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The Structure of Absorptive Regge Cuts in the πp Charge-Exchange Reaction*

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We analyze the $\pi^- p$ charge-exchange reaction above 5 GeV in a Regge-pole-plus-cut model. The absorption generating the cut is parametrized according to a geometrical picture in terms of a strength, radius, and diffuseness, and dispersion is introduced by a phase proportional to the absorption. The input Regge pole has no nonsense-wrong-signature zeros. We find the rescattering to be strongly energy-dependent and quite different in shape from elastic scattering near 6 GeV. However, as the energy increases to 18 GeV we find it approaching elastic scattering in shape and magnitude. The phase of the rescattering amplitude is crucial to understanding the structure of the polarization. At 6 GeV our scattering amplitudes, and, in particular, their pattern of zeros, agree with recent model-independent analyses, and differ from the results of most previous absorptive-model calculations. In the geometrical picture which emerges, the imaginary parts of the partial-wave amplitudes are peripheral, in accord with ideas proposed by Harari and others. Moreover, the real part of the helicity-flip amplitude is also peripheral, but the real part of the helicity-nonflip amplitude is not.

I. INTRODUCTION

We investigate the strength, energy dependence, and impact-parameter dependence of absorptive Regge cuts in a model in which the input Regge poles have no nonsense-wrong-signature zeros. Our results are based on a detailed analysis of the πp charge-exchange (CEX) reaction $\pi^- p \to \pi^0 n$.

Absorptive Regge cuts may be viewed as rescattering corrections. Their dominant effect is to reduce partial-wave amplitudes with small impact parameter, b, so that the resultant scattering is mainly peripheral. Several features of high-energy reactions find a convincing explanation in such a geometric picture, namely, the crossover of particle and antiparticle differential cross sections at $t=-0.2$ $(\text{GeV}/c)^2$ in πp, Kp, and pp elastic scattering, sharp forward peaks in reactions in which the π may be exchanged, and diffractive-like structures in various two-body reactions, both elastic and inelastic.[1,2] Additional support for this kind of description has been given by Harari who has used arguments based on duality to infer that quantum-number exchange occurs peripherally, at least in the imaginary parts of the scattering amplitudes.[2]

Elastic rescattering corrections alone do not provide strong enough absorption to account quantitatively for the data. Henyey, Kane, Pumplin, and Ross[1] included the contribution of inelastic diffractively produced intermediate states by using an effective rescattering amplitude

$$F_{\text{eff}} = F_{\text{el}} + F_{\text{inel}} = \lambda F_{\text{el}} , \tag{1}$$

where λ is a constant enhancement factor adjusted to fit the data. The resulting fits are in fair agreement with the data. However, as has been emphasized by Fox,[3] in general the model predicts too slow a shrinkage of the differential cross section in the region of the secondary maximum, where the Regge cut dominates. Stated another way, the effective Regge trajectory predicted, α_{eff}, is too flat. Another failing is that in $\pi^- p$ CEX scattering the position of the dip is predicted to move to smaller $|t|$ as the energy increases whereas the data seem to show it moving to larger $|t|$. In addition, recent measurements show that the polarization in $\pi^- p \to \pi^0 n$ remains positive out to $-t=1$ $(\text{GeV}/c)^2$, in disagreement with most fits of this type. It is a general feature of this prescription that the polarization changes sign around $-t=0.2$ to 0.4 $(\text{GeV}/c)^2$ and is large and negative near $-t=0.6$ $(\text{GeV}/c)^2$.

There is in fact no reason to believe that λ should be independent of t or s,[4] and the difficulties detailed above indicate that it is not. In this paper we study the effective rescattering in the process $\pi^- p \to \pi^0 n$. We use an optical model for the rescattering which is characterized by the radius of the interaction, the dimension (diffuseness) of the scattering boundary, and an effective strength (opacity) for the interaction. Dispersion is introduced through a phase proportional to the absorption. As we will show, the existence of a real part of the rescattering amplitude is essential to understanding the structure of the spin amplitudes. However, the data at high energies are insufficient to determine its detailed shape.

We find, by making quantitative fits to the $\pi^- p$ CEX differential cross section, polarization, and $\text{Im}F(t=0)$ data above 5 GeV/c, that this model can account for all these data, and that the parameters are well determined. The amplitudes we find are in agreement with the recent amplitude analysis of Halzen and Michael,[5] and, in particular, with the pattern of zeros they find. The data require a definite energy dependence for the rescattering parameters, namely, the radius decreases, the diffuseness increases, and the opacity is roughly constant as the energy increases. Moreover, this energy dependence is such that at 18 GeV we find $F_{\rm eff} \simeq F_{\rm el}$ implying that $F_{\rm inel}$ is vanishing with increasing energy. This puts in question the hypothesis that the rescattering is enhanced predominantly through diffractively produced intermediate states.

We formulate the model in Sec. II and present the results of our fits in Sec. III. We discuss the resulting amplitudes and compare them to the predictions of other models, as well as to the results of model-independent amplitude analysis in Sec. IV.

II. ABSORPTIVE REGGE CUTS

We work with s-channel helicity amplitudes $F_{\mu\mu'}(s, t)$, where μ and μ' are the helicities of the initial and final nucleons, respectively, and s and t are the usual Mandelstam variables. The measured quantities, differential cross section and polarization, are given by

$$\frac{d\sigma}{dt} = \frac{M}{16\pi s k^2}(|F_{++}|^2 + |F_{+-}|^2) \tag{2}$$

and

$$P = -\frac{2\,\mathrm{Im}(F_{++}F_{+-}^*)}{|F_{++}|^2 + |F_{+-}|^2}, \tag{3}$$

where M is the nucleon mass, and k the center-of-mass momentum. In addition, isotopic-spin invariance combined with the optical theorem relates the imaginary part of the helicity nonflip amplitude, F_{++}, at zero momentum transfer to the total cross sections for π^+p and π^-p scattering as follows:

$$\mathrm{Im}F_{++}(s, t=0) = \frac{k}{2\sqrt{2}\,\pi}[\sigma_T(\pi^+p) - \sigma_T(\pi^-p)]. \tag{4}$$

Our input Regge amplitude for the exchange of the ρ is constructed to have the following properties:

(i) Regge asymptotic behavior, i.e., $F \propto (s-u)^{\alpha(t)}$, with signature factor $(1 - e^{-i\pi\alpha})$,

(ii) no nonsense-wrong-signature zeros[6],

(iii) poles at the recurrences of the ρ, and

(iv) the residues at the ρ mass characterized by the well-known vector and tensor couplings.

