PHYSICAL REVIEW D VOLUME 5, NUMBER 6 15 MARCH 1972

Construction of Multiperipheral Dynamical Equations and Inclusive Sum Rules*

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(Received 25 October 1971; revised manuscript received 20 December 1971)

Multiperipheral-like integral equations are derived from inclusive sum rules by making direct approximations on inclusive cross sections. As a result, the physical basis of multiperipheral dynamics is clarified. Furthermore, the Regge behavior for exclusive processes and the scaling property of inclusive cross sections can be considered as consequences of our dynamical approximations.

I. INTRODUCTION

The importance of sum rules[1] for inclusive cross sections has been emphasized recently by Veneziano.[2] He has demonstrated that these linear relations between discontinuities of multiparticle amplitudes are equivalent to a whole set of *nonlinear* unitarity equations. The purpose of this paper is to demonstrate that dynamical equations can be derived from these sum rules by direct approximations on the inclusive cross sections. These equations resemble the ordinary multiperipheral integral equations,[3,4] and arguments are given to indicate that they are actually more general.

The necessary approximations are suggested by the apparent lack of long-range correlation effects and the observed strong cutoff in transverse momenta in high-energy particle productions. Similar but *stronger* assumptions are needed in the ordinary formulation of multiperipheral dynamics.[3,4] The virtues of our present approach are the following: (1) Approximations are made only on experimental observables; thus any error committed can in principle be controlled. (2) Since we do not work with general production amplitudes, we avoid certain unnecessary dynamical approximations as well as achieve great kinematic simplifications. (3) It provides a more definite procedure for injecting the concept of short-range correlations. (4) It could lead to new techniques for studying properties of inclusive cross sections. Since we are able to derive general integral equations, which include multiperipheral equations as special cases, this approach exhausts the complete dynamical contents of the multiperipheralism. In particular, both the Regge behavior for exclusive processes and the scaling property of inclusive processes[5] are *consequences* of our dynamical considerations.

We introduce notations and review inclusive sum rules in Sec. II and then state our dynamical approximations in Sec. III. These general assumptions are sufficient for us to construct dynamical equations based on the sum rules. However, in order to avoid kinematic complications, we shall employ more specific assumptions in Sec. IV to derive a Chew-Goldberger-Low-type multi-Regge integral equation. A simpler integral equation for the Reggeon-Reggeon absorptive amplitude will also be introduced. Finally, we contrast the present approach with the conventional multiperipheral dynamics in Sec. V.

II. INCLUSIVE SUM RULES AND KINEMATICS

Let $T_{n,n}$ be the connected part of the scattering amplitude for the process

$$a+b+1'+2'+\cdots+n'\to a'+b'+1+2+\cdots+n,$$

and let $2iD_n$ be the discontinuity of $T_{n,n}$ in the missing-mass variable squared

$$M^2=\left(p_a+p_b-\sum_{i=1}^{n}p_i\right)^2.$$

It has been shown[6] that D_n, in the forward limit of $p_a=p_{a'}$, $p_b=p_{b'}$, $p_{i'}=p_i$, is directly related to the n-particle inclusive cross section by

$$\frac{d\sigma}{\prod_{i=1}^{n}d^4p_i\delta^+(p_i^2-\mu^2)}=(2\pi)^{-3n}\Delta^{-1/2}(s_{ab},m_a^2,m_b^2)\times D_n(p_a;p_1,\ldots,p_n;p_b),\tag{2.1}$$

where

$$\Delta(x,y,z)=x^2+y^2+z^2-2(xy+yz+zx).$$

Inclusive sum rules are linear relations between D_n and D_{n+1}, $n=1,2,\ldots$. For instance, in the case of D_2 and D_3, the sum rule reads[7]

$$(p_\alpha-p_\beta)_\mu D_2(p_a;p_1,p_2;p_b)=(p_\alpha-p_\beta)_\mu D_2^{(0)}+(2\pi)^{-3}\int d^4p_3\delta^+(p_3^2-\mu^2)(p_3)_\mu\times D_3(p_a;p_1,p_2,p_3;p_b),\tag{2.2}$$

where

$$p_\alpha = p_a + p_b, \quad p_\beta = p_1 + p_2,$$

and $D_2^{(0)}$ is the single-particle-state contribution to the discontinuity D_2:

$$D_2^{(0)}(p_a; p_1, p_2; p_b) = \pi |T(a, b \to 1, 3, 2)|^2 \times \delta^+((p_\alpha - p_\beta)^2 - \mu^2). \qquad (2.3)$$

