{H}_s(q',E)\\&+(p'^2+q'^2-z)I_s(p',q'{:}z)+\delta_{1s'}\,\phi_\gamma(p')\delta(q'-(E+\gamma^2)^{1/2})]\,.\end{aligned}\tag{12}$$

These equations are still exact, if $\hat{H}_s$ and I_s are obtained by solving Eq. (8).

For data of finite accuracy, the information contained in the breakup spectrum is exhausted by determining a finite number of coefficients in the expansion of the spectrum in terms of any set complete on the interval $0<q^2<E$. The Legendre polynomials $P_n(1-2q^2/E)$ provide such a set, so we assume that

$$A^s(q{:}z)=\sum_{n=0}^{N_s}A^s_n(z)P_n(1-2q^2/E)\,.\tag{13}$$

We project Eq. (10) onto this set and obtain a matrix equation of dimension $(N_1+N_2+N_3+4)$ for the constants $A^s_n(z)$ and $T_\gamma(z)$. By inserting this representation back into Eq. (4) and (5), we find that the cross section for breakup is proportional to

$$\frac{d\sigma}{d\Omega_p\,d\Omega_q\,dp^2}\propto p(E-p^2)^{1/2}|T(p)|^2,\tag{14}$$

where

$$\begin{aligned}&T(p)=\tau_1(p)\sum_{n=0}^{N_1}A^s_n(E)P_n(1-2P^2/E)+\sum_{s=2,3}\sum_{n=0}^{N_s}\frac{A^s_n(E)}{\sin2\mu_{1s}}\int_{\xi_p-\mu_{1s}}^{\min(\xi_p+\mu_{1s},\,\pi-\xi_p-\mu_{1s})}d\phi\,\tau_s(E\cos\phi)P_n(1-2\cos^2\phi),\\&\xi_p=\tan^{-1}(1-E/p^2)^{1/2}\,.\end{aligned}\tag{15}$$

Thus, if we know the two-body phase shifts $\delta_s(p)$ over the *finite* energy range $0<p^2<E$, we can describe a breakup spectrum of arbitrary complexity in terms of a finite set of parameters $A^s_n(E)$. The number of parameters is determined by the complexity of the data to be confronted. Clearly the A^s_n provide the equivalent of a phase-shift analysis for three-body breakup. Note that resonances in the direct channel (or the tails of bound-state or virtual-state poles) are explicitly exhibited in $\tau_1(p)$ and the kinematic reflections of similar structure in the other two channels in the $\tau_s(E\cos\phi)$, with *specified* phase relations. Equation (15) therefore provides a generalization of the Watson-Migdal final-state expression to include all three Faddeev channels, and is capable of confronting data of arbitrary complexity. For example, using the effective-range approximation

$$\tau(p)=(\beta-\alpha)/(p-i\beta)(p+i\alpha)$$

for the *n*-*p* and *n*-*n* $^1S_0$ and the *n*-*p* $^3S_1$ amplitudes in the $J=\frac{1}{2}^+$ state of *n*-*d* breakup already exhibits all the observed features of the energy spectrum at a single angle,[3] although of course not the angular variation, using only 3 real constants A^s_0. The over-all normalization of the breakup spectrum fixes one of these three constants from Eq. (4) if the inelasticity parameter of the state in question is known.

Although we have provided a phenomenology capable of fitting (when generalized to include angular momentum and spin) any breakup experiment, actual data analysis directly for the A^s_n would obviously encounter all the woes of ambiguous solutions encountered in two-particle phase-shift analyses in a much more severe form, and might well prove hopeless without theoretical guidance. We, therefore, sketch a systematic approach for refining the analysis. To obtain a first approximation for the A_n^s, we note that the wave-function dependence in Eq. (10) does not contribute to the singularities of T, on the physical cut, since

$$\phi_\gamma(\vec{p}) = N_\gamma + (p'^2 + q'^2 - q^2 + \gamma^2) f_\gamma(\vec{p}), \quad N_\gamma \approx (2\gamma)^{1/2}$$

$$F^s_{z-q^2}(\vec{p}) = 1 + (p'^2 + q'^2 - z) f^s_{z-q^2}(\vec{p}),$$

and the zero-range approximation $f_\gamma = 0 = f^s$ still gives all the correct physical singularities to T. But if we make this approximation, and keep only

$$\delta_{1s} N_\gamma \delta(q - (E+\gamma^2)^{1/2})$$

in the driving terms, Eq. (10) becomes an explicit matrix equation for the A_n^s which requires only knowledge of known (or knowable) two-body phase shifts and binding energies. Thus, solving this matrix equation is a reasonable *dynamical* first approximation with no unknown constants. If this equation succeeds in reproducing a particular breakup experiment to the accuracy available, we can learn nothing more from that experiment in an unambiguous way.

Provided the three-body data are rich enough, and the zero-range approximation fails, we can then introduce off-shell effects (two-body wave functions f) and see if the fit to experiment can be improved. A specific method for parametrizing these wave functions which preserves the fit to the two-body on-shell data and insures the orthogonality of wave functions at different energies has been described elsewhere.[3] Such a parametrization allows the $H_s(q)$ to be computed in the unobservable region $q^2 > E$, and also allows I_s to be computed *under the assumption* that there are no three-body forces. Such an assumption is highly questionable,[3,4,13] but until we have actually determined the two-body wave functions from three-body breakup, we will be unable to proceed very far in the calculation of I_s even under this assumption. It is, therefore, crucial to find out where I_s significantly affects our results, and where it can be very crudely approximated. Since I_s is restricted to a finite region, it may well be possible to expand it in the hyperspherical harmonics[18,19]; the long-range pieces which usually frustrate such expansions in breakup problems are clearly removed by the separation of the exterior function H_s from the interior wave function I_s.

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