tum electrodynamics seems to confirm the validity of these assumptions.

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$$W_3=\frac{-1}{2\pi}\,\mathrm{Im}\,H_1\,,\quad W_4=\frac{-1}{2\pi\nu}\,\mathrm{Im}\,H_2\,.$$

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Theoretical Difficulties with the Regge-Eikonal Model*

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Dynamical models are used to discuss the theoretical justification of the Regge-eikonal formula for scattering amplitudes. A generalized ladder-diagram model with Reggeon exchange replacing single-particle exchange yields the eikonal result only if the Reggeon amplitude has no mass-shell dependence, since the intermediate-state particles are far from their mass shells at high energy. This result is contrasted with a nonperturbative model of a Regge-pole amplitude which has strong mass-shell dependence. Next, a theory in which the coupling of the Reggeons to the external particles is nonplanar is examined. The double-Reggeon-exchange amplitude does not satisfy the eikonal representation. The perturbation-theory model of Cicuta and Sugar is shown to break down if higher-order corrections to the residue function are calculated. The conclusion of the paper is that there does not exist any theoretical justification for considering the Regge-eikonal model to be a valid description of high-energy scattering.

I. INTRODUCTION

In recent years it has become apparent that the simple model in which a few Regge poles dominate scattering amplitudes at high energies is incapable of adequately describing scattering data.[1] This failure to agree with experiment suggests either the Regge-pole model must be modified or the whole concept of Regge poles should be abandoned. Although the second alternative has its devotees, most proponents of Regge theory feel that the natural step is to introduce Regge cuts. Cuts in the angular momentum plane were first discussed almost ten years ago,[2] but not until the pure pole model failed did anyone take their importance seriously. There exist a number of phenomenological models which combine poles and cuts.[1] Most of them are successful in fitting the data. In this paper we concentrate on one of them, the Regge-eikonal model.[3] Not only is it simple to describe, but it appears to be a straightforward extrapolation of potential-theory results on high-energy scattering amplitudes. The question is whether potential theory can be trusted. It should be possible to develop relativistic models which lead to the Regge-eikonal picture. What are the limits of these models, if they exist?

The Regge-eikonal model states that at high energy the scattering amplitude takes on the form

$$A(s,t)=-is\int d^2b\, e^{i\vec{q}\cdot\vec{b}}(e^{i\chi(s,b)}-1)\,, \qquad (1)$$

where $q^2=-t$. By itself (1) says nothing; it is just another representation of the scattering amplitude. The crucial step in the Regge-eikonal model comes in identifying $\chi(s,b)$ with the Fourier transform of single-Reggeon exchange:

$$\chi(s,b)=\frac{1}{s}\int\frac{d^2k}{(2\pi)^2}e^{-i\vec{k}\cdot\vec{b}}\beta(k^2)s^{\alpha(k^2)}\,. \qquad (2)$$

The amplitude for single-Regge-pole exchange is $\beta(t)s^{\alpha(t)}$, where $\alpha(t)$ is the trajectory function. When the exponential in (1) is expanded in a power series, the first term corresponds to the exchange of a single Regge pole. The second term yields the Reggeon-Reggeon cut, while the third term contains the cut from triple-Reggeon exchange, and so forth. Equations (1) and (2) are similar to the eikonal approximation in potential theory where $\chi(s,b)$ is essentially the two-dimensional Fourier transform of the Born approximation.[4] According to the eikonal picture, the two particles scatter off each other many times, yet each retains its original identity and remains close to its mass shell. To be valid in the relativistic domain these conditions must be satisfied at high energy.

The Regge-eikonal model has certain attractive theoretical features. In most dynamical models of Regge trajectories, there is no restriction on the magnitude of $\alpha(k^2=0)$. If $\alpha(0)>1$, single-Regge-pole exchange violates the Froissart bound. However, as Chang and Yan[5] have shown, the Regge-eikonal amplitude satisfies the Froissart bound, whatever the magnitude of $\alpha(0)$. Secondly, the Regge-eikonal model forces approximate s-channel unitarity on t-channel pole exchange and, in the process, provides a unique prescription for introducing the accompanying Regge cuts. Unfortunately, in spite of these attractive features, theoretical justification of the model is based mainly on analogy with potential theory, and potential theory has proved to be a notoriously poor guide to the relativistic version of the complex angular momentum plane. The Schrödinger equation does not incorporate multiparticle unitarity or crossing and does not yield Regge cuts or daughter poles. Moreover, the usefulness of potential theory at high energy is questionable.

What is the theoretical justification of the relativistic version of the Regge-eikonal model? Note that this is not the same as asking whether certain field theories eikonalize. The fact that some field theories eikonalize certainly suggest that there is no basic incompatibility between relativity theory and the eikonal representation. Moreover, the reason why some theories do not eikonalize may be related to the difficulty of justifying the Regge-eikonal model.[6] Chang and Yan give a heuristic derivation of the model in a ϕ^3 theory, while Cicuta and Sugar[7] obtain the Regge-eikonal form in the same theory by carefully summing diagrams containing nested ladders. Their work, however, is strictly a weak-coupling, leading-logarithm calculation. Muzinich, Tiktopoulos, and Treiman[8] have shown that in certain situations the eikonal formula does not emerge, at least not in the form envisaged by Chang and Yan.[5] Cheng and Wu[9] and Islam[10] have discussed a nonplanar diagram consisting of Regge-pole and single-particle exchange. They find that the eikonal approximation fails in this case.

Theoretical models have to be subjected to two tests: Does the model sum to the eikonal form, and does the eikonal sum actually represent the high-energy limit of the model?

In our study of these questions we use off-mass-shell techniques. When the high-energy particles which undergo the scattering occur in intermediate states, they are represented by Feynman propagators. This approach is consistent both with other attempts to justify the Regge-eikonal form and with potential-theory derivations of the eikonal approximation. Our conclusion is that there does not exist at this time any theoretical justification for considering the Regge-eikonal model to be an accurate description of the mechanism of high-energy scattering. This result is based upon the investigation of two general theoretical models which can be forced to yield the Regge-eikonal formula, but only with the imposition of certain artificial constraints which are believed to be incorrect. We are thus in accord with other investigations on the same subject,[8-10] though our reasons are different and our results are more general. The Regge-eikonal model requires that at high energy either the scattering particles stay close to their mass shells or there be no mass-shell dependence in off-mass-shell scattering amplitudes.

The first condition is not met by any of the theoretical models, and the second is unlikely to be true of strongly interacting, composite hadrons. The model also requires that fragmentation of the incident particles be neglected at high energies. In other words, processes in which the incident particles break up and transfer a sizable fraction of their energy to other particles must be unimportant. Just as fragmentation is suppressed in quantum electrodynamics,[6] it can be suppressed in the Regge-eikonal model by a proper choice of the Regge-pole amplitude to be eikonalized.

The first class of models we consider follows very closely the ideas developed by Arnold[3] in his original paper on the Regge-eikonal model. Two particles interact in all possible ways through Regge-pole exchange. This is just the generalized ladder-diagram model of ϕ^3 theory with Regge-pole exchange replacing elementary-particle exchange. Since the residue function $\beta(t)$ for single-Reggeon exchange is expected to depend on the mass of the scattering particles, this dependence is included in the models. In fact, nonperturbative, field-theoretic models of Regge poles lead to just such a behavior. The eikonal result in field-theory models turns out to be crucially dependent on the fact that the only mass-shell dependence is in the particle propagators. This result is particularly evident in a detailed study of double-Reggeon exchange.