The particular form we choose is

$$F^{\rho}_{\mu\mu'}(s, t) = \frac{1 - e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))\Gamma(\frac{1}{2}(\alpha+1))} \times \left(\frac{s-u}{2s_0}\right)^{\alpha(t)}\left(\frac{-t}{m_\rho^2}\right)^{|\mu-\mu'|/2} G_{\mu\mu'}, \tag{5a}$$

where we take the trajectory α to be linear and constrained to pass through the ρ,

$$\alpha(t) = 1 + \alpha'(t - m_\rho^2). \tag{5b}$$

The factor $\Gamma(\frac{1}{2}(\alpha+1))$ removes the nonsense-wrong-signature zeros. The angular-momentum-conserving factor has been taken out explicitly. Equating the residues of these amplitudes to the usual Born term gives the coefficients $G_{\mu\mu'}$ in terms of the $\rho\pi\pi$ and ρNN coupling constants,

$$\frac{2}{\pi\alpha'}\left(\frac{s - u(t=m_\rho^2)}{2s_0}\right)(-1)^{|\mu-\mu'|/2}G_{\mu\mu'} = (V_{\rho\pi\pi})_\nu (V_{\rho NN})^\nu, \tag{6}$$

where the $V_{\rho\pi\pi}$ and $V_{\rho NN}$, the conventional $\rho\pi\pi$ and ρNN couplings, are given by

$$(V_{\rho\pi\pi})^\nu = g_{\rho\pi\pi}(q+q')^\nu \tag{7}$$

and

$$(V_{\rho NN})^\nu = \bar{u}(p', \mu')\left((G_V + G_T)\gamma^\nu - \frac{G_T}{2M}(p+p')^\nu\right)u(p, \mu). \tag{8}$$

The q and q' are, respectively, the momenta of the initial and final pion, and $u(p, \mu)$ and $\bar{u}(p', \mu')$ are the helicity spinors for the initial and final nucleon. We fix $(g_{\rho\pi\pi})^2/4\pi = 2.1$ in our calculations, corresponding to a ρ width of 110 MeV. On the basis of ρ-meson dominance of the nucleon isovector form factor,[7] we expect $G_T/G_V = 3.7$ and $G_V^2/4\pi = 1.05$. The input Regge-pole amplitude which we use has four adjustable parameters: α', s_0, G_V, and G_T/G_V.

The Sopkovich prescription for applying elastic rescattering corrections to Regge-pole amplitudes consists of making the partial-wave expansion of the Regge-pole amplitude and then multiplying the partial-wave amplitudes by the square root of the S matrix for elastic scattering in the initial and in the final states.[8] If the elastic scattering is assumed the same for the initial and final states and if it is taken to be helicity nonflip, the total scattering amplitude can be written as

$$F_{\mu\mu'}(s, t) = \frac{1}{k}\sum_J (J+\tfrac{1}{2}) f^J_{\rm pole}(\mu, \mu') e^{2i\delta^J_{\rm el}} d^J_{\mu'\mu}(\theta), \tag{9}$$

where θ is the s-channel center-of-mass scattering angle, the $\delta^J_{\rm el}$ are the elastic phase shifts, and $f^J_{\rm pole}$ are the partial-wave projections of the Regge-pole amplitude. If we write $F_{\mu\mu'}$ as the sum of a Regge-pole and a Regge-cut term, the Regge-cut partial-wave amplitudes are

$$f^J_{\rm cut}(\mu, \mu') = f^J_{\rm pole}(\mu, \mu')(-1 + e^{2i\delta^J_{\rm el}}) \equiv -if^J_{\rm pole}(\mu, \mu')f^J_{\rm el}. \tag{10}$$

Since elastic scattering, $f_{\rm el}^J$, is predominantly imaginary, this cut amplitude interferes destructively with the Regge-pole amplitude.

In addition, a cut generated by rescattering through inelastic states should also be present, and different procedures have been used to take it into account. Henyey *et al.*[1] assumed that this inelastic contribution is proportional to the elastic amplitude so that in their model the cut is given by

$$f_{\rm cut}^J(\mu,\mu')=-i\lambda(\mu,\mu')f_{\rm pole}^J(\mu,\mu')f_{\rm el}^J\,, \tag{11}$$

where the $\lambda(\mu,\mu')$ are constants, independent of s, which are determined by fitting the data. The λ represents a phenomenological way of accounting for the enhanced absorption. Their assumption that λ can be taken constant, independent of s and J, is based in part on the conjectured dominance of inelastic states which can be reached by Pomeranchukon exchange from either the initial or final state. On the other hand, Roth and Renninger[1] used an energy-independent rescattering with phase, which corresponds roughly to using only a ρ-Pomeranchukon cut. Despite differences in their treatment of the rescattering, these and similar models suffer from the same difficulties. Typically, they find that the data require $\lambda\simeq 1.5-3$,[9] implying the diffraction-dissociation amplitude is in many cases equal to or greater than the elastic amplitude. The data on diffraction dissociation suggest that its contribution should be approximately 10–30% of the elastic rescattering, so use of λ's greater than 1.3 is perhaps not justified.[4,10]

Another difficulty in the model is that it predicts the wrong energy dependence. This becomes clear when one computes the effective trajectory assuming a single Regge-pole exchange so that

$$\frac{d\sigma}{dt}=\beta(t)s^{2\alpha_{\rm eff}(t)-2}\,. \tag{12}$$

The experimental $\alpha_{\rm eff}$ is approximately linear in the entire measured range of negative t and has no structure. It is, in fact, similar to the trajectory one assumes for a simple ρ Regge pole.[11] The $\alpha_{\rm eff}$ obtained from strong-absorption models agrees roughly with the data at small $|t|$ but has a bump in the region of the dip, $t\simeq -0.6$ (GeV/c)^2, and is too flat at larger $|t|$.[3] The bump is a consequence of the prediction, in these models, that the dip should move to smaller values of $|t|$, but this is not observed.

A third difficulty with these strong-absorption models is that they predict a zero in the polarization between $t=-0.2$ and -0.4 (GeV/c)^2 with a large negative polarization near $t=-0.5$ (GeV/c)^2. The new CERN polarization data[12] at 5 and 8 GeV/c indicate that the πp CEX polarization has no zero out to $t=-1$ (GeV/c)^2 at 5 GeV/c and is large and positive near $t=-0.5$ (GeV/c)^2 at both energies. These various difficulties suggest that rescattering corrections other than those given by elastic scattering or their simple multiplicative enhancement are present and are important at the energies at which these analyses have been done, namely, energies corresponding to laboratory momenta of 5–20 GeV/c.

