so that

$$f_1(z) \underset{z\to\infty}{\sim} c_1 \exp\{[-1+(1-2\alpha/\pi)^{1/2}](\ln z)/2\}.$$

In terms of the variable t this gives

$$|f_1(s,t)| \underset{t\to\infty}{\sim} c_1(s)\exp\{-(\alpha/2\pi)\ln t\}.$$

The other equations may be solved in a similar manner.

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Statistics of a Composite Photon Formed of Two Fermions*

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A modification of the author's earlier version of the neutrino theory of photons is presented. A composite-photon distribution is obtained which is similar enough to Planck's distribution to satisfy experimental results. Linearly polarized photon operators are constructed and the theory is shown to describe truly neutral photons, thus removing an objection that had been raised against the earlier version by Berezinskii.

I. INTRODUCTION

The "neutrino theory of light" as developed by Jordan and Kronig was postulated on the basis that exact Bose statistics must be satisfied. In satisfying the Bose statistics it was necessary to postulate a Raman effect wherein one neutrino could simulate a photon. For this reason it was also postulated that there was no interaction between the neutrino and antineutrino. Work on this theory was brought to a halt in 1938 when Pryce[1] showed that Bose commutation relations and invariance under rotation of the coordinate system could not be satisfied simultaneously for a neutrino theory of photons. (The Jordan-Kronig theory satisfied Bose commutation relations but it was not invariant under spatial rotation.) Since 1957 there has been some renewed interest in a composite-photon theory.[2-12] In an early paper[7] the author has suggested that the requirement of Bose statistics for the theory be dropped. This allows one to circumvent Pryce's theorem. In a rather straightforward manner a composite-photon theory was constructed by describing the annihilation and creation of photons in terms of neutrino and antineutrino processes. The electric and magnetic fields so formed were shown to satisfy Maxwell's equations, and thereby the polarization problems of the old Jordan-Kronig theory were solved. Also the operations of space inversion and charge conjugation were defined in terms of the neutrino operators in such a way that the electric and magnetic fields transform in the usual manner under these symmetry operations.

Getting back to statistics, one notes than an integral-spin particle composed of two fermions (such as the deuteron) does not obey Bose statistics.[13,14] The commutation relations[7,14] for composite particles (formed of two fermions) are similar to Bose commutation relations, and the non-Bose terms become important only when the fermion wave functions of the particles overlap. In most cases the wave functions of nuclei do not overlap sufficiently to give observable effects. If the photon is composed of two fermions it will similarly *not* obey exact Bose statistics. (This is true for any composite particle formed from a finite number of fermions.[15])

If the composite photon does not obey Bose statistics, it is necessary to show that this does not lead to disagreement with experimental observations. In particular, either Planck's distribution (or a radiation distribution sufficiently similar to satisfy the experiments) must result from the theory. By an additional condition in the previous paper[7] the author attempted to derive an energy distribution for the composite photon. More recently Berezinskii[12] has correctly shown that this additional condition is not relativistically invariant. In his paper Berezinskii further argues that certain commutation relations must be satisfied for the photon to be neutral and to allow construction of linearly polarized photons. We construct linearly polarized photon operators in Sec. II whose commutation relations differ from Berezinskii's, but whose transformation properties under charge conjugation nevertheless show that the theory describes strictly neutral photons.

In order to obtain a satisfactory radiation distribution for the composite photon, a modification of the earlier theory[7] was necessary. Although we have assumed a neutrino-antineutrino interaction, we also assumed that the neutrinos, interacting with an electron for example, combined to form a photon only when their momenta were *parallel*. In this paper we will generalize the theory and allow the neutrinos interacting with an electron to have both *parallel* and *antiparallel* momenta. This allows more photons to be in the low-energy part of the distribution.

II. CONSTRUCTION OF PHOTON FIELD FROM NEUTRINO FIELD

We will use the representation with

$$\vec{\sigma}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix},\quad \vec{\alpha}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & -\vec{\sigma}\end{pmatrix},$$
$$\vec{\gamma}=i\begin{pmatrix}0 & \vec{\sigma}\\ -\vec{\sigma} & 0\end{pmatrix},\quad \gamma_4=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\qquad (1)$$
$$\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}.$$

The Dirac equation for $m=0$ is

$$\gamma_\mu p_\mu \psi = 0. \qquad (2)$$

Letting

$$\psi = U(\vec{n})e^{i(\vec{p}\cdot\vec{x}-pt)}, \qquad (3)$$

we obtain four normalized solutions for U for positive $\vec{n}$ $(=\vec{p}/p)$:

$$U^{+1}_{+1}(\vec{n})=\left(\frac{1+n_3}{2}\right)^{1/2}\begin{bmatrix}1\\ \frac{n_1+in_2}{1+n_3}\\ 0\\ 0\end{bmatrix}, \qquad (4)$$

$$U^{-1}_{-1}(\vec{n})=\left(\frac{1+n_3}{2}\right)^{1/2}\begin{bmatrix}\frac{-n_1+in_2}{1+n_3}\\ 1\\ 0\\ 0\end{bmatrix}, \qquad (5)$$

$$U^{+1}_{-1}(\vec{n})=\left(\frac{1+n_3}{2}\right)^{1/2}\begin{bmatrix}0\\ 0\\ \frac{-n_1+in_2}{1+n_3}\\ 1\end{bmatrix}, \qquad (6)$$

$$U^{-1}_{+1}(\vec{n})=\left(\frac{1+n_3}{2}\right)^{1/2}\begin{bmatrix}0\\ 0\\ 1\\ \frac{n_1+in_2}{1+n_3}\end{bmatrix}. \qquad (7)$$

The subscript on U refers to spin state while the superscript refers to energy state. Thus, the helicity operator $(S=\vec{\sigma}\cdot\vec{n})$ has eigenvalues of +1 for the spinors of (4) and (7) and −1 for the spinors of (5) and (6). Similarly the energy operator $(W_{\text{op}}=\vec{\alpha}\cdot\vec{n})$ has eigenvalues of +1 for (4) and (6) and −1 for (5) and (7).