The derivation of these sum rules can be found in Refs. 1 and 2, and they become obvious when the D_n's are considered as (unnormalized) energy-momentum density functions. With p_a, p_b, p_1, p_2 fixed, D_3 describes the (relative) probability density for finding a particle with four-momentum p_3; and Eq. (2.2) simply expresses the energy-momentum conservation conditions. Conversely, if all D_n's satisfy inclusive sum rules, Veneziano[2] has demonstrated that Eq. (2.1) follows. These linear relations then become statements of unitarity; and they can obviously be used as a starting point for dynamics based on the direct-channel unitarity.

It turns out that if we use a two-particle-correlation approximation, we need only to work with D_2 and D_3. It is convenient to first convert Eq. (2.2) into an invariant equation by dotting the vector $p_\alpha - p_\beta$, and we obtain

$$D_2(p_a; p_1, p_2; p_b) = D_2^{(0)} + \tfrac{1}{2}(2\pi)^{-3} \int d^4p_3 \delta^+(p_3^{\,2} - \mu^2)\left(\frac{M^2 - M'^2 + \mu^2}{M^2}\right) D_3(p_a; p_1, p_2, p_3; p_b), \qquad (2.4)$$

where

$$M^2 = (p_\alpha - p_\beta)^2, \quad M'^2 = (p_\alpha - p_\beta - p_3)^2. \qquad (2.5)$$

Equation (2.4) indicates that D_2 is a "weighted" average of D_3 over the phase space of p_3. The weighting factor is simplified due to the experimental observation that transverse momentum of every particle produced at high energies is small so that both D_2 and D_3 have a built-in cutoff in $\vec{q}_i^{\perp 2}$. This can best be done by parametrizing the phase space by collinear variables[8] (the rapidity variables in particular). Let $p_i = (\mu_i^\perp \cosh y_i, \vec{q}_i^\perp, \mu_i^\perp \sinh y_i)$, $\mu_i^\perp = (\mu^2 + \vec{q}_i^{\perp 2})^{1/2}$, where $\vec{q}_a^\perp = \vec{q}_b^\perp = 0$, and $y_a \gg y_b$. Equation (2.4) can then be written as

$$D_2 = D_2^{(0)} + \tfrac{1}{2}(2\pi)^{-3} \int_{y_b}^{y_a} \frac{dy_3}{2} \int d\vec{q}_3^\perp \left(\frac{M^2 - M'^2 + \mu^2}{M^2}\right) D_3, \qquad (2.6)$$

and the limits are further restricted by the energy-momentum conservations and the experimentally observed cutoff in $|\vec{q}_3^\perp|$.

We shall concentrate in this paper on the region

$$s_{ab}, \quad M^2 \to \infty, \qquad (2.7)$$

with $0 \leq M^2/s_{ab} \leq \Delta^2$, $t_1 = (p_a - p_1)^2$, $t_2 = (p_b - p_2)^2$ held at very small values. This is often referred to as the double-fragmentation region.[9] In the limit $\Delta \to 0$, it becomes the "di-triple" Regge region.[9] In terms of the rapidity variables, the limit (2.7) corresponds to $|\vec{q}_1^\perp|$, $|\vec{q}_2^\perp|$ small and $y_a - y_1 = O(\Delta)$, $y_2 - y_b = O(\Delta)$, as $Y = y_a - y_b \simeq \ln s_{ab} \to \infty$. The integration volume in (2.6) is roughly a cylinder with its length increasing with s_{ab} as $Y - O(\Delta)$.

