Relations Among the Structure Functions of Deep-Inelastic Neutrino-Nucleon Scattering*

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Light-cone current commutators are used to derive relations between the structure functions of the vector-vector part of neutrino-nucleon scattering and the structure functions of the vector-axial-vector part.

Reasonable expressions for the commutator of currents restricted to a lightlike surface have been derived from the quark model.^{1,2} These expressions involve bilocal operators and it is possible to derive sum rules which relate the structure functions of inelastic lepton-nucleon scattering to the matrix elements of these bilocal operators.³ In particular, if the matrix elements of the bilocal operators are written out in terms of form factors, then, by using the light-cone version¹ of the Bjorken-Johnson-Low (BJL) limit,⁴ the deep-inelastic limits of the structure functions become simply Fourier transforms of the form factors. In Ref. 3 it was found that the structure functions of a scattering process involving only vector currents were related to the form factors in a one-to-one manner.

A basic assumption of the light-cone-commutator method is that the commutators are invariant under $SU(3) \times SU(3)$ symmetry. Therefore the commutator of a (conserved) axial-vector current with a vector current involves the same bilocal operators as the commutator of two vector currents and the matrix element of the commutator involves the same form factors. Deep-inelastic neutrino-nucleon scattering, for example, will have structure functions for the vector-axial-vector (VA) cross terms as well as the structure functions for the vector-vector or axial-vector-axial-vector terms. However, the relations between the structure functions and the form factors will involve only the form factors from vector-vector scattering. Thus the structure functions for the VA cross terms are all related, through the form factors, to the structure functions of the VV terms.

In the case where the nucleon spin is summed over, the relation is well known. It was first derived from the parton model by Llewellyn Smith⁵ and has been derived from light-cone commutators by several people.^{2,6,7} In the present work we shall find the relations among the spin-dependent structure functions as well. These results are interesting in that they can be compared with other models, most of which give the same answer for the spin-independent vector-vector structure functions but differ in the spin-dependent and/or vector-axial-vector structure functions.⁸ In addition, if electron scattering should verify the essential correctness of the light-cone-commutator approach then these relations can provide a handle on neutrino scattering.

There are four independent invariant amplitudes for vector-spinor \rightarrow vector-spinor scattering in the forward direction, two spin-dependent and two spin-independent. For spinor-vector \rightarrow spinor-axial-vector scattering there are also four but only one is independent of spin.⁹ The time-ordered product of two currents $J_a^{\mu}(x) = V_a^{\mu}(x) - A_a^{\mu}(x)$ can be written as

$$T_{ab}^{\mu\nu}(p,q) \equiv i \int d^{4}x \, e^{iq \cdot x} \langle p, s | (J_{a}^{\mu}(x)J_{b}^{\nu}(0))_{+} | p, s \rangle$$

$$= -\left(g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q^{2}}\right) T_{L}^{ab}(q^{2},\nu) + \left(p^{\mu}p^{\nu} - \frac{\nu}{q^{2}}(p^{\mu}q^{\nu} + p^{\nu}q^{\mu}) + \frac{\nu^{2}}{q^{2}}g^{\mu\nu}\right) T_{2}^{ab}(q^{2},\nu)$$

$$+ i\epsilon^{\mu\nu\alpha\beta}s_{\alpha}q_{\beta}T_{3}^{ab}(q^{2},\nu) + iq \cdot s\epsilon^{\mu\nu\alpha\beta}p_{\alpha}q_{\beta}T_{4}^{ab}(q^{2},\nu) + \frac{1}{2}i\epsilon^{\mu\nu\alpha\beta}p_{\alpha}q_{\beta}T_{5}^{ab}(q^{2},\nu)$$

$$+ q \cdot s\left(g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q^{2}}\right) T_{6}^{ab}(q^{2},\nu) + q \cdot s\left(p^{\mu}p^{\nu} - \frac{\nu}{q^{2}}(p^{\mu}q^{\nu} + p^{\nu}q^{\mu}) + \frac{\nu^{2}}{q^{2}}g^{\mu\nu}\right) T_{7}^{ab}(q^{2},\nu)$$