For negative momentum $(-\vec{n})$ we obtain the relations

$$\begin{aligned}U^{+1}_{+1}(-\vec{n})&=U^{-1}_{-1}(\vec{n}),\\ U^{-1}_{-1}(-\vec{n})&=U^{+1}_{+1}(\vec{n}),\\ U^{+1}_{-1}(-\vec{n})&=U^{-1}_{+1}(\vec{n}),\\ U^{-1}_{+1}(-\vec{n})&=U^{+1}_{-1}(\vec{n}).\end{aligned} \qquad (8)$$

We will use the notation that ν_1 is the neutrino (positive-energy state) with spin *parallel* to its momentum and ν_2 is the neutrino (positive-energy state) with spin *antiparallel* to its momentum. Let $a_1(k,\vec{n})$, $c_1(k,\vec{n})$, $a_2(k,\vec{n})$, and $c_2(k,\vec{n})$ be the annihilation operators with momentum $k\vec{n}$ for ν_1, $\bar{\nu}_1$, ν_2, and $\bar{\nu}_2$, respectively. Then, the general neutrino field in terms of particles and antiparticles (not holes) is

$$\psi(\vec{x},t)=\int d\vec{k}[a_1(k,\vec{n})U_{+1}^{+1}(\vec{n})e^{i(\vec{k}\cdot\vec{x}-kt)}+c_1^\dagger(k,\vec{n})U_{-1}^{-1}(-\vec{n})e^{-i(\vec{k}\cdot\vec{x}-kt)}+a_2(k,\vec{n})U_{-1}^{+1}(\vec{n})e^{i(\vec{k}\cdot\vec{x}-kt)}+c_2^\dagger(k,\vec{n})U_{+1}^{-1}(-\vec{n})e^{-i(\vec{k}\cdot\vec{x}-kt)}], \quad (9)$$

where † is used to designate Hermitian conjugate.

We define the annihilation operators for right and left circularly polarized photons in terms of the neutrino operators respectively as

$$\chi(p,\vec{n})=\int_{p/2}^{\infty}dk\,\phi^\dagger(k)c_1(|\tfrac{1}{2}p-k|,-\vec{n})a_1(|\tfrac{1}{2}p+k|,\vec{n})+\int_{-p/2}^{p/2}dk\,\phi^\dagger(k)c_2(|\tfrac{1}{2}p+k|,\vec{n})a_1(|\tfrac{1}{2}p-k|,\vec{n})+\int_{p/2}^{\infty}dk\,\phi^\dagger(k)c_2(|\tfrac{1}{2}p+k|,\vec{n})a_2(|\tfrac{1}{2}p-k|,-\vec{n}) \quad (10)$$

and

$$\omega(p,\vec{n})=\int_{p/2}^{\infty}dk\,\phi^\dagger(k)c_2(|\tfrac{1}{2}p-k|,-\vec{n})a_2(|\tfrac{1}{2}p+k|,\vec{n})+\int_{-p/2}^{p/2}dk\,\phi^\dagger(k)c_1(|\tfrac{1}{2}p+k|,\vec{n})a_2(|\tfrac{1}{2}p-k|,\vec{n})+\int_{p/2}^{\infty}dk\,\phi^\dagger(k)c_1(|\tfrac{1}{2}p+k|,\vec{n})a_1(|\tfrac{1}{2}p-k|,-\vec{n}), \quad (11)$$

where $\phi(k)$ is as yet an unspecified function of k.

These photon operators are more general than those used in a previous paper[7] in which only the integrals from $-p/2$ to $p/2$ were taken. Physically, we are now allowing the neutrinos (that are annihilated when a photon is annihilated) to have *antiparallel* as well as *parallel* momentum. This modification is important in determining the photon energy distribution; the photon commutation relations are non-Bose in either case. It would have been more general to take $\phi=\phi(p,k)$; previously we had used[7] $\phi(p,k)=1/\sqrt{p}$.

Comparing Eqs. (8), (9), and (10), one notes that the spinor combinations that go along with $\chi(p,\vec{n})$ of Eq. (10) are of the form

$$[U_{-1}^{-1}(\vec{n})]^\dagger O_{\text{int}}U_{+1}^{+1}(\vec{n}),$$
$$[U_{-1}^{+1}(\vec{n})]^\dagger O_{\text{int}}U_{+1}^{+1}(\vec{n}),$$
and
$$[U_{-1}^{+1}(\vec{n})]^\dagger O_{\text{int}}U_{-1}^{+1}(-\vec{n}). \quad (12)$$

The possible choices for O_{int} are

$$O_S=\gamma_4,\quad O_V=\gamma_4\gamma_\mu,\quad O_T=i\gamma_4(\gamma_\lambda\gamma_\mu-\gamma_\mu\gamma_\lambda),\quad O_A=i\gamma_4\gamma_\mu\gamma_5,\quad O_P=\gamma_4\gamma_5. \quad (13)$$

The only nonvanishing terms resulting from substituting Eq. (13) into Eq. (12) can be put in the form

$$[U_{-1}^{+1}(\vec{n})]^\dagger\gamma_\mu U_{+1}^{+1}(\vec{n}).$$

Similarly, for $\omega(p,\vec{n})$ of Eq. (11) we have only terms of the form

$$[U_{+1}^{+1}(\vec{n})]^\dagger\gamma_\mu U_{-1}^{+1}(\vec{n}).$$

For convenience let

$$u(\vec{n})=U_{+1}^{+1}(\vec{n})$$
and
$$v(\vec{n})=U_{-1}^{+1}(\vec{n}). \quad (14)$$

It should be noted that $u(\vec{n})$ and $v(\vec{n})$ refer to positive-energy states with spin parallel and antiparallel to the direction of propagation, respectively.

We now form the electric and magnetic fields [see Eq. (32) and Eq. (34) of Ref. 7]:

$$\vec{E}(\vec{x},t)=\frac{-i}{2\sqrt{2\pi}}\int_0^\infty d\vec{p}\,p^{1/2}\{[\chi(p,\vec{n})v^\dagger(\vec{n})\vec{\gamma}u(\vec{n})+\omega(p,\vec{n})u^\dagger(\vec{n})\vec{\gamma}v(\vec{n})]e^{i(\vec{p}\cdot\vec{x}-pt)}-[\chi^\dagger(p,\vec{n})u^\dagger(\vec{n})\vec{\gamma}v(\vec{n})+\omega^\dagger(p,\vec{n})v^\dagger(\vec{n})\vec{\gamma}u(\vec{n})]e^{-i(\vec{p}\cdot\vec{x}-pt)}\}, \quad (15)$$

$$\vec{H}(\vec{x},t)=\frac{-1}{2\sqrt{2\pi}}\int_0^\infty d\vec{p}\,p^{1/2}\{[\chi(p,\vec{n})v^\dagger(\vec{n})\vec{\gamma}u(\vec{n})-\omega(p,\vec{n})u^\dagger(\vec{n})\vec{\gamma}v(\vec{n})]e^{i(\vec{p}\cdot\vec{x}-pt)}+[\chi^\dagger(p,\vec{n})u^\dagger(\vec{n})\vec{\gamma}v(\vec{n})-\omega^\dagger(p,\vec{n})v^\dagger(\vec{n})\vec{\gamma}u(\vec{n})]e^{-i(\vec{p}\cdot\vec{x}-pt)}\}. \quad (16)$$