III. DYNAMICAL APPROXIMATIONS

We next state a "sufficient" set of dynamical approximations which will allow us to construct integral equations. They are meant to be an illustration on the essential ingredients that are necessary to turn the *exact* sum rules into *approximate* dynamical equations. These approximations will be made precise in Sec. IV and they will also be examined more critically later when we contrast the present approach with the conventional multiperipheral dynamics.

(a) Strong ordering. The integration region in Eq. (2.6) can be divided into three regions, A: $|y_1 - y_3| \simeq O(\Delta)$, B: $|y_3 - y_2| \simeq O(\Delta)$, C: $y_3 \notin A, B$, such that the contribution from the region C in (2.6), *in the limit (2.7),* can be neglected.

(b) Two-Particle correlation. D_3 can be approximated[10] by

$$D_3 \simeq F(p_a; p_1, p_3) D_2(p_a - p_1; p_3, p_1; p_b) \text{ in } A, \qquad (3.1a)$$

$$D_3 \simeq D_2(p_a; p_1, p_3; p_b - p_2) F(p_b; p_2, p_3) \text{ in } B. \qquad (3.1b)$$

Both approximations are motivated by the apparent lack of long-range correlation effects at high energy and the observed strong cutoff in transverse momenta. Approximation (a) can be understood by noting that, for $|\vec{q}_i^\perp| \simeq 0$, the weighting factor $(M^2 - M'^2 + \mu^2)/M^2$ in the integrand of (2.6) is of the order 1, 0, 1 in the regions A, C, B, respectively. We see that *the cutoff in transverse momenta automatically provides a kinematical cutoff for the y_3 integration.* (a) corresponds to replacing this smooth damping by a sharp θ-function cutoff. In practice, this choice is awkward, and, as we shall see in Sec. IV, it can be reformulated with the help of some rapidly decreasing functions, such as Regge residues. Its counterpart in the ordinary multiperipheral dynamics is the *arbitrary* neglect of the so-called "crossed-graph." Our approach makes this concept precise, and it can in principle be checked more directly by experiments.[11] This point will be elaborated further in Sec. V.

Approximation (b) specifies the nature of the short-range correlations. Eq. (3.1) is, strictly speaking, attainable only in the limit $\Delta \to 0$. In order to keep Δ small but finite, we should interpret (3.1) as a vector equation, with D_2 and D_3 being column vectors, i.e., we need to keep terms to higher orders in (M'^2/M^2). In the case of the multi-Regge model, this corresponds to keeping lower-angular-momentum branch points so that the integral equation derived becomes a coupled-channel problem. This, of course, will not cause any conceptual difficulty, aside from increasing the notational inconvenience. We shall, therefore, keep ourselves to a single-channel analysis in what follows, although the content is actually more general. The physical basis of the two-particle-correlation approximation will be discussed in Sec. V.

IV. CHEW-GOLDBERGER-LOW EQUATION

The multi-Regge integral equation proposed by Chew, Goldberger, and Low[3] (CGL) can be derived by a specific choice of the two-particle correlation function. Before doing so, we first take care of the problem of replacing the θ-function cutoff in the y_3 integration. We note that in the limit (2.7), the amplitude $T(a, b \to 1, 3, 2)$ (which enters into $D_2^{(0)}$) has a double-Regge expansion

$$T(a, b\to 1, 3, 2) \simeq G(t_1)[(p_1+p_3)^2/\mu^2]^{\alpha(t_1)}\beta(t_1, \cos\phi_{12}, t_2) \times [(p_3+p_2)^2/\mu^2]^{\alpha(t_2)} G(t_2), \qquad (4.1)$$

where $\cos\phi_{12} = \hat{p}_1^\perp \cdot \hat{p}_2^\perp$ and is related to the Toller angle.[4] $G(t_1)$ and $\beta(t_1, \cos\phi_{12}, t_2)$ are single- and double-Regge vertices, respectively. They are known to be rapidly decreasing functions of t_1 and t_2, as $t_1, t_2 \to -\infty$, and can thus be used to damp out unwanted contributions in (2.6).