$$+ \left[\left(s^{\mu} - \frac{q \cdot s}{q^{2}}q^{\mu}\right)\left(p^{\nu} - \frac{\nu}{q^{2}}q^{\nu}\right) + \left(p^{\mu} - \frac{\nu}{q^{2}}q^{\mu}\right)\left(s^{\nu} - \frac{q \cdot s}{q^{2}}q^{\nu}\right)\right] T_{8}^{ab}(q^{2},\nu)$$

$$+ if_{abc}\frac{1}{q^{2}}\Gamma_{c}\left(g^{\mu\nu\nu} - q^{\mu}p^{\nu} - q^{\nu}p^{\mu}\right) + if_{abc}\frac{1}{q^{2}}\Gamma_{c}^{A}\left(g^{\mu\nu}q \cdot s - q^{\mu}s^{\nu} - q^{\nu}s^{\mu}\right), \qquad(1)$$

$$s^{0} = \mathbf{\tilde{p}} \cdot \mathbf{\hat{n}}, \quad \mathbf{\tilde{s}} = m \,\mathbf{\hat{n}} + \frac{\mathbf{\tilde{p}} \cdot \mathbf{\hat{n}}}{E + m} \,\mathbf{\tilde{p}}.$$
 (2)

The terms with $T_L^{ab}(q^2, \nu)$ and $T_2^{ab}(q^2, \nu)$ are the spin-independent, VV + AA contribution; $T_3^{ab}(q^2, \nu)$ and $T_4^{ab}(q^2, \nu)$ mark the spin-dependent, VV + AA contribution. The term with $T_5^{ab}(q^2, \nu)$ is the only spin-independent, VA + AV part, while $T_6^{ab}(q^2, \nu)$, $T_7^{ab}(q^2, \nu)$, and $T_8^{ab}(q^2, \nu)$ give the spin-dependent, VA + AV terms.

The final two terms in (1) are the *t*-channel poles, where we have defined Γ_c and Γ_c^A in terms of the matrix elements of the vector and axial-vector currents,

$$\langle p | V_c^{\mu}(0) | p \rangle = p^{\mu} \Gamma_c,$$

$$\langle p | A_c^{\mu}(0) | p \rangle = s^{\mu} \Gamma_c^A.$$
(3a)
(3b)

The cross section in the laboratory frame for the scattering of a neutrino with a nucleon whose spin is aligned parallel (+) or antiparallel (-) to the direction of motion of the incoming neutrino is written in terms of the imaginary parts, $W_i(q^2, \nu)$, of the $T_i(q^2, \nu)$ as

$$\frac{d\sigma^{\nu\rho(4)}}{dE'd\Omega} = \frac{G^2}{16\pi^3} \frac{E'}{mE} \left[-q^2 W_L(q^2,\nu) + (E^2 + E'^2 + \frac{1}{2}q^2) m^2 W_2(q^2,\nu) \pm m q^2(E + E'\cos\theta) W_3(q^2,\nu) \right. \\ \left. \pm m^2 q^2(E + E')(E - E'\cos\theta) W_4(q^2,\nu) - \frac{1}{2}m q^2(E + E') W_5(q^2,\nu) \mp m q^2(E - E'\cos\theta) W_6(q^2,\nu) \right. \\ \left. \pm m^3(E - E'\cos\theta)(E^2 + E'^2 + \frac{1}{2}q^2) W_7(q^2,\nu) \mp 2m^2 EE'(1 + \cos\theta) W_8(q^2,\nu) \right],$$
(4)

where E is the energy of the original neutrino, E' is the energy of the final electron, θ is the scattering angle, m is the mass of the nucleon, G is the Fermi coupling constant, $q^2 = -2EE'(1 - \cos\theta)$, and $\nu = m(E - E')$. The electron mass has been set equal to zero.¹⁰ In writing (4) we have neglected the Cabibbo angle (since $\sin\theta_{c} \approx 0.05 \approx 0$) and so have only the $\Delta S = 0$ cross section. For the scattering of an antineutrino we replace $W_i(\nu, q^2)$ by $-W_i(-\nu, q^2)$ for i = L, 2, 4, 8 and by $+W_i(-\nu, q^2)$ for i = 3, 5, 6, 7.

