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# Nature of Operator Schwinger Terms, Fixed Poles, and Scale-Invariance Breaking at Short Distances

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We give general arguments that if there are operator Schwinger terms in the equal-time commutator of the time and space components of the electromagnetic current these must be of both scalar and tensor nature. The contribution of the tensor term is isolated and shown to be proportional to $-(1/\pi)\int_0^\infty d\omega \tilde{F}_2(\omega)$. $\tilde{F}_2(\omega)$ is the Bjorken scale function less its leading scaling Regge parts. The existence of such a term is then related to the presence of a fixed pole at $j=0$ in the real part of the amplitude $\nu T_2$. This seems to be substantiated by the data. From the point of view of light-cone (or short-distance) operator-product expansions, such a term is shown to lead to scale-invariance breaking. The specific term expressing this breaking is then given and is shown not to disturb scaling in the variable $\omega$.

## I. INTRODUCTION

Recent interest in expansions of products of operators (or commutators) near the light cone stems from the realization that such products control the behavior of amplitudes in the Bjorken scaling limit. These expansions incorporate in a simple manner invariance under scale transformations and the operators carry their physical canonical dimensions. They are natural generalizations of the short-distance expansions proposed by Wilson.[1]

We shall be interested in the product expansions for the electromagnetic current found by various authors[2-4] to be consistent with the Bjorken scaling of the deep-inelastic electron-proton structure functions. In particular, we shall be interested in the presence of any scale-invariance-breaking terms. We do this by looking at the structure of the (time-space) equal-time commutators and especially the nature of the operator Schwinger terms implied by these expansions. In particular, it is found that they admit only the existence of a scalar term of the form $R\partial_i\delta^3(x)$; tensor-type terms of the form $S_{ij}\partial_j\delta^3(x)$ are not allowed. Such tensor terms represent scale-invariance breaking. These points are discussed in Secs. II and III and the problem conveniently formulated in Sec. IV.

Using general analyticity and positivity arguments, we show in Sec. V that if there are operator Schwinger terms they must include tensor terms. This is done by deriving a lower bound on the Schwinger-term contribution that comes solely from its tensor part. The tensor contribution is then isolated and is found to be

$$-\frac{1}{\pi}\int_0^\infty d\omega\, \tilde{F}_2(\omega),$$

where $\tilde{F}_2(\omega)$ is the scale function less its (scaling) leading Regge parts.

It is found in Sec. VI that the existence of a fixed pole at $j=0$ in the amplitude $\nu T_2(\nu, q^2)$ leads also to a tensor Schwinger term with the same contribution isolated above. Experimentally this contribution tends to be nonzero.

The interpretation that the fixed-pole and tensor Schwinger-term contribution is a scale-invariance-breaking effect is given in Sec. VII and the singularity of the scale-invariance-breaking term is specified for the commutator light-cone expansion.

In the Appendix we show, following Wilson, how the short-distance (or light-cone) expansions specify the equal-time commutator.

## II. SCALE INVARIANCE AND OPERATOR-PRODUCT EXPANSIONS

It was suggested by Wilson[1] that the product of two local operators may be described, for short distances, by a series over local operators with singular $c$-number functions as coefficients. An ordinary product $A(x)B(y)$ may then, for $(x-y)$ approaching zero, take the form

$$A(x)B(y)=\sum_n C_n(x-y)O_n(x+y)\,. \qquad (1)$$

The expansion in general involves an infinite set of operators $O_n(x+y)$. However, depending on the singularity of the functions $C_n(x-y)$, only a finite number contribute to any finite order of $(x-y)$. The singularities of the functions $C_n(x-y)$ reveal the singular nature of the product and may be of the form

$$C_n(x-y)=[(x-y)^2-i\epsilon(x_0-y_0)]^{-P},$$

where $P$ is any real number. Logarithmic singularities of the form $\{\ln[(x-y)^2 - i\epsilon(x_0-y_0)]\}^S$ are also allowed.

The usefulness of expansions such as that of Eq. (1) is dependent upon knowledge of the singularities of the $c$-number functions. These may be determined if one accepts, following Wilson, two fundamental assumptions. The first is that for $(x-y)$ approaching zero the expansion is invariant under scale transformations. The second is that the "dimensions" of the operators $A(x)$, $B(y)$, and $O_n(x+y)$ under such transformations are all known or at least specified.

With these assumptions one finds that, if $d_A$, $d_B$, and $d_n$ are the dimensions, respectively, of $A(x)$, $B(y)$, and $O_n(x+y)$, then the dimension of $C_n(x-y)$ (in units of mass or inverse length) must be

$$d(C_n) = d_A + d_B - d_n . \tag{2}$$

This implies that for $(x-y)$ approaching zero the singularity of $C_n(x-y)$ must be of the form

$$C_n(x-y) = [(x-y)^2 - i\epsilon(x_0-y_0)]^{-(d_A+d_B-d_n)/2} . \tag{3}$$

Dimensionless logarithmic terms may also be present. For the dimensions $d_A$, $d_B$, and $d_n$ one may take the natural choice, namely, the canonical physical dimensions the corresponding operators have in free-field theory.

An expansion for the commutator of two operators may be obtained by subtracting the Hermitian adjoint of the product expansion from itself. Taking all operators $O_n(x+y)$ to be Hermitian, one obtains

$$[A(x), B(y)] = \sum_n E_n(x-y) O_n(x+y) , \tag{4}$$

where

$$E_n(x-y) = \operatorname{Im} C_n(x-y) . \tag{5}$$

The $c$-number functions $E_n(x-y)$ may then have singularities of the form

$$E_n(x-y) = [(x-y)^2 - i\epsilon(x_0-y_0)]^{-P} - [(x-y)^2 + i\epsilon(x_0-y_0)]^{-P} . \tag{6}$$

These functions vanish for $(x-y)$ spacelike, thus ensuring the vanishing of the commutator required by microcausality.

Operator commutator expansions for short distances specify fully the equal-time commutator. This is demonstrated in the Appendix. In particular, the singularity of $E_n(x-y)$ determines whether Schwinger terms exist or not and what their exact nature is. Since the nature of this singularity relies so heavily on the assumptions of scale invariance and canonical dimensions, it should be clear then that the nature of the Schwinger term is a probe for the validity of these assumptions. This indeed will be our main tool in investigating scale symmetry-breaking effects in processes involving expansions for the electromagnetic current.

## III. PRODUCT EXPANSIONS FOR CONSERVED NEUTRAL VECTOR CURRENTS

In order to satisfy current conservation and Lorentz covariance explicitly, one may, following Brandt and Preparata and others,[5] write for the product of two conserved vector currents an expression of the form

$$j_\mu(x) j_\nu(0) = (\partial_\mu \partial_\nu - g_{\mu\nu}\Box) E_0(x^2) R_0(x,0) + i\epsilon_{\mu\nu\alpha\beta}\partial^\alpha E_1(x^2) R_1^\beta(x,0) + (g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\alpha\nu}\partial_\beta\partial_\mu - g_{\alpha\mu}\partial_\beta\partial_\nu + g_{\alpha\mu}g_{\beta\nu}\Box) E_2(x^2) R_2^{\alpha\beta}(x,0) . \tag{7}$$

$E_0(x^2)$, $E_1(x^2)$, and $E_2(x^2)$ are singular $c$-number functions and the operators $R_0(x,0)$, $R_1^\beta(x,0)$, and $R_2^{\alpha\beta}(x,0)$ are, respectively, a scalar, a four-vector, and a symmetric second-rank tensor.

In order that the expression be valid also near the light cone, these operators must generally be of the form

$$R_0(x,0) = \sum_{n=0}^{\infty} x^{\alpha_1} \cdots x^{\alpha_n} R_{0,\alpha_1 \cdots \alpha_n}(0) , \tag{8}$$

$$R_1^\beta(x,0) = \sum_{n=0}^{\infty} x^{\alpha_1} \cdots x^{\alpha_n} R^\beta_{1,\alpha_1 \cdots \alpha_n}(0) , \tag{9}$$

$$R_2^{\alpha\beta}(x,0) = \sum_{n=0}^{\infty} x^{\alpha_1} \cdots x^{\alpha_n} R^{\alpha\beta}_{2,\alpha_1 \cdots \alpha_n}(0) . \tag{10}$$

Here $R_{0,\alpha_1\cdots\alpha_n}(0)$, $R^\beta_{1,\alpha_1\cdots\alpha_n}(0)$, and $R^{\alpha\beta}_{2,\alpha_1\cdots\alpha_n}(0)$ are higher-rank tensors whose dimension increases by unity with the addition of each Lorentz index $\alpha_i$, namely, the increase by unity of the maximum spin.