First, we introduce new variables $Q_a = p_a$, $Q_1 = p_a - p_1$, $Q_2 = p_b - p_2$, $Q_b = p_b$, and define new functions B_2, $B_2^{(0)}$, for $t_1 = Q_1^2$ and $t_2 = Q_2^2$ small, by

$$D_2(p_a; p_1, p_2; p_b) \simeq |G(t_1)|^2 B_2(Q_a, Q_1; Q_2, Q_b)|G(t_2)|^2, \qquad (4.2)$$

$$D_2^{(0)}(p_a; p_1, p_2; p_b) \simeq |G(t_1)|^2 B_2^{(0)}(Q_a, Q_1; Q_2, Q_b)|G(t_2)|^2. \qquad (4.3)$$

The CGL equation corresponds to the choice that the correlation function is given by a "helicity-pole" contribution,[12] in terms of which Eq. (3.1) becomes

$$D_3(p_a; p_1, p_2, p_3; p_b) \simeq |G(t_1)|^2 H(Q_a, Q_1, Q_3) \times |\beta(t_1, \cos\phi_{13}, t_3)|^2 \times B_2(Q_1, Q_3; Q_2, Q_b)|G(t_2)|^2, \qquad (4.4a)$$

$$D_3(p_a; p_1, p_2, p_3; p_b) \simeq |G(t_1)|^2 B_2(Q_a, Q_1; Q_3, Q_2) \times |\beta(t_3, \cos\phi_{32}, t_2)|^2 \times H(Q_3, Q_2, Q_b)|G(t_2)|^2, \qquad (4.4b)$$

in regions A and B, respectively. In (4.4a), we have $Q_3 = p_a - p_1 - p_3$, $\cos\phi_{13} = \hat{Q}_1^\perp \cdot \hat{Q}_3^\perp$, and, in (4.4b), $Q_3 = p_b - p_2 - p_3$, $\cos\phi_{32} = \hat{Q}_3^\perp \cdot \hat{Q}_2^\perp$, and

$$H(Q_x, Q_y, Q_z) = [(Q_x - Q_z)^2/\mu^2]^{\alpha(Q_y^2)}. \qquad (4.5)$$

Since these two regions do not overlap, there are no difficulties in defining B_2 and in introducing Q_3. We next make the ansatz that the definition of B_2 can be extended to all values of its arguments analytically, and assume that this extension leads to a smooth function in the region C. We see that when substituting (4.4) back into Eq. (2.6) the θ-function cutoff can be removed by virtue of the rapid damping of the factor $|\beta|^2$. This will lead to an integral equation for B_2; and our ansatz can then be verified *a posteriori* upon solving this equation.

We now return to a four-vector integration and find[13]

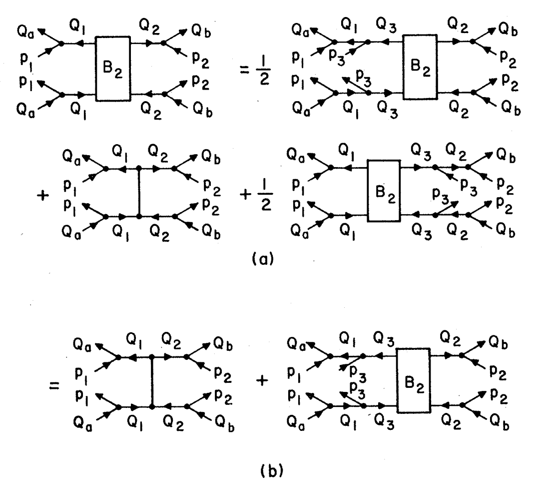

FIG. 1. (a) Schematic representation of a multiperipheral-like integral equation with two-particle correlations. (b) The same integral equation in the conventional "one-sided" form.