It is easy to see, by writing (4) in terms of the variables $\omega = -q^2/2\nu$ and $x = \nu/mE$, that each term in (4) increases linearly with E as E goes to infinity. Such behavior is well known for the spin-independent terms.

To derive the relations we use the light-cone BJL theorem,^{1,3}

$$T^{\mu\nu}_{ab}(p,q) \underset{q^{-\to\infty}}{\sim} \text{polynomial} - \frac{1}{q^{-}} \int dx^{-} d^{2}x_{\perp} e^{iq^{+}x^{-}} e^{-i\overline{q}_{\perp}\cdot\overline{x}_{\perp}} \langle p | [J^{\mu}_{a}(x), J^{\nu}_{b}(0)] | p \rangle,$$
(5)

with the commutators¹

$$\begin{bmatrix} V_a^+(x), V_b^+(0) \end{bmatrix}_{x^+=0} = \begin{bmatrix} A_a^+(x), A_b^+(0) \end{bmatrix}_{x^+=0}$$

$$= if \qquad V_a^+(0)\delta(x^-)\delta^2(x^-)$$
(6)

$$= i \int_{abc} V_{c}(0) O(x_{\perp}) O(x_{\perp}),$$

$$[V_a^+(x), A_b^+(0)]_{x^+=0} = i f_{abc} A_c^+(0) \delta(x^-) \delta^2(x_\perp),$$

 $[V_a^+(x), V_b^-(0)]_{x^+=0} = [A_a^+(x), A_b^-(0)]_{x^+=0}$

.

$$= if_{abc}V_{c}^{-}(0)\delta(x^{-})\delta^{2}(x_{\perp})$$

$$- \frac{1}{2}if_{abc}\left\{\partial_{-}[\epsilon(x^{-})\delta^{2}(x_{\perp})\overline{\upsilon_{c}}(x|0)] + \frac{1}{2}\partial_{i}[\epsilon(x^{-})\delta^{2}(x_{\perp})\overline{\upsilon_{c}}(x|0)] - \frac{1}{2}\partial_{i}\epsilon^{ij}[\epsilon(x^{-})\delta^{2}(x_{\perp})\overline{\alpha}_{jc}(x|0)]\right\}$$

$$- \frac{1}{2}id_{abc}\left\{\partial_{-}[\epsilon(x^{-})\delta^{2}(x_{\perp})\overline{\upsilon_{c}}(x|0)] + \frac{1}{2}\partial_{i}[\epsilon(x^{-})\delta^{2}(x_{\perp})\overline{\upsilon_{c}}(x|0)] + \frac{1}{2}\partial_{i}\epsilon^{ij}[\epsilon(x^{-})\delta^{2}(x_{\perp})\partial_{jc}(x|0)]\right\},$$
(8)

$$\begin{split} \left[V_{a}^{i}(x), A_{b}^{-}(0) \right]_{x^{+}=0} &= \left[A_{a}^{i}(x), V_{b}^{-}(0) \right]_{x^{+}=0} \\ &= i f_{abc} A_{c}^{-}(0) \delta(x^{-}) \delta^{2}(x_{\perp}) \\ &- \frac{1}{2} i f_{abc} \left\{ \partial_{-} \left[\epsilon(x^{-}) \delta^{2}(x_{\perp}) \partial_{c}^{-}(x \mid 0) \right] + \frac{1}{2} \partial_{i} \left[\epsilon(x^{-}) \delta^{2}(x_{\perp}) \partial_{c}^{i}(x \mid 0) \right] - \frac{1}{2} \epsilon^{ij} \partial_{i} \left[\epsilon(x^{-}) \delta^{2}(x_{\perp}) \overline{\upsilon}_{cj}(x \mid 0) \right] \right\} \\ &- \frac{1}{2} i d_{abc} \left\{ \partial_{-} \left[\epsilon(x^{-}) \delta^{2}(x_{\perp}) \overline{a}_{c}^{-}(x \mid 0) \right] + \frac{1}{2} \partial_{i} \left[\epsilon(x^{-}) \delta^{2}(x_{\perp}) \partial_{a}^{i}(x \mid 0) \right] + \frac{1}{2} \epsilon^{ij} \partial_{i} \left[\epsilon(x^{-}) \delta^{2}(x_{\perp}) \overline{\upsilon}_{jc}(x \mid 0) \right] \right\}, \end{split}$$

