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Bounds on K_{13} -Decay Form Factors

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We rederive certain bounds on K_{13} decay parameters found recently by Li, Pagels, and Okubo and obtain some new results with a simpler method based on a direct application of the maximum-modulus theorem for holomorphic functions. For an arbitrary value of the momentum-transfer variable t in the complex cut t plane, we find that the domain of values which can be taken by the form factor d(t) of the divergence of the weak strangeness-changing vector current $V_{\mu}^{(\kappa)}$ is bounded by a circle in the plane (Red(t), Imd(t)). We express the radius and the position of the center of this circle in terms of $f_+(0)$ and of the propagator $\Delta(t)$ of the divergence of $V_{\mu}^{(K)}$.

I. INTRODUCTION

Recently Li, Pagels,^{1,2} and Okubo³ have established rigorous bounds on K_{13} -decay form factors, some of the results obtained being the best bounds one can obtain under the given input information. In this paper we shall rederive their results and obtain some new ones with a more straightforward method based on a direct application of the maximum-modulus theorem for holomorphic functions.

We shall consider the form factor, d(t), of the divergence of the weak strangeness-changing vector current, $V_{\mu}^{(K^+)}$, responsible for K_{I3} decays:

$$\frac{1}{2}d(t) = \langle \pi^{0}(p) | i \partial_{\mu} V_{\mu}^{(K^{+})}(0) | K^{+}(k) \rangle$$
$$= \frac{1}{2} [(m_{K}^{2} - m_{\pi}^{2})f_{+}(t) + tf_{-}(t)], \qquad (1)$$

where $t = (p - k)^2$, m_K and m_{π} are the kaon and pion masses, and the f_{\pm} are defined by

$$\langle \pi^{0}(p) | V_{\mu}^{(K^{+})}(0) | K^{+}(k) \rangle = \frac{1}{2} [(k+p)_{\mu} f_{+}(t) + (k-p)_{\mu} f_{-}(t)].$$
(2)

As shown in Ref. 1, d(t) is bounded over the unitarity cut starting at $t_0 \equiv (m_K + m_\pi)^2$ by the spectral function $\rho(t)$ of the propagator of the divergence of $V_{\mu}^{(K^+)}$ through the inequality

$$\left|\frac{d(t')}{m_{\kappa}^{2}}\right| \leq \frac{8\pi}{m_{\kappa}^{2}\sqrt{3}} \frac{\sqrt{t'} \rho^{1/2}(t')}{[(t'-t_{0})(t'-t_{1})]^{1/4}} \equiv g(t'), \quad t' \geq t_{0}$$
(3)

where

 $t_1 \equiv (m_{\kappa} - m_{\pi})^2$

and $\rho(t')$ is given by $\Delta(t) \equiv \int d^4x \ e^{iqx} \langle 0 | \ T(\partial_\mu V^{(K^+)}_\mu(x) \ \partial_\nu V^{(K^-)}_\nu(0)) | \ 0 \rangle$ $\equiv \int_{t_0}^{\infty} \frac{\rho(t')dt'}{t'-t} \quad (q^2 = t) \; .$ (4)

The domain of holomorphy Σ of the functions d(t), $\Delta(t)$ is just the whole complex t plane cut along the real axis from t_0 to ∞ , the boundary region of Σ being formed by the upper and lower borders of the cut.

In the derivation of the subsequent bounds, in addition to the maximum-modulus theorem for holomorphic functions, we shall make use of the fact that $\Delta(t)$ has no zeros inside the domain Σ because of the positivity of the spectral function $\rho \left[\rho(t') \ge \mathbf{0} \right].$

II. BOUNDS ON d(t) AND $f_{\perp}(0)$

First of all we shall consider the analytic function constructed with the aid of g(t') defined in Eq. (3):

$$G(t) = \exp\left(\frac{1}{\pi} (t_0 - t)^{1/2} \int_{t_0}^{\infty} \frac{\ln g(t') dt'}{(t' - t)(t' - t_0)^{1/2}}\right).$$
 (5)