The vectors $\vec{E}$ and $\vec{H}$ are real as $\vec{E}^\dagger=\vec{E}$ and $\vec{H}^\dagger=\vec{H}$ and they satisfy Maxwell's equations.[16] To write the fields in terms of linear photon operators, let

$$\xi(p,\vec{n})=(1/\sqrt{2})[\omega(p,\vec{n})+\chi(p,\vec{n})], \quad (17)$$

$$\eta(p,\vec{n})=(i/\sqrt{2})[\omega(p,\vec{n})-\chi(p,\vec{n})], \quad (18)$$

$$\vec{\epsilon}_1(\vec{n}) = \tfrac{1}{2}[v^\dagger(\vec{n})\vec{\gamma}u(\vec{n}) + u^\dagger(\vec{n})\vec{\gamma}v(\vec{n})], \tag{19}$$

$$\vec{\epsilon}_2(\vec{n}) = (i/2)[v^\dagger(\vec{n})\vec{\gamma}u(\vec{n}) - u^\dagger(\vec{n})\vec{\gamma}v(\vec{n})]. \tag{20}$$

With the use of (17)–(20), Eqs. (15) and (16) become

$$\vec{E}(\vec{x}, t) = \frac{-i}{2\pi}\int_0^\infty d\vec{p}\, p^{1/2}\{[\xi(p,\vec{n})\vec{\epsilon}_1(\vec{n}) + \eta(p,\vec{n})\vec{\epsilon}_2(\vec{n})]e^{i(\vec{p}\cdot\vec{x}-pt)} - [\xi^\dagger(p,\vec{n})\vec{\epsilon}_1(\vec{n}) + \eta^\dagger(p,\vec{n})\vec{\epsilon}_2(\vec{n})]e^{-i(\vec{p}\cdot\vec{x}-pt)}\} \tag{21}$$

and

$$\vec{H}(\vec{x}, t) = \frac{i}{2\pi}\int_0^\infty d\vec{p}\, p^{1/2}\{[\xi(p,\vec{n})\vec{\epsilon}_2(\vec{n}) - \eta(p,\vec{n})\vec{\epsilon}_1(\vec{n})]e^{i(\vec{p}\cdot\vec{x}-pt)} - [\xi^\dagger(p,\vec{n})\vec{\epsilon}_2(\vec{n}) - \eta^\dagger(p,\vec{n})\vec{\epsilon}_1(\vec{n})]e^{-i(\vec{p}\cdot\vec{x}-pt)}\}. \tag{22}$$

We note that $\vec{\kappa}(\vec{n}) = (1/\sqrt{2})u^\dagger(\vec{n})\vec{\gamma}v(\vec{n})$ is a self-orthogonal complex unit vector [i.e., $\vec{\kappa}(\vec{n})\cdot\vec{\kappa}(\vec{n}) = \vec{\kappa}^*(\vec{n})\cdot\vec{\kappa}^*(\vec{n}) = 0$; $\vec{\kappa}(\vec{n})\cdot\vec{\kappa}^*(\vec{n}) = 1$] while $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ are real unit vectors. The photons are transversely polarized since[16]

$$\vec{n}\cdot\vec{\kappa}(\vec{n}) = \vec{n}\cdot\vec{\kappa}^*(\vec{n}) = 0, \tag{23}$$

$$\vec{n}\times\vec{\kappa}(\vec{n}) = i\vec{\kappa}(\vec{n}), \tag{24}$$

and

$$\vec{\kappa}(\vec{n})\times\vec{\kappa}^*(\vec{n}) = i\vec{n}. \tag{25}$$

III. COMMUTATION RELATIONS FOR PHOTON OPERATORS

The neutrinos are assumed to obey Fermi-Dirac commutation relations:

$$\begin{aligned}[][a_1(k,\vec{n}), a_1^\dagger(k',\vec{n}')]_+ &= [a_2(k,\vec{n}), a_2^\dagger(k',\vec{n}')]_+ \\ &= [c_1(k,\vec{n}), c_1^\dagger(k',\vec{n}')]_+ = [c_2(k,\vec{n}), c_2^\dagger(k',\vec{n}')]_+ = \delta(k-k')\delta(\vec{n}-\vec{n}')\end{aligned} \tag{26}$$

while all other combinations anticommute.

With the use of (26) one can determine the commutation relations for the circularly polarized photon operators of Eqs. (10) and (11):

$$[\chi(p,\vec{n}),\ \chi(q,\vec{n}')]_- = 0, \tag{27}$$

$$[\chi(p,\vec{n}),\ \chi^\dagger(q,\vec{n}')]_- = \delta(\vec{n}-\vec{n}')[\delta(p-q) - \alpha_{12}(p,q,\vec{n})], \tag{28}$$

$$[\omega(p,\vec{n}),\ \omega(q,\vec{n}')]_- = 0, \tag{29}$$

$$[\omega(p,\vec{n}),\ \omega^\dagger(q,\vec{n}')]_- = \delta(\vec{n}-\vec{n}')[\delta(p-q) - \alpha_{21}(p,q,\vec{n})], \tag{30}$$

$$[\chi(p,\vec{n}),\ \omega(q,\vec{n}')]_- = 0, \tag{31}$$

$$[\chi(p,\vec{n}),\ \omega^\dagger(q,\vec{n}')]_- = -\delta(\vec{n}+\vec{n}')\beta_{12}(p,q,\vec{n}), \tag{32}$$

$$[\omega(p,\vec{n}),\ \chi^\dagger(q,\vec{n}')]_- = -\delta(\vec{n}+\vec{n}')\beta_{21}(p,q,\vec{n}), \tag{33}$$

where

$$\begin{aligned}\alpha_{12}(p,q,\vec{n}) = &\int_{\sup(0,q-p)}^\infty dk\,\phi^\dagger(\tfrac{1}{2}p+k)\phi(p-\tfrac{1}{2}q+k)[c_1^\dagger(|q-p-k|,-\vec{n})c_1(|k|,-\vec{n}) + a_2^\dagger(|q-p-k|,-\vec{n})a_2(|k|,-\vec{n})] \\ &+ \int_{\sup(0,p-q)}^\infty dk\,\phi^\dagger(\tfrac{1}{2}p-k)\phi(p-\tfrac{1}{2}q-k)[a_1^\dagger(|q-p+k|,\vec{n})a_1(|k|,\vec{n}) + c_2^\dagger(|q-p+k|,\vec{n})c_2(|k|,\vec{n})] \\ &+ \int_{p/2}^{\sup(p/2,q-p/2)} dk\,\phi^\dagger(k)\phi(\tfrac{1}{2}p-\tfrac{1}{2}q+k)[c_2^\dagger(|q-\tfrac{1}{2}p-k|,\vec{n})c_1(|\tfrac{1}{2}p-k|,-\vec{n}) \\ &\qquad + a_1^\dagger(|q-\tfrac{1}{2}p-k|,\vec{n})a_2(|\tfrac{1}{2}p-k|,-\vec{n})] \\ &+ \int_{q/2}^{\sup(q/2,p-q/2)} dk\,\phi^\dagger(\tfrac{1}{2}p-\tfrac{1}{2}q-k)\phi(k)[c_1^\dagger(|\tfrac{1}{2}q-k|,-\vec{n})c_2(|p-\tfrac{1}{2}q-k|,\vec{n}) \\ &\qquad + a_2^\dagger(|\tfrac{1}{2}q-k|,-\vec{n})a_1(|p-\tfrac{1}{2}q-k|,\vec{n})]\end{aligned} \tag{34}$$