In order to study the nature of the additional rescattering correction necessary, we introduce a simple optical model for the effective rescattering amplitude. We write

$$f_{\rm cut}^J(\mu,\mu')=-if_{\rm pole}^J(\mu,\mu')f_{\rm eff}^J\,, \tag{13}$$

where we have chosen $f_{\rm eff}^J$ to have the eikonal form[13]

$$f_{\rm eff}^J=(1-e^{i\chi(b)})/i\,, \tag{14}$$

where $b=(J+\frac{1}{2})/k$ is the impact parameter. We take the eikonal $\chi(b)$ to give absorption of the form

$$e^{-{\rm Im}\chi(b)}=\frac{1-c(1+e^{-r/d})}{1+e^{(b-r)/d}}\,. \tag{15}$$

This function is the familiar Fermi function which appears in nuclear physics[14] and has been used in high-energy physics by Dar, Watts, and Weisskopf.[15] This expression effectively parametrizes the absorption as that due to a distribution of absorbing material of radius r, edge dimension (diffuseness) d, and strength (opacity) c. The parameter c is normalized in such a way that $f_{\rm cut}^J=-cf_{\rm pole}^J$ at $b=0$ when the rescattering is purely absorptive. It is clear that unitarity requires c to be less than one.

We introduce dispersion through a phase proportional to the absorption,

$$\mathrm{Re}\chi(b)=a\,\frac{\mathrm{Im}\chi(b)}{\mathrm{Im}\chi(0)}\,. \tag{16}$$

We expect the rescattering to be predominantly absorptive in nature, as in elastic scattering. The dispersive part is, however, important in explaining the CEX polarization, as we discuss later. Since the polarization is not well known we do not expect the dispersive part to be well determined in detail. It turns out that a knowledge of the average phase of the rescattering is sufficient to understand the qualitative features of the polarization. Our model for generating phase is motivated mainly by the simplicity of the physical picture it gives.

We have made the simplifying assumption that $f_{\rm eff}^J$ does not flip helicity and that it is, in fact, independent of helicity. Thus $f_{\rm eff}^J$ depends on four parameters, r, d, c, and a, which may be energy-dependent. We have no model for the energy dependence of these parameters and assume no relation of these parameters to elastic scattering but rather determine them by fitting only the πp CEX data. As we discuss in Sec. III the πp CEX data are sufficient to determine quite well these parameters and their energy dependence. We remark that this type of optical potential can also give good fits to the elastic scattering, the differential cross section, the imaginary part of the forward amplitude, and the ratio of real to imaginary parts at $t=0$. In this way we can compare the absorption needed to explain the CEX reaction to the imaginary part of the elastic optical potential.

III. RESULTS

Using the form for the input Regge pole described in Sec. II, we fit the πp CEX data to determine the effective rescattering. As argued above, we expect the rescattering to have a different energy dependence from elastic scattering in order to remedy the failings of previous strong-cut absorption models. It is instructive to first look at the case where the absorption is energy-independent (our solution A). As we will show, this case has the qualitative features of previous strong-cut absorption-model calculations,[1] even though it is a different formulation. Here we find that the difficulty with the large negative polarization near $t=-0.5$ $(\mathrm{GeV}/c)^2$ predicted in previous calculations is resolved by our treatment of the phase. We then show that allowing the rescattering amplitude to vary smoothly with energy in strength and shape, our solution B, eliminates the remaining difficulties with previous strong-cut models. The absorption needed is different from what has been tried heretofore and we will discuss its probable source.

Before presenting our results we briefly describe the techniques we used in our data analysis. All partial-wave projections used to calculate the Regge-cut amplitudes were done numerically, with 15 terms kept in the expansion (the corrections due to the remaining terms were negligible even at 18 GeV/c). The exact partial-wave expansions in terms of the d^J functions were used. Best fits were found by using a modified quadratic-search method[16] to minimize χ^2.

The data on the reaction $\pi^- p \to \pi^0 n$ that we analyzed consisted of differential cross-section measurements at laboratory momenta of 6, 10, 13, and 18 GeV/c,[17] polarization measurements at 5, 6, 8, and 11 GeV/c,[12,18] and the imaginary part of the forward amplitude at these energies as determined from the total cross sections for $\pi^{\pm} p$ scattering.[19] In minimizing the χ^2, we normalized the data taking into account the quoted systematic errors.[20]

The fit to the differential cross section and polarization for solution A, energy-independent rescattering, is presented in Figs. 1 and 2. The quality of the fit is fair, corresponding to a χ^2 per degree of freedom of 3. The parameters of the fit are listed in Table I.

These results for the differential cross section are very similar to the fits of Henyey *et al.*[1] and also of Roth and Renninger.[1] The dip at $t=-0.6$ $(\mathrm{GeV}/c)^2$ is reproduced, as well as the turnover in forward direction which is characteristic of the dominance of the helicity-flip amplitude. The imaginary part of the helicity-nonflip amplitude has a zero at $t\simeq -0.2$ $(\mathrm{GeV}/c)^2$ which correctly gives the crossover of $\pi^{\pm} p$ elastic differential cross sections. The failures of the fit occur systematically for values of $|t|>0.4$ $(\mathrm{GeV}/c)^2$ and arise principally from having too slow an energy dependence there. The dip moves to smaller $|t|$ at high energy and is sharper than indicated by the data.

The similarity of the differential cross sections in these three calculations is somewhat surprising

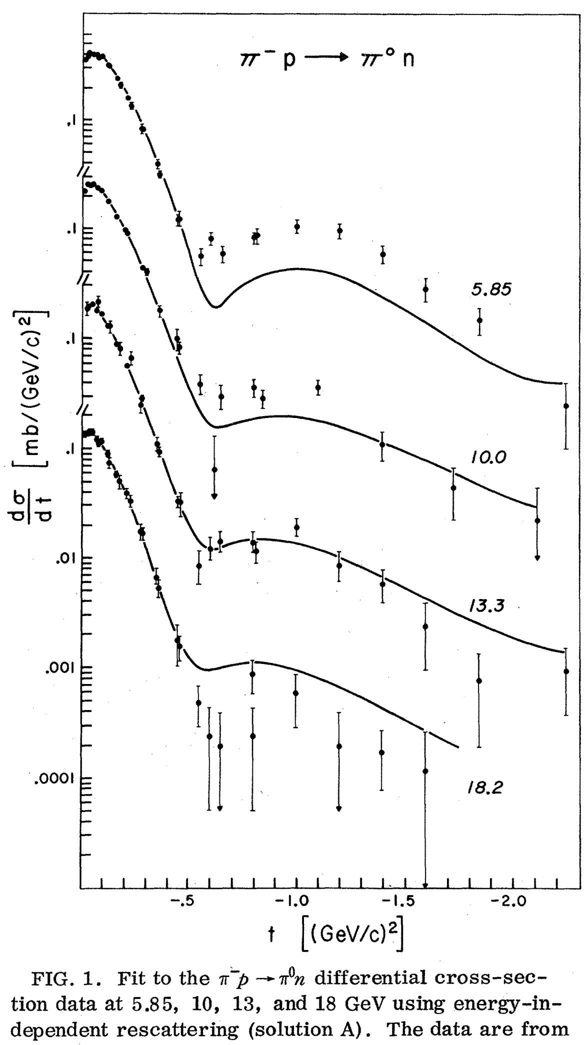

FIG. 1. Fit to the $\pi^- p \to \pi^0 n$ differential cross-section data at 5.85, 10, 13, and 18 GeV using energy-independent rescattering (solution A). The data are from Ref. 17 and the parameters are listed in Table I.