As can immediately be seen, the function G(t) is holomorphic, has no zeros in the domain Σ , and is of modulus g(t') for $t' \ge t_0$ along the cut. Now considering the function

$$M(t) \equiv d(t)/m_{\rm K}^2 G(t) , \qquad (6)$$

we see that, because of the properties of d(t) and G(t), M(t) is holomorphic in the domain Σ (the cut t plane) and, by Eq. (3), is of modulus ≤ 1 on the cut. According to the maximum-modulus theorem for holomorphic functions, from the inequality $|M(t')| \leq 1$ on the boundary region it follows that $|M(t)| \leq 1$ for any t inside the holomorphy region. So one has for any t in the cut t plane the inequality

$$\left|\frac{d(t)}{m_{K}^{2}}\right| \leq |G(t)|,$$

or, using the definition of g(t'), Eq. (3),

$$\left|\frac{d(t)}{m_{k}^{2}}\right| \leq |I(t)| [|Z(t)|]^{1/2}, \qquad (7)$$

where I(t) is a known "kinematical" function given by

$$I(t) = \exp\left[\frac{1}{\pi} (t_0 - t)^{1/2} \int_{t_0}^{\infty} \ln\left(\frac{8\pi\sqrt{t'}}{\sqrt{3}[(t' - t_0)(t' - t_1)]^{1/4}}\right) \times \frac{dt'}{(t' - t)(t' - t_0)^{1/2}}\right],$$
(8)

and Z(t) is defined by

$$Z(t) = \exp\left(\frac{1}{\pi} \left(t_0 - t\right)^{1/2} \int_{t_0}^{\infty} \frac{\ln[\rho(t')/m_K^4] dt'}{(t' - t)(t' - t_0)^{1/2}}\right).$$
(9)

Looking at Z(t) as given by Eq. (9), we see that it is a holomorphic function in the above-considered

domain, and its modulus on the cut (for $t' \ge t_0$) is $\rho(t')/m_{\kappa}^4$. On the border we have

$$Z(t')| = \rho(t')/m_{K}^{4} = \mathrm{Im}\Delta(t')/\pi m_{K}^{4}$$

$$\leq |\Delta(t')|/\pi m_{K}^{4} \quad (t' \geq t_{0}). \quad (10)$$

By Eq. (10) the function $\pi m_K^4 Z(t)/\Delta(t)$, which is holomorphic inside the region $\Sigma [\Delta(t)$ has no zeros in Σ], is of modulus ≤ 1 on the contour (for $t = t' \geq t_0$). So again, by the maximum-modulus theorem we have

$$|Z(t)| \leq |\Delta(t)| / \pi m_K^4 \tag{11}$$

for any t in the complex cut t plane. Equations (7) and (11) then give us the desired result,

$$\left|\frac{d(t)}{m_{K}^{2}}\right| \leq |I(t)| \frac{|\Delta(t)|^{1/2}}{\sqrt{\pi} m_{K}^{2}}, \qquad (12)$$

for an arbitrary t in the cut t plane. In the above inequality the momentum-transfer dependence of the form factor d(t) occurring in K_{13} decays is bounded by the corresponding t dependence of the propagator $\Delta(t)$. Once $\Delta(t)$ is taken as given, the exact bound expressed by Eq. (12) is the best one which can be obtained under the given input. [This can be seen from the fact that among all possible functions $\Delta(t)$, the one with $\operatorname{Re}\Delta(t') = 0$ along the cut is not excluded, so that no information has been lost in the last inequality from Eq. (10).]

For $t < t_0$ a simple computation yields

$$I(t) = \frac{8\pi}{\sqrt{3}} \frac{1 + \delta + [(1+\delta)^2 - t/m_K^2]^{1/2}}{[(1+\delta)^2 - t/m_K^2]^{1/4} [2\sqrt{\delta} + [(1+\delta)^2 - t/m_K^2]^{1/2}]^{1/2}} \quad (\delta \equiv m_\pi/m_K) .$$
(13)

At t=0 we reobtain from Eqs. (12) and (13) the main result of Refs. 2 and 3,

$$|f_{+}(0)| \leq \frac{16\sqrt{\pi} (1+\delta)^{1/2}}{\sqrt{3} (1+\sqrt{\delta})(1-\delta^{2})} \frac{[\Delta(0)]^{1/2}}{m_{K}^{2}}.$$
 (14)

With the estimation given in Refs. 1-3 for $\Delta(0)$, Eq. (14) says that $|f_+(0)| \le 1.01$, which is reasonable in view of the Ademollo-Gatto theorem.

