ularly when combined with a longitudinal analysis. The correlation of any pair of final-state particles can be thus studied. For example, the correlation of particles in differing charge states should provide information on interactions and resonances in the final state. Correlations of leading forward- and backward-moving particles will test fundamental assumptions of hadronic models. This freedom should also be of great utility in delineating the properties intrinsic to, and connections between, the secondaries assignable to the fragmentation and pionization regions on the basis of longitudinal momenta.

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PHYSICAL REVIEW D VOLUME 5, NUMBER 1 1 JANUARY 1972

Wide-Angle $\pi^- p \to \pi^0 n$ Data and the ρ Trajectory*

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New data on the $\pi^- p \to \pi^0 n$ differential cross section in the wide-angle region are compared with an asymptotic form of the Regge amplitude valid in that limit. The ρ trajectory is approximately linear in t out to $t \approx -7$ GeV^2, but shows indications of preferring a less steep dependence such as $\alpha_{1/2}(t) = 2.11 - 2.65(-t)^{1/2}$ in the region $2 \lesssim -t \lesssim 7$ GeV^2.

Using data obtained by Brockett *et al.*,[1] Barger and Phillips have recently reported[2] that the $\pi^- p \to \pi^0 n$ differential cross section in the wide-angle region may be parametrized as

$$\frac{d\sigma}{dt}(\nu, t) = r(t)\nu^{2\alpha(t)-2}, \quad (1)$$

where $\nu = (s-u)/4M$ is the crossing-antisymmetric energy variable. Based upon a fit of this type, they show that $\alpha(t) = 0.55 + t$ is in fair agreement with the data out to $t = -5$ GeV^2.

It has been pointed out,[3] however, that the approximation $P_{\alpha(t)}(-\cos\theta_t) \sim \nu^{\alpha(t)}$, from which (1) follows for small t, is not valid in the wide-angle region. The reason is that

$$-\cos\theta_t = (s-u)/[(t-4\mu^2)(t-4m^2)]^{1/2} \quad (2)$$

is not large when $-t$ is comparable to s. (For the data of Ref. 1, for example, $1.3 < -\cos\theta_t < 4$.) The usual asymptotic expansion of a Regge-pole term is therefore not valid, and consequently the $\alpha(t)$ obtained by Barger and Phillips does not correspond in any simple way with the ρ Regge trajectory.

Instead, we have shown that a more suitable approximation to the scattering amplitude of a single Regge pole, valid in the wide-angle region if $\alpha(t) \lesssim -\frac{1}{2}$, is given by

$$F(x,t) = R(t)[x(1+x)]^{-1/4} \times \exp\{[\alpha(t)+\tfrac{1}{2}]\operatorname{arc\,cosh}(2x-1)\}, \quad (3)$$

where $2x-1=\cos\theta_t$. It can be shown by expanding $F(x,t)$ as a power series in $\cos\theta_t$ that (3) differs considerably from the Barger-Phillips form. A measurement of the ρ Regge trajectory can thus be made by comparing the differential cross section with (3) at fixed t, as a function of x. Since x is a known function of s and t via (2), this procedure is equivalent to the usual comparison at fixed s, except that it uses a more meaningful variable. The ratio $|F(x_1,t)|^2/|F(x_2,t)|^2$, which one can obtain directly from experimental differential cross sections, is independent of the residue $R(t)$ and thus provides a direct measurement of $\alpha(t)$. The results of such a comparison, using the data of Ref. 1 at incident momenta 3.67 and 4.83 GeV/c, plus preliminary data at 5.90 GeV/c by the same group,[4] are shown as data points in Fig. 1; they differ from the values obtained by Barger and Phillips in that they are somewhat higher at larger values of $-t$.

In order to contrast the two amplitudes and to compare linear and nonlinear parametrizations of $\alpha(t)$, we have fitted the differential cross section corresponding to the amplitude (3) to these data. Using a linear parametrization of $\alpha(t)$, we find a best fit with

$$\alpha_1(t) = (0.39 \pm 0.11) + (0.93 \pm 0.05)t, \quad (4)$$

which is compared with the "direct measurements" in Fig. 1. This trajectory is consistent with the ρ trajectory measured at near-forward angles, although it could be interpreted as becoming slightly flatter.

The results of Barger and Phillips, on the other hand, seem to indicate a possible steepening of their $\alpha(t)$ for $-t \gtrsim 2$ GeV^2. To verify this point, we fitted the same data to a single power of $\cos\theta_t$, as in (1), obtaining a somewhat poorer fit with

$$\alpha_{BP}(t) = (0.76 \pm 0.02) + (1.12 \pm 0.01)t\ . \quad (5)$$

We also tested the linearity of $\alpha(t)$ in (3) by using nonlinear parametrizations of it. Adding a term quadratic in t produced no significant change in either the fit or the trajectory. Using a trajectory linear in $(-t)^{1/2}$ rather than t produced appreciable improvement, however, the best fit having