The second class of models we consider looks for the Regge-eikonal amplitude to emerge from a summation of nested amplitudes along the lines proposed by Chang and Yan[5] and developed in detail by Cicuta and Sugar.[7] The Cicuta-Sugar[7] result depends on having a ϕ^3 theory so that direct and exchanged particles are identical. The fragmentation graphs are then equivalent to the direct graphs. Moreover, their perturbative calculation breaks down if higher-order corrections to the residue functions are calculated. For double-Reggeon exchange, the eikonal formula predicts an amplitude of the form

$$A_2(s,t)=\frac{i}{2(2\pi)^2}\int d^2k\,\beta(k^2)\beta((q-k)^2)s^{\alpha(k^2)+\alpha((q-k)^2)-1}, \tag{3}$$

where $q^2=-t$. On the other hand, we find that double-Reggeon exchange with nonplanar coupling does not lead to amplitudes of the form given by (3), but rather we obtain

$$A_2(s,t)=\int d^2k\,F(k^2,q^2,(q-k)^2)s^{\alpha(k^2)+\alpha((q-k)^2)-1}. \tag{4}$$

Equation (4) differs from (3) by the lack of factorization of the integrand. This factorization is a crucial step in the summation procedures necessary to produce the eikonal result.

In Sec. II we develop generalized ladder-diagram models with Regge-pole exchange. Particular emphasis is placed on the conditions necessary for the eikonal amplitude to emerge. These requirements are compared with the results of a nonperturbative field-theoretical calculation of the full off-mass-shell scattering amplitude for single-Regge-pole exchange. Section III is concerned with models wherein the Regge-eikonal formula is supposed to emerge from the nonplanar coupling of Regge poles to the scattering particles. Here we concentrate on double-Regge-pole exchange and examine the conditions under which the factorization predicted by the eikonal amplitude in (3) actually appears. The ϕ^3 perturbation-theory model of the Regge-eikonal amplitude is examined in detail and its limitations discussed. The Appendixes contain calculations too involved to include in the text of the paper.

II. GENERALIZED LADDER DIAGRAMS WITH REGGEON EXCHANGE

The eikonal model arises in quantum electrodynamics (QED) as the high-energy limit of the sum of a restricted class of diagrams in electron-electron scattering.[11] One such class, known as generalized ladder diagrams, is made up of all diagrams in which a photon emitted from one electron line is absorbed by the other. The resulting eikonal function, $\chi(s,b)$ [see Eq. (1)], is the two-dimensional Fourier transform of the high-energy limit of single-photon exchange. This result suggests that the Regge-eikonal amplitude might be obtained from a sum of generalized ladder diagrams where the Nth-order term in the sum contains all possible permutations of N-Reggeon exchanges. If the analogy to QED is valid, the eikonal function will be the transform of the single-Reggeon-exchange amplitude with all four legs on the mass shell. Let us construct such a model and see if the eikonal result emerges.

The off-mass-shell scattering amplitude for the reaction $p_1+p_2\to p_1'+p_2'$, where all particles are spinless, is a function of six variables: $s=(p_1+p_2)^2$, $t=(p_1-p_1')^2$, and the four squared masses p_1^2, p_2^2, $p_1'^2$, $p_2'^2$. In the large-s domain the amplitude for single-Reggeon exchange, Fig. 1, has the form

$$F(p_1^2,p_2^2,p_1'^2,p_2'^2;s,t)=\beta(p_1^2,p_1'^2,t)\beta(p_2^2,p_2'^2,t)Z(t)s^{\alpha(t)}. \tag{5}$$

Signature and pole factors are lumped into $Z(t)$, and the factorization of the residue function is ex-

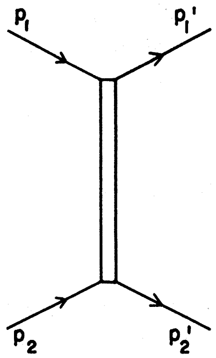

FIG. 1. The single-Reggeon-exchange amplitude.

plicitly displayed. As a first step we calculate the large-s limit of the two second-order diagrams in Fig. 2 and compare the result with the eikonal prediction of Eq. (3). The calculation is outlined in Appendix A, and we present the answer here. The high-energy limit of the box diagram (A) is

$$F_A(s,t)=c\int d^{2-4N}k[\beta(\infty,m^2,k^2)\beta(\infty,m^2,(q-k)^2)]^2 \times Z(k^2)Z((q-k)^2)s^{\alpha(k^2)+\alpha((q-k)^2)-2N-1}\ln s\,, \tag{6}$$

while the crossed graph (B) is given by

$$F_B(s,t)=(-1)^{2N+1}F_A(s,t)\,. \tag{7}$$

We have assumed that, as $p_1{}^2\to\infty$, $\beta(p_1{}^2,m^2,k^2)\to\beta(\infty,m^2,k^2)/p_1{}^{2N}$. N need not be an integer. The momentum transfer $t=-q^2=(p_1-p_1')^2$. Equation (6) contains an integral over $2-4N$ dimensions. Continuous dimensional integration has been introduced and discussed elsewhere.[12] It is sufficient for our purposes here to note that the integral can be given a definite meaning.

The important feature of (6) is that although the

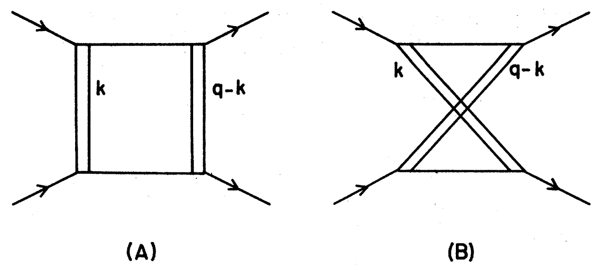

FIG. 2. The two double-Reggeon-exchange diagrams with the direction of the loop momentum indicated.

integral factorizes in the manner required by the Regge-eikonal model, the asymptotic behavior of the amplitude depends on the residue function $\beta(p_1{}^2,p_1'^2,t)$ far from the mass shell. In other words, the particles in the intermediate states in Fig. 2 are far from their mass shells, and the whole rationale for the Regge-eikonal model comes into question. A specific difficulty with the amplitude in (6) is that the branch point for the Regge cut is displaced by $2N$ to the left from $2\alpha(0)-1$, the generally accepted position.[2] Finally we note that the leading behavior of each diagram is of the form $s^{\alpha_c}\ln s$. The logarithm does not cancel between the two diagrams unless N is an integer. Thus, we find that this generalized ladder-diagram model cannot lead to the Regge-eikonal model unless the residue functions are assumed to have no mass-shell dependence. On the other hand, we show below that residue functions should have a strong mass-shell dependence.

Another way to arrive at these same conclusions is to attempt to calculate the asymptotic behavior of the box and crossed diagrams by means of the standard approximation of dropping terms quadratic in the loop momentum.[13] Then we find

$$F_A+F_B=\int d^4k s^{\alpha(k^2)+\alpha((q-k)^2)}[\beta(k^2)]^2Z(k^2)[\beta((q-k)^2)]^2Z((q-k)^2) \times\left(\frac{1}{(-2p_1\cdot k)^{2N+1}}\frac{1}{(2p_2\cdot k)^{2N+1}}+\frac{1}{(-2p_1\cdot k)^{2N+1}}\frac{1}{(-2p_2'\cdot k)^{2N+1}}\right). \tag{8}$$

For simplicity we have assumed

$$\beta(p^2,m^2,t)=\beta(t)/(p^2-m^2)^{2N}\,.$$

The reader can easily convince himself that the difference between $N=0$ and $N\neq 0$ is significant when it comes to combining denominators to obtain the factorized form characteristic of field theories. On the other hand, if $N=0$, the only mass-shell dependence of the generalized ladder diagram is in the propagators, and this model can be analyzed and summed just like QED.