$$B_2(Q_a, Q_1; Q_2, Q_b) = \tfrac{1}{2}(2\pi)^{-3}\int d^4Q_3 H(Q_a, Q_1, Q_3)|\beta(t_1, \cos\phi_{13}, t_3)|^2\delta^+((Q_1-Q_3)^2-\mu^2)B_2(Q_1, Q_3; Q_2, Q_b)$$
$$+B_2^{(0)}(Q_a, Q_1; Q_2, Q_b)$$
$$+\tfrac{1}{2}(2\pi)^{-3}\int d^4Q_3 B_2(Q_a, Q_1; Q_3, Q_2)\delta^+((Q_2-Q_3)^2-\mu^2)|\beta(t_3, \cos\phi_{32}, t_2)|^2 H(Q_3, Q_2, Q_b), \quad (4.6)$$

where

$$B_2^{(0)}(Q_a, Q_1; Q_2, Q_b) = \pi H(Q_a, Q_1, Q_2)|\beta(t_1, \cos\phi_{12}, t_2)|^2\delta^+((Q_1+Q_2)^2-\mu^2)H(Q_1, Q_2, Q_b). \quad (4.7)$$

Equation (4.6) is a "two-sided" integral equation, schematically represented by Fig. 1(a). Either by symmetry or by direct iteration, one can show that those two integrals on the right-hand side of (4.6) are identically equal. We can thus rewrite (4.6) in a more conventional "one-sided" form [Fig. 1(b)]:

$$B_2(Q_a, Q_1; Q_2, Q_b) = B_2^{(0)}(Q_a, Q_1; Q_2, Q_b)$$
$$+(2\pi)^{-3}\int d^4Q_3 H(Q_a, Q_1, Q_3)|\beta(t_1, \cos\phi_{13}, t_3)|^2\delta^+((Q_1-Q_3)^2-\mu^2)B_2(Q_1, Q_3; Q_2, Q_b). \quad (4.8)$$

Aside from the fact that both Q_a and Q_b are continued off the mass shell along helicity poles, this is precisely the CGL integral equation.

Equation (4.8) possesses the key characteristics of all (forward) multiperipheral integral equations: The kernel $H|\beta|^2\delta^+$ is invariant under simultaneous Lorentz transformation of Q_a, Q_1, Q_3 (with Q_2 and Q_b fixed). Using the same reasoning as Amati, Bertocchi, Fubini, Stanghellini, and Tonin (ABFST) and CGL, and noting the symmetry of B_2, we may conclude that, as $s_{ab}, M^2 \to \infty$, B_2 is of the form

$$B_2 \sim (M^2)^{\alpha_V(0)} b_2(t_1, p_a\cdot Q_2/Q_1\cdot Q_2, p_b\cdot Q_1/Q_1\cdot Q_2, t_2), \quad (4.9)$$

where $\alpha_V(0)$ is the largest eigenvalue of the homogeneous equation. Experience also tells us that B_2 is a rapidly decreasing function of t_1 and t_2, thus justifying our initial ansatz. Furthermore, by applying the same analysis to (D_1, D_2) and (D_0, D_1), we may proceed to demonstrate the Regge behavior and the scaling property of exclusive and inclusive processes, respectively. Since this has been discussed in great length elsewhere,[5,14] we shall not pursue it here.

We close Sec. IV by deriving another integral equation for the forward "reduced" Reggeon-Reggeon absorptive part $\tilde{A}(Q_1, Q_2)$, which is directly related to the two-particle inclusive distribution in the di-triple Regge region.[15] For pedagogical reasons, we shall write the integral equation in terms of another (unreduced) function[4] $A(Q_1, Q_2)$ defined by

$$\tilde{A} = [(Q_a+Q_2)^2/\mu^2]^{-2\alpha(t_1)} B_2 [(Q_1+Q_b)^2/\mu^2]^{-2\alpha(t_2)}, \quad (4.10)$$

$$A = [(Q_1+Q_2)^2/\mu^2]^{2\alpha(t_1)+2\alpha(t_2)}\tilde{A}. \quad (4.11)$$

Starting from (4.8) and making the kinematic approximation

$$H(Q_a, Q_1, Q_3) = [(Q_a-Q_3)^2/\mu^2]^{2\alpha(t_1)} \simeq [(\kappa_{13}/\mu^2)(Q_a+Q_2)^2/(Q_1+Q_2)^2]^{2\alpha(t_1)}, \quad (4.12)$$