$$(9)$$

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(7)

where $\mathfrak{V}_{c}^{\mu}(x|0)$, $\overline{\mathfrak{V}}_{c}^{\mu}(x|0)$, $\alpha_{c}^{\mu}(x|0)$, and $\overline{\alpha}_{c}^{\mu}(x|0)$ are the bilocal operators defined in Ref. 3. The form factors are defined by

$$\langle p | \mathbf{v}_{c}^{\mu}(x | 0) | p \rangle = p^{\mu} V_{1}^{c}(x^{2}, x \cdot p) + x^{\mu} V_{2}^{c}(x^{2}, x \cdot p),$$

$$(10)$$

$$\langle p | \mathbf{a}_{c}^{\mu}(x | 0) | p \rangle = s^{\mu} A_{1}^{c}(x^{2}, x \cdot p) + p^{\mu} x \cdot s A_{2}^{c}(x^{2}, x \cdot p)$$

$$|\mathbf{a}_{c}^{\mu}(x|0)|p\rangle = s^{\mu}A_{1}^{\nu}(x^{2}, x \cdot p) + p^{\mu}x \cdot sA_{2}^{\nu}(x^{2}, x \cdot p)$$

 $+x^{\mu}x \cdot sA_{3}^{c}(x^{2}, x \cdot p)$ (11)

with similar definitions for $\overline{\upsilon}_{c}^{\mu}(x|0)$ in terms of $\overline{V}_i^c(x^2, x \cdot p)$ and $\overline{\alpha}_i^{\mu}(x|0)$ in terms of $\overline{A}_i^c(x^2, x \cdot p)$.

The scaling limits $(\nu \rightarrow \infty, q^2 \rightarrow \infty, -q^2/2\nu \equiv \omega \text{ fixed})$ of the structure functions are

$$W_L^{ab}(q^2,\nu) \rightarrow \frac{-1}{2\omega} \left(F_L^{ab}(\omega) + \frac{1}{q^2} G_L^{ab}(\omega) \right), \tag{12a}$$

$$\nu W_i^{ab}(q^2, \nu) \rightarrow F_i^{ab}(\omega) + \frac{1}{q^2} G_i^{ab}(\omega), \quad i = 2, 3, 5, 6, 8$$

(12b)

$$\nu^2 W_i^{ab}(q^2, \nu) \rightarrow F_i^{ab}(\omega) + \frac{1}{q^2} G_i^{ab}(\omega), \quad i = 4, 7.$$
 (12c)

Using $F_i^{ab}(\omega) = F_i^{(ab)}(\omega) + iF_i^{[ab]}(\omega)$ we have our results:

$$F_L^{ab}(\omega) = 0, \qquad (13)$$

$$F_{2}^{[ab]}(\omega) = \omega f_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} V_{1}^{c}(0,\alpha), \qquad (14a)$$

$$F_{5}^{(ab)}(\omega) = 2d_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} V_{1}^{c}(0, \alpha), \qquad (14b)$$

$$F_{2}^{(ab)}(\omega) = i\omega d_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \overline{V}_{1}^{c}(0, \alpha), \qquad (15a)$$

$$F_{5}^{[ab]}(\omega) = 2if_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \overline{V}_{1}^{c}(0,\alpha), \qquad (15b)$$

$$\omega F_{6}^{ab}(\omega) + F_{8}^{ab}(\omega) = 0, \qquad (16)$$

$$F_3^{[ab]}(\omega) = \frac{1}{2} i f_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \overline{A}_1^c(0,\alpha) , \qquad (17a)$$

$$F_{8}^{(ab)}(\omega) = i\omega d_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \overline{A}_{1}^{c}(0,\alpha), \qquad (17b)$$