If we arbitrarily set $F(p_1{}^2,p_2{}^2,p_1'^2,p_2'^2;s,t)$ equal to its value when all legs are on the mass shell and use the resulting amplitude as the Born approximation to a generalized ladder-diagram series, the Regge-eikonal model will emerge. We emphasize that this restriction to the mass shell, although it agrees with the intuitive picture behind the eikonal model, is an artificial one; it should emerge from the dynamics rather than be used as input. The ladder sum can be performed in many ways; we omit the details.[14] The answer is just (1) with

$$\chi(s, b) = \int \frac{d^2k}{(2\pi)^2} e^{-i\vec{k}\cdot\vec{b}} [\beta(m^2, m^2, k^2)]^2 Z(k^2) s^{\alpha(k^2)-1}. \tag{9}$$

Although (1) and (9) have the desired Regge-eikonal form, we must check to make sure that they represent the true high-energy behavior of the set of diagrams. Treiman and Tiktopoulos[6] have shown that, although the corresponding expression in QED is valid, the eikonal amplitude in a ϕ^3 theory does not represent the correct asymptotic behavior. The basic problem concerns the importance of fragmentation processes in which the energy is transmitted across the rungs of the ladder rather than only along the sides. A standard analysis of the high-energy behavior of Feynman diagrams shows that the eikonal formula with (9) is correct if either (a) $\alpha(0) \geq 1$ or (b) $\beta(t)$ vanishes faster than any power of t as $|t| \to \infty$. In case (a) the summation procedure succeeds for the same reason it does in QED.[6] The eikonal path is enhanced relative to fragmentation paths by a power of s at each vertex. Moreover, the Nth term in the expansion of the exponential in (1) is of the same order or larger than the first term so that the series makes sense. In case (b), the contributions of noneikonal paths to the asymptotic s behavior vanish faster than any power of s due to the behavior of the residue functions. On the other hand, eikonal paths lead to a power-law behavior in s.

We have here a dynamical model of the Regge-eikonal amplitude, if mass-shell effects are neglected and either condition (a) or (b) is satisfied. What do theoretical models of Regge poles say about the reasonableness of these restrictions?

The simplest theoretical model of a Regge pole is an infinite sum of planar ladder diagrams.[15] However, a perturbation-theory treatment of the model is not adequate for a discussion of the behavior of residue functions. Recent work[12] on nonperturbative approximations to the partial-wave Bethe-Salpeter equation can be used to obtain representations of the off-mass Regge pole amplitude which have reasonable properties. Solutions to the partial-wave equation, when combined with a Sommerfeld-Watson transformation, yield the following amplitude for single-Reggeon exchange[16]:

$$F(p_1^2, p_2^2, p_1'^2, p_2'^2; s, t) = -\frac{1}{2i} \frac{\Gamma(-\alpha)}{G_\alpha'(t)} \frac{K_\alpha(p^2) K_\alpha(p'^2)}{K_\alpha(0)} s^\alpha, \tag{10}$$

where $\alpha = \alpha(t)$, $4p^2 = (p_1 + p_1')^2 = 2(p_1^2 + p_1'^2) - 2t$, and $4p'^2 = (p_2 + p_2')^2$. The Regge trajectory $\alpha(t)$ is generated by unitarizing a fundamental interaction $K(p^2)$ in the ladder-approximation Bethe-Salpeter equation. For example, in a ϕ^3 theory $K(p^2) = g^2(p^2 + \lambda^2)^{-1}$. The modified kernel, $K_\alpha(p^2)$, which appears in (10) is defined by

$$K_\alpha(p^2) = \frac{1}{\Gamma(-\alpha)} \int_0^\infty y^{-\alpha-1} K(y + p^2)\, dy. \tag{11}$$

For single-particle exchange in a ϕ^3 theory

$$K_\alpha(p^2) = g^2 \frac{\Gamma(\alpha+1)}{(p^2 + \lambda^2)^{\alpha+1}}.$$

Regge trajectories are given, in this approximation, by solutions to the equation $1 = G(\alpha(t), t)$,[12] and

$$G_\alpha'(t) = \frac{\partial}{\partial l} G(l, t) \bigg|_{l=\alpha(t)}.$$

The important feature of the amplitude in (10) is its explicit dependence on the masses of the external particles. In perturbation theory the residue function is a constant in leading order.

If $K(p^2)$ represents single-particle exchange, we find that as $s \to \infty$, the Regge behavior $s^{\alpha(k^2)}$ is exactly canceled by the mass-shell dependence of the residue functions and the second-order amplitude does not contain moving Regge cuts. This result is not surprising since planar diagrams with planar ladder exchange do not have Regge cuts. If, instead of single-particle exchange, we consider a theory in which the fundamental particles making up the Regge pole interact through a kernel $K(p^2)$ which vanishes faster than any power as $p^2 \to \infty$, we would find that $K_\alpha(p^2)$ vanishes faster than any power and the double-Reggeon amplitude would not have power behavior as $s \to \infty$. Such a kernel is interesting because it generates trajectories which have $\alpha(-\infty) = -\infty$. In any case, the representation (10) for a Regge-pole amplitude suggests that it is unreasonable to use the mass-shell limit of (5) in an off-mass-shell calculation. The single-Regge-pole-exchange amplitude should have a strong dependence on the effective masses of the external particles.

The conclusion of this section is that the eikonal form of the scattering amplitude emerges from a sum of generalized ladder diagrams with Regge-pole exchange replacing particle exchange only if the fundamental exchange amplitude is independent of the effective masses of the scattering particles. Although the eikonal form is obtained with this restriction, the model does not provide theoretical justification of the Regge-eikonal formula for scattering amplitudes. Rather it provides a demonstration of the close connection between mass-shell behavior and the eikonal representation. Moreover, an analysis of a nonperturbative model of a Regge pole shows that the assumption is unwarranted. However, the generalized ladder-diagram model is not the broadest possible model. In a field-theory sense, diagram (A) in Fig. 2 is planar and should not contain cuts. The nested ladder-diagram model of Cicuta and Sugar[7] is not of the type considered here. For this reason, Sec. III is devoted to an analysis of diagrams containing nested Regge poles with nonplanar couplings. Conceivably the pinch analysis necessary to evaluate the asymptotic behavior of nonplanar diagrams could result in a suppression of the mass-shell dependence of the residue functions.

III. MULTI-REGGEON EXCHANGE WITH NONPLANAR COUPLING

Although in Sec. II it was shown that generalized ladder diagrams do not sum to the eikonal result, there still remains the question of what happens when the coupling of Regge poles to external particles is nonplanar. Regge cuts emerge from an analysis of a different type of singularity. The ϕ^3 model with nested ladders[5,7] is of this type, and it leads to the Regge-eikonal formula. Although perturbation theory was used in that calculation, we wish to avoid perturbation theory here. To this end we write down the amplitude for a general, nonplanar, double-Reggeon-exchange diagram and analyze its high-energy behavior. The graph under consideration is shown in Fig. 3.[17] Again we assume the off-mass-shell Reggeon-exchange amplitude has the form given by (5). The limit as $s\to\infty$ of the nonplanar diagram in Fig. 3 is

$$F(s,t)=c\int d^2k\, s^{\alpha(k^2)+\alpha((k-q)^2)-1}[G((k-q)^2,k^2,q^2)]^2\,. \tag{12}$$

The details of the calculation are given in Appendix B. Unlike the amplitude generated by double-Reggeon exchange in Sec. II, this amplitude has the correct asymptotic behavior. In fact, (12) is just the impact-factor representation of the scattering amplitude discussed by Cheng and Wu.[18] Whether

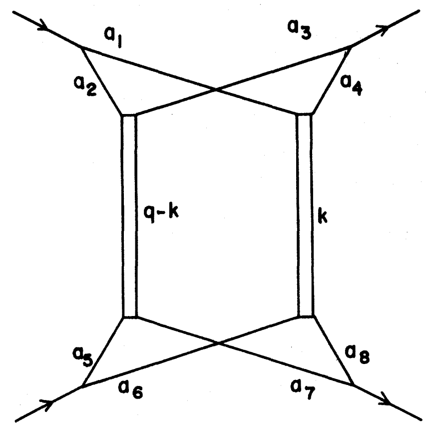

FIG. 3. The double-Reggeon-exchange amplitude with nonplanar coupling to the external a particles. The single-particle lines are labeled by the corresponding Feynman parameters.