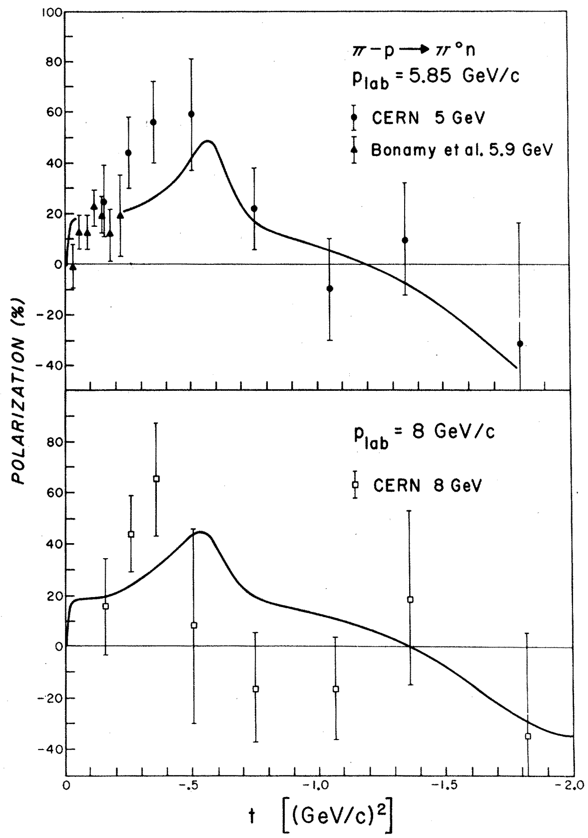

FIG. 2. Fit to the polarization data for solution A at 5–6 and 8 GeV. The data are from Refs. 11 and 18 and the parameters are given in Table I.

since the calculations differ in various details. Roth and Renninger used an energy-independent rescattering amplitude with constant phase and Gaussian shape whereas our solution has constant phase but corresponds to scattering off an absorbing disk of radius 0.65 F with very sharp edge. Henyey *et al.*[1] used a multiple of elastic scattering, with the energy dependence determined from the elastic scattering data and a constant phase determined from the elastic scattering phase at $t=0$.

Our fit to the polarization has the qualitative features found in the new CERN measurements,[12] i.e., it is positive out to $t=-1$ $(\mathrm{GeV}/c)^2$ with a maximum around $t=-0.5$ $(\mathrm{GeV}/c)^2$. This contrasts with the large negative polarization near $t=-0.5$ $(\mathrm{GeV}/c)^2$ predicted in the previous calculations, which were made before the new CERN measurements were available. The markedly different polarization can be traced to the value of the phase of the rescattering amplitude used in the different calculations. We find a constant phase of 30° (from the imaginary axis) for our rescattering amplitude compared with the 10° that Henyey *et al.*[1] use based on the forward

TABLE I. Parameters for best solution with energy-independent rescattering (A) and best over-all solution with energy-dependent rescattering (B). Estimated errors on parameters are also given.

Rescattering parameters	r_0 (F)	r_1 (F)	d_0 (F)	d_1 (F)	c_0	c_1	a	a_1
A	0.64 ± 0.015	...	0.001 ± 0.18	...	0.461 ± 0.032	...	−0.349 ± 0.018	...
B	0.590 ± 0.015	−0.143 ± 0.019	0.066 ± 0.006	0.211 ± 0.026	0.639 ± 0.026	0.0 ± 0.08	...	−2.09 ± 0.09

Regge parameters	α' $[(\mathrm{GeV}/c)^{-2}]$	s_0 $[(\mathrm{GeV}/c)^2]$	$G_V^2/4\pi$	G_T/G_V
A	0.915 ± 0.029	0.103 ± 0.017	4.46 ± 0.32	7.73 ± 0.19
B	0.910 ± 0.022	0.120 ± 0.018	3.69 ± 0.28	7.40 ± 0.15

elastic scattering data and the 10° that Roth and Renninger find in their fit. However, the phase of elastic scattering may vary with t and so, if a constant phase is used, this phase should be some average phase of elastic scattering, say for $|t| < 0.6$ $(\mathrm{GeV}/c)^2$. Barger and Phillips,[21] for example, found the phase on the nonflip elastic scattering amplitude to vary from 10° at $t=0$ to approximately 20° at $t=-0.2$ $(\mathrm{GeV}/c)^2$, 25° at $t=-0.45$ $(\mathrm{GeV}/c)^2$, and then decreasing again, giving an average phase over the range of $-t=0$ to 0.6 $(\mathrm{GeV}/c)^2$ of approximately 20°.

Note that for energy-independent rescattering the polarization is only weakly energy-dependent. We will discuss the polarization in greater detail in Sec. IV, where we will discuss the mechanism for removing the negative spike in the polarization and the possible energy dependence of the polarization in the region of the dip in the differential cross section.

The remaining difficulties of the strong-cut-model calculations have to do with the energy dependence of the rescattering. To determine what energy dependence is required by the data we allow the four parameters characterizing the rescattering to vary with energy. We parametrize the energy dependence of the radius, diffuseness, and opacity logarithmically, as follows:

$$r = r_0 + r_1 \ln(s/20), \qquad (17a)$$

$$d = d_0 + d_1 \ln(s/20), \qquad (17b)$$

and

$$c = c_0 + c_1 \ln(s/20). \qquad (17c)$$

For the range of energies considered here the constant term represents the central value, and the coefficient of the ln term the range, for each of these parameters. The phase parameter a is not well determined by the available data so we write for it

$$a = a_1/\sqrt{s}\,, \quad a_1 \text{ const}, \qquad (17d)$$

as we might expect if the real part of the rescattering is dominated by the P' trajectory.

The parameters found in the energy-dependent search are listed in Table I as solution B.[22] The fit to the differential cross section and polarization is shown in Figs. 3 and 4. It can be seen that the problems with the previous strong-cut-model fits have been dealt with: The differential cross section is well fit to $t=-2.25$ $(\mathrm{GeV}/c)^2$ at all energies, the polarization remains positive to $t=-1$ $(\mathrm{GeV}/c)^2$, and the dip no longer moves to smaller $|t|$ with increasing energy. The over-all χ^2 is 177 for 155 pieces of data and 11 search parameters (corresponding to a χ^2 per degree of freedom of 1.2), with the different types of data equally well fit.