For the short-distance behavior $(x_\mu \to 0)$ only a finite number of terms will contribute to Eqs. (8), (9), and (10) to any finite order in $x$.

We shall be mainly interested in the electromagnetic current $J_\mu(x)$ and the product expansion found by many authors[2-4] to be consistent with the Bjorken scaling behavior for the deep-inelastic electron-proton structure functions. In particular, we wish to probe for any scale-invariance-breaking effects by studying the nature of Schwinger terms implied by the singularities of this expansion. The expansion reads as follows:

$$J_\mu(x)J_\nu(0) = (\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\frac{1}{x^2 - i\epsilon x_0}R_0(x, 0) + (g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\alpha\nu}\partial_\beta\partial_\mu - g_{\alpha\mu}\partial_\beta\partial_\nu + g_{\alpha\mu}g_{\beta\nu}\Box)\ln(-x^2 + i\epsilon x_0)R_2^{\alpha\beta}(x, 0)\,. \tag{11}$$

Using the fact that

$$\operatorname{Im}(x^2 - i\epsilon x_0)^{-1} = \pi\epsilon(x_0)\delta(x^2)\,, \tag{12}$$

$$\operatorname{Im}\ln(-x^2 - i\epsilon x_0) = \pi\epsilon(x_0)\theta(x^2)\,, \tag{13}$$

one obtains an expansion for the commutator

$$[J_\mu(x), J_\nu(0)] = (\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\pi\epsilon(x_0)\delta(x^2)R_0(x, 0) + (g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\alpha\nu}\partial_\beta\partial_\mu - g_{\alpha\mu}\partial_\beta\partial_\nu + g_{\alpha\mu}g_{\beta\nu}\Box)\pi\epsilon(x_0)\theta(x^2)R_2^{\alpha\beta}(x, 0)\,. \tag{14}$$

The singularity $\pi\epsilon(x_0)\delta(x^2)$ in the first term on the right-hand side and $\pi\epsilon(x_0)\theta(x^2)$ in the second term are essential for demonstrating the scaling behavior in the Bjorken limit. From scale invariance one concludes that the dimension of the operator $R_0(x, 0)$ must be two and that of $R_2^{\alpha\beta}(x, 0)$ must be four. Thus from Eqs. (8) and (9) we find that the lowest-dimension operators contributing to $R_0(x, 0)$ and $R_2^{\alpha\beta}(x, 0)$ must be two and four, respectively. This is consistent with renormalizable field-theory models when free-field dimensions determine the dimensions of the operators mentioned above and where $J_\mu$ has dimension three.[6]

A behavior of a current commutator as displayed in Eq. (14) specifies fully the nature of the equal-time commutators of the electromagnetic current components. This here is precisely the nature of the operator Schwinger term. Using the results of the Appendix, we find the following result for the time-time and time-space commutator:

$$[J_0(0, \vec{x}), J_0(0)] = 0\,, \tag{15}$$

$$[J_i(0, \vec{x}), J_0(0)] = (2\pi^2 i)\partial_i\delta^3(\vec{x})R_0(0)\,. \tag{16}$$

We thus see that the Schwinger term is controlled by the singularity $\epsilon(x_0)\delta(x^2)$ and is a scalar operator of dimension two. Notably absent is any tensor contribution from $R_2^{\alpha\beta}(x, 0)$ due to the singularity $\epsilon(x_0)\theta(x^2)$. Thus the absence of such a term in the real world is a crucial test of the expansion of Eq. (14), in other words, a test of the assumptions of "scale invariance" and "canonical dimensionality." Indeed, it has been pointed out by Brown[3] and equivalently by Brandt[2] that the absence of such a tensor Schwinger term is essential in making possible the connection between the behavior of commutators near the light cone and the Bjorken scaling limit when one uses generalized representations such as that of Deser-Gilbert-Sudarshan or Jost-Lehmann-Dyson for the invariant electromagnetic structure functions in position space [see Eq. (21) below]. They are led to this, after making exchanges of limits and integrals, via smoothness requirements for their spectral functions. On the other hand, Jackiw, Van Royen, and West[4] assume the absence of tensor Schwinger terms in obtaining the singularity of the commutator on the light cone of Eq. (14) starting from the Bjorken scale functions.

Having presented the necessary background, we shall now proceed to study the nature of the opertor Schwinger terms, if they exist, in pursuance of our main quest as to whether scale invariance is strictly respected or broken.

We note here that $c$-number Schwinger terms do not appear in our discussion as these are inconsistent with scale invariance unless they are infinite.[6] However, since we shall be concerned only with connected parts of amplitudes, their contribution may simply be ignored.

## IV. SCHWINGER TERMS AND THE FORWARD VIRTUAL COMPTON SCATTERING AMPLITUDE

Consider the spin-averaged connected forward scattering amplitude given by

$$T_{\mu\nu}(q, P) = +i\int d^4x\, e^{iq\cdot x}\langle P|T(J_\mu(x)J_\nu(0))|P\rangle + S_{\mu\nu}(q, P)\,. \tag{17}$$

$S_{\mu\nu}(q, p)$ is the seagull term which contributes only to $T_{ij}$ and is a polynomial in $q^2$ and $P$. One may decompose $T_{\mu\nu}(q, P)$ as follows:

$$T_{\mu\nu}(q, P) = [q^2P_\mu P_\nu - \nu(P_\mu q_\nu + q_\mu P_\nu) + \nu^2 g_{\mu\nu}]M_1(q^2, \nu) + (q_\mu q_\nu - g_{\mu\nu}q^2)M_2(q^2, \nu)\,, \tag{18}$$

or, keeping in accord with convention,

$$T_{\mu\nu}(q,P)=\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)T_1(q^2,\nu)+\left(P_\mu-\frac{\nu}{q^2}q_\mu\right)\left(P_\nu-\frac{\nu}{q^2}q_\nu\right)T_2(q^2,\nu),\tag{19}$$

where $\nu=q\cdot P=q^0P^0-\vec{q}\cdot\vec{P}$. Comparing Eq. (18) with Eq. (19) one has

$$M_1=\frac{1}{q^2}T_2,\qquad M_2=\frac{1}{q^2}\left(T_1+\frac{\nu^2}{q^2}T_2\right).\tag{20}$$

In configuration space we may write for the spin-averaged proton matrix element of the commutator[5]:

$$\frac{1}{2\pi}\langle P|[J_\mu(x),J_\nu(0)]|P\rangle=-[\Box P_\mu P_\nu-(P\cdot\partial)(P_\mu\partial_\nu-P_\nu\partial_\mu)+(P\cdot\partial)^2g_{\mu\nu}]\hat{V}_1(x^2,x\cdot P)-(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)\hat{V}_2(x^2,x\cdot P).\tag{21}$$

Then if one has for $x^2\to 0$

$$\langle P|R_0(x,0)|P\rangle=f_0(x\cdot P)+O(x^2),\tag{22}$$

$$\langle P|R_2^{\alpha\beta}(x,0)|P\rangle=g^{\alpha\beta}f(x\cdot P)+P^\alpha P^\beta f_2(x\cdot P)+O(x^2),\tag{23}$$

then using Eq. (14) in Eq. (21), one obtains[5]

$$\hat{V}_2(x^2,x\cdot P)\underset{x^2\to 0}{\sim}-\epsilon(x_0)\delta(x^2)f_0(x\cdot P),\tag{24}$$

$$\hat{V}_1(x^2,x\cdot P)\underset{x^2\to 0}{\sim}-\epsilon(x_0)\theta(x^2)f_2(x\cdot P).\tag{25}$$

Now

$$\mathrm{Im}M_i=V_i(q^2,\nu)=\int d^4x\,e^{iq\cdot x}\hat{V}_i(x^2,x\cdot P).\tag{26}$$

If we define

$$W_i(q^2,\nu)=\mathrm{Im}T_i(q^2,\nu),\tag{27}$$

then we obtain

$$\mathrm{Im}M_1=\frac{1}{q^2}W_2,\tag{28}$$

$$\mathrm{Im}M_2=\frac{1}{q^2}\left(W_1+\frac{\nu^2}{q^2}W_2\right).\tag{29}$$

Thus from Eqs. (23), (25), and (26) we see that for $x\to 0$ the singularity of the tensor term $[R_2^{\alpha\beta}(x,0)]$ in the commutator expansion controls the high-$q$ behavior of $\mathrm{Im}M_1$ and hence $M_1(q^2,\nu)$. Thus the appearance of a tensor Schwinger term will reflect itself in the appearance of a Schwinger term in $M_1(q^2,\nu)$ (or equivalently $T_2$). By this we mean that if we consider $T_{0i}(q^2,\nu)$ in the Bjorken-Johnson-Low limit (BJL limit) defining the equal-time commutator ($q_0\to i\infty$, $\vec{q}$ fixed), we obtain a contribution from $M_1$ (or $T_2$) of order $1/q_0$.