$$F_{3}^{(ab)}(\omega) = \frac{1}{2}d_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} A_{1}^{c}(0,\alpha), \qquad (18a)$$

$$F_8^{[ab]}(\omega) = \omega f_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} A_1^c(0,\alpha) , \qquad (18b)$$

$$F_4^{[ab]}(\omega) = \frac{1}{2}i f_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \alpha \overline{A}_2^c(0,\alpha), \qquad (19a)$$

$$F_{7}^{(ab)}(\omega) + F_{8}^{(ab)}(\omega) = i\omega d_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \alpha \overline{A}_{2}^{c}(0,\alpha) ,$$
(19b)

$$F_{4}^{(ab)}(\omega) = \frac{1}{2}d_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\,\omega\,\alpha} \alpha A_{2}^{c}(0,\alpha), \qquad (20a)$$

$$F_{7}^{[ab]}(\omega) + F_{8}^{[ab]}(\omega) = \omega f_{abc} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \alpha A_{2}^{c}(0,\alpha) ,$$
(20b)

and

$$G_{L}^{ab}(\omega) = -8i\omega^{3} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \alpha \left[d_{abc} \overline{V}_{2}^{c}(0,\alpha) + f_{abc} V_{2}^{c}(0,\alpha) + f_{abc} V_{2}^{c}(0,\alpha) \right], \quad (21)$$

$$\omega G_{6}^{ab}(\omega) + G_{8}^{ab}(\omega) = -4i\omega^{3} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega\alpha} \alpha^{2}$$

$$\mathcal{G}_{6}^{c}(\omega) + \mathcal{G}_{8}^{c}(\omega) = -4i\omega^{c} \int_{-\infty}^{\infty} d\alpha \ e^{-i\omega \alpha} \alpha^{c} \times \left[d_{abc} \overline{A}_{3}^{c}(0, \alpha) + f_{abc} A_{3}^{c}(0, \alpha) \right].$$
(22)

Equation (13) follows from the fact that there is no q-number Schwinger term on the right-hand side of (6).¹ In the same way if there were a Schwinger term in (7) of the form $\partial_{-}\partial_{-}[P^{ab}(x|0)]$ $\times \epsilon(x^{-})\delta^{2}(x_{\perp})]$, where $P^{ab}(x|0)$ is a pseudoscalar whose matrix element can be written as $x \cdot s$ $\times P^{ab}(x^2, x \cdot p)$, then the right-hand side of (16) would be the Fourier transform of $x \cdot pP^{ab}(0, x \cdot p)$.

Equations (13), (14), (15), (17a), (18a), (19a), (20a), and (21) have been derived before.^{1,2,3} Equations (16), (17b), (18b), (19b), (20b), and (22) are new. These can be used to derive many sum rules. One that is well $known^{2,5-7}$ used (15) to relate $F_2(\omega)$ for electron-nucleon scattering with $F_5(\omega)$ for neutrino-nucleon scattering (neglecting the Cabibbo angle):

$$6[F_{2}^{ep}(\omega) - F_{2}^{en}(\omega)] = \omega[F_{5}^{\nu n}(\omega) - F_{5}^{\nu p}(\omega)].$$
(23)

In the same way (18) and (20) give

$$6\omega [F_{3}^{ep}(\omega) - F_{3}^{en}(\omega)] = [F_{8}^{vn}(\omega) - F_{8}^{vp}(\omega)], \qquad (24)$$

$$6\omega [F_4^{ep}(\omega) - F_4^{en}(\omega)] = [F_7^{\nu n}(\omega) + F_8^{\nu n}(\omega) - F_7^{\nu p}(\omega) - F_8^{\nu p}(\omega)].$$
(25)

Equation (18a) implies³

$$d_{abc} \Gamma_{c}^{A} = \frac{2}{\pi} \int_{0}^{1} d\omega F_{3}^{(ab)}(\omega), \qquad (26)$$

where Γ_c^A is the matrix element of the axial-vector current as defined in (3b). From (18b), (20b), and (16) we have

$$f_{abc}\Gamma_c^A = \frac{1}{\pi} \int_0^1 \frac{d\omega}{\omega} F_8^{[ab]}(\omega)$$
(27a)

$$= -\frac{1}{\pi} \int_{0}^{1} d\omega F_{6}^{[ab]}(\omega)$$
 (27b)

$$= -\frac{1}{\pi} \int_0^1 \frac{d\omega}{\omega} F_7^{[ab]}(\omega) \,. \tag{27c}$$