The rescattering partial-wave amplitude f^J_{eff} that we find has the following behavior as the energy increases:

(i) Its diffuseness increases,

(ii) its radius decreases, and

(iii) its opacity (strength at $b=0$) remains constant.

The absorptive part of f^J_{eff} is responsible for the dip in differential cross section and is therefore well determined by the data. It is plotted in Fig. 5 as a function of impact parameter $b=(J+\frac{1}{2})/k$. The dots indicate the discrete J values which enter into the calculation. The dispersive part of the rescattering amplitude, $\mathrm{Re}f^J_{\mathrm{eff}}$, is determined mainly by the polarization in the vicinity of $t=-0.5$ $(\mathrm{GeV}/c)^2$, as discussed in connection with solution A, and is therefore much less well determined. In our model

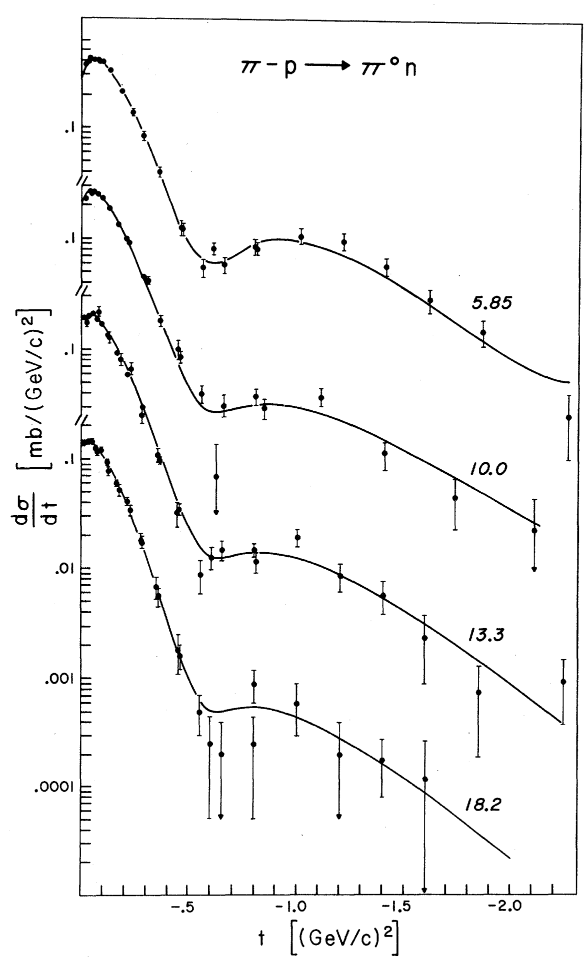

FIG. 3. Our best over-all fit to the $\pi^- p \to \pi^0 n$ differential cross section using energy-dependent rescattering (solution B). The parameters for the solution are listed in Table I.

$\mathrm{Re} f^J_{\mathrm{eff}}$ is similar in shape to $\mathrm{Im} f^J_{\mathrm{eff}}$ and much smaller in size. We find the phase of the rescattering amplitude $F(s, t)$ to be 27° at $t = 0$, 23° at $t = -0.3$ $(\mathrm{GeV}/c)^2$, and 18° at $t = -0.6$ $(\mathrm{GeV}/c)^2$ which corresponds to an average phase of approximately 23° from the imaginary axis, which is quite similar to the 20° found for elastic scattering by Barger and Phillips[20] in the same range of t, at $p_{\mathrm{lab}} = 6$ GeV/c. However, as we show below, the shape of f^J_{eff} at this energy is very different from elastic scattering.

In order to compare f^J_{eff} with f^J_{el}, we used our parametrization to fit elastic scattering. The data used were differential cross sections out to $t = -1$ $(\mathrm{GeV}/c)^2$, the total cross sections, and the ratio of the real to the imaginary part of the forward amplitude.[23] The P, P' exchanges were thus lumped into our optical potential and the helicity-flip amplitudes neglected. In Fig. 6 we compare the resulting f^J_{el} with f^J_{eff} at 5.85 and 18.2 GeV/c. We recall that according to the conjecture of Henyey, Kane, Pumplin, and Ross[1,24] rescattering is the sum of elastic and inelastic contributions,

$$f^J_{\mathrm{eff}} = f^J_{\mathrm{el}} + f^J_{\mathrm{inel}}\,, \tag{18}$$

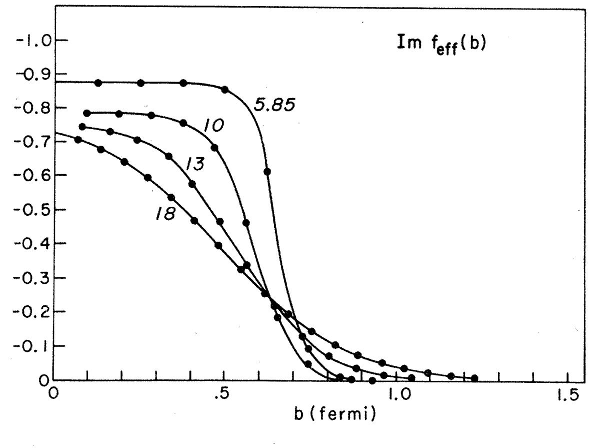

FIG. 5. The imaginary part of the effective rescattering amplitude, f^J_{eff}, corresponding to solution B at 5.85, 10, 13, and 18 GeV/c. The dots represent the discrete values of J $[b = (J+\frac{1}{2})/k]$ which enter into the partial-wave sum.

with f^J_{inel} proportional to f^J_{el}. In our result it can be seen that f^J_{inel} does not have the same shape as f^J_{el}, and furthermore disappears at higher energy. In fact, a simultaneous fit to elastic and CEX data at 18 GeV/c, with f^J_{eff} set equal to f^J_{el}, gave a χ^2 per degree of freedom of $\simeq 1$. We do not regard this simultaneous fit at one energy as conclusive since the errors in $d\sigma/dt$ for πp CEX reactions are large (and possibly overestimated) and since there are no polarization measurements for πp CEX reactions at that energy. However, the trends established by