(12) constitutes the second term in an eikonal expansion depends on whether $G((k-q)^2,k^2,q^2)=g(k^2)g((k-q)^2)$. To answer this question we must look at the detailed structure of $G((k-q)^2,k^2,q^2)$. First, however, we note that if the Regge-eikonal model must be based on a theory of nested Reggeon exchanges, the attractive physical picture behind the model is undermined. Which of the internal lines in Fig. 3 is to be identified with the external particles? Which line carries most of the energy? There is no way to decide, and the idea that the incident particle passes through the scattering process without losing its identity is questionable. Thus, if the eikonal model can be recovered from (12), it will be at the expense of the intuitive ideas which originally motivated it.

For a nonplanar model to eikonalize, the function $G((k-q)^2,k^2,q^2)$ must factorize into a product of a function of k^2 and a function of $(k-q)^2$. [See Eq. (3).] The explicit functional form of $G((k-q)^2,k^2,q^2)$ is

$$G=\int_0^\infty \frac{da_1da_2da_3da_4dx_1dx_2dx_1'dx_2'}{\Sigma}e^{iQ}\delta(\bar a_1\bar a_4-\bar a_2\bar a_3)\,\rho(x_1,x_2,(k-q)^2)\rho(x_1',x_2',k^2)\left(\frac{\bar a_4}{\bar a_3+\bar a_4}\right)^{\alpha(k^2)}\left(\frac{\bar a_1}{\bar a_1+\bar a_2}\right)^{\alpha((k-q)^2)}, \tag{13}$$

where

$$Q=k^2\frac{\bar a_1\bar a_3}{\bar a_1+\bar a_3}+(k-q)^2\frac{\bar a_2\bar a_4}{\bar a_2+\bar a_4}-q^2\frac{\bar a_1\bar a_4}{\Sigma}+p_1{}^2\frac{(\bar a_1+\bar a_3)(\bar a_2+\bar a_4)}{\Sigma}-m^2(a_1+a_2+a_3+a_4)\,,$$

$$\bar a_1=a_1+x_1',\quad \bar a_3=a_3+x_2',\quad \bar a_2=a_2+x_1,\quad \bar a_4=a_4+x_2\,,$$

and

$$\Sigma=\bar a_1+\bar a_2+\bar a_3+\bar a_4\,.$$

The δ function in the integrand of (13) has been used to rearrange the factors in Q. Almost by inspection it is possible to conclude that (13) does not factorize, since the integrand does not factorize into the form

$\bar{g}(a_1, a_3, x_1, x_2)\times\bar{g}(a_2, a_4, x_1', x_2')$. Although the residue function $\rho(x_1', x_2', k^2)$ and the coefficient of k^2 in Q depend on the same set of variables a_1, a_3, x_1', x_2', as they should if the integral is to factorize, neither the coefficients of q^2 and $p_1^2=p_1'^2$ in Q nor the term $[\bar{a}_4/(\bar{a}_3+\bar{a}_4)]^{\alpha(k^2)}=[\bar{a}_2/(\bar{a}_1+\bar{a}_2)]^{\alpha(k^2)}$ factorizes correctly. Moreover, the δ function in the integrand of (13) prevents factorization. The fact that $G((k-q)^2, k^2, q^2)$ does not separate is independent of the mass-shell dependence of the residue functions. However, in the absence of any mass-shell dependence, the result can be made more explicitly. The four-dimensional integral in (12) can be reduced to a single integral.

$$G=4\rho(k^2)\rho((k-q)^2)\int_0^1 dz\,\frac{z^{\alpha(k^2)}(1-z)^{\alpha((k-q)^2)}}{(A^2+4AB)^{1/2}}\ln\left(\frac{A+(A^2+4AB)^{1/2}}{2(AB)^{1/2}}\right), \tag{14}$$

where

$$A=k^2z+(k-q)^2(1-z)-q^2z(1-z), \quad B=p_1^2z(1-z)-m^2+i\epsilon.$$

Clearly double-Reggeon exchange with nonplanar couplings does not lead to an amplitude which corresponds to the appropriate term in the expansion of an eikonal amplitude.[19] On the other hand, the ϕ^3 model with nested ladders is of the form considered here, and it leads to an eikonal amplitude.

The answer to this paradox lies in the difference between inserting ladder diagrams that have already been summed into Fig. 3 and treating nonplanar graphs with finite ladders and summing the result. In a ϕ^3 theory fixed poles in the angular momentum plane are summed to give a moving pole. The residues of these fixed poles receive contributions both from the interior ladder diagrams and from the crossed lines coupling the ladders to external particles. To see how the Regge-eikonal result emerges in a ϕ^3 perturbation-theory calculation, we present a quick derivation of this model. Starting with (B5) of Appendix B, we find the Mellin transform of an amplitude containing the exchange of ladders with N and M rungs is given by

$$F_{NM}(\alpha,t)=\frac{2\pi i}{\alpha+1}(g^2)^{N+M+2}c\int\frac{d\tau_N d\tau_M'\prod_{i=1}^{8}da_i}{\Delta_N^2\Delta_M'^2\Sigma_1\Sigma_2}\int d^2k\,\delta(\bar{a}_1\bar{a}_4-\bar{a}_2\bar{a}_3)\delta(\bar{a}_5\bar{a}_8-\bar{a}_6\bar{a}_7)\frac{f^{\alpha+1}e^{iQ}}{(\bar{a}_3+\bar{a}_4)^{\alpha+1}(\bar{a}_7+\bar{a}_8)^{\alpha+1}}, \tag{15}$$

where $f=\bar{a}_3\bar{a}_8f_N+\bar{a}_4\bar{a}_7f_M$, $\Sigma_1=\sum_{i=1}^4\bar{a}_i$, $\Sigma_2=\sum_{i=5}^8\bar{a}_i$, and we have used (B8) to convert part of the Feynman-parameter integral back to an integral over the two-dimensional loop momentum k. The precise form of Q is unimportant at this stage; Δ_N and Δ_M' are the determinants of the N- and M-rung ladders and $d\tau_N$ and $d\tau_M'$ represent the integrals over the Feynman parameters for the two ladders. An N-rung ladder diagram with its legs off the mass shell, such as shown in Fig. 3, has the integral representation

$$F_N=c'(g^2)^N\int\frac{d\tau_N e^{i\psi_N}}{\Delta_N^2}, \tag{16}$$

where

$$\psi_N=A_N+p_1^2x_1B_N+p_1'^2x_1C_N+p_2^2x_ND_N+p_2'^2x_NE_N+(p_1+p_2)^2f_N+(p_1-p_2')^2G_N.$$

The only important function is $\Delta_Nf_N=x_1x_2\cdots x_N$. The other functions A_N, B_N, etc. do not vanish if one of the $x_i=0$. The representation of F_N in (16) has the same form as that developed in Appendix A for the general Regge-pole amplitude in (5). Given ψ_N, we see from Fig. 4 and the definition of $\bar{a}_1$ used in Appendix B that

$$\begin{aligned}\bar{a}_1&=a_1+x_1'B_M', & \bar{a}_5&=a_5+x_ND_N,\\ \bar{a}_2&=a_2+x_1B_N, & \bar{a}_6&=a_6+x_M'D_M',\\ \bar{a}_3&=a_3+x_1'C_M', & \bar{a}_7&=a_7+x_NE_N,\\ \bar{a}_4&=a_4+x_1C_N, & \bar{a}_8&=a_8+x_M'E_M'.\end{aligned} \tag{17}$$

Since u and u' in Appendix B are the conjugate variables to the momentum transfer across the amplitudes, they are identified with f_N and f_M, respectively.