Similarly the existence of a scalar Schwinger term reflects itself in a similar contribution by $M_2$ (or $T_1$).

We have in the BJL limit

$$T_{0i}=q_0q_i\lim_{\mathrm{BJL}}M_2(q^2,\nu)+(\vec{q}\cdot\vec{P}q_0P_i-q_0q_iP_0^2)\lim_{\mathrm{BJL}}M_1(q^2,\nu)\tag{30}$$

or

$$T_{0i}=q_0q_i\lim_{\mathrm{BJL}}\frac{T_1(q^2,\nu)}{q^2}+q_0P_i(\vec{q}\cdot\vec{P})\lim_{\mathrm{BJL}}\frac{T_2(q^2,\nu)}{q^2}.\tag{31}$$

A tensorial Schwinger-term contribution is here indicated by the factor $P_i(\vec{q}\cdot\vec{P})$, for this is the Fourier transform of the matrix element of a term of the form $S_{ij}\partial_j\delta^3(\vec{x})$. Scalar Schwinger terms are simply indicated by the factor $q_i$.

Upon examining Eq. (30) we see that a tensor Schwinger-term contribution exists if

$$\lim_{\mathrm{BJL}}M_1(q^2,\nu)=\frac{C_1}{(q^0)^2}\tag{32}$$

or equivalently

$$\lim_{\mathrm{BJL}}T_2=C_1,\tag{33}$$

where $C_1$ is a nonzero constant. A scalar Schwinger term exists and is provided for in our expansion if

$$\lim_{\mathrm{BJL}}M_2(q^2,\nu)=\frac{C_2}{(q^0)^2}\tag{34}$$

or equivalently

$$\lim_{\mathrm{BJL}}T_1(q^2,\nu)=C_2,\tag{35}$$

where $C_2$ is a nonzero constant. We should point out here that Eq. (32) also provides for such a term; however, because of Eq. (34) the existence of this term does not imply Eq. (32).

Our problem is thus reduced to studying the behavior of $T_{0i}$ in the BJL limit and seeing whether the equal-time commutator receives contributions from $M_1$ or $T_2$. We could, of course, have started by this statement after Eqs. (18) and (19). However, the intermediate steps were intended to make the connection between the space-time and momentum-space statements clearer.

## V. ANALYTICITY, POSITIVITY, AND THE EXISTENCE OF OPERATOR SCHWINGER TERMS

Consider the forward virtual Compton scattering amplitude $T_{\mu\nu}(q,P)$. Its absorptive part receives contributions from two classes of states.[7] The Class-I states are the usual $s$-channel states and the so-called $z$-graph states. The Class-II states are those that couple directly to the current, contribute only for timelike $q$, and are hence physically distinct from the Class-I states. Separating the Born term $B_{\mu\nu}(q, P)$, we may write

$$T_{\mu\nu}(q, P) = B_{\mu\nu}(q, P) + T^{\mathrm{I}}_{\mu\nu}(q, P) + T^{\mathrm{II}}_{\mu\nu}(q, P). \quad (36)$$

The Born term contains form factors that vanish rapidly for $q^2 \to -\infty$ and hence does not contribute to Schwinger terms.

Using the analyticity property of $T_{00}(q, P)$ in the variable $q^0$ and the positivity of the absorptive-part contribution of the Class-I states, it was shown by Bitar and Khuri[7] that $T^{\mathrm{I}}_{00}(q, \nu)$ satisfies a lower bound of the form

$$\lim_{\mathrm{BJL}} T^{\mathrm{I}}_{00} \geq \frac{C}{(q^0)^2}, \quad C \neq 0. \quad (37)$$

No such statement could be made regarding $T^{\mathrm{II}}_{00}$. Thus, unless the unknown behavior of $T^{\mathrm{II}}_{00}$ cancels precisely (and in all frames) the behavior exhibited in Eq. (37), the lower bound will be satisfied by the full amplitude, namely,

$$\lim_{\mathrm{BJL}} T_{00} \geq \frac{C}{(q^0)^2}. \quad (38)$$

If the lower bound is saturated, operator Schwinger terms exist; for then, using the decomposition,

$$T_{00} = \vec{q}^{\,2} M_2(q^2, \nu) + [(\vec{q}\cdot\vec{P})^2 - \vec{q}^{\,2}(P^0)^2] M_1(q^2, \nu), \quad (39)$$

we may conclude that either

$$\lim_{\mathrm{BJL}} M_2(q^2, \nu) = \frac{C_2}{(q^0)^2} \quad (40)$$

or

$$\lim_{\mathrm{BJL}} M_1(q^2, \nu) = \frac{C_1}{(q^0)^2} \quad (41)$$

or both. If the bound is satisfied via Eq. (41), both a scalar and a tensor Schwinger term will exist. If the bound is satisfied via Eq. (40), then only a scalar Schwinger term will appear. The latter case is the one allowed by the light-cone expansion under consideration. Thus this expansion is in the class of theories where the cancellation between Class-II and Class-I states occurs for the invariant amplitude $M_1(q^2, \nu)$, but not for the invariant amplitude $M_2(q^2, \nu)$. The cancellation is also such that the lower bound is saturated.

If the bound is not saturated the equality signs in Eqs. (40) and (41) become "greater than" signs. As was pointed out in Ref. 7, and may be seen by inspection, this implies the nonexistence of the BJL definition of the time-space equal-time commutator.

With these properties for the light-cone expansion at hand, let us see whether they are plausible. We intend indeed to give arguments that if the lower bound of Eq. (38) is saturated, this is not likely to happen without the participation of the invariant amplitude $M_1(q^2, \nu)$, that is, without the existence of tensor Schwinger terms. Using the positivity of $\mathrm{Im}T^{\mathrm{I}}_{00}$, we shall first derive a lower bound for the constant $C$ appearing in Eq. (37). We give here only the main steps of a derivation presented in Ref. 7. We have

$$T^{\mathrm{I}}_{00}(q_0, |\vec{q}\,|) = \frac{1}{\pi}\int_{q^0_t}^{\infty} q_0' \, dq_0' \, \frac{\mathrm{Im}T^{\mathrm{I}}_{00}(q_0', |\vec{q}\,|)}{q_0'^2 - q_0^{\,2}}, \quad (42)$$

where the threshold is at

$$q^0_t = [(M+\mu)^2 + \vec{P}^2 + \vec{q}^{\,2}]^{1/2} - (M + \vec{P}^2)^{1/2}.$$

$M$ is the nucleon mass and $\mu$ the mass of the $\pi$ meson.