and

$$\beta_{12}(p,q,\vec{n})=\int_0^\infty dk\{\phi^\dagger(q+\tfrac{1}{2}p+k)\phi(\tfrac{1}{2}q+k)[a_1^\dagger(|k|,\vec{n})a_1(|q+p+k|,\vec{n})+c_2^\dagger(|k|,\vec{n})c_2(|q+p+k|,\vec{n})]$$
$$+\phi^\dagger(\tfrac{1}{2}p+k)\phi(p+\tfrac{1}{2}q+k)[c_1^\dagger(|q+p+k|,-\vec{n})c_1(|k|,-\vec{n})+a_2^\dagger(|q+p+k|,-\vec{n})a_2(|k|,-\vec{n})]\}$$
$$+\int_0^q dk\,\phi^\dagger(\tfrac{1}{2}p+k)\phi(\tfrac{1}{2}q-k)[a_2^\dagger(|q-k|,-\vec{n})a_1(|p+k|,\vec{n})+c_1^\dagger(|q-k|,-\vec{n})c_2(|p+k|,\vec{n})]$$
$$+\int_0^p dk\,\phi^\dagger(\tfrac{1}{2}p-k)\phi(\tfrac{1}{2}q+k)[a_2^\dagger(|q+k|,-\vec{n})a_1(|p-k|,\vec{n})+c_1^\dagger(|q+k|,-\vec{n})c_2(|p-k|,\vec{n})],\qquad(35)$$

with $\sup(a,b)$ being the larger of a and b. The expectation values of $\alpha_{12}(p,q,\vec{n})$ and $\beta_{12}(p,q,\vec{n})$ are

$$\langle\alpha_{12}(p,q,\vec{n})\rangle=\delta(p-q)\int_0^\infty dk\{\phi^\dagger(\tfrac{1}{2}p+k)\phi(\tfrac{1}{2}p+k)[c_1^\dagger(|k|,-\vec{n})c_1(|k|,-\vec{n})+a_2^\dagger(|k|,-\vec{n})a_2(|k|,-\vec{n})]$$
$$+\phi^\dagger(\tfrac{1}{2}p-k)\phi(\tfrac{1}{2}p-k)[a_1^\dagger(|k|,\vec{n})a_1(|k|,\vec{n})+c_2^\dagger(|k|,\vec{n})c_2(|k|,\vec{n})]\}\qquad(36)$$

and

$$\langle\beta_{12}(p,q,\vec{n})\rangle=0.\qquad(37)$$

In obtaining these commutation relations we have taken $\phi(k)$ to satisfy

$$\int_{-\infty}^\infty dk\,|\phi(k)|^2=1\quad\text{and}\quad\phi(-k)=\phi(k).\qquad(38)$$

Equations (27), (29), and (31) are obtained by using the fact that in these cases all the fermion operators anticommute and therefore an even number of interchanges will result in the photon operators commuting. Equation (28) is obtained as follows:

$$[\chi(p,\vec{n}),\ \chi^\dagger(q,\vec{n}')]_-$$
$$=\delta(\vec{n}-\vec{n}')\int_{p/2}^\infty dk\int_{q/2}^\infty dk'\phi^\dagger(k)\phi(k')\{[c_1(|\tfrac{1}{2}p-k|,-\vec{n})a_1(|\tfrac{1}{2}p+k|,\vec{n}),a_1^\dagger(|\tfrac{1}{2}q+k'|,\vec{n}')c_1^\dagger(|\tfrac{1}{2}q-k'|,-\vec{n}')]_-$$
$$+[c_2(|\tfrac{1}{2}p+k|,\vec{n})a_2(|\tfrac{1}{2}p-k|,-\vec{n}),a_2^\dagger(|\tfrac{1}{2}q-k'|,-\vec{n}')c_2^\dagger(|\tfrac{1}{2}q+k'|,\vec{n}')]_-\}$$
$$+\delta(\vec{n}-\vec{n}')\int_{-p/2}^{p/2}dk\int_{-q/2}^{q/2}dk'\phi^\dagger(k)\phi(k')[c_2(|\tfrac{1}{2}p+k|,\vec{n})a_1(|\tfrac{1}{2}p-k|,\vec{n}),a_1^\dagger(|\tfrac{1}{2}q-k'|,\vec{n}')c_2^\dagger(|\tfrac{1}{2}q+k'|,\vec{n}')]_-$$
$$+\delta(\vec{n}-\vec{n}')\int_{p/2}^\infty dk\int_{-q/2}^{q/2}dk'\phi^\dagger(k)\phi(k')\{c_1(|\tfrac{1}{2}p-k|,-\vec{n})c_2^\dagger(|\tfrac{1}{2}q+k'|,\vec{n}')[a_1(|\tfrac{1}{2}p+k|,\vec{n}),a_1^\dagger(|\tfrac{1}{2}q-k'|,\vec{n}')]_+$$
$$+a_2(|\tfrac{1}{2}p-k|,-\vec{n})a_1^\dagger(|\tfrac{1}{2}q-k'|,\vec{n}')[c_2(|\tfrac{1}{2}p+k|,\vec{n}),c_2^\dagger(|\tfrac{1}{2}q+k'|,\vec{n}')]_+\}$$
$$+\delta(\vec{n}-\vec{n}')\int_{q/2}^\infty dk'\int_{-p/2}^{p/2}dk\,\phi^\dagger(k)\phi(k')\{c_2(|\tfrac{1}{2}p+k|,\vec{n})c_1^\dagger(|\tfrac{1}{2}q-k'|,-\vec{n}')[a_1(|\tfrac{1}{2}p-k|,\vec{n}),a_1^\dagger(|\tfrac{1}{2}q+k'|,\vec{n}')]_+$$
$$+a_1(|\tfrac{1}{2}p-k|,\vec{n})a_2^\dagger(|\tfrac{1}{2}q-k'|,-\vec{n}')[c_2(|\tfrac{1}{2}p+k|,\vec{n}),c_2^\dagger(|\tfrac{1}{2}q+k'|,\vec{n}')]_+\}.\qquad(39)$$