The Dashen-Fubini-Gell-Mann sum rule in the deep-inelastic region follows from (14a)¹¹:

$$\int_0^1 d\omega \frac{1}{\omega} F_2^{[ab]}(\omega) = \pi f_{abc} \Gamma^c , \qquad (28)$$

where Γ^{c} is defined in (3a). Now we also get a sum rule from $(14b)^{12}$:

$$\int_0^1 d\omega F_5^{(ab)}(\omega) = 2\pi d_{abc} \Gamma^c .$$
⁽²⁹⁾

In the same way the other sum rules derived in

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Ref. 3 for $F_2^{ab}(\omega)$, $F_3^{ab}(\omega)$, and $F_4^{ab}(\omega)$ now have analogs in terms of $F_5^{ab}(\omega)$, $F_6^{ab}(\omega)$, $F_7^{ab}(\omega)$, and $F_8^{ab}(\omega)$.

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⁹In this paper we are only interested in the deep-in-

elastic (scaling) region where, presumably, all masses can be neglected and all currents are conserved. The number of invariant amplitudes is determined by counting helicity amplitudes for spin-1, spin- $\frac{1}{2}$ scattering into spin 1, spin $\frac{1}{2}$. There are 12 independent helicity amplitudes, four of which remain in the forward direction; $T_{1,\frac{1}{2};1,\frac{1}{2}}, T_{0,\frac{1}{2};0,\frac{1}{2}}, T_{-1,\frac{1}{2};-1,\frac{1}{2}}$, and $T_{1,\frac{1}{2};0,-\frac{1}{2}}$. When we sum over the spin of the nucleon we have two amplitudes for the vector-vector case, $T_{1,\frac{1}{2};1,\frac{1}{2}} + T_{1,-\frac{1}{2};1,-\frac{1}{2}}$ $= T_{1,\frac{1}{2};1,\frac{1}{2}} + T_{-1,\frac{1}{2};-1,\frac{1}{2}} \text{ and } T_{0,\frac{1}{2};0,\frac{1}{2}} + T_{0,-\frac{1}{2};0,-\frac{1}{2}} = 2T_{0,\frac{1}{2};0,\frac{1}{2}}, \text{ but only one amplitude for the vector-axial$ vector case because $T_{0,\frac{1}{2};0,\frac{1}{2}} + T_{0,-\frac{1}{2};0,-\frac{1}{2}} = 0$. ¹⁰The fact that we have treated the axial-vector current

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High-Energy Limit of Lowest-Order Weak Amplitudes with Electromagnetic **Radiative Corrections in the Ladder Approximation**

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The asymptotic lowest-order weak amplitude for the process $\nu_{\mu} + \overline{\nu}_{e} \rightarrow \mu^{-} + e^{+}$ is found to be modified by a factor $\exp[-(\alpha/2\pi) \ln t]$ on account of radiative corrections in the ladder approximation. This is in conformity with the result obtained earlier by Li in the perturbation theory.

I. INTRODUCTION

It is well known that cross sections for the purely leptonic weak scattering processes, such as $\nu_{\mu} + e^{-} \rightarrow \mu^{-} + \nu_{e}, \ \nu_{\mu} + \overline{\nu}_{e} \rightarrow \mu^{-} + e^{+}, \text{ etc., increase}$ quadratically with the center-of-mass energy of the system, in the lowest order in the weak interaction in the current × current theory.¹ Recently Li² has investigated the effect of electromagnetic

radiative corrections to the high-energy behavior of lowest-order weak amplitudes in perturbation theory. He finds that the damping thus obtained, though calculated to all orders in α , does not produce a significant change in the behavior of these amplitudes at the available experimental energy resolutions. His method essentially consists in replacing all the fermion propagators in the matrix element for the *n*th-order Feynman diagram by