Since we are interested in the behavior of $T^{\mathrm{I}}_{00}$ in the limit $q_0 \to i\infty$, we put $q_0 = i\lambda$ and consider the limit $\lambda \to \infty$. We thus have

$$T^{\mathrm{I}}_{00}(q_0, |\vec{q}\,|) = \frac{1}{\pi}\int_{q^0_t}^{\infty} q_0' \, dq_0' \, \frac{\mathrm{Im}T^{\mathrm{I}}_{00}(q_0', |\vec{q}\,|)}{q_0'^2 + \lambda^2}. \quad (43)$$

Owing to the positivity of $\mathrm{Im}T^{\mathrm{I}}_{00}(q_0', |\vec{q}\,|)$ one has the trivial lower bound

$$T^{\mathrm{I}}_{00}(q_0, |\vec{q}\,|) \geq \frac{1}{\pi[a^2(q^0_t)^2 + \lambda^2]} \times \int_{q^0_t}^{aq^0_t} q_0' \, dq_0' \, \mathrm{Im}T^{\mathrm{I}}_{00}(q_0', |\vec{q}\,|), \quad (44)$$

where $a$ is some large positive number. It is possible to choose $|\vec{P}|$ and $|\vec{q}\,|$ large enough in Eq. (44) so that the integral is over a spacelike region. In this region $\mathrm{Im}T^{\mathrm{II}}_{00}$ is zero and $\mathrm{Im}T^{\mathrm{I}}_{00} = \mathrm{Im}T_{00}$. Putting $\vec{q}\cdot\vec{P} = 0$ for simplicity,[8] we then find

$$T^{\mathrm{I}}_{00}(q_0, |\vec{q}\,|) \geq \frac{\vec{q}^{\,2}}{\pi[a^2(q^0_t)^2 + \lambda^2]} \times \int_{q^0_t}^{aq^0_t} q_0' \, dq_0' [\mathrm{Im}M_2 - (P^0)^2 \mathrm{Im}M_1], \quad (45)$$

with

$$\nu = q^0 P^0, \quad q^2 = \frac{\nu^2}{(P^0)^2} - \vec{q}^{\,2},$$

$$T^{\mathrm{I}}_{00}(q_0, |\vec{q}\,|) \geq \frac{\vec{q}^{\,2}}{\pi[a^2(q^0_t)^2 + \lambda^2]} \times \int_{q^0_t P^0}^{aq^0_t P^0} \nu \, d\nu \left( -\mathrm{Im}M_1(\nu, q^2) + \frac{1}{(P^0)^2}\mathrm{Im}M_2(\nu, q^2) \right). \quad (46)$$

For large $\lambda\to\infty$ this inequality gives us a lower bound for $C$ in terms of an integral at fixed $\vec{q}^2$. To get a fixed-$q^2$ bound, we take the limit $P_0\to\infty$. The limit is legitimate since the integral is over a compact region. As $P^0\to\infty$ we find

$$q^2\to -\vec{q}_2 \tag{47}$$

and

$$q_t^0 P^0 \to \tfrac{1}{2}(2M\mu+\vec{q}^2+\mu^2)+O(1/P^0). \tag{48}$$

Thus the limits of the integral remain finite in this limit and the contribution of $\mathrm{Im}M_2(\nu, q^2)$ is damped out by the $1/(P^0)^2$ factor leading to

$$T_{00}^{\mathrm{I}}(q_0, |\vec{q}|)\geq \frac{(-\vec{q}^2)}{\pi\lambda^2}\int_{\nu_t}^{a\nu_t}\nu d\nu\, \mathrm{Im}M_1(\nu, q^2=-\vec{q}^2). \tag{49}$$

In the above, $(-\vec{q}^2)\mathrm{Im}M_1(\nu, q^2=-\vec{q}^2)$ is equivalent to $\mathrm{Im}T_2=W_2(\nu, q^2=-\vec{q}^2)$. Thus we find that

$$C\geq \frac{1}{\pi}\int_{\nu_t}^{a\nu_t}\nu d\nu\, W_2(q^2=-\vec{q}^2, \nu). \tag{50}$$

Equation (49) or (50) is then a clear indication that in the limit $P_0\to\infty$ the contribution to the operator Schwinger term is controlled by the invariant amplitude $M_1$. Thus follows our assertion that such a term is unlikely to exist without the cooperation of $M_1$. The integral in Eq. (50) is a measure of the existence of such a term. The constant $a$ may, of course, be taken arbitrarily large. We shall see now that, assuming that the electroproduction structure functions scale, we shall be able to subtract from $W_2$ its leading term with Regge behavior and get expressions for the Schwinger term contributed by $M_1$ in terms of integrals over all $\nu$.

Consider the invariant amplitudes $M_1$ and $M_2$ in the Regge limit. If $\alpha_i(0)$ are the leading Regge trajectories, one has for large $\nu$ the behavior

$$M_1 \underset{\nu\to\infty}{\sim}\sum_i \beta_i(q^2)\nu^{\alpha_i-2}\frac{e^{i\pi\alpha_i}\pm 1}{\sin\pi\alpha_i}, \tag{51}$$

$$M_2 \underset{\nu\to\infty}{\sim}\sum_i \beta_i(q^2)\nu^{\alpha_i}\frac{e^{i\pi\alpha_i}\pm 1}{\sin\pi\alpha_i}. \tag{52}$$

For all $\alpha_i>0$ we then define the modified amplitudes

$$\tilde{M}_1=M_1(q^2,\nu)-\sum_{\alpha_i>0}\beta_i(q^2)\nu^{\alpha_i-2}\frac{e^{i\pi\alpha_i}\pm 1}{\sin\pi\alpha_i}, \tag{53}$$

$$\tilde{M}_2=M_2(q^2,\nu)-\sum_{\alpha_i>0}\beta_i(q^2)\nu^{\alpha_i}\frac{e^{i\pi\alpha_i}\pm 1}{\sin\pi\alpha_i}. \tag{54}$$

Correspondingly we may write

$$\mathrm{Im}\tilde{M}_1=\mathrm{Im}M_1-\sum_{\alpha_i>0}\beta_i(q^2)\nu^{\alpha_i-2}, \tag{55}$$

$$\mathrm{Im}\tilde{M}_2=\mathrm{Im}M_2-\sum_{\alpha_i>0}\beta_i(q^2)\nu^{\alpha_i}. \tag{56}$$

Consider now the Regge parts in the Bjorken scaling limit $(B)$ $\nu\to\infty$, $q^2\to-\infty$, at $\omega=\nu/(-q^2)$ fixed. In this limit it is known experimentally that $\nu W_2(q^2,\nu)$ scales. In other words, using $W_2=q^2\,\mathrm{Im}M_1(q^2,\nu)$, one has

$$\lim_B \nu q^2\, \mathrm{Im}M_1(q^2,\nu)=f(\nu/q^2). \tag{57}$$

For this to be satisfied by the Regge term, one must have

$$\beta_i(q^2)\underset{q^2\to\infty}{\sim}(q^2)^{-\alpha_i}. \tag{58}$$

If this is the case, then we find

$$\lim_{\mathrm{BJL}}\beta_i(q^2)\nu^{\alpha_i-2}\to\frac{1}{(q^0)^2}\left(\frac{P^0}{q^0}\right)^{\alpha_i}. \tag{59}$$

Thus for all $\alpha_i>0$ the Regge term does not contribute to order $1/(q^0)^2$ in $M_1$ and therefore does not contribute to the tensorial Schwinger term. Therefore the contribution must come from the modified amplitude $\tilde{M}_1$. A similar argument applies for $\tilde{M}_2$.

One sees immediately from Eq. (59) that the presence of a pole at $\alpha_i=0$ does indeed lead to the $1/(q^0)^2$ behavior needed for a Schwinger term. There is no physical justification for such poles and we assume they do not exist. We hasten to say though that these are not to be confused with fixed poles that contribute only to the *real* part of the invariant amplitudes, which seem to exist and are responsible for tensorial Schwinger terms, as we shall see later.

Define now, using Eq. (39),

$$\tilde{T}_{00}=\vec{q}^2\tilde{M}_2(q^2,\nu)+[(\vec{q}\cdot\vec{P})^2-\vec{q}^2(P^0)^2]\tilde{M}_1(q^2,\nu), \tag{60}$$

just like $T_{00}$; this satisfies an unsubtracted dispersion relation in $q_0$ for fixed $\vec{q}$. If a subtraction is needed, the lower bound in $q^0$ will be satisfied beyond its limit $Cq_0^{-2}$ and all operator Schwinger terms will be infinite.

We thus have

$$\tilde{T}_{00}=\frac{1}{\pi}\int_{q_0^t}^{\infty}dq_0'q_0'\frac{1}{q_0'^2-q_0^2}\times\{\vec{q}^2\mathrm{Im}\tilde{M}_2+[-\vec{q}^2P_0^2+(\vec{q}\cdot\vec{P})^2]\mathrm{Im}\tilde{M}_1\}. \tag{61}$$

The Born term, as usual, is not considered. Now

$$\nu=q_0P_0-\vec{q}\cdot\vec{P},\quad q^2=\left(\frac{\nu+\vec{q}\cdot\vec{P}}{P_0}\right)^2-\vec{q}^2. \tag{62}$$

Again, since we are interested in the limit $q_0\to i\infty$ we write $q^0=i\lambda$ and look at the limit $\lambda\to\infty$.