Considering the first term,

$$[c_1(|\tfrac{1}{2}p-k|,-\vec{n})a_1(|\tfrac{1}{2}p+k|,\vec{n}),a_1^\dagger(|\tfrac{1}{2}q+k'|,\vec{n})c_1^\dagger(|\tfrac{1}{2}q-k'|,\vec{n})]_-$$
$$=\delta(|\tfrac{1}{2}p-k|-|\tfrac{1}{2}q-k'|)\delta(|\tfrac{1}{2}p+k|-|\tfrac{1}{2}q+k'|)$$
$$-\delta(|\tfrac{1}{2}p-k|-|\tfrac{1}{2}q-k'|)a_1^\dagger(|\tfrac{1}{2}q+k'|,\vec{n})a_1(|\tfrac{1}{2}p+k|,\vec{n})-\delta(|\tfrac{1}{2}p+k|-|\tfrac{1}{2}q+k'|)c_1^\dagger(|\tfrac{1}{2}q-k'|,-\vec{n})c_1(|\tfrac{1}{2}p-k|,-\vec{n}),\qquad(40)$$

by use of Eq. (26). The first term of Eq. (39) is then

$$\delta(\vec{n}-\vec{n}')\Big[\int_{p/2}^\infty dk\,\delta(p-q)|\phi(k)|^2-\int_{p/2}^\infty dk\,\phi^\dagger(k)\phi(\tfrac{1}{2}q-\tfrac{1}{2}p+k)a_1^\dagger(|q-\tfrac{1}{2}p+k|,\vec{n})a_1(|\tfrac{1}{2}p+k|,\vec{n})$$
$$+\int_{\sup(p/2,q-p/2)}^\infty dk\,\phi^\dagger(k)\phi(\tfrac{1}{2}p-\tfrac{1}{2}q+k)c_1^\dagger(|q-\tfrac{1}{2}p-k|,-\vec{n})c_1(|\tfrac{1}{2}p-k|,-\vec{n})\Big].$$

With similar operations for the other terms and by use of Eq. (38) one obtains Eq. (28). Equation (30) is obtained by noting that interchange of subscripts 1 and 2 converts χ to ω [see Eqs. (10) and (11)]. By similar manipulations one obtains Eqs. (32) and (33).

The commutation relations for the linearly polarized photon operators [defined in Eqs. (17) and (18)] are obtained from Eqs. (27)–(33):

$$[\xi(p,\vec{n}),\ \xi(q,\vec{n}')]_- = 0\,, \tag{41}$$

$$[\xi(p,\vec{n}),\ \xi^\dagger(q,\vec{n}')]_- = \delta(\vec{n}-\vec{n}')\{\delta(p-q)-\tfrac{1}{2}[\alpha_{12}(p,q,\vec{n})+\alpha_{21}(p,q,\vec{n})]\} - \tfrac{1}{2}\delta(\vec{n}+\vec{n}')[\beta_{12}(p,q,\vec{n})+\beta_{21}(p,q,\vec{n})]\,, \tag{42}$$

$$[\eta(p,\vec{n}),\ \eta(q,\vec{n}')]_- = 0\,, \tag{43}$$

$$[\eta(p,\vec{n}),\ \eta^\dagger(q,\vec{n}')]_- = \delta(\vec{n}-\vec{n}')\{\delta(p-q)-\tfrac{1}{2}[\alpha_{12}(p,q,\vec{n})+\alpha_{21}(p,q,\vec{n})]\} + \tfrac{1}{2}\delta(\vec{n}+\vec{n}')[\beta_{12}(p,q,\vec{n})+\beta_{21}(p,q,\vec{n})]\,, \tag{44}$$

$$[\xi(p,\vec{n}),\ \eta(q,\vec{n}')]_- = 0\,, \tag{45}$$

$$[\xi(p,\vec{n}),\ \eta^\dagger(q,\vec{n}')]_- = \tfrac{1}{2}i\{\delta(\vec{n}-\vec{n}')[\alpha_{21}(p,q,\vec{n})-\alpha_{12}(p,q,\vec{n})] + \delta(\vec{n}+\vec{n}')[\beta_{12}(p,q,\vec{n})-\beta_{21}(p,q,\vec{n})]\}\,, \tag{46}$$

$$[\eta(p,\vec{n}),\ \xi^\dagger(q,\vec{n}')]_- = \tfrac{1}{2}i\{\delta(\vec{n}-\vec{n}')[\alpha_{12}(p,q,\vec{n})-\alpha_{21}(p,q,\vec{n})] + \delta(\vec{n}+\vec{n}')[\beta_{12}(p,q,\vec{n})-\beta_{21}(p,q,\vec{n})]\}\,. \tag{47}$$

These commutation relations for the circularly and linearly polarized photon operators are obviously non-Bose owing to the terms involving $\alpha_{12}(p,q,\vec{n})$, $\alpha_{21}(p,q,\vec{n})$, $\beta_{12}(p,q,\vec{n})$, and $\beta_{21}(p,q,\vec{n})$. In their proof that a neutrino theory of photons is impossible, both Pryce[1] and Berezinskii[12] require that the commutators of Eqs. (46) and (47) equal zero. We question the need for this requirement; see Sec. VI.

IV. PHOTON ENERGY DISTRIBUTION

The method and notation used in this section are similar to those of Refs. 14 and 17. We define the following one-composite-particle Green's functions:

$$G^>(\vec{R},t;\vec{R}',t') = (1/i)^2\langle\chi(\vec{R},t)\chi^\dagger(\vec{R}',t')\rangle \tag{48}$$

and

$$G^<(\vec{R},t;\vec{R}',t') = (1/i)^2\langle\chi^\dagger(\vec{R}',t')\chi(\vec{R},t)\rangle\,. \tag{49}$$

For systems at finite temperature the expectation value of an operator S can be calculated with the aid of the grand canonical ensemble

$$\langle S\rangle = \mathrm{Tr}[e^{-\beta H}S]/\mathrm{Tr}[e^{-\beta H}]\,, \tag{50}$$

where $\beta = 1/kT$. From Eqs. (49) and (50),

$$G^<(\vec{R},0;\vec{R}',t) = \frac{\mathrm{Tr}[(1/i)^2e^{-\beta H}\chi^\dagger(\vec{R}',t')\chi(\vec{R},0)]}{\mathrm{Tr}[e^{-\beta H}]}\,. \tag{51}$$

Using the cyclic invariance of the trace and inserting $e^{-\beta H}e^{\beta H}$, we obtain

$$G^<(\vec{R},0;\vec{R}',t') = (1/i)^2\langle e^{\beta H}\chi(\vec{R},0)\,e^{-\beta H}\chi^\dagger(\vec{R}',t)\rangle\,. \tag{52}$$

Using the prescription for converting Schrödinger operators into Heisenberg operators yields

$$G^<(\vec{R},0;\vec{R}',t') = (1/i)^2\langle\chi(\vec{R},-i\beta)\chi^\dagger(\vec{R}',t')\rangle = G^>(\vec{R},-i\beta;\vec{R}',t')\,. \tag{53}$$