The bound given in Eq. (12) was obtained by taking as input the knowledge of all the momentumtransfer dependence of the propagator $\Delta(t)$. The authors of Refs. 1-3 take as input only the value of $\Delta(t)$ at zero momentum transfer, $\Delta(0)$, so that their bound for the momentum-transfer dependence of d(t) (for $t < t_0$) looks different from our Eq. (12), obtained with $\Delta(t)$ taken as known. In the following we shall rederive Eq. (2.25) of Ref. 3 which expresses a bound on the form factor d(t) for $t < t_0$ when only $\Delta(0)$ is taken as given. In order to do that, we shall first consider the variable transformation $t \to \tau$:

$$t = \tau (1 - T/t_0) + T, \qquad (15)$$

where T is a real point of the t plane such that $T \le t_0$. Through this transformation, which takes the point T of the t plane into the origin of the τ plane, the threshold t_0 of the cut in the t plane is mapped onto the same point t_0 in the τ plane, and the origin of the t plane is mapped onto the point

 $\tau_0 = -T/(1 - T/t_0)$.

We shall use the notation

$$\overline{\Delta}(\tau) \equiv \Delta(t(\tau)), \quad \overline{Z}(\tau) \equiv Z(t(\tau)).$$
(16)

Then

$$\overline{\Delta}(0) = \Delta(T), \quad \overline{Z}(0) = Z(T), \quad (17)$$

and

$$\overline{Z}(\tau) = \exp\left(\frac{1}{\pi} (t_0 - \tau)^{1/2} \int_{t_0}^{\infty} \frac{\ln[\rho(t'(\tau'))/m_K^4] d\tau'}{(\tau' - \tau)(\tau' - t_0)^{1/2}}\right),$$
(18)

$$\overline{\Delta}(\tau) = \int_{t_0}^{\infty} \frac{\rho(t'(\tau')) d\tau'}{\tau' - \tau} .$$
(19)

The quantity $\Delta(0)$, which is taken as known, can be expressed in the form

$$\Delta(0) = \int_{t_0}^{\infty} \frac{\rho(t'(\tau')) d\tau'}{\tau' - \tau_0}$$
(20)

or, equivalently, by defining

$$R(\tau') \equiv \frac{\rho(t'(\tau'))\tau'}{\tau' - \tau_0}, \qquad (21)$$

 \mathbf{as}

$$\Delta(0) = \int_{t_0}^{\infty} \frac{R(\tau')d\tau'}{\tau'} \,. \tag{22}$$

Now let us introduce the new functions $\overline{Z}_R(\tau)$ and $\overline{\Delta}_R(\tau)$:

$$\overline{Z}_{R}(\tau) \equiv \exp\left(\frac{1}{\pi} (t_{0} - \tau)^{1/2} \int_{t_{0}}^{\infty} \frac{\ln[R(\tau')/m_{K}^{4}] d\tau'}{(\tau' - \tau)(\tau' - t_{0})^{1/2}}\right),$$
(23)

$$\overline{\Delta}_{R}(\tau) \equiv \int_{t_{0}}^{\infty} \frac{R(\tau')d\tau'}{\tau' - \tau} .$$
(24)

By reasoning (now in the τ plane) similar to that used in deriving Eq. (11) from Eq. (10), we have

$$\left|\overline{Z}_{R}(\tau)\right| \leq \left|\overline{\Delta}_{R}(\tau)\right| / m_{K}^{4} \pi , \qquad (25)$$

$$\left|\overline{Z}_{R}(0)\right| \leq \Delta(0)/m_{K}^{4}\pi, \qquad (26)$$

where we took into account the relation

$$\Delta(t=0) = \overline{\Delta}_R (\tau=0) . \tag{27}$$

Using Eqs. (18), (21), and (23) we have

$$\overline{Z}(0) = P(T)\overline{Z}_R(0), \qquad (28)$$

where

$$P(T) \equiv \exp\left(\frac{\sqrt{t_0}}{\pi} \int_{t_0}^{\infty} \frac{\ln[(\tau' - \tau_0)/\tau'] d\tau'}{\tau'(\tau' - t_0)^{1/2}}\right)$$
$$= \frac{\left[\sqrt{t_0} + (t_0 - \tau_0)^{1/2}\right]^2}{4t_0}$$
$$= \frac{1}{4} \frac{\left[1 + (1 - T/t_0)^{1/2}\right]^2}{1 - T/t_0} .$$
(29)