where $\kappa_{13} = \mu^2 + (\vec{Q}_1^\perp - \vec{Q}_3^\perp)^2$, a straightforward manipulation then yields [Fig. 2(a)]

$$A(Q_1; Q_2) = A^{(0)}(Q_1; Q_2)$$
$$+(2\pi)^{-3}\int d^4Q_3\,\delta^+((Q_1-Q_3)^2-\mu^2)|\beta(t_1, \cos\phi_{13}, t_3)|^2[(\kappa_{13}/\mu^2)(Q_1+Q_2)^2/(Q_3+Q_2)^2]^{2\alpha(t_1)}A(Q_3; Q_2), \quad (4.13)$$

with

$$A^{(0)}(Q_1; Q_2) = \pi|\beta(t_1, \cos\phi_{12}, t_2)|^2\delta^+((Q_1+Q_2)^2-\mu^2).$$

We can also write a two-sided equation for A to exhibit the symmetry of the problem [Fig. 2(b)]. A standard analysis[4] then shows that A has an asymptotic behavior[16]

$$g(t_1)[(Q_1+Q_2)^2]^{\alpha_V(0)}g(t_2),$$

as well as a lower term associated with the branch cut at

$$J = 2\alpha(0) - 1.$$

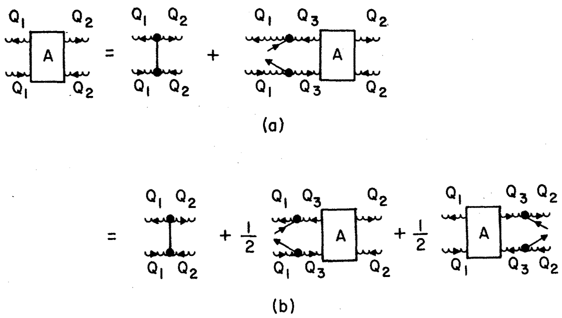

FIG. 2. (a) Integral equation for a Reggeon-Reggeon absorptive part. (b) The same integral equation in the "two-sided" form.

One immediate consequence of this result is that the interplay between this branch point and the output Regge pole demands the vanishing of the triple-Pomeranchon contribution to inclusive cross sections at the forward limit, if $\alpha_P(0)=1$.

V. DISCUSSION

To clarify further the advantage of our present approach, we briefly review the assumptions necessary for the formulation of the multiperipheral models. The key theoretical input is the unitarity relations, e.g.,

$$\operatorname{Im} T_{2,2}(p_a, p_b \to p_a, p_b) = \tfrac{1}{2}\sum_{n=2} \frac{1}{n!}\int d\Phi_n |T_{2,n}|^2, \tag{5.1}$$

where

$$d\Phi_n = (2\pi)^4 \delta^4\left(p_a + p_b - \sum_{i=1}^{n} q_i\right) \prod_{j=1}^{n}\left(\frac{d^4 q_j}{(2\pi)^3}\delta^+(q_j^2-\mu^2)\right), \tag{5.2}$$

and we assume all particles are identical. The importance of the production mechanisms in understanding the dynamics of two-particle amplitude has long been recognized; however, progress has been slow because of the difficulties of handling many-particle systems. Multiperipheralism is a scheme in which we rely heavily on the experimental information that the mean transverse momentum of any particle produced at high energy is small and nearly independent of the total energy. This provides a kinematic simplification because particles produced can now be ordered sequentially according to their longitudinal moments, and the amplitude $T_{2,n}$ in (5.1) for the reaction

$$a + b \to 1 + 2 + \cdots + n \tag{5.3}$$

is large only if momenta q_i's satisfy the conditions

(i) $|\vec{q}_i^{\perp}|^2$ small, for all $i = 1, 2, \ldots,$

so that there exists a permutation $\{\lambda_i\} = \{i\}$, and

$$\text{(ii)}\quad p_b^{\parallel} < q_{\lambda_1}^{\parallel} < q_{\lambda_2}^{\parallel} < \cdots < q_{\lambda_n}^{\parallel} < p_a^{\parallel}. \tag{5.4}$$

This phenomenon is often referred to as a strong ordering, and it is generally believed that this will allow us to make a meaningful approximation where production amplitudes are subdivided into products of functions, each one depending on only a small number of neighboring variables in (5.4). However, this procedure is not as trivial as it seems because (a) the integration in (5.1) covers the *whole* phase space, and (b) a single factorizable approximation is highly unrealistic. We shall explain the point (a) first.