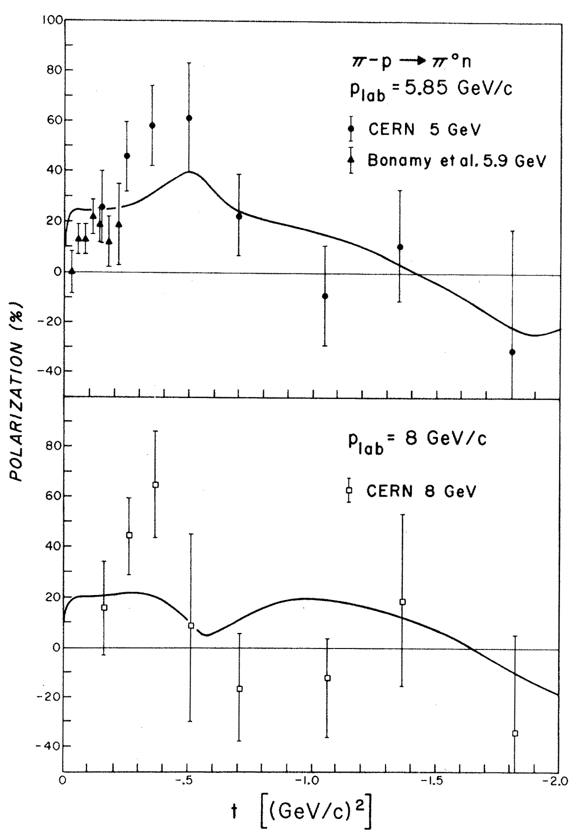

FIG. 4. Fit to the polarization data corresponding to solution B at 5–6 and 8 GeV. Parameters for the fit are given in Table I.

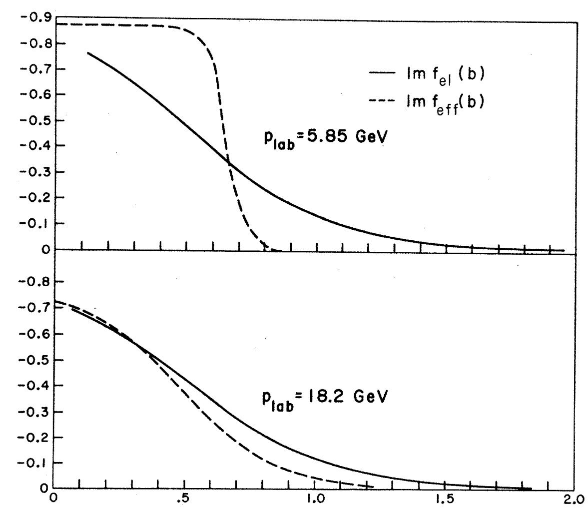

FIG. 6. The effective rescattering amplitude, f^J_{eff}, corresponding to solution B compared at 5.85 and 18.2 GeV/c with the elastic scattering amplitude f^J_{el} found by fitting elastic scattering with the same parametrization as used for f^J_{eff}.

the four-energy fit do suggest that at high energy "strong" cuts become "weak," in the sense that the corrections to elastic rescattering fall with energy so that only elastic rescattering remains. This conclusion has experimentally verifiable consequences, which we return to in Sec. IV after we have discussed the structure of the amplitudes that we find.

IV. DISCUSSION

In this section we first discuss the shape and energy dependence of the partial-wave projections of our scattering amplitudes. We compare our results with traditional absorption models and with the speculations concerning the structure of these partial-wave amplitudes put forth by Harari,[2] Dar,[25] Ross,[1,24] Chu and Hendry,[26] and their collaborators. We then discuss the structure of our scattering amplitudes as a function of momentum transfer and compare our results with the model-independent amplitude analysis at 6 GeV of Halzen and Michael.[5]

The real and imaginary parts of the partial-wave projections, $f^J_{+\pm}$, are shown in Fig. 7 as a function of the impact parameter b. We point out the following features:

(a) The imaginary parts of both flip and nonflip amplitudes are dominated by partial waves in a band centered about 0.8 F. The low partial waves are suppressed and it is essentially the most peripheral partial waves which contribute.

(b) The real part of the flip amplitude is also peripheral, in the sense described above, while the real part of the nonflip amplitude is not.

(c) The width of the band of b values from which the scattering amplitude gets significant contributions increases with energy.

In the various strong-cut or absorption models both the real and imaginary parts of the amplitudes are peripheral. In these models the rescattering is taken to be predominantly imaginary, and negative in our convention, so that the cut, given by

$$f^J_{\text{cut}} = -i f^J_{\text{pole}} f^J_{\text{el}}\,, \tag{19}$$

is 180° out of phase with the pole. Since f^J_{el} is large only for $b = (J+\frac{1}{2})/k$ less than some radius r, determined by the slope of elastic scattering, the cut cancels the pole for $b < r$ leaving the total, pole-plus-cut amplitude peripheral. If, however, f^J_{el} has a non-negligible real part, the phase of the cut will change. Explicitly, we have

$$\text{Re} f^J_{\text{cut}} = \text{Re} f^J_{\text{pole}}\,\text{Im} f^J_{\text{el}} + \text{Im} f^J_{\text{pole}}\,\text{Re} f^J_{\text{el}} \tag{20a}$$