Before we proceed we make the following remark. Anticipating our result that a Schwinger term is generated by $\tilde{M}_1(q^2,\nu)$ we assume that it has the form

$$\lim_{\text{BJL}} \tilde{M}_1(q^2,\nu)=\frac{1}{(q^0)^2}[A_1+B_1\vec{q}\cdot\vec{P}+O((\vec{q}\cdot\vec{P})^2)+\cdots], \tag{63}$$

where $A_1$ and $B_1$ are constants. A similar expression holds possibly for $\tilde{M}_2$ with constants $A_2$ and $B_2$.

Substituting in $T_{0i}$ as given in Eq. (30), we find

$$\lim_{\text{BJL}} T_{0i}=\frac{1}{q^0}(\vec{q}\cdot\vec{P}P_i-q_iP_0^2)[A_1+B_1\vec{q}\cdot\vec{P}+O((\vec{q}\cdot\vec{P})^2)+\cdots]+\frac{1}{q^0}q_i[A_2+B_2\vec{q}\cdot\vec{P}+O((\vec{q}\cdot\vec{P})^2)+\cdots]. \tag{64}$$

Thus the tensorial Schwinger-term contribution is determined by the constant $A_1$ [coefficient of $-1/q_0=\vec{q}\cdot\vec{P}P_i(-A_1)$ as seen from Eq. (17)]. The constant $B_1$ determines Schwinger terms with higher than one derivative of the δ function due to the presence of the factor $\vec{q}\cdot\vec{P}$ multiplying it. The constant $A_1$ also gives the contribution of $\tilde{M}_1$ to the scalar Schwinger term as it also multiplies $q_iP_0^2$. Indeed, if $\tilde{M}_2$ does not contribute to the Schwinger term ($A_2=0$) the constant $A_1$ determines both the scalar and tensor Schwinger terms. Changing variables, using Eq. (62), we find

$$\tilde{T}_{00}=[\vec{q}^2P_0^2+(\vec{q}\cdot\vec{P})^2]\frac{1}{\pi}\int_{P_0q_0^t}^{\infty}\frac{d\nu(\nu+\vec{q}\cdot\vec{P})}{P_0^2}\frac{\operatorname{Im}\tilde{M}_1}{(\nu+\vec{q}\cdot\vec{P})^2/P_0^2+\lambda^2}+\vec{q}^2\frac{1}{\pi}\int_{P_0q_0^t}^{\infty}\frac{d\nu(\nu+\vec{q}\cdot\vec{P})}{P_0^2}\frac{\operatorname{Im}\tilde{M}_2}{(\nu+\vec{q}\cdot\vec{P})^2/P_0^2+\lambda^2}. \tag{65}$$

The integrals represent $\tilde{M}_1$ and $\tilde{M}_2$, respectively. Clearly the terms with $\vec{q}\cdot\vec{P}$ in the numerator will contribute to the constants $B_1$, $B_2$, and higher terms defined in Eq. (63). These then do not contribute to the term $A_1$ and therefore we shall neglect them. Thus we are then concerned with

$$\tilde{T}_{00}=\frac{1}{\pi P_0^2}[-\vec{q}^2P_0^2+(\vec{q}\cdot\vec{P})^2]\int_{P_0q_0^t}^{\infty}\frac{d\nu\,\nu\operatorname{Im}\tilde{M}_1(\nu,q^2)}{(\nu+\vec{q}\cdot\vec{P})^2/P_0^2+\lambda^2}+\frac{\vec{q}^2}{\pi P_0^2}\int_{P_0q_0^t}^{\infty}d\nu\,\nu\frac{\operatorname{Im}\tilde{M}_2(\nu,q^2)}{(\nu+\vec{q}\cdot\vec{P})^2/P_0^2+\lambda^2}. \tag{66}$$

Define $\vec{P}=(P_0^2-M^2)^{1/2}\hat{P}$. Since the imaginary parts are void of their leading Regge behavior one expects that taking the $\lambda\to\infty$ limit in Eq. (66) would give us the leading term of $O(1/\lambda^2)$ and thus determine $A_1$ and $A_2$. We will then get sum rules for these terms at fixed $\vec{q}^2$. To get a fixed-$q^2$ sum rule we take the limit $P_0\to\infty$ and again expect it to be interchangeable with the integration. In this limit the contribution of $M_2$ vanishes, leading to $A_2=0$ in this limit. We are left with

$$\lim_{P^0\to\infty}\tilde{T}_{00}=\frac{1}{\pi\lambda^2}[-\vec{q}^2+(\vec{q}\cdot\hat{P})^2]\int_{\nu_t}^{\infty}\nu d\nu\operatorname{Im}\tilde{M}_1(\nu,q^2), \tag{67}$$

evaluated at $q^2=[-\vec{q}^2+(\vec{q}\cdot\hat{P})^2]<0$, spacelike. Since $\operatorname{Im}\tilde{M}_1(\nu,q^2)\sim\nu^{-2}$ at best, then the integral is at worst logarithmically divergent. This justifies our exchanges of limits and integrations. Since $\operatorname{Im}\tilde{M}_2\sim$ const at best, the justification for neglecting its contribution in the infinite-momentum limit is not well founded. However, we are not as concerned with the value of $A_2$ as with the value of $A_1$ determined in Eq. (67). Using the relationship

$$W_2(\nu,q^2)=q^2\operatorname{Im}M_1(\nu,q^2),$$

if we define $\tilde{W}_2$ as $W_2$ minus its leading Regge-pole contributions, we find that in the infinite-momentum limit we have

$$\lim_{P^0\to\infty}\tilde{T}_{00}=\frac{1}{\pi\lambda^2}\int_{\nu_t}^{\infty}d\nu\,\nu\tilde{W}_2(\nu,q^2=-\vec{q}^2+(\vec{q}\cdot\hat{P})^2). \tag{68}$$

Comparing Eq. (67) with Eq. (60) we find in the infinite-momentum limit ($\lambda^2=-q_0^2$)

$$A_1=\frac{-1}{\pi P_0^2}\int_{\nu_t}^{\infty}\nu d\nu\operatorname{Im}\tilde{M}_1(\nu,q^2=\vec{q}^2+(\vec{q}\cdot\hat{P})^2) \tag{69}$$

or equivalently

$$A_1=-\frac{1}{\pi}\frac{1}{-\vec{q}^2P_0^2+(\vec{q}\cdot\vec{P})^2}\times\int_{\nu_t}^{\infty}d\nu\,\nu\tilde{W}_2(\nu,q^2=-\vec{q}^2+(\vec{q}\cdot\hat{P})^2). \tag{70}$$

Thus again we see that $\tilde{M}_1$ does contribute to a tensor Schwinger term. Substituting $A_1$ as given in Eq. (69) or (70), in Eq. (64), we find the infinite-momentum frame

$$\lim_{P_0\to\infty}\lim_{\text{BJL}}T_{0i}=\frac{1}{q^0}(\vec{q}\cdot\hat{P}\hat{P}_i-q_i)\left(\frac{-1}{\pi}\right)\times\int_{\nu_t}^{\infty}\nu d\nu\operatorname{Im}\tilde{M}_1(\nu,q^2=-\vec{q}^2+(\vec{q}\cdot\hat{P})^2) \tag{71}$$

giving the Schwinger term as an integral over $\operatorname{Im}\tilde{M}_1$, or

$$\lim_{P_0\to\infty}\lim_{\text{BJL}} T_{0i} = \frac{1}{q^0}(\vec{q}\cdot\hat{P}\hat{P}_i - q_i) \times\left(\frac{-1}{\pi}\right)\int_{\nu_t}^{\infty} d\nu\left(\frac{\nu}{q^2}\tilde{W}_2(\nu,q^2)\right)_{q^2=-\vec{q}^2+(\vec{q}\cdot\hat{P})^2}. \tag{72}$$

In the above expressions the contribution of $\tilde{M}_2$ is zero. The value of the integral appearing in Eq. (72) then determines whether or not there is a tensor Schwinger term (or a scalar term for that matter). The integral may be evaluated for large $\vec{q}^{\,2}$ in terms of a modified scaling function.