The usual procedure (which has never been spelled out in published articles) is to first approximate

$$T_{2,n}(p_a + p_b \to q_1 + q_2 + \cdots + q_n) \simeq \sum_{\mathcal{P}(\{\lambda_i\})} T_{2,n}^{\text{s.o.}}(p_a; q_{\lambda_n}, q_{\lambda_{n-1}}, \ldots, q_{\lambda_1}; p_b), \tag{5.5}$$

where $T_{2,n}^{\text{s.o.}}$ is *large* only in the region defined by (5.4) and it is strongly *damped* once one moves out of this region. This is supposed to be accomplished by properly choosing the "cell" function in the factorizable approximation for $T_{2,n}^{\text{s.o.}}$, e.g., the rapid vanishing of Regge residues as the momentum-transfer variables become large and negative. $\mathcal{P}(\{\lambda_i\})$ represents the $n!$ possible orderings of $q_i^{\parallel}$'s; and "s.o." stands for strong ordering. Substituting (5.5) into (5.1) and using the fact that all particles are identical, we find

$$\operatorname{Im} T_{2,2} = \tfrac{1}{2}\sum_{n=2}^{\infty}\int d\Phi_n |T_{2,n}^{\text{s.o.}}|^2 + X, \tag{5.6}$$

$$X = \tfrac{1}{2}\sum_{n=2}^{\infty}\frac{1}{n!}\int d\Phi_n X_n, \tag{5.7}$$

where X_n is the sum of $n!(n!-1)$ cross-product terms. Clearly, if the term X can be ignored, an integral equation for $\operatorname{Im} T_{2,2}$ can then be constructed if a factorizable approximation is made for $|T_{2,n}^{\text{s.o.}}|^2$. This is usually assumed to be the case. However, the rapid increase of the number of terms in X_n makes this assumption somewhat dubious.

We would like to contrast the above procedure with our approximation (a). Although they apply to two slightly different functions, the physics involved is clearly related. In the ordinary approach, if an error has been committed, it is not clear where the source is because it could have been the result of the removal of X, or because (5.5) may be incorrect. To the extent that avoiding the details of production amplitudes is the essence of studying the inclusive processes, we find our approach much more direct and precise. We never have to talk about $T_{2,n}$, and approximations are made only once on physical observables. Furthermore, if it turns out that the contribution from the region C cannot be ignored, its magnitude can be obtained from experiments. With this knowledge on hand, it can then be grouped into the inhomogeneous term of the integral equation, and its effect can then be analyzed. This presumabably is the case if Pomeranchon is not a factorizable singularity. Since we are at the present only interested in the *structure* of our dynamical equations, we shall not discuss this question here.

The second point (b) is common to both approaches. One normally assumes $T_{2,n}^{s.o.}$ is given by the multi-Regge expansion appropriate for the limit (5.4). If only the leading trajectory is kept, one is naturally led to a single-channel problem, which is identical to our choice of keeping only one helicity pole. This turns out to be a bad approximation because the average adjacent subenergy $(q_{\lambda_i}+q_{\lambda_{i+1}})^2$ is never too large. The remedy clearly lies in either keeping several nonleading singularities, or making a more realistic factorization approximation involving more than two particles. One such attempt is the use of the ABFST model with a high-energy tail in the π-π amplitude.