The Green's functions depend on $\vec{R}$ and $\vec{R}'$ only through the combination $\vec{r} = \vec{R}-\vec{R}'$ and upon t and t' through the combination $\tau = t-t'$. Thus (53) can be written

$$G^<(\vec{r},\tau) = G^>(\vec{r},\tau-i\beta)\,. \tag{54}$$

The Fourier transform of the Green's function is defined by

$$G^>(\vec{p},\omega) = (i)^2\int d\vec{r}\int d\tau\, e^{-\vec{p}\cdot\vec{r}+i\omega\tau}G^>(\vec{r},\tau)\,, \tag{55}$$

with a similar equation for $G^<(\vec{p},\omega)$. Taking the Fourier transform of (54) results in

$$G^<(\vec{p},\omega) = e^{-\beta\omega}G^>(\vec{p},\omega) \tag{56}$$

or

$$G^<(\vec{p},\omega) = \frac{1}{e^{\beta\omega}-1}[G^>(\vec{p},\omega)-G^<(\vec{p},\omega)]\,. \tag{57}$$

Moreover,

$$G^<(\vec{p},\omega) = \int d\tau\, e^{i\omega\tau}\langle\chi^\dagger(\vec{p},0)\chi(\vec{p},\tau)\rangle \tag{58}$$

and

$$G^>(\vec{p},\omega) = \int d\tau\, e^{i\omega\tau}\langle\chi(\vec{p},\tau)\chi^\dagger(\vec{p},0)\rangle\,, \tag{59}$$

where $\chi(\vec{p},\tau)$ is the Fourier transform of $\chi(\vec{r},\tau)$. Since $\chi(\vec{p},0)$ removes a free particle with momentum $\vec{p}$, it must remove energy p (as the rest mass is zero) from the system. Thus,

$$\chi(\vec{p},t) = e^{iHt}\chi(\vec{p},0)\,e^{-iHt} = e^{-ipt}\chi(\vec{p},0)\,. \tag{60}$$

Substituting (60) into (58) and (59) and then substituting those results into (57) yields

$$G^{<}(\vec{p},\omega)=\frac{2\pi\delta(\omega-p)}{e^{\beta\omega}-1}\times\langle[\chi(\vec{p},0)\chi^{\dagger}(\vec{p},0)-\chi^{\dagger}(\vec{p},0)\chi(\vec{p},0)]\rangle\,. \tag{61}$$

Writing the left-hand side as $\int d\tau\, e^{i\omega\tau}G^{<}(\vec{p},\tau)$, integrating over all ω, and interchanging order of integration on the left-hand side results in

$$G^{<}(\vec{p},\tau=0)=\frac{1}{e^{\beta p}-1}\langle[\chi(\vec{p},0),\ \chi^{\dagger}(\vec{p},0)]_{-}\rangle\,. \tag{62}$$

Note that $G^{<}(\vec{p},\tau=0)$ is the expectation value of the operator representing the density of composite particles with momentum $\vec{p}$. Therefore, from Eq. (62) it is apparent that if the χ's obeyed Bose commutation relations, Planck's distribution would be obtained. From Eqs. (28) and (36) we obtain

$$\langle[\chi(p,\vec{n}),\ \chi^{\dagger}(p,\vec{n})]_{-}\rangle=1-\int_0^{\infty}dk\,\{\phi^{\dagger}(\tfrac{1}{2}p+k)\phi(\tfrac{1}{2}p+k)[N_{\bar{\nu}1}(|k|)+N_{\nu2}(|k|)]+\phi^{\dagger}(\tfrac{1}{2}p-k)\phi(\tfrac{1}{2}p-k)[N_{\bar{\nu}2}(|k|)+N_{\nu1}(|k|)]\}\,. \tag{63}$$

Dropping the labels 1 and 2 for the neutrinos, changing the integration variable in (63), and then substituting into (62) results in

$$N_{\gamma}(p)=\frac{1}{e^{\beta p}-1}\left\{1-\int_{-\infty}^{\infty}dk\,|\phi(k)|^2[N_{\nu}(|\tfrac{1}{2}p+k|)+N_{\bar{\nu}}(|\tfrac{1}{2}p+k|)]\right\}. \tag{64}$$

The neutrinos are taken to have the Fermi-Dirac distribution

$$N_{\nu}(k)=N_{\bar{\nu}}(k)=\frac{1}{Be^{\beta k}+1}\,. \tag{65}$$

We can obtain an approximate value for B (which is a function of β) for the range of β in which this second term of Eq. (64) can be neglected as follows. Integrating Eq. (64) over momentum and real space, we obtain the total number of photons

$$N_{\gamma}=\frac{2V}{(2\pi)^3}(4\pi)\int_0^{\infty}dp\frac{p^2}{e^{\beta p}-1}\,,$$

so the photon density is

$$\frac{N_{\gamma}}{V}=\frac{2(1.202)}{\pi^2\beta^3}\,. \tag{66}$$

Doing the same for Eq. (65), we obtain the neutrino density

$$\frac{N_{\nu}}{V}=\frac{1}{\pi^2\beta^3}\int_0^{\infty}dx\frac{x^2}{Be^{x}+1}\,. \tag{67}$$

Equating the photon and neutrino densities, we obtain $B\approx 0.73$ from a digital computation.

If we knew $\phi(k)$, the energy distribution of our composite photon would be determined. For convenience let $\phi(k)$ have a Gaussian distribution

$$\phi(k)=(2/\pi k_0^{\,2})^{1/4}e^{-k^2/k_0^{\,2}}\,. \tag{68}$$

Note that Eq. (68) is consistent with Eq. (38). Substituting (65) and (68) into (64) results in

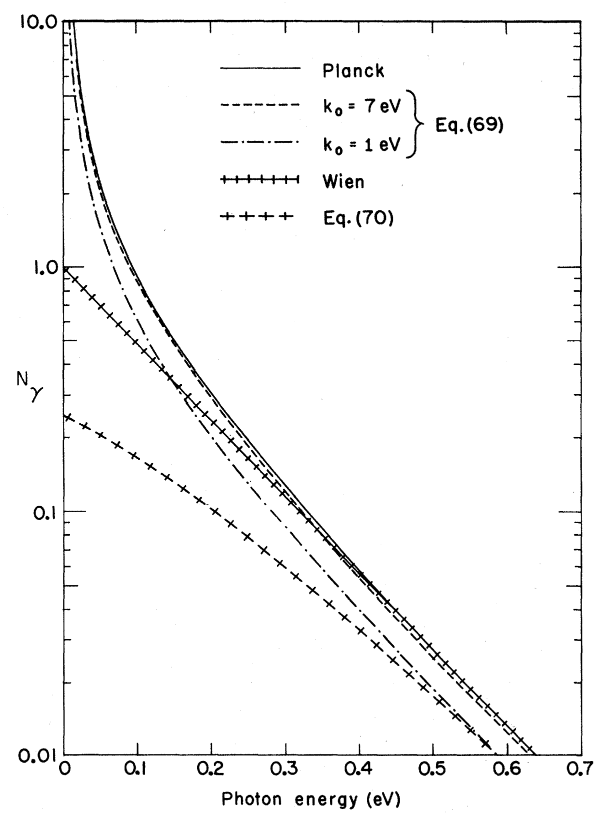

FIG. 1. The photon energy distribution for $T=1600$ °K as calculated from Eq. (69) with $k_0=1$ and 7 eV and from Eq. (70). Also shown for comparison are the Planck and Wien distributions.