The poles of $F_{NM}(\alpha,t)$ arise from those regions of Feynman-parameter space where the integral representation diverges. These regions are determined by the function

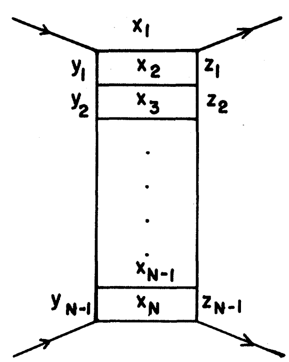

FIG. 4. An N-rung ladder diagram with its Feynman parameters explicitly labeled.

$$f=\frac{x_1\cdots x_N}{\Delta_N}(a_3+x_1'C_M')(a_8+x_M'E_M')+\frac{x_1'\cdots x_M'}{\Delta_M'}(a_4+x_1C_N)(a_7+x_NE_N), \tag{18}$$

and include not only $x_1=x_2=\cdots=x_N=x_1'=\cdots x_M'=0$ but also four disjoint regions of the integral over the a_i parameters. These regions are

(a) $a_1=a_3=a_5=a_7=0$;

(b) $a_1=a_3=a_6=a_8=0$;

(c) $a_2=a_4=a_5=a_7=0$;

(d) $a_2=a_4=a_6=a_8=0$.

For example, near $\alpha=-3$ the contribution of region (a) is given by

$$F_{NM}(\alpha,t)=-\pi ic(g^2)^{N+M+2}\int\frac{d^2k\,d\tau\,d\tau'}{\Delta_N{}^2\Delta_M'^2}da_2da_3da_4da_6da_7da_8\frac{a_4a_8e^{iQ}}{(a_2+a_4)(a_6+a_8)}$$
$$\times\left(\frac{x_1\cdots x_N}{\Delta_N}a_8(a_3+x_1'C_M')+\frac{x_1'\cdots x_M'}{\Delta_M'}a_4(a_7+x_NE_N)\right)^{\alpha+1}. \tag{19}$$

The δ functions in (15) were used to eliminate the a_1 and a_5 integrations. Next, if $a_3=\rho_1(1-y_1)$, $x_1'=\rho_1y_1$, $a_7=\rho_N(1-y_N)$, and $x_N=\rho_ny_N$, we find that (19) becomes

$$F_{NM}(\alpha,t)=c'(g^2)^{N+M+2}\int d^2k\int_0^\infty dx_1\cdots dx_{N-1}dx_2'\cdots dx_M'$$
$$\times\int_0^\infty\rho_1{}^{\alpha+2}\rho_N{}^{\alpha+2}d\rho_1d\rho_N\int_0^1dy_1dy_N\frac{a_4a_8e^{iQ}}{(a_2+a_4)(a_6+a_8)\Delta_N{}^2\Delta_M'^2}$$
$$\times\left(\frac{x_1\cdots x_{N-1}}{\Delta_N}y_Na_8(1-y_1+y_1C_M')+\frac{y_1x_2'\cdots x_M'}{\Delta_M'}a_4(1-y_N+y_NE_N)\right)^{\alpha+1}. \tag{20}$$

Poles of $F_{NM}(\alpha,t)$ at $\alpha=-3$ come from the $\rho_1,\rho_N,x_1,\ldots,x_{N-1},x_2',\ldots,x_M',y_1,y_N$ integrations. Thus, we set these variables equal to zero wherever they occur and use the integral

$$\int_0^\epsilon(Ax_1\cdots x_N+By_1\cdots y_M)^{\alpha+1}dx_1\cdots dx_Ndy_1\cdots dy_M\approx\frac{1}{(\alpha+3)^{N+M-1}}\frac{\Gamma(N+M-1)}{\Gamma(N)\Gamma(M)}\frac{1}{AB} \tag{21}$$

to obtain

$$F_{NM}(\alpha,t)=c'(g^2)^{N+M+2}\int d^2k\frac{(N+M-2)!}{(N-1)!\,(M-1)!}\frac{1}{(\alpha+3)^{N+M+1}}\int\frac{da_2da_4}{a_2+a_4}\int\frac{da_6da_8}{a_6+a_8}\int d\tau_Nd\tau_M'\frac{e^{iQ}}{\Delta_N\Delta_M'}. \tag{22}$$

When $x_i=x_j'=a_1=a_3=a_5=a_7=0$, the exchanged ladders are fully contracted so that the determinants Δ_N and Δ_M' factorize,

$$\Delta_N=\prod_{i=1}^{N-1}(y_i+z_i),\quad\Delta_M'=\prod_{i=1}^{M-1}(y_i'+z_i'),$$

and Q takes on the form

$$Q=k^2\left(\sum_{i=1}^{M-1}\frac{y_i'z_i'}{y_i'+z_i'}+\frac{a_6a_8}{a_6+a_8}\right)-m^2\left(\sum_{i=1}^{M-1}(y_i'+z_i')+(a_6+a_8)\right)+(k-q)^2\left(\sum_{i=1}^{N-1}\frac{y_iz_i}{y_i+z_i}+\frac{a_2a_4}{a_2+a_4}\right)-m^2\left(\sum_{i=1}^{N-1}(y_i+z_i)+(a_2+a_4)\right). \tag{23}$$

Note that only in a ϕ^3 theory will all the masses in Q be identical. In general, the masses from the ladders differ from those which arise from intermediate states in the nonplanar coupling scheme. If $K(s)$ is defined by

$$K(s)=g^2\int_0^\infty\frac{dydz}{y+z}\exp\left\{i\left[s\frac{yz}{y+z}-m^2(y+z)\right]\right\}, \tag{24}$$

the final form of $F_{NM}(\alpha,t)$ is

$$F^a_{NM}(\alpha,t)=c'g^2\int d^2k\,\frac{(N+M-2)!}{(N-1)!(M-1)!}\,\frac{[K(k^2)]^M[K((q-k)^2)]^N}{(\alpha+3)^{N+M+1}}. \tag{25}$$

The superscript a indicates that (25) is the contribution of region (a). Regions (b), (c), and (d) are similarly evaluated. The four terms are added together and the result summed over all N and M to give

$$F(\alpha,t)=c'g^2\int d^2k\sum_{N=1}^{\infty}\sum_{M=1}^{\infty}\frac{(N+M-2)!}{(N-1)!(M-1)!}\,\frac{[K(k^2)]^{M-1}[K((k-q)^2)]^{N-1}}{(\alpha+3)^{N+M+1}}[K(k^2)+K((q-k)^2)]^2$$

$$=c'g^2\int d^2k\left\{\frac{1}{\alpha+3-K(k^2)-K((k-q)^2)}-\frac{1}{(\alpha+3)^2}[\alpha+3+K(k^2)+K((k-q)^2)]\right\}. \tag{26}$$

Keeping just the cut term and going from the Mellin transform of the scattering amplitude to the amplitude itself, we find

$$F(s,t)=c''g^2\int d^2k\,s^{\alpha(k^2)+\alpha((k-q)^2)-1}, \tag{27}$$

where

$$\alpha(k^2)=-1+K(k^2).$$

Certainly (27) has the Regge-eikonal structure. The residue function $\beta(k^2)$ is a constant.