As we have emphasized, the important fact to remember is that as long as a *meaningful factorization approximation* can be made, and as long as the contribution from the region C is *asymptotically* negligible, the "Regge" pole structure of our solution is then guaranteed. Modifications from either the region C or a more realistic factorization approximation will only change the numerical details of our results, but not the general features. (In terms of the partial-wave integral equation, these modifications can change the locations and the strengths of the J-plane singularities of the kernel, but they will not affect the Fredholm nature of the integral equation.) The usefulness of the inclusive sum rules does not merely lie in their ability to rederive multiperipheral-like integral equations but in the fact that they are exact relations that can be used to discuss dynamical questions such as in the pionization region. This and other related questions will be discussed elsewhere.

ACKNOWLEDGMENTS

I am grateful to R. Brower, C. DeTar, K. Kang, G. Veneziano, and J. Weis for interesting discussions.

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*Work supported in part by the U. S. Atomic Energy Commission (Report No. NYO-2262TA-254).

[1]T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1072 (1971); C. E. DeTar, D. Freedman, and G. Veneziano, Phys. Rev. D 4, 906 (1971).

[2]G. Veneziano, Phys. Letters 36B, 397 (1971).

[3]D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento 26, 896 (1962); L. Bertocchi, S. Fubini, and M. Tonin, *ibid.* 25, 626 (1962); G. F. Chew, M. L. Goldberger, and F. E. Low, Phys. Rev. Letters 22, 208 (1969); G. F. Chew and C. DeTar, Phys. Rev. 180, 1577 (1969); A. H. Mueller and I. J. Muzinich, Ann. Phys. (N.Y.) 57, 20 (1970); 57, 500 (1970).

[4]M. L. Goldberger, C.-I Tan, and J. M. Wang, Phys. Rev. 184, 1920 (1969); D. Silverman and C.-I Tan, Phys. Rev. D 1, 3479 (1970); S. Pinsky and W. I. Weisberger, *ibid.* 2, 1640 (1970).

[5]D. Silverman and C.-I Tan, Phys. Rev. D 3, 991 (1971); N. F. Bali, A. Pignotti, and D. Steele, *ibid.* 3, 1167 (1971).

[6]H. P. Stapp, Phys. Rev. D 3, 3177 (1971); C.-I Tan, *ibid.* 4, 2412 (1971).

[7]The single-particle-state term was inadvertently left out in Refs. 1 and 2.

[8]C.-I Tan, Phys. Rev. D 3, 790 (1971); C. DeTar, *ibid.* 3, 128 (1971); K. Wilson, Acta. Phys. Austr. 17, 37 (1963).

[9]C. Jen, K. Kang, P. Shen, and C.-I Tan, Phys. Rev. Letters 27, 754 (1971); Ann. Phys. (N.Y.) (to be published).

[10]In order to construct integral equations, it is necessary to allow D_2 to go off the mass shell in p_a and p_b. Our choice in Sec. IV will be to go off along helicity poles.

[11]Alternatively, we can add the contribution from C to the inhomogeneous term of (2.6). This approach can provide us with a technique for handling long-range correlations.

[12]C. DeTar, C. E. Jones, F. E. Low, C.-I Tan, J. H. Weis, and J. H. Young, Phys. Rev. Letters 26, 675 (1971).

[13]Consistent with the spirit of our approximations, the factor $(M^2-M'^2+\mu^2)/M^2$ has been set to unity. If it is left in the integrals, it will not change the "scaling" property of the kernel. However, it will lead to a modification on the lower-helicity-pole structure.

[14]Properties of two-particle distributions will be discussed using the conventional multiperipheral approach in a forthcoming paper (S.Y. Mak and C.-I Tan).

[15]C. Jen, K. Kang, P. Shen, and C.-I Tan, Phys. Rev. Letters 27, 754 (1971).

[16]$g(t)$ is proportional to the triple-Regge coupling. Since the "longitudinal" direction is implicit in the definition of A, it can in general depend on $\cos\phi_{12} \equiv \hat{Q}_1^{\perp}\cdot\hat{Q}_2^{\perp}$. This will be the case if the leading singularity is not a pole. [See F. E. Low, D. Freedman, C. E. Jones, and J. H. Young, Phys. Rev. Letters 26, 1197 (1971).]