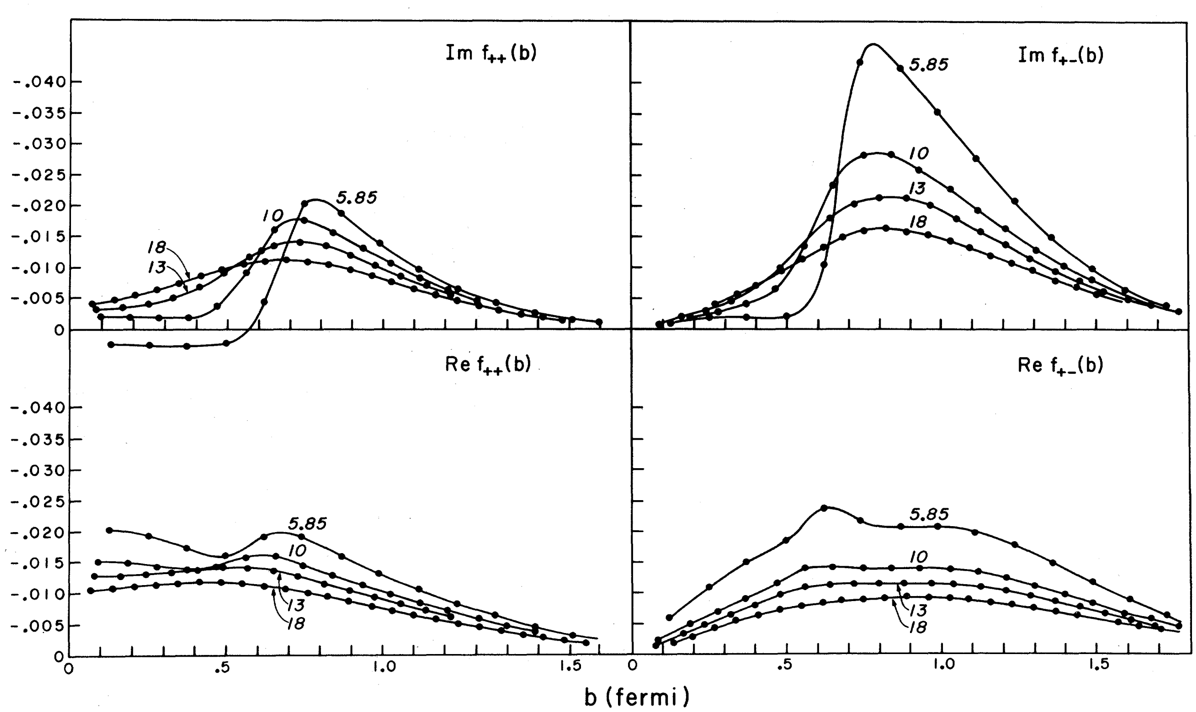

FIG. 7. The real and imaginary parts of the partial-wave projections of the scattering amplitudes $F_{++}(s,t)$ and $F_{+-}(s,t)$ corresponding to solution B. The dots represent the discrete values of J $[b=(J+\frac{1}{2})/k]$ which enter into the partial-wave sum.

and

$$\mathrm{Im} f^J_{\mathrm{cut}} = \mathrm{Im} f^J_{\mathrm{pole}} \, \mathrm{Im} f^J_{\mathrm{el}} - \mathrm{Re} f^J_{\mathrm{pole}} \, \mathrm{Re} f^J_{\mathrm{el}} \,. \qquad (20\mathrm{b})$$

The real and imaginary parts of the ρ Regge-pole partial-wave amplitudes have the same sign. Then, if $\mathrm{Re} f^J_{\mathrm{el}}/\mathrm{Im} f^J_{\mathrm{el}}$ is negative, the term proportional to $\mathrm{Re} f^J_{\mathrm{el}}$ enhances $\mathrm{Im} f^J_{\mathrm{cut}}$ and reduces $\mathrm{Re} f^J_{\mathrm{cut}}$. The sign of the ratio of the real to the imaginary parts of the effective rescattering amplitude is in fact negative, and, as we have seen, the effect of this is to strengthen the cut in the imaginary part and weaken it in the real part of the scattering amplitude. This explains why the imaginary parts are more peripheral than the real parts.

Note that the pole-plus-cut amplitude can have a sign opposite to the primitive pole amplitude, as occurs here in the imaginary part of the nonflip amplitude at 5.85 GeV/c. This is what Ross, Henyey, and Kane call "overabsorption," and this would be unphysical for a purely absorptive rescattering. In our case the overabsorption is just a reflection of the significant real part of the rescattering amplitude. Our amplitude, in fact, satisfies the unitarity bounds, so that the overabsorption is in no way unphysical.

Harari[2] has studied partial-wave structure from the standpoint of duality. He finds that duality leads to the prediction that the imaginary parts of the scattering amplitudes should be peripheral. Less firmly he finds that the real part of the flip amplitude is peripheral, while the real part of the nonflip amplitude need not be. Our results agree with all these predictions.

Dar and his co-workers[25] have considered an extension of the absorption model which assumes that the modulus of the partial-wave amplitudes is peripheral and that the phase of the partial-wave amplitudes can be determined using theorems relating the phase of an amplitude to its asymptotic energy dependence. Our results are in apparent contradiction with such a model. At 5.85 GeV/c the phase of the rescattering amplitude is well determined and we find that the real part of the nonflip partial-wave amplitudes is definitely not peripheral. However, at higher momenta, say above 13 GeV/c, sufficient polarization data do not exist to determine well the phase of the rescattering, and so in this region we cannot rule out that both the real and imaginary parts of the nonflip amplitude are peripheral.

Property (c) of the partial-wave amplitudes differs from the result of Dar, Watts, and Weisskopf[15] who found a radius which increases with energy. The difference arises from their use of elementary-particle exchange as input, while we use a Regge-pole amplitude, which has increasing radius.

In this work we have assumed πp CEX scattering is dominated by the exchange of a single trajectory, the ρ, together with rescattering corrections, and therefore our model is appropriate only at higher energies. There is evidence, however, that this geometrical picture continues to hold at lower energies. Chu and Hendry[26] have described πp elastic and CEX scattering between 2.7 and 5 GeV/c by parametrizing the partial-wave amplitudes as smooth functions of the impact parameter. The functional forms were chosen to account for speci-

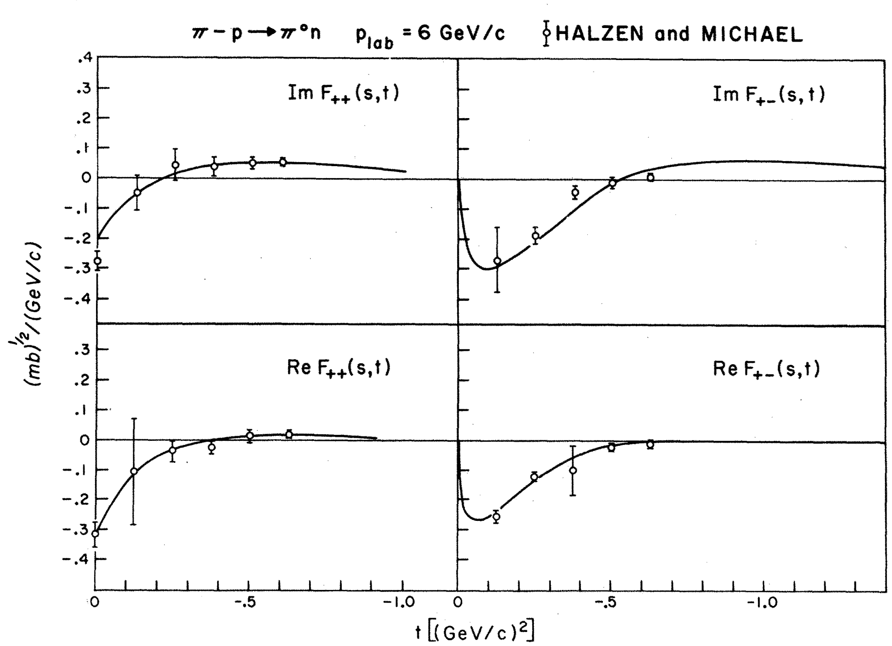

FIG. 8. The real and imaginary parts (Ref. 27) of the scattering amplitudes $F_{++}(s,t)$ and $F_{+-}(s,t)$ at 6 GeV/c compared with these quantities as determined by Halzen and Michael (Ref. 1) using a model-independent analysis based on differential cross-section and polarization data in $\pi^{\pm}p$ elastic scattering and $\pi^- p$ CEX scattering and on a recent measurement of R in $\pi^- p$ elastic scattering.

fic physical effects expected to be present. In particular, they included a term of the Breit-Wigner form centered about impact parameter b_c. They show that this term is responsible for the dips in the CEX differential cross section at $t=0$ and -0.6 $(\mathrm{GeV}/c)^2$ and also the one at $u=-0.2$ $(\mathrm{GeV}/c)^2$. They find $b_c = 0.8$ to 0.9 F and energy-independent in the range of energies considered, in agreement with our results based solely on high-energy data.