The reason for going into some of the details of this perturbation-theory model is to be able to discuss the limitations of the calculation. The model breaks down if (i) higher-order terms are systematically calculated, (ii) the basic interaction which binds particles in the ladders to a Regge pole is modified, (iii) the particles on the sides of the ladders are not identical to those that occur as intermediate states in the coupling of the Regge poles to the external particles. The most serious failure of the model is the fact that it breaks down in higher order. Higher-order corrections in the Mellin-transform approach to perturbation theory are obtained from summing the residues of less singular terms. (In more conventional words, higher-order corrections are down by powers of $\ln s$ from the leading term in each order of the coupling constant.) The first correction to (25) comes from the coefficient of $(\alpha+3)^{-N-M}$. A complete calculation of this term is very difficult. However, it suffices for our purposes to note that at least one contribution to the residue of $(\alpha+3)^{-N-M}$ comes from a region of Feynman-parameter space where none of the parameters a_1, a_2, a_3, a_4 are zero. If these parameters are nonzero, the model becomes equivalent to those considered at the beginning of this section. The integral does not factorize into a function of k^2 and $(k-q)^2$ as required by the Regge-eikonal formula. In other words, the first-order correction to the residue function of the Regge cut in (27) does not have the factorization properties required for the eikonal representation.

Although single-particle-exchange models of Regge poles are conceptually simple, since they are based on a conventional field theory, recent investigations[12,20] have shown that more realistic trajectories are obtained when other interactions are substituted for single-particle exchange. If ladder diagrams with such interactions are used here, there will be a mismatch between the poles from the ladders and those from the nonplanar coupling. For example, if the single-particle propagator $(q^2+\lambda^2)^{-1}$ is modified by a form factor $F(q^2)$ which vanishes like q^{-2N} as $q^2\to\infty$, the ladder diagrams sum to trajectories which have $\alpha(t=-\infty)=-N-1$. A calculation of the Regge cut such as that done above requires summation of poles at $\alpha=-2N-3$. However, the singularities which contract the lines coupling the ladder diagrams to the external particles occur at $\alpha=-N-3$. Perturbation theory breaks down since the leading singularity at $\alpha=-N-3$ does not sum to a Regge cut. A nonperturbative calculation would give a cut to the right of this point at the expense of the eikonal formula.

The final point to notice about the perturbation-theory calculation is that if the particles on the sides of the ladder diagrams have masses different from those of the particles coupling the ladders to the external particles, then (26) is replaced by

$$F(\alpha,t)=c'\int d^2k\sum_{N=1}^{\infty}\sum_{M=1}^{\infty}\frac{[K_1(k^2)]^{M-1}[K_1((k-q)^2)]^{N-1}}{(\alpha+3)^{N+M+1}}\,\frac{(N+M-2)!}{(N-1)!(M-1)!}[K_2(k^2)+K_2((k-q)^2)]^2$$

$$=c'\int\frac{d^2k[K_2(k^2)+K_2((k-q)^2)]^2}{(\alpha+3)^2[\alpha+3-K_1(k^2)-K_1((k-q)^2)]}. \tag{28}$$

$K_1(k^2)$ and $K_2(k^2)$ are different functions due to the different masses involved. The residue of the pole in (28) does not factorize into a product of a function of k^2 and a function of $(k-q)^2$. The Regge-eikonal form of the scattering amplitude could not be derived in a $\psi^*\psi\phi$ field theory, even in perturbation theory.

We conclude this section and the paper with the disappointing result that there is no satisfactory theoretical justification of the Regge-eikonal formula for high-energy scattering amplitudes.[21] Double-Regge-pole exchange with nonplanar coupling fails because the resulting amplitude does not show the correct factorization properties. The basic reason for this is that there is no well-defined path along which the energy flows from incoming to outgoing particle. The only theoretical calculation of the Regge-eikonal formula, that of Cicuta and Sugar,[7] is too limited to be used as justification for the model. The most that can be said is that the scattering amplitude satisfies an impact-parameter representation[18] at high energy.

ACKNOWLEDGMENT

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APPENDIX A

The first step in evaluating the asymptotic behavior of the amplitudes represented by Fig. 2 is the assumption that the residue functions in (5) have the following representation:

$$\beta(p_1^2,p_2^2,t)=\int_0^\infty dx\,dy\,\rho(x,y,t)\,e^{ix(p_1^2+i\epsilon)}\,e^{iy(p_2^2+i\epsilon)}. \tag{A1}$$

Such a representation exists if a dispersion relation in the variables p_1^2, p_2^2 exists. If in the limit $p_1^2\to\infty$, $\beta(p_1^2,p_2^2,t)\to p_1^{-2N}\bar\beta(\infty,p_2^2,t)$, then $\rho(x,y,t)\to x^{N-1}\bar\rho(0,y,t)$ as $x\to 0$. In fact,

$$\bar\beta(\infty,p^2,t)=\Gamma(N)i^N\int_0^\infty \bar\rho(0,y,t)\,e^{iy(p^2+i\epsilon)}. \tag{A2}$$

Next, we write

$$\begin{aligned}\rho(x,y,t)\rho(z,w,t)Z(t)s^{\alpha(t)}&\\ =\frac{1}{2\pi i}\int_C d\beta\int_0^\infty \frac{u^\beta du\, e^{iu(s+i\epsilon)}}{\beta+\alpha(t)+1}\,\frac{Z(t)\rho(x,y,t)\rho(z,w,t)}{\Gamma(1+\beta)(i)^\beta}&\\ =\frac{1}{2\pi i}\int_C d\beta\int_0^\infty u^\beta du\int_0^\infty dv\,H(x,y,z,w,\beta,v)&\\ \times e^{i(s+i\epsilon)u}\,e^{iv(t+i\epsilon)}.&\end{aligned} \tag{A3}$$

The contour C encloses the path traced out by the Regge trajectory in the complex angular momentum plane. The detailed nature of the t transform in (A3) is unimportant. Equations (A1) and (A3), together with the identity

$$\frac{1}{a+i\epsilon}=\frac{1}{i}\int_0^\infty e^{ix(a+i\epsilon)}dx,$$

allow us to write the Feynman integral corresponding to each diagram in Fig. 2 in the form

$$F(s,t)=c\int d^4k\,d\tau\,d\tau'\,dp\,dq\,u^\beta H(\tau)u'^{\beta'}H(\tau')\times\exp\{i[Q+(k-b)^2\Delta]\}, \tag{A4}$$

where $Q=A+(fs+gt)/\Delta$, $d\tau=dx\,dy\,dz\,dw\,du\,dv\,d\beta$, and p and q are the parameters corresponding to the particle propagators. The k integration in (A4) is easily performed to yield

$$F(s,t)=c'\int\frac{d\tau\,d\tau'\,dp\,dq}{\Delta^2}u^\beta H(\tau)u'^{\beta'}H(\tau')\,e^{iQ}. \tag{A5}$$

The asymptotic behavior of $F(s,t)$ is most easily calculated by taking the Mellin transform of (A5) with respect to s and looking for the leading singularity in the α plane.

$$\begin{aligned}F(\alpha,s)&=\int_0^\infty s^{-\alpha-1}F(s,t)\\ &=\Gamma(-\alpha)i^\alpha c'\int\frac{d\tau\,d\tau'\,dp\,dq}{\Delta^{\alpha+2}}e^{iQ}f^\alpha\\ &\quad\times u^\beta H(\tau)u'^{\beta'}H(\tau').\end{aligned} \tag{A6}$$

For diagram (A), the box diagram,

$$\begin{aligned}&\Delta_A=\bar p+\bar q+v+v',\\ &f_A=(u+u')\Delta_A+\bar p\bar q,\\ &\bar p=z+p+x',\quad \bar q=w+q+y',\end{aligned} \tag{A7}$$

while for the crossed diagram

$$\begin{aligned}&\Delta_B=\bar p+\bar q+v+v'+u+u',\\ &f_B=(u+u')(v+v')-\bar p\bar q,\\ &\bar p=z+p+x',\quad \bar q=w'+q+y.\end{aligned} \tag{A8}$$