Now, writing Eq. (7) for t = T in the equivalent form

$$\left|\frac{d(T)}{m_{K}^{2}}\right| \leq |I(T)| \left[\left|\overline{Z}(\tau=0)\right|\right]^{1/2}$$
(30)

and using Eqs. (26), (28), and (29), we finally have

$$\left| \frac{d(T)}{m_{K}^{2}} \right| \leq \frac{4\sqrt{\pi}}{\sqrt{3}} \frac{\sqrt{\Delta(0)}}{m_{K}^{2}} \frac{\left[1 + (1 - T/t_{0})^{-1/2} \right]^{2}}{\left\{ 1 + \left[2\sqrt{\delta} / (1 + \delta) \right] (1 - T/t_{0})^{-1/2} \right\}^{1/2}} (T < t_{0}), \quad (31)$$

which is exactly the result of Okubo³ given in his

Eq. (2.25).

III. A BOUND ON d'(0)

Taking as given quantities $\Delta(0)$ and $\Delta'(0)$ [the values of the propagator $\Delta(t)$ and of its first derivative with respect to t at zero momentum transfer], we shall find here the best possible bound (under this given input) for d'(0), the derivative of the divergence form factor d(t) at t=0.

We first recall the connection between d'(0) and the parameters $\xi \equiv f_{-}(0)/f_{+}(0)$, $\lambda_{+} \equiv m_{\pi}^{2}f_{+}'(0)/f_{+}(0)$ measured in K_{13} decays:

$$d'(0) = f_{+}(0) \left(\xi + \frac{m_{K}^{2} - m_{\pi}^{2}}{m_{\pi}^{2}} \lambda_{+}\right).$$
(32)

To get the desired result more quickly we prefer now to work on the unit circle of a new complex plane z. The needed conformal transformation which maps the domain Σ onto the unit circle in the z plane is

$$t = 4t_0 z / (1+z)^2 . (33)$$

The upper and lower borders of the cut in the t plane map onto the upper and lower semicircles, and the points z = 1, z = 0, z = -1 correspond, respectively, to the points $t = t_0$, t = 0, $t = \infty$.

We will make use of the following known result: If f(z) is a holomorphic function in the unit circle $|z| \le 1$ such that $|f(z')| \le 1$ on the boundary region [hence $|f(z)| \le 1$ for z interior as well], then

$$|f'(0)| \leq 1 - |f(0)|^2$$
 (34)

Once $f_{+}(0)$ is taken as known, Eq. (34) represents the best bound on |f'(0)| because one can find a certain particular function f(z) satisfying all the above requirements for which the inequality (34) becomes just an equality. Such a function is, for example, $[z+f(0)]/[1+zf^*(0)]$.

Equation (34) can be deduced immediately, for instance, by defining the new function

$$f_1(z) \equiv \frac{1}{z} \frac{f(z) - f(0)}{1 - f^*(0)f(z)}.$$
(35)

From Eq. (35) we see that

$$|f_1(z)| \leq 1 \quad \text{for} \quad |z| \leq 1. \tag{36}$$

[The inequality clearly holds for |z| = 1 and, because $f_1(z)$ is holomorphic inside the circle, it holds for $|z| \le 1$ as well.] Now, using the expression for f(z) in terms of $f_1(z)$ from Eq. (35) and taking the derivative at z = 0, one has

$$f'(0) = f_1(0) [1 - |f(0)|^2]$$

This relation [together with Eq. (36) for z=0] then proves the inequality (34) which we are going to use in the following.