It is known that in the scaling limit $\nu\to\infty$, $q^2\to-\infty$, $-\nu/q^2=\omega$ fixed, one has

$$\nu W_2(\nu,q^2)\to F_2(\omega). \tag{73}$$

If we then define

$$\nu\tilde{W}_2(\nu,q^2)\to\tilde{F}_2(\omega), \tag{74a}$$

then

$$\tilde{F}_2(\omega)=\theta(\omega-1)F_2(\omega)-\sum_{\alpha_i>0}\gamma_i\,\omega^{\alpha_i-1}. \tag{74b}$$

Consider now the integral in Eq. (72),

$$A_1=-\frac{1}{\pi}\int_{\nu_t}^{\infty}d\nu\frac{\nu}{q^2}\tilde{W}_2(\nu,q^2)\bigg|_{q^2=-\vec{q}^2+(\vec{q}\cdot\hat{P})^2} \tag{75}$$

going to large values of $\vec{q}$, and changing variables to $\omega=-\nu/q^2$, we find

$$A_1=+\frac{1}{\pi}\int_0^{\infty}d\omega\,\tilde{F}_2(\omega). \tag{76}$$

Therefore the value of this integral is the measure of existence of a tensor Schwinger term of magnitude $-A_1$.

The integral may be evaluated from experimental data. It is found to be nonzero. Indeed, it is found to determine (for large $q^2$) the residue of a fixed pole at $j=0$ in the real part of $\nu T_2=\nu q^2 M_1$. We shall see in Sec. VI that such a fixed pole indeed generates a tensor Schwinger term with a strei gth given by the integral of Eq. (76).

## VI. FIXED POLE IN $\nu T_2$ AND THE TENSOR SCHWINGER TERM

It was pointed out some time ago that the data on deep-inelastic electron-proton scattering seem to indicate the existence of a fixed pole at $j=0$ in the *real* part of the amplitude $\nu T_2(q^2,\nu)=\nu q^2 M_1(q^2,\nu)$. We shall show here that this leads us to a Schwinger term of the tensor type. Note that we have already seen from the discussion of Sec. V that Regge poles contributing to both the real and imaginary parts of $\nu T_2$ do lead to such a term if $\alpha_i=0$. The argument depends on the fact that the imaginary part $\nu W_2(\nu,q^2)$, to which these poles would contribute, scales. Such an argument, however, is not applicable to fixed poles contributing to the real part of $\nu T_2$. We shall see that the connection between such poles and the Schwinger term does, however, depend on the behavior of the scale function $\tilde{F}(\omega)$.

We review briefly here the arguments of Rajaraman and Rajasekaran[9] for the fixed pole and its properties.

Using Eq. (53) for $\tilde{M}_1$ then in the absence of any poles with $\alpha_i\geq 0$, one has the superconvergence relation

$$\int_0^{\infty}d\nu\,\nu\tilde{W}_2(\nu,q^2)=0, \tag{77}$$

where as usual $\tilde{W}_2=\text{Im}\tilde{T}_2$ and $\tilde{T}_2=q^2\tilde{M}_1$. In the Bjorken scaling limit one has from Eq. (77)

$$\int_0^{\infty}d\omega\,\tilde{F}_2(\omega)=0, \tag{78}$$

where $\nu\tilde{W}_2\to\tilde{F}_2(\omega)$ given in Eq. (74a) above. Equation (78) may be checked using experimental data. The left-hand side is reported to have a value approximately unity in Ref. 9. This discrepancy may be eliminated if a fixed pole at $j=0$ exists in the real part of $\nu\tilde{T}_2$. That is, for large $\nu$ one has

$$\nu\tilde{T}_2\underset{\nu\to\infty}{\sim}\frac{f(q^2)}{\nu}+O\left(\frac{1}{\nu^{1+\epsilon}}\right). \tag{79}$$

In this case Eqs. (77) and (78) are modified as follows:

$$\int_0^{\infty}d\nu\,\nu\tilde{W}_2(q^2,\nu)=-\pi f(q^2) \tag{80}$$

and

$$\int_0^{\infty}\tilde{F}_2(\omega)d\omega=+\pi\lim_{-q^2\to\infty}\frac{f(q^2)}{q^2}. \tag{81}$$

There will be no contradiction with experiment if for large $q^2$ one has

$$f(q^2)\underset{-q^2\to\infty}{\sim}aq^2. \tag{82}$$

One then has

$$\frac{+1}{\pi}\int_0^{\infty}\tilde{F}_2(\omega)d\omega=a. \tag{83}$$

We thus see that the residue of this fixed pole for large $q^2$ is controlled by the same integral that determines the tensor Schwinger term. We can see this directly. Using Eq. (79) and substituting in Eq. (31), we find

$$\lim_{\text{BJL}}T_{0i}=q_0q_i\lim_{\text{BJL}}\frac{T_1}{q^2}+\frac{1}{q_0}(a)\left(\frac{1}{P_0^{\,2}}\right)P_i(\vec{q}\cdot\vec{P}). \tag{84}$$

In the limit $P_0\to\infty$ we get a term of the form

$$-\frac{1}{q_0}[\hat{P}_i(\vec{q}\cdot\hat{P})(-a)],$$

where the bracketed term is the tensor Schwinger term with coefficient $-a$. Here, as before, the constant

$$+a=+A_1=+\frac{1}{\pi}\int_0^\infty d\omega\,\tilde{F}_2(\omega) \tag{85}$$

is the determining factor.

From the preceding discussion we see that the existence of a fixed pole at $j=0$ in $\nu T_2$ does not lead to a tensor Schwinger term unless its residue behaves for large $q^2$ as $aq^2$. If the behavior is less than linear in $q^2$, the Schwinger term does not exist and the integral in Eq. (85) is zero. If the behavior is of the form $(q^2)^{1+\epsilon}$, the residue of the Schwinger term is infinite. Various authors[9,10] have discussed the consequences of the polynomial character of the residue of the fixed pole. We point out here that we do not require such a behavior. We only require a linear behavior in $q^2$ for large $q^2$.

## VII. NATURE OF SCALE-INVARIANCE BREAKING

We have seen in Sec. III that a light-cone expansion for the product of two electromagnetic currents does not allow for tensor-operator Schwinger terms in the equal-time time-space commutator. This is because the expansion respects scale invariance and admits operators with canonical physical dimensions. On the other hand, we find in the first part of Sec. IV, on the basis of general analyticity and positivity and barring unlikely cancellations, that operator Schwinger terms exist and are also of the tensor type. In this case one may isolate the contribution of such a term and find that it is proportional to the integral

$$I=\frac{1}{\pi}\int_0^\infty d\omega\,\tilde{F}_2(\omega)\,.$$

This integral has been evaluated earlier and is found to be of the order of unity. Indeed, its nonvanishing has led some authors to postulate the existence of a fixed pole in the real part of $\nu T_2(\nu,q^2)$ with a residue that behaves linearly in $q^2$ for large $q^2$. As we have seen in Sec. V, this indeed leads naturally to the tensor Schwinger term. Thus we may conclude that the existence of such a term is governed by the nonvanishing of the integral

$$I=\frac{1}{\pi}\int_0^\infty d\omega\,\tilde{F}_2(\omega).$$

As pointed out in Sec. II, this has consequences for scale-invariant canonical product expansions on the light cone and on the connection usually made between these expansions and the scaling limit.