The leading singularity in the α plane for both diagrams comes from the region where u, u', $\bar p$, $\bar q$ are small. It is assumed that the residue functions $\beta(p_1^2,p_2^2,t)$ vanish as function of t fast enough to make the v, v' integrations unimportant for the crossed graph. $H(x,y,z,w,\beta,v)$ is replaced by its value in the limit $z,w\to 0$:

$$H(\tau)\underset{z,w\to 0}{\sim}(wz)^{N-1}\bar H(\tau).$$

Taking a similar limit for $H(\tau')$ with respect to x',y' and then changing variables to $p=\bar p p$, $z=\bar p z$, $x'=\bar p(1-p-z)$, $q=\bar q q$, $w=\bar q w$, $y'=\bar q(1-q-w)$, $u=\bar u u$, $u'=\bar u(1-u)$, we find

$$F_A(\alpha,t)=\Gamma(-\alpha)i^{\alpha}c'\int_0^{\epsilon}\bar{u}^{1+\beta+\beta'}(\bar{p}\bar{q})^{2N}d\bar{u}\,d\bar{p}\,d\bar{q}\times e^{i\bar{Q}}\frac{H(\tau)H(\tau')}{\bar{\Delta}^{\alpha+2}}(\bar{u}\bar{\Delta}+\bar{p}\bar{q})^{\alpha}\times\frac{\Gamma(N)^4}{\Gamma(2N+1)^2}\frac{\Gamma(1+\beta)\Gamma(1+\beta')}{\Gamma(2+\beta+\beta')}. \tag{A9}$$

The integrals

$$\int_0^1 x^{\beta}(1-x)^{\beta'}dx=\frac{\Gamma(1+\beta)\Gamma(1+\beta')}{\Gamma(2+\beta+\beta')},$$

$$\int_0^1 dz\,z^{N-1}\int_0^{1-z}dp\,(1-z-p)^{N-1}=\frac{[\Gamma(N)]^2}{\Gamma(2N+1)}$$

were used in deriving (A9). Next we note that

$$\int_0^{\epsilon}\bar{u}^{1+\beta+\beta'}(\bar{p}\bar{q})^{2N}(\bar{u}\bar{\Delta}+\bar{p}\bar{q})^{\alpha}d\bar{u}\,d\bar{p}\,d\bar{q}\approx\frac{1}{(\alpha+3+\beta+\beta'+2N)^2}\frac{\Gamma(2+\beta+\beta')\Gamma(2N+1)}{\Gamma(\beta+\beta'+2N+3)\bar{\Delta}^{2+\beta+\beta'}}, \tag{A10}$$

so that

$$F_A(\alpha,t)=\frac{c'[\Gamma(N)]^4}{\Gamma(2N+1)}\int\frac{e^{i\bar{Q}}\bar{H}(\tau)\bar{H}(\tau')(\bar{\Delta})^{2N-1}}{(\alpha+3+2N+\beta+\beta')^2}\times\frac{\Gamma(1+\beta)\Gamma(1+\beta')}{(i)^{\beta+\beta'+2N+3}}d\tau\,d\tau'. \tag{A11}$$

We have set $\alpha=-\beta-\beta'-2N-3$ wherever possible. Since $\bar{\Delta}=v+v'$ and $\bar{Q}=q^2vv'/(v+v')-m^2(x+y+w+z')$, (A11) represents a bubble diagram in $2-4N$ dimensions. In other words, the v and v' integrations can be eliminated and (A11) written in the form

$$F_A(\alpha,t)=\frac{c''}{\Gamma(2N+1)}\times\int\frac{d^{2-4N}k\,[\bar{\beta}(m^2,\infty,k^2)\bar{\beta}(m^2,\infty,(k-q)^2)]^2}{[\alpha+2N+1-\alpha(k^2)-\alpha((k-q)^2)]^2}\times Z(k^2)Z((k-q)^2). \tag{A12}$$

Both (A2) and (A3) were used to replace the integral representations of the residue functions by the functions themselves. The continuous dimensional integration in (A12) can be given meaning for all N.[12] To recover the large-s behavior of $F(s,t)$ from (A12) it is necessary to invert the Mellin transform by means of the relation

$$F(s)=\frac{1}{2\pi i}\int_{\bar{c}}d\alpha\,s^{\alpha}F(\alpha).$$

The contour $\bar{c}$ runs parallel to the Imα axis and has Re$\alpha=-\epsilon$, $0<\epsilon<1$. Since $F_A(\alpha,t)$ in (A12) has a pole in the α plane, its inverse Mellin transform is just Eq. (6) of the text.

The large-s behavior of the crossed graph is extracted in exactly the same way. The only difference is the minus sign in the definition of f_B in (A8). This minus sign propagates through to the integral in (A10) to give

$$F_B(\alpha,s)=(-1)^{2N+1}F_A(\alpha,s). \tag{A13}$$

This completes the derivation of the high-energy limits of the diagrams in Fig. 2.

APPENDIX B

The techniques used to evaluate high-energy limits in Appendix A are also applicable to the amplitude represented by Fig. 3. Each Regge-pole amplitude is assumed to have the form given in (5). Only an outline of the derivation is presented here. Equations (A1) and (A3) are used for each Regge-pole amplitude in Fig. 3, and the solid lines are given by standard Feynman propagators. The analysis here is complicated by the fact that three loop integrations are involved compared with only one for each diagram in Fig. 2. We have

$$F(s,t)=c\int d\tau\,d\tau'\,da_i u^{\beta}u'^{\beta'}H(\tau)H(\tau')\,e^{i\psi}d^4k_1d^4k_2d^4k_3, \tag{B1}$$

where $\psi=k_iA_{ij}k_j+2b_ik_i+C$. In (B1) the unprimed variables refer to the left-hand amplitude in Fig. 3. The Feynman parameters a_i, $i=1,\ldots,8$ correspond to the eight propagators which are written in exponential form. After the loop integrations are performed by transforming ψ to a diagonal quadratic form, the coefficient of s in the exponential can be isolated. Thus, upon performing a Mellin transformation on (B1), we find

$$F(\alpha,t)=c\Gamma(-\alpha)\int d\tau\,d\tau'\,da_iu^{\beta}u'^{\beta'}H(\tau)H(\tau')\frac{(if)^{\alpha}}{\Delta^{\alpha+2}}e^{iQ}. \tag{B2}$$

The coefficient of s, f/Δ, is the important quantity.

$$f=(\bar{a}_1\bar{a}_4-\bar{a}_2\bar{a}_3)(\bar{a}_6\bar{a}_7-\bar{a}_5\bar{a}_8)+uu'\left[(v+v')\sum_{i=1}^{8}\bar{a}_i+(\bar{a}_3+\bar{a}_4+\bar{a}_7+\bar{a}_8)(\bar{a}_1+\bar{a}_2+\bar{a}_5+\bar{a}_6)\right]$$
$$+u[(v+v')(\bar{a}_6+\bar{a}_8)(\bar{a}_1+\bar{a}_3)+\bar{a}_1\bar{a}_6(\bar{a}_3+\bar{a}_4+\bar{a}_7+\bar{a}_8)+\bar{a}_3\bar{a}_8(\bar{a}_1+\bar{a}_2+\bar{a}_5+\bar{a}_6)]$$
$$+u'[(v+v')(\bar{a}_2+\bar{a}_4)(\bar{a}_5+\bar{a}_7)+\bar{a}_2\bar{a}_5(\bar{a}_3+\bar{a}_4+\bar{a}_7+\bar{a}_8)+\bar{a}_4\bar{a}_7(\bar{a}_1+\bar{a}_2+\bar{a}_5+\bar{a}_6)]. \tag{B3}$$