$$N_\gamma(p)=\frac{1}{e^{\beta p}-1}\times\left[1-\frac{2}{k_0}\left(\frac{2}{\pi}\right)^{1/2}\int_{-\infty}^{\infty}dk\,\frac{\exp(-2k^2/k_0^{\,2})}{B\exp(|\tfrac{1}{2}p+k|\beta)+1}\right]. \quad (69)$$

If we had only allowed parallel neutrino momentum as in the earlier theory, the composite-photon distribution would have been [with $\phi(p,k)=1/\sqrt{p}$]

$$N_\gamma(p)=\frac{1}{e^{\beta p}-1}\left(1-\frac{2}{p}\int_0^p dk(Be^{\beta k}+1)^{-1}\right). \quad (70)$$

In Fig. 1 we have plotted the following for $T=1600\,°\mathrm{K}$: (1) the Planck distribution; (2) Eq. (69) for $k_0=1$ eV, 7 eV; (3) the Wien distribution; (4) Eq. (70). First we note that the curve of Eq. (70) is lower than the Wien distribution and could not possibly satisfy the experimental results. As k_0 approaches infinity the composite-photon distribution [Eq. (69)] approaches the Planck distribution. However, the experiments of Refs. 18–21 could be reasonably satisfied with $k_0\gtrsim 7$ eV. Since the second term of Eq. (69) is not a function of the product (λT), it was necessary to compare it with temperature and wavelength variations. The experimental data are usually normalized to the theoretical curve so it is the variations with energy (or wavelength) and temperature that are important in determining the fit. Comparing the areas under the curves in Fig. 1 shows that neglecting the second term of Eq. (64) in obtaining B is a good approximation for $k_0\gtrsim 7$ eV, and $T\leqslant 1600\,°\mathrm{K}$. (This was checked by a numerical computation.)

V. PARITY, CHARGE CONJUGATION, AND ROTATION TRANSFORMATIONS

The transformation of $\vec{E}$ and $\vec{H}$ under the parity P, charge conjugation C, and rotations about $\vec{n}$ operators were given in a previous paper,[7] so here we will only show that the recent modifications do not change that result.

The parity operator was defined[7] such that

$$Pa_1(k,\vec{n})P^{-1}=\epsilon_P a_2(k,-\vec{n}), \quad (71)$$

$$Pa_2(k,\vec{n})P^{-1}=\epsilon_P a_1(k,-\vec{n}), \quad (72)$$

$$Pc_1(k,n)P^{-1}=\epsilon_P^* c_2(k,-\vec{n}), \quad (73)$$

$$Pc_2(k,\vec{n})P^{-1}=\epsilon_P^* c_1(k,-\vec{n}), \quad (74)$$

and the charge conjugation operator such that

$$Ca_1(k,\vec{n})C^{-1}=\epsilon_C c_2(k,\vec{n}), \quad (75)$$

$$Ca_2(k,\vec{n})C^{-1}=\epsilon_C c_1(k,\vec{n}), \quad (76)$$

$$Cc_1(k,\vec{n})C^{-1}=\epsilon_C^* a_2(k,\vec{n}), \quad (77)$$

$$Cc_2(k,\vec{n})C^{-1}=\epsilon_C^* a_1(k,\vec{n}). \quad (78)$$

The transformation of $\chi(p,\vec{n})$ of Eq. (10) is

$$\begin{aligned}P\chi(p,\vec{n})P^{-1}=&\int_{p/2}^{\infty}dk\,\phi^\dagger(k)Pc_1(|\tfrac{1}{2}p-k|,-\vec{n})P^{-1}Pa_1(|\tfrac{1}{2}p+k|,\vec{n})P^{-1}\\&+\int_{-p/2}^{p/2}dk\,\phi^\dagger(k)Pc_2(|\tfrac{1}{2}p+k|,\vec{n})P^{-1}Pa_1(|\tfrac{1}{2}p-k|,\vec{n})P^{-1}\\&+\int_{p/2}^{\infty}dk\,\phi^\dagger(k)Pc_2(|\tfrac{1}{2}p+k|,\vec{n})P^{-1}Pa_2(|\tfrac{1}{2}p-k|,-\vec{n})P^{-1}.\end{aligned}$$

By use of Eqs. (71)–(74) we obtain

$$P\chi(p,\vec{n})P^{-1}=\omega(p,-\vec{n}). \quad (79)$$

Similarly, we obtain the transformation equations for the other photon operators, $\alpha_{12}(p,q,\vec{n})$ of Eq. (34) and $\beta_{12}(p,q,\vec{n})$ of Eq. (35). The results are as follows:

$$P\omega(p,\vec{n})P^{-1}=\chi(p,-\vec{n}), \quad (80)$$

$$P\xi(p,\vec{n})P^{-1}=\xi(p,-\vec{n}), \quad (81)$$

$$P\eta(p,\vec{n})P^{-1}=-\eta(p,-\vec{n}), \quad (82)$$

$$P\alpha_{12}(p,q,\vec{n})P^{-1}=\alpha_{21}(p,q,-\vec{n}), \quad (83)$$

$$P\beta_{12}(p,q,\vec{n})P^{-1}=\beta_{21}(p,q,-\vec{n}), \quad (84)$$

$$C\chi(p,\vec{n})C^{-1}=-\chi(p,\vec{n}), \quad (85)$$

$$C\omega(p,\vec{n})C^{-1}=-\omega(p,\vec{n}), \quad (86)$$

$$C\xi(p,\vec{n})C^{-1}=-\xi(p,\vec{n}), \quad (87)$$

$$C\eta(p,\vec{n})C^{-1}=-\eta(p,\vec{n}), \quad (88)$$

$$C\alpha_{12}(p,q,\vec{n})C^{-1}=\alpha_{12}(p,q,\vec{n}), \quad (89)$$

$$C\alpha_{21}(p,q,\vec{n})C^{-1}=\alpha_{21}(p,q,\vec{n}), \quad (90)$$

$$C\beta_{12}(p,q,\vec{n})C^{-1}=\beta_{12}(p,q,\vec{n}), \quad (91)$$

$$C\beta_{21}(p,q,\vec{n})C^{-1}=\beta_{21}(p,q,\vec{n}). \quad (92)$$

With the above transformations for the photon operators it can be shown[7] that $\vec{E}$ and $\vec{H}$ transform in the usual way. Also, it follows by inspection that the photon commutation relations Eqs. (27)–(33) and Eqs. (41)–(47) are invariant under P and

C transformations.