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We come back again to the t plane, noting that

$$\frac{d}{dz}\Big|_{z=0} = 4t_0 \frac{d}{dt}\Big|_{t=0},$$
(37)

so that for any function of t, f(t) [f(t) = f(z(t))]holomorphic in Σ and of modulus $|f(t')| \leq 1$ on the cut $(t' \ge t_0)$, we have the inequality

$$\left|\frac{df(t)}{dt}\right|_{t=0} \le \frac{1}{4t_0} \left[1 - |f(0)|^2\right].$$
(38)

Looking at Eq. (12) we see that the theorem (38)

$$\left|\frac{1}{m_{K}^{2}-m_{\pi}^{2}}\left(\xi+\frac{m_{K}^{2}-m_{\pi}^{2}}{m_{\pi}^{2}}\lambda_{+}\right)-\frac{V'(0)}{V(0)}\right| \leq \frac{1}{4(m_{K}+m_{\pi})^{2}}\left(\left|\frac{V(0)}{f_{+}(0)(1-m_{\pi}^{2}/m_{K}^{2})}\right|\right)$$

with

$$\frac{V'(0)}{V(0)} = \frac{I'(0)}{I(0)} + \frac{1}{2} \frac{\Delta'(0)}{\Delta(0)} .$$
(42)

 $\Delta(0)$ and $\Delta'(0)$ are envisaged here as known quantities, while I(0) and I'(0) can be immediately found from the concrete expression, Eq. (8), of I(t). Equation (41) represents the best bound on d'(0)compatible with the given input information [represented practically by the knowledge of $\Delta(0)$ and $\Delta'(0)$].

IV. A RESTRICTION ON THE DOMAIN OF POSSIBLE VALUES OF d(t) AT GIVEN $f_{\perp}(0) \text{ AND } \Delta(t)$

Up to this point we did not take $f_{+}(0)$ as given, but we have established a bound on it, Eq. (14). As far as $f_{+}(0)$ is not taken as given, the only input being $\Delta(t)$, Eq. (12) represents the strongest restriction we can have on d(t). However, if we take $f_{+}(0)$ [that is, d(0)] as a known number [and consider also that all the momentum-transfer dependence of the propagator $\Delta(t)$ is given, as has been done in getting Eq. (12)], we can obtain a better bound on d(t) than the one expressed by Eq. (12). This will be shown in the following.

We shall come back now to the unit circle in the z plane, and we shall write Eq. (12) in the form

$$|f(z)| \leq 1. \tag{43}$$

where

$$f(z) \equiv d(z)/V(z) . \tag{39'}$$

We have used the obvious notations d(z) = d(t(z)), $V(z) \equiv V(t(z))$. V(z) [from Eq. (40)] and the quantity m defined by

$$m \equiv f(0) = d(0)/V(0)$$
$$= \frac{(m_{\kappa}^{2} - m_{\pi}^{2})f_{+}(0)\sqrt{\pi}}{I(0)\sqrt{\Delta(0)}}, \qquad (44)$$

should be applied for

 $f(t) \equiv d(t)/V(t)$,

with

$$V(t) = \pi^{-1/2} I(t) [\Delta(t)]^{1/2} .$$
(40)

The final result is

$$\left| \frac{d'(0)}{d(0)} - \frac{V'(0)}{V(0)} \right| \leq \frac{1}{4t_0} \left[\left| \frac{V(0)m_K^2}{d(0)} \right| - \left| \frac{d(0)}{m_K^2 V(0)} \right| \right]$$

or (41a)

$$\left| \leq \frac{1}{4(m_{K}+m_{\pi})^{2}} \left(\left| \frac{V(0)}{f_{+}(0)(1-m_{\pi}^{2}/m_{K}^{2})} \right| - \left| \frac{f_{+}(0)(1-m_{\pi}^{2}/m_{K}^{2})}{V(0)} \right| \right), \quad (41b)$$

where

$$I(0) = \frac{16\pi}{\sqrt{3}} \frac{(1+\delta)^{1/2}}{1+\sqrt{\delta}} , \quad \delta \equiv \frac{m_{\pi}}{m_{K}}$$

are viewed here as given. We ask now what is the range of values which can be taken by a certain holomorphic function f(z) defined inside the unit circle in the z plane and satisfying the condition that $|f(z)| \leq 1$ for $|z| \leq 1$, when, in addition, one knows its value at the origin f(0) = m ($|m| \le 1$, of course). The answer to this question is the following.⁴ In the plane ($\operatorname{Re} f(z), \operatorname{Im} f(z)$), f(z) can be found in the smaller circle Γ defined by