In (B3) $\bar{a}_i$ is defined to be equal to a_i plus the parameter related to the mass-shell dependence of the residue function at the end of that line. In other words, $\bar{a}_i = a_i + x_j$. The parameters u, u', v, v' are defined in Appendix A. [See (A3).] If form factors are used in the coupling of internal to external particles, they can be accommodated by a suitable modification of the definition of $\bar{a}_i$. The first term in the expression for f can be positive or negative, and its vanishing at an interior point in Feynman-parameter space generates the Regge cut. Therefore, since

$$\int_{-\delta}^{\delta} dx \int_{-\delta}^{\delta} dy (xy + \bar{f})^{\alpha} \approx \frac{2\pi i}{\alpha+1} \bar{f}^{\alpha+1} + \cdots , \tag{B4}$$

$$F(\alpha, t) = \frac{2\pi i^{\alpha+1}}{\alpha+1} c \int \frac{d\tau\, d\tau' da_i}{\Delta^{\alpha+2}} u^{\beta} H(\tau) u'^{\beta'} H(\tau') e^{iQ} (Auu' + Bu + Cu')^{\alpha+1} \delta(\bar{a}_1\bar{a}_4 - \bar{a}_2\bar{a}_3)\delta(\bar{a}_5\bar{a}_8 - \bar{a}_6\bar{a}_7) . \tag{B5}$$

The dots in (B4) denote terms which do not lead to a Regge cut, and (B5) is that portion of $F(\alpha, t)$ which contains a cut in α.

The functions A, B, C, are determined from (B3). The leading singularity, or at least the one that leads to the Regge cut, comes from the region of small u, u'. The rightmost singularity of $F(\alpha, t)$ is determined by

$$\int_0^{\epsilon} du\, du' u^{\beta} u'^{\beta'} (Auu' + Bu + Cu')^{\alpha+1} \approx \frac{1}{\alpha + 3 + \beta + \beta'} \frac{\Gamma(1+\beta)\Gamma(1+\beta')}{\Gamma(2+\beta+\beta')} \frac{1}{B^{1+\beta} C^{1+\beta'}} . \tag{B6}$$

The next step is to note that when $u = u' = 0$, $\bar{a}_1\bar{a}_4 = \bar{a}_2\bar{a}_3$, and $\bar{a}_5\bar{a}_8 = \bar{a}_6\bar{a}_7$,

$$\Delta = \frac{(\bar{a}_3 + \bar{a}_4)(\bar{a}_7 + \bar{a}_8)}{\bar{a}_4\bar{a}_8} [(v + v')(\bar{a}_2 + \bar{a}_4)(\bar{a}_6 + \bar{a}_8) + \bar{a}_2\bar{a}_6(\bar{a}_3 + \bar{a}_4 + \bar{a}_7\bar{a}_8) + \bar{a}_2\bar{a}_8(\bar{a}_3 + \bar{a}_4) + \bar{a}_4\bar{a}_6(\bar{a}_7 + \bar{a}_8)] , \tag{B7}$$

and

$$B = \frac{\bar{a}_3\bar{a}_8}{(\bar{a}_3 + \bar{a}_4)(\bar{a}_7 + \bar{a}_8)} \Delta , \quad C = \frac{\bar{a}_7\bar{a}_4}{(\bar{a}_3 + \bar{a}_4)(\bar{a}_7 + \bar{a}_8)} \Delta . \tag{B8}$$

The leading singularity in $F(\alpha, t)$ is given by

$$F(\alpha, t) = -2\pi c \int \frac{d\tau d\tau' da_i}{\alpha + 3 + \beta + \beta'} \frac{\delta(\bar{a}_1\bar{a}_4 - \bar{a}_2\bar{a}_3)\delta(\bar{a}_6\bar{a}_7 - \bar{a}_5\bar{a}_8)}{\Delta} H(\tau)H(\tau')\, e^{iQ} \frac{\Gamma(1+\beta)\Gamma(1+\beta')}{i^{2+\beta+\beta'}} \frac{[(\bar{a}_3 + \bar{a}_4)(\bar{a}_7 + \bar{a}_8)]^{2+\beta+\beta'}}{(\bar{a}_3\bar{a}_8)^{1+\beta}(\bar{a}_7\bar{a}_4)^{1+\beta'}} . \tag{B9}$$

From the fact that Δ^{-1} occurs to the first power, we conclude that the integral over Feynman parameters can be converted in part to a two-dimensional Euclidean integral over a loop momentum k. This result is to be contrasted with the $(2-4N)$-dimensional integral which appeared at this stage in Appendix A. When the integral is expressed in terms of the momentum integration, the β and β' contour integrations are carried out to obtain

$$F(\alpha, t) = c' \int \frac{d^2k [G((k-q)^2, k^2, q^2)]^2}{\alpha + 1 - \alpha(k^2) - \alpha((k-q)^2)} , \tag{B10}$$

where $G((k-q)^2, k^2, q^2)$ is given in Eq. (13) of the text. When (B10) is used in an inverse Mellin-transform integral and the residue of the pole in the α plane isolated, Eq. (12) is the result.

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[16]Although Eq. (10) is obtained from a first-rank separable approximation to the Bethe-Salpeter equation, its asymptotic dependence on p^2 and p'^2 is maintained in higher-rank approximations.

[17]The diagram in Fig. 3 is often referred to as the Mandelstam diagram, since it is of the type analyzed by Mandelstam in his original discussion of Regge cuts. (See Ref. 2.) There are other more complicated diagrams which contain the double-Reggeon cut. In principle the analysis of this section could be applied to them. The conclusion about the failure of the eikonal representation would remain unaffected.

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Effects of a Neutral Intermediate Boson in Semileptonic Processes*

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The observable effects of a neutral intermediate vector boson in semileptonic processes are considered in the context of a proposed renormalizable theory of the weak and electromagnetic interactions. With strange particles neglected, this theory allows the calculation of neutral-current form factors in terms of charged-current form factors and electromagnetic form factors. The results are neither confirmed nor refuted by present data. One possible method of incorporating the strange particles is briefly discussed.

I. INTRODUCTION

It now appears that a renormalizable theory of the weak and electromagnetic interactions may be constructed from a Yang-Mills theory with spontaneous breaking of gauge invariance.[1] There are many possible theories of this general type, but for the present it seems best to concentrate on one particularly simple model,[2] which requires the smallest possible number of unobserved particles. This proposed model has so far been worked out in detail only as it applies to the area of the weak and electromagnetic interactions of leptons, and within this area involves the usual leptons, photons, a new massive scalar meson, charged massive vector mesons W, and a neutral massive vector meson Z. The neutral vector meson produces striking effects in neutrino-electron scattering, effects which are just on the verge of observability.[3]

This paper will deal with the possible observable effects of the Z boson in semileptonic processes, especially neutrino scattering. The difficult part of this problem has to do with the role of the strange particles. In Sec. II the strange particles are simply ignored altogether, and we find a rather natural extension of the proposed model to semileptonic reactions, which allows the form factors for any neutral-current process to be calculated from the form factors for the corresponding charge-exchange and electron-scattering processes. This theory is applied to neutrino scattering in Sec. III. The chief result, valid for not too large values of the momentum transfer, is that

$$0.15 \leq \frac{\sigma(\nu' + p \to \nu' + p)}{\sigma(\nu' + n \to \mu^- + p)} \leq 0.25 \qquad (1.1)$$

as compared with an experimental value[4] (or upper limit) 0.12 ± 0.06.

The strange particles are considered in Sec. IV. It is found that a four-quark model of Glashow, Maiani, and Iliopoulos[5] naturally explains the absence of neutral strangeness-changing currents, but leads to new terms in the neutral strangeness-conserving current, which could in principle alter predictions like (1.1).

In summary, there is no obvious obstacle to the