Under a rotation of the coordinate system through an angle θ about $\vec{n}$, the neutrino operators transform as follows:

$$R_\theta a_1(k,\vec{n})R_\theta^{-1} = e^{-i\theta/2}a_1(k,\vec{n}), \quad (93)$$

$$R_\theta a_2(k,\vec{n})R_\theta^{-1} = e^{i\theta/2}a_2(k,\vec{n}), \quad (94)$$

$$R_\theta c_1(k,\vec{n})R_\theta^{-1} = e^{i\theta/2}c_1(k,\vec{n}), \quad (95)$$

$$R_\theta c_2(k,\vec{n})R_\theta^{-1} = e^{-i\theta/2}c_2(k,\vec{n}), \quad (96)$$

$$R_\theta a_1(k,-\vec{n})R_\theta^{-1} = e^{i\theta/2}a_1(k,-\vec{n}), \quad (97)$$

$$R_\theta a_2(k,-\vec{n})R_\theta^{-1} = e^{-i\theta/2}a_2(k,-\vec{n}), \quad (98)$$

$$R_\theta c_1(k,-\vec{n})R_\theta^{-1} = e^{-i\theta/2}c_1(k,-\vec{n}), \quad (99)$$

$$R_\theta c_2(k,-\vec{n})R_\theta^{-1} = e^{i\theta/2}c_2(k,-\vec{n}). \quad (100)$$

We thus see from Eqs. (10), (11), (17), and (18) that the photon operators transform so that

$$R_\theta \chi(p,\vec{n})R_\theta^{-1} = e^{-i\theta}\chi(p,\vec{n}), \quad (101)$$

$$R_\theta \omega(p,\vec{n})R_\theta^{-1} = e^{i\theta}\omega(p,\vec{n}), \quad (102)$$

$$R_\theta \xi(p,\vec{n})R_\theta^{-1} = \xi(p,\vec{n})\cos\theta + \eta(p,\vec{n})\sin\theta, \quad (103)$$

$$R_\theta \eta(p,\vec{n})R_\theta^{-1} = \eta(p,\vec{n})\cos\theta - \xi(p,\vec{n})\sin\theta. \quad (104)$$

With the above transformations for the photon operators, it can be shown[7] that $\vec{E}$ and $\vec{H}$ transform as vectors under a rotation R_θ of the coordinate system.

VI. DISCUSSION

Firstly, we have attempted to show that a neutrino theory of photons *is* possible. To form a composite-photon theory, one must give up exact Bose statistics. Besides Pryce's theorem there are other reasons why exact Bose commutation relations cannot be obtained for a composite photon. To cancel out the α_{12} and α_{21} terms in the commutation relations [Eqs. (28) and (30)], Jordan[22] postulated that the absorption of a photon of momentum $\vec{p}$ could be simulated by a Raman effect of neutrinos or antineutrinos (i.e., one neutrino or antineutrino with momentum $\vec{p}+\vec{k}$ is absorbed while another of the same energy state, opposite spin, and momentum $\vec{k}$ is emitted) as well as the simultaneous absorption of a neutrino-antineutrino pair. This Raman effect provided the additional terms to give Bose commutation relations. Nowadays, this Raman effect in which a single neutrino simulates a photon is experimentally ruled out as it would easily have been observed in the inverse-beta-decay experiments.

We have assumed that the neutrino obeys Fermi statistics. One might assume parafermion statistics for the neutrino in hopes of obtaining Bose statistics for a composite photon. Berezinskii has shown[23] that this does not work for a composite photon, and he has further arguments[23] against using Jordan's Raman-effect hypothesis in which the effects of a photon are simulated by a single neutrino.

We also differ from Jordan's theory in assuming a neutrino-antineutrino interaction to be necessary. Indeed, without the interaction (or Raman-effect hypothesis) one cannot obtain a satisfactory photon distribution, since the neutrino pair being absorbed or emitted by an electron can only have parallel momentum. This would lead to a composite-photon distribution differing greatly from Planck's distribution and with the experiment (see Fig. 1). However, if the neutrinos being absorbed or emitted in pairs by an electron are allowed to have either parallel or antiparallel momentum, the resulting distribution can be similar to Planck's distribution and experiment (see Sec. IV). Unfortunately, the exact form of the distribution depends on an unknown function or parameter, so an experimental test to decide between this distribution and Planck's distribution is not possible at present.

Berezinskii[12] has claimed that the commutators in Eqs. (32), (33), (46), and (47) must equal zero so that: (1) the photon is neutral and (2) construction of linearly polarized photons is possible. We have shown in Sec. V that our theory describes neutral photons in the usual sense even though the commutators in these equations do not vanish. Also in Sec. II we have constructed linearly polarized photons.

The consequences of the theory for quantum electrodynamics have not been examined in this paper. We do not know of any experimental result that rules out the non-Bose commutation relations of Sec. III. This is not to say that the difference between these commutation relations and Bose commutation relations cannot be detected by experiment. On the contrary, we feel that this difference in commutation relations will lead to experimental tests to determine if the photon is a composite particle.

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Compton Scattering and Fixed Poles in Parton Field-Theoretic Models*

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We extend a class of parton models to a fully gauge-invariant theory for the full Compton amplitude. The existence of local electromagnetic interactions is shown to always give rise to a constant real part in the high-energy behavior of the amplitude $T_1(\nu, q^2)$. In the language of Reggeization this is interpreted as a fixed pole at $J=0$ in T_1 and νT_2, with residue polynomial in the photon mass squared.

Recent inelastic electroproduction experiments (which essentially measure the imaginary part of the forward off-shell Compton amplitude) hint at a composite nature for the nucleon. This has been represented by parton models involving pointlike (possibly field-theoretic) constituents, but up to the present time these concepts have only been applied to the scaling, incoherent impulse approximation, region. Gauge invariance and the low-energy theorem place further restrictions upon such theories, and in this note we report the extension of parton field-theoretic ideas to a discussion of the full Compton amplitude. In particular we shall see that such models always give rise to a real part at high energies additional to that expected from the Regge behavior of the imaginary part. This extra real part should be identified with the "fixed pole"[1] of conventional Regge analysis. Evidence for such a fixed pole for on-shell photons has been found phenomenologically from dispersion relations.[2] In addition we find that the "fixed pole" appears as a constant real part, C, in T_1 independent of q^2, and appears in νT_2 in the form $-Cq^2/\nu$.[3]

If the proton were as simple as the nucleus, then the high-energy behavior of the forward Compton amplitude would follow directly from the coherent impulse approximation. At $\nu=0$, the Compton amplitude on a nucleus is given by the Thomson limit[4] $f_1(0)=-(Z^2\alpha/M_{\text{nucleus}})$ whereas at energies high compared to the binding energy, but below threshold for photoproduction of mesons, the forward amplitude is given by the coherent sum of the individual nucleon amplitudes,

$$f_1(\nu)\to -\sum_{i=1}^{Z}\frac{\alpha}{\omega_i}\quad(\omega_i\simeq m_i).$$

In fact, for the case of a composite proton the analogous high-energy behavior would be given by the coherent sum of "seagull" terms for the individual proton constituents (quarks, bare hadrons) and the formulas (7), (11) we give later correspond to this picture.

Field theory gives us the clearest example of a fully covariant, gauge-invariant Compton amplitude which can also incorporate the composite na-