Recent measurements of the R parameter in $\pi^- p$ elastic scattering at $p_{\mathrm{lab}} = 6$ GeV/c have made it possible to reconstruct the helicity amplitudes in a model-independent way. In Fig. 8 we compare the amplitudes arrived at by our fit with the results of the analysis of Michael and Halzen.[27] The agreement is remarkably good, with the possible exception of $\mathrm{Im}F_{++}$ for $|t| < 0.1$ $(\mathrm{GeV}/c)^2$ where our analysis gives this amplitude smaller in magnitude and varying more slowly with t than apparently required by the data. The position of the zeros is of special importance. For example, the zero of the imaginary part of the nonflip amplitude near $t=-0.2$ $(\mathrm{GeV}/c)^2$ is responsible for the crossover of the $\pi^- p$ and $\pi^+ p$ elastic differential cross sections. If both real and imaginary parts of the nonflip amplitude were peripheral, they would both have a zero at $t=-0.2$ $(\mathrm{GeV}/c)^2$, with the result that the polarization would go through zero at this point. In fact the zero of the real part of the nonflip amplitude occurs at $t=-0.35$ $(\mathrm{GeV}/c)^2$ and the polarization stays positive. The predictions of a strong negative polarization at $t=-0.5$ $(\mathrm{GeV}/c)^2$ in previous calculations can thus be traced to insufficient separation of the position of the zeros. The mechanism which separates the zeros in our treatment is the presence of a substantial real part to the rescattering amplitude, opposite in sign to the imaginary part. As discussed above, this enhances the cut in the imaginary part, weakening it in the real part. In Sec. III we pointed out that the amount of real part we need is not inconsistent with the phase of elastic scattering, since it is some average phase which is important. In previous strong-cut or absorption models where the phase of the rescattering is ignored, or else its t dependence is not taken into account, the real and imaginary parts of the cut are comparable and the zeros of the real and imaginary parts are close together.

The energy dependence of our amplitudes also differs from that found in previous strong-cut calculations. As mentioned in our discussion in Sec. III our results suggest that the "strong" cut becomes "weak." In the region where the rescattering decreases rapidly with energy we expect the crossover zero to move to larger values of $|t|$. In fact, we find the zero in $\mathrm{Im}F_{++}$ moving from $t=-0.2$ $(\mathrm{GeV}/c)^2$ at 5.85 GeV/c to $t=-0.325$ $(\mathrm{GeV}/c)^2$ at 18 GeV/c. Similarly, the zero of $\mathrm{Re}F_{++}$ moves from $t=-0.35$ $(\mathrm{GeV}/c)^2$ to $t=-0.4$ $(\mathrm{GeV}/c)^2$ in the same energy interval. Ultimately, elastic rescattering dominates and the rescattering ceases to be strongly energy-dependent. Our results suggest that this occurs at approximately 20 GeV. Above that energy the zero must move to smaller $|t|$ as the energy increases further, since the ρ exchange amplitude and the ρ-Pomeranchukon-cut amplitude with which it interferes destructively to produce the zero both shrink, with the pole shrinking more rapidly. The energy dependence of the phase of the rescattering amplitude is not well determined at present because there is little polarization data above 6 GeV/c. Our result that the rescattering approaches elastic rescattering, however, leads to the interesting prediction that as elastic scattering becomes more nearly imaginary at high energies, we expect πp CEX polarization to develop a zero near the crossover zero and to become large and negative around $t=-0.5$ $(\mathrm{GeV}/c)^2$.

The amplitudes we have obtained are in accord with all experimental data, elastic and CEX, for $|t| \lesssim 0.6$ (GeV/c). It has been argued by Ringland and Phillips[28] that the strong-cut prescription must fail beyond that region. Their argument is that the strong-cut prescription leads to a simple zero in the real part of the $I_t = 1$ flip amplitude. Thus the Henyey *et al.*[1] strong-cut calculation, taken together with the Barger and Phillips[21] result for the $I_t = 0$ amplitudes [which include information on the phase of the amplitudes from finite-energy sum rules (FESR)] fails to reproduce the double zeros observed in the $\pi^\pm p$ elastic polarizations at $t \simeq -0.6$ $(\mathrm{GeV}/c)^2$. The situation is only slightly better for the $I_t = 1$ amplitudes which we obtain here, the polarizations being of the right sign but remaining too small beyond $|t| \simeq 0.8$ $(\mathrm{GeV}/c)^2$. However, we found that with our ρ-exchange amplitudes only a minor change in the phase of the Barger and Phillips amplitude, e.g., requiring the real part of the nonflip amplitude to go through zero at $t \simeq -0.85$ $(\mathrm{GeV}/c)^2$, is sufficient to give excellent agreement with experiment.[29] As Ringland and Phillips themselves point out, this sort of behavior is supported by their reevaluation of FESR integrals to higher energies.

V. CONCLUSIONS

Starting from the assumption that the structure of πp CEX reactions is due to rescattering (strong-cut model), we have determined the properties of the rescattering needed. Using an optical-potential form for the rescattering we have obtained a very good fit (χ^2 per degree of freedom of 1.2) to the

CEX data in a range of laboratory energies from 6 to 18 GeV. The amplitudes obtained are in excellent agreement with model-independent analysis at 6 GeV using the recent determination of the R parameter in $\pi^- p$ elastic scattering.

In previous strong-cut-model calculations the rescattering amplitude was assumed to have the same shape as elastic scattering. Our investigation shows that the rescattering is strongly energy-dependent and quite different in shape from elastic scattering near 6 GeV. However, as the energy increases to 18 GeV we find it approaching elastic scattering in shape and magnitude.

It would be interesting to understand the dynamics of the rescattering at lower energies. The fact that at high energy the rescattering approaches elastic scattering suggests that diffractive rescattering is not enhanced significantly. The existence of energy dependence suggests that Regge-Regge cuts play a dominant role in the enhancement of the rescattering at lower energies. The importance of these at intermediate energies has recently been pointed out by Harari and by Henyey, Kane, and Scanio in another context.[30]

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