Let us look again at Eq. (14):

$$\begin{aligned}[J_\mu(x),J_\nu(0)]=&\;(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)\pi\epsilon(x_0)\delta(x^2)R_0(x,0)\\&+(g_{\mu\nu}\partial_\alpha\partial_\beta-g_{\alpha\nu}\partial_\beta\partial_\mu-g_{\alpha\mu}\partial_\beta\partial_\nu+g_{\alpha\mu}g_{\beta\nu}\Box)\\&\times\pi\epsilon(x_0)\theta(x^2)R_2^{\alpha\beta}(x,0)\,.\end{aligned} \tag{86}$$

The singularity of the second term does not provide for a tensor Schwinger term. This means that the singularity has to be modified. The simplest such modification would be a term of the form

$$\begin{aligned}S_{\mu\nu}=&\;(g_{\mu\nu}\partial_\alpha\partial_\beta-g_{\alpha\nu}\partial_\beta\partial_\mu-g_{\alpha\mu}\partial_\beta\partial_\nu+g_{\alpha\mu}g_{\beta\nu}\Box)\\&\times\pi\epsilon(x_0)\delta(x^2)S_2^{\alpha\beta}(x,0)\,,\end{aligned} \tag{87}$$

where

$$S_2^{\alpha\beta}(x,0)=\sum_n x^{\alpha_1}\cdots x^{\alpha_n}S_{2,\alpha_1\cdots\alpha_n}^{\alpha\beta}(0) \tag{88}$$

and

$$\langle P|S_2^{\alpha\beta}(0)|P\rangle\propto I=\frac{1}{\pi}\int_0^\infty d\omega\,\tilde{F}_2(\omega). \tag{89}$$

In a theory where the lowest dimension of a second-rank symmetric tensor is four, an expression like Eq. (87) for $S_{\mu\nu}$ generally violates scale invariance. Thus it is appropriate to consider it as a scale-invariance-breaking term. The parameter controlling this breaking would then be the integral

$$I=\frac{1}{\pi}\int_0^\infty d\omega\,\tilde{F}_2(\omega).$$

Of course, if one is willing to admit, following Wilson, that dimensions change with interaction, then it is possible to consider a case where Eq. (87) is scale-invariant; the case being where the operator (or operators) $S_2^{\alpha\beta}$ originally having canonical dimension four, would in an interacting world have dimension two. The change in dimension thus must be strictly two. A fractional change would, of course, sustain scale-invariance breaking. Such a scheme might look artificial but, though unlikely, is still possible.

## VIII. DISCUSSION

As mentioned earlier, various authors have derived the singularity structure of Eq. (87) upon assuming scaling. Such a derivation admittedly relies on a cavalier exchange of limits and integration. Allowing such an exchange clearly then leads to the absence of tensor Schwinger terms. One may see this from the fact that $T_2(\nu,q^2)$ satisfies an unsubtracted dispersion relation in $\nu$ at fixed $-q^2$. We have

$$T_2(\nu,q^2)=\frac{1}{\pi}\int_{-q^2/2}^\infty d\nu'\,\frac{\nu' W_2(\nu',q^2)}{\nu'^2-\nu^2}\,. \tag{90}$$

Changing variables to $\omega=-\nu/q^2$, we find that

$$T_2(\nu,q^2)=\frac{1}{\pi q^2}\int_2^\infty d\omega'\,\frac{\overline{F}_2(\omega',q^2)}{\omega'^2-\omega^2}\,. \tag{91}$$

Scaling requires that for any fixed *finite* $\omega'$ one has

$$\lim_{q^2\to-\infty} \overline{F}_2(\omega', q^2) = F_2(\omega') . \tag{92}$$

Now in the BJL limit $q^2\to-\infty$ and $\omega\to i0$, so that if the integral in Eq. (91) is uniformly convergent one may conclude that

$$\lim_{\text{BJL}} T_2(\nu, q^2) = 0 . \tag{93}$$

Thus the only possibility for a Schwinger term to arise ($\lim_{\text{BJL}} T_2 = \text{const}$) would be that the function $\overline{F}_2(\omega', q^2)$ grows indefinitely for simultaneous large values of $\omega'$ *and* $-q^2$. Only such a growth in the region of large $-q^2$ and large $\omega'$ in the integral of Eq. (91) could render Eq. (93) invalid. It is not difficult to construct functions which grow indefinitely for large $\omega'$ *and* large $-q^2$ whereas they vanish for large $-q^2$ at any fixed *finite* $\omega'$.[11]

It is also in order to point out here that in spite of the seemingly straightforward connection between a $j=0$ fixed-pole (in $T_2$) contribution of the form $aq^2/\nu^2$ and the tensor Schwinger term, such a connection could be lost in the full amplitude through a conspiracy with lower-lying poles. The bare Born-term contribution $aq^2/(\nu^2-\frac{1}{4}q^4)$ is a good example of such a conspiracy. This conspiracy should be happening in the light-cone-expansion model of Eq. (84). For it has been shown by Mack[12] that the $\epsilon(x_0)\theta(x^2)$ singularity does lead to a scaling $j=0$ fixed pole; nevertheless no Schwinger term arises. Such a Schwinger term here arises only from a family of scaling fixed poles with leading member at $j=1$ generated by the scale-invariance-breaking singularity $\epsilon(x_0)\delta(x^2)$. Thus in this model the connection between the isolated $j=0$ contribution and the Schwinger term serves to relate the strengths of these two singularities. The precise connection in this model may be obtained by carefully isolating the two contributions. This point is under further study.

One final word on the scalar Schwinger-term contribution. As we have seen in Sec. IV, if $M_2$ does not contribute to it, this contribution is the same in magnitude as that of the tensor term. In this case they both come from the invariant amplitude $M_1$. Such an equality is upheld in the infinite-momentum limit. In general, however, the contribution to it comes from both $R_0(0)$ and $S_2^{\alpha\beta}(0)$ ($M_2$ and $M_1$) in Eq. (87), $S_2^{\alpha\beta}(0)$, thus providing a scale-invariance-breaking correction. In the infinite-momentum limit, however, this term seems to be dominant. This throws some doubt on the validity of sum rules appearing in the literature[4] estimating the scalar term. For in those estimates the contribution of tensor terms was assumed to be zero.


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## APPENDIX

We discuss here the calculation of equal-time commutators from short-distance expansions following Wilson.[1]

If $E_n(x)$ is the singular $c$-number functions, then, using the definition of the $\delta$ function, we have

$$E_n(x) = F_n(x_0)\delta^3(\vec{x}) + \vec{F}_n(x_0)\cdot\vec{\nabla}\delta^3(\vec{x}) + \cdots , \tag{A1}$$

with

$$F_n(x_0) = \int d^3x\, E_n(x_0, \vec{x}) , \tag{A2}$$

$$\vec{F}_n(x_0) = -\int d^3x\, \vec{x} E_n(x_0, \vec{x}) , \quad \text{etc.} \tag{A3}$$

In the limit $x_0\to 0$, $F_n(x_0)$ and $\vec{F}_n(x_0)$ give, respectively, the regular part and the Schwinger term of the equal-time commutator.

In evaluating integrals of the form given in Eqs. (A2) and (A3) we are usually faced with expressions of the form

$$f_{\mu\cdots\nu}(x_0) = \int d^3x \left(\frac{1}{(x^2-i\epsilon x_0)^P} - \frac{1}{(x^2+i\epsilon x_0)^P}\right) x_\mu\cdots x_\nu . \tag{A4}$$

Expressions with logarithms may be treated similarly using

$$\ln x^2 = \frac{1}{P}\frac{d}{dP}(x^2)^P\bigg|_{P=0}$$

or similar such expressions.

If the $x_\mu$ factors in the numerator are such that any component of $\vec{x}$ is left unsquared, the integral is zero from rotational invariance. Using spherical coordinates, one then first does the angular integrations and is left with

$$f_{\mu\cdots\nu}(x_0) \propto \int_0^\infty r^2 dr\, F_{\mu\cdots\nu}(r^2, x^0) \times\left(\frac{1}{(r-x_0-\frac{1}{2}i\epsilon)^P(r+x_0+\frac{1}{2}i\epsilon)^P} - \text{c.c.}\right). \tag{A5}$$

In the complex $r$ plane, and for noninteger $P$, the integrand displays cuts from $x_0$ to $+\infty$ and $-x_0$ to $-\infty$. The integral in (A5) is then given by

$$f_{\mu\cdots\nu}(x_0) \propto \int_{C_1} r^2 dr\, F_{\mu\cdots\nu}(r^2, x^0)\frac{1}{(r-x^0)^P(r+x_0)^P} , \tag{A6}$$

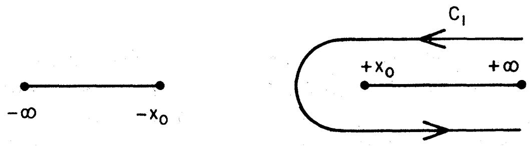


FIG. 1. Contour $C_1$ for Eq. (A6).