$$|f(z) - \gamma(z)| \le \rho_0(z) \tag{45}$$

[included in the circle $|f(z)| \leq 1$] with the center $\gamma(z)$ and radius $\rho_0(z)$ given by

$$\gamma(z) = \frac{1}{m^*} \frac{|m|^2 - |zm|^2}{1 - |zm|^2}, \qquad (46)$$

$$\rho_0(z) = |z| \frac{1 - |m|^2}{1 - |zm|^2} . \tag{47}$$

This result can be easily proved on the basis of Eq. (35). Indeed, the new defined function $f_1(z)$ is now free of any restriction, except that the modulus is not greater than 1 for any z, $|z| \le 1$, unlike the case of the function f(z) which satisfied the supplementary condition f(0) = m with $m (|m| \le 1)$ given. Now, as one can check immediately by doing a little algebra, when $f_1(z)$ is in the unit circle $|f_1(z)| \leq 1$, the function f(z) in which we are interested will be somewhere in the circle Γ with the center $\gamma(z)$ and radius $\rho_0(z)$ given by Eqs. (46) and (47). So, using Eqs. (45) and (39') we have our desired result

$$|d(z) - V(z)\gamma(z)| \leq \rho_0(z) |V(z)| \quad \text{for} \quad |z| \leq 1.$$
(48)

To come back to the t variable, we have only to remember that

(39)

$$z = \frac{1 - (1 - t/t_0)^{1/2}}{1 + (1 - t/t_0)^{1/2}}$$
(33')

[V(t) is given by Eq. (40)].

Equation (48), when $f_{+}(0)$ and $\Delta(t)$ are known quantities, gives all possible restrictions on d(t), for any t in the complex cut t plane, not only for the modulus |d(t)|, but for the corresponding phase as well [or, in other terms, for both the real and imaginary parts of d(t).⁵ The bound furnished by Eq. (48) on the divergence form factor d(t) could become of practical use in discussing the K_{13} data when a certain detailed knowledge of the momentum-transfer dependence of the propagator $\Delta(t)$ will be available.

We note also that, conversely, when the experimental K_{13} data are taken into account, our new results give important conditions on $\Delta(t)$ which could be very helpful in studying other physical problems involving this propagator.

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⁵The use of the reality condition on the form factor $d(t) [d^{*}(t^{*}) = d(t)]$ does not improve the bound expressed

by Eq. (48) for real t, in particular for t belonging to the physical K_{13} decay region in which we are mainly inter-

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Approach to the Inclusion of Spin in the Veneziano Model

for $K^{-}p \rightarrow \overline{K}^{0}\pi^{-}p$ and $K^{-}p \rightarrow K^{*-}p$

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A model combining the U(6, 6) supermultiplet scheme with the generalized Veneziano model as a means of including spin in the latter is applied to the process $K^- p \to \overline{K}^0 \pi^- p$. A fourpoint reduction of the amplitude, describing the subprocess $K^- p \rightarrow K^{*-}(890) p$, is obtained and its predictions compared with available data. Extremely favorable agreement with the data is obtained.

I. INTRODUCTION

In a recent paper¹ on $K^-p \rightarrow \pi^-\pi^+\Lambda$, a method for including both spin and unitary spin in the generalized Veneziano model was employed. The prescription² involved combining the U(6, 6) supermultiplet formalism with the Veneziano spinless amplitude in such a way that, for nonzero-spin external particles, one obtains a well-defined kinematic factor for each contributing diagram. In Sec. II we give further details of the model and derive the $K^-p \rightarrow \overline{K}{}^0\pi^-p$ amplitude. In Sec. III we extract the $K^*(890)$ contribution in the $\overline{K}{}^0\pi^-$ channel and obtain the four-point amplitude for the subprocess

 $K^- p \rightarrow K^{*}(890) p$. The main results and predictions of the model are discussed in Sec. IV.

II. MODEL

The generalized Veneziano amplitude for 3-particle production processes is, in general, a sum of 12 diagrams, corresponding to the noncyclic permutations of the external particles. To take account of external particles with nonzero spins. we modify the Veneziano amplitude by writing it in the form

$$A = \sum_{i=1}^{12} F_i B_5^i, \qquad (1)$$