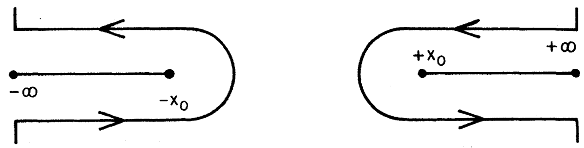


FIG. 2. Contour $C$ for Eq. (A7).

where $C_1$ is the contour shown in Fig. 1. Using the fact that the numerator is an even function of $r$, one may then, by changing variables $r \to -r$, show that

$$f_{\mu\cdots\nu}(x_0)[\exp(-i2\pi P)-1] \propto \int_C r^2 dr\, F_{\mu\cdots\nu}(r^2, x^0) \times\left(\frac{1}{(r-x_0)^P(r+x_0)^P}\right), \tag{A7}$$

where $C$ is the contour given in Fig. 2. Thus in the absence of other singularities in $F_{\mu\cdots\nu}(r^2, x^0)$ and the vanishing of the integral over the great circle, $f_{\mu\cdots\nu}(x_0)=0$ for noninteger $P$.

For $P$ integer the cut in Fig. 1 becomes a pole and the integral may be evaluated by the residue theorem.

*Example*. We calculate the following commutators implied by the expansion of Eq. (11):

$$\text{(I)}\quad J_0(x)J_0(0) = -\vec{\partial}^2\left(\frac{1}{x^2-i\epsilon x_0}\right)(R_0 + x^{\alpha_1}R_{0,\alpha_1}+\cdots) + (\partial_\alpha\partial_\beta - 2g_{\alpha 0}\partial_\beta\partial_0 + g_{\alpha 0}g_{\beta 0}\Box) \times \ln(-x^2+i\epsilon x_0)(R_2^{\alpha\beta} + x^{\alpha_1}R^{\alpha\beta}_{2,\alpha_1}+\cdots). \tag{A8}$$

In evaluating the integrals of Eq. (A2) the contribution of the first term in (A8) is zero as it is a total derivative. In the second term, only those parts without a spatial derivative contribute. These lead to zero.

The Schwinger term given by Eq. (A3) gets zero from the first term in (A8) because of the double total divergence. In the second term only terms with one total divergence contribute. One is then left with integrals of the form

$$\int_{C_2} r^2 dr \frac{-2x_0}{(r-x_0)(r+x_0)} \tag{A9}$$

coming from one time derivative. Evaluating this at the pole and taking the limit $x_0 \to 0$ one obtains zero. Thus we obtain

$$[J_0(\vec{x},0), J_0(0)] = 0. \tag{A10}$$

For the time-space commutator we have

$$J_i(x)J_0(0) = \partial_i\partial_0\frac{1}{x^2-i\epsilon x_0}(R_0 + x^{\alpha_1}R_{0,\alpha_1}+\cdots) + (-\partial_i\partial_\beta g_{\alpha 0} - g_{\alpha i}\partial_\beta\partial_0 + g_{\alpha i}g_{\beta 0}\Box) \times\ln(-x^2+i\epsilon x_0)(R_2^{\alpha\beta}+x^{\alpha_1}R^{\alpha\beta}_{2,\alpha_1}+\cdots). \tag{A11}$$

The regular commutator of Eq. (A2) gets zero from the first term because of the total divergence $\partial_i$, and then only terms with $\partial_0\partial_0$ might contribute in the second term. These, however, differentiate $\ln(-x^2+i\epsilon x_0)$ and eventually lead to a null contribution. Due again to the weak singularity of the second term, it gives zero contribution to the Schwinger term. Thus, here we see the effect of dimensionality of $R_2^{\alpha\beta}$ on the nature of the Schwinger term. On the other hand, we find a contribution from the first due to the singularity $(x^2-i\epsilon x_0)^{-1}$ present there. For we then have

$$\vec{F}_i(x_0) = 4\pi\int_C r^2 dr\left(\frac{2x_0}{(r-x_0)^2(r+x_0)^2}\right)\hat{i}. \tag{A12}$$

$\hat{i}$ is a unit vector in the direction of $x_i$. From the residue theorem we have

$$\lim_{x_0\to 0}\vec{F}_i(x_0) = 4\pi\times\hat{i}\,2\pi i\left(2x_0\frac{1}{2}\frac{d}{dr}\left.\frac{r^2}{(r+x_0)^2}\right|_{r=x_0}\right) = \hat{i}\,2\pi i\times 4\pi(\tfrac{1}{4}) = \hat{i}\,2\pi^2 i. \tag{A13}$$

Thus one has

$$[J_i(\vec{x},0), J_0(0)] = i2\pi^2 R_0(0)\partial_i\delta^3(\vec{x}). \tag{A14}$$

Notice that if the singularity of the second term was also $1/(x^2-i\epsilon x_0)$, a Schwinger term would arise from $R_2^{00}$ and the term $-\partial_i\partial_\beta g_{\alpha 0}$ on the right-hand side of Eq. (A11) and a tensor term of the form $R_2^{ij}\partial_j\delta^3(\vec{x})$ from the term $-g_{\alpha i}\partial_\beta\partial_0$ next to it. This is where scale-invariance breaking occurs.

In a theory of fermion quarks interacting with spinless bosons $R_0(0)$ of Eq. (A14) may, for example, be a term of the form $\phi^2$ and $R_2^{\alpha\beta}(0)$ may be $\phi\partial^\alpha\partial^\beta\phi(0)$.

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# Breaking the Symmetry of the Baryons and the Mesons*

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We present an approach to the breaking of SU(3) × SU(3) symmetry which emphasizes the Nambu-Goldstone realization of this symmetry. Here the octet $\pi$, $K$, $\eta$ are the ground-state mesons. For matrix elements which are not analytic in the symmetry-breaking parameters, we can establish exactly the leading behavior in symmetry breaking. An example of this is a chiral-limit theorem on the meson decay constants $f_K/f_+(0)f_\pi - 1 = [3(m_K^2 - m_\pi^2)/64\pi^2 f_\pi^2] \times \ln[64\pi^2 f_\pi^2/3(m_K + m_\pi)^2] + O(\lambda)$. For matrix elements which are analytic to leading order in symmetry breaking, we advance the hypothesis of threshold dominance of the Goldstone-boson-pair states. When this hypothesis is applied to the mass splittings of the ground-state mesons there results an eigenvalue problem to which the unique nontrivial solution corresponds to octet enhancement. This is independent of any assumption about the Hamiltonian symmetry breaking. When we apply these ideas to the baryon mass splittings we again obtain octet solutions corresponding to tadpole-model results and a new result $\frac{3}{10}(f/d)_B = (f/d)_A/[3(f/d)_A^2 - 1]$ relating the baryon mass $f/d$ ratio to the axial-vector–baryon $f/d$ ratio, in good agreement with experiment. We also discuss electromagnetic mass shifts in this context and advocate that for $\Delta I = 1$ mass shifts the Cottingham formula diverges (and should be abandoned). If the Cottingham formula diverges for $\Delta I = 1$ mass shifts, then we no longer have Dashen's sum rule $\mu_{K^+}^2 - \mu_{K^0}^2 = \mu_{\pi^+}^2 - \mu_{\pi^0}^2$. An alternative, finite approach for $\Delta I = 1$ mass shifts is suggested and developed.

## I. INTRODUCTION

This paper is devoted to a study of the breaking of the SU(3) symmetry of the strong interactions. This is undertaken with the recognition that it is low-energy Goldstone-boson-pair states that dominate symmetry-breaking matrix elements.

The primary assumption on which we base this work is that in the absence of symmetry breaking the Hamiltonian for the strong interactions is SU(3)×SU(3)-invariant,[1] but the vacuum state is just SU(3)-symmetric. Coleman's theorem[2] then requires that the SU(3) symmetry of the vacuum state be manifest for all physical states so that they may be classified according to the irreducible representations of SU(3). But the vacuum symmetry, SU(3), is not the same as the Hamiltonian symmetry, SU(3)×SU(3). In this instance the Goldstone theorem[3] requires the existence of an octet of massless pseudoscalar ground-state mesons. These are identified with the octet $\pi$, $K$, and $\eta$. That the physical pseudoscalar mesons are massive is to be accounted for by the presence of symmetry-breaking terms in the Hamiltonian. Explicit symmetry breaking is also responsible for removing the SU(3) degeneracy of other states. In the absence of such symmetry-breaking terms, however, the ground-state mesons are strictly