$$\alpha_{1/2}(t) = (2.11 \pm 0.15) - (2.65 \pm 0.12)(-t)^{1/2}\ . \quad (6)$$

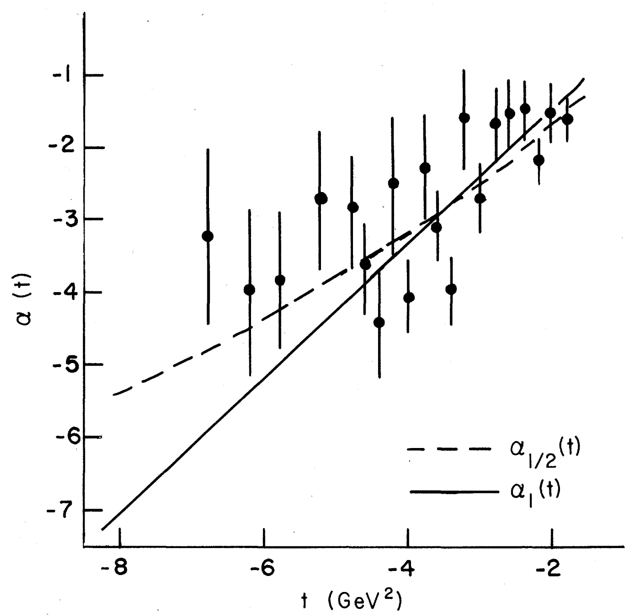

FIG. 1. Measured values of the ρ Regge trajectory, obtained by comparing data at different s values for *fixed t*, are shown as data points. The curves show trajectories obtained by fitting the wide-angle Regge amplitude to all the data, using a simply parametrized residue.

As shown also in Fig. 1, this trajectory is almost identical with (4) in the region $2 \lesssim -t \lesssim 5$ GeV^2, while for larger $-t$ it becomes somewhat flatter, in better agreement with the "measured" values.

In all of these fits we used the simplest possible parametrization of the residue, $R(t)=Ae^{bt}$, and consequently the fits did not reproduce very well the detailed structure in t of the data. The χ^2 values are correspondingly rather high, namely, 219, 268, and 168 for the fits yielding (4), (5), and (6), respectively, with 72 data points and four parameters. (The goodness of fit can be improved with more complicated residues, but the results then become less sensitive to the trajectories.) By comparing these values, however, we may conclude that:

(a) A single-Regge-pole term (3) is in better agreement with the data than a single power of $\cos\theta_t$.

(b) The ρ trajectory, although consistent with linearity in t, seems to prefer a less rapid decrease for large $-t$.

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Bounds on K_{l3}-Decay Form Factors

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We rederive certain bounds on K_{l3} decay parameters found recently by Li, Pagels, and Okubo and obtain some new results with a simpler method based on a direct application of the maximum-modulus theorem for holomorphic functions. For an arbitrary value of the momentum-transfer variable t in the complex cut t plane, we find that the domain of values which can be taken by the form factor $d(t)$ of the divergence of the weak strangeness-changing vector current $V_\mu^{(K)}$ is bounded by a circle in the plane $(\text{Re}d(t), \text{Im}d(t))$. We express the radius and the position of the center of this circle in terms of $f_+(0)$ and of the propagator $\Delta(t)$ of the divergence of $V_\mu^{(K)}$.

I. INTRODUCTION

Recently Li, Pagels,[1,2] and Okubo[3] have established rigorous bounds on K_{l3}-decay form factors, some of the results obtained being the best bounds one can obtain under the given input information. In this paper we shall rederive their results and obtain some new ones with a more straightforward method based on a direct application of the maximum-modulus theorem for holomorphic functions.

We shall consider the form factor, $d(t)$, of the divergence of the weak strangeness-changing vector current, $V_\mu^{(K^+)}$, responsible for K_{l3} decays:

$$\tfrac{1}{2}d(t) = \langle \pi^0(p) | i\partial_\mu V_\mu^{(K^+)}(0) | K^+(k)\rangle = \tfrac{1}{2}[(m_K{}^2 - m_\pi{}^2)f_+(t) + tf_-(t)], \quad (1)$$

where $t = (p-k)^2$, m_K and m_π are the kaon and pion masses, and the $f_\pm$ are defined by

$$\langle \pi^0(p) | V_\mu^{(K^+)}(0) | K^+(k)\rangle = \tfrac{1}{2}[(k+p)_\mu f_+(t) + (k-p)_\mu f_-(t)]. \quad (2)$$

As shown in Ref. 1, $d(t)$ is bounded over the unitarity cut starting at $t_0 \equiv (m_K + m_\pi)^2$ by the spectral function $\rho(t)$ of the propagator of the divergence of $V_\mu^{(K^+)}$ through the inequality

$$\left|\frac{d(t')}{m_K{}^2}\right| \leq \frac{8\pi}{m_K{}^2\sqrt{3}} \frac{\sqrt{t'}\,\rho^{1/2}(t')}{[(t'-t_0)(t'-t_1)]^{1/4}} \equiv g(t'), \quad t' \geq t_0 \quad (3)$$

where

$$t_1 \equiv (m_K - m_\pi)^2$$

and $\rho(t')$ is given by

$$\Delta(t) \equiv \int d^4x\, e^{iqx} \langle 0 | T(\partial_\mu V_\mu^{(K^+)}(x)\ \partial_\nu V_\nu^{(K^-)}(0)) | 0\rangle \equiv \int_{t_0}^{\infty} \frac{\rho(t')dt'}{t'-t} \quad (q^2 = t). \quad (4)$$

The domain of holomorphy Σ of the functions $d(t)$, $\Delta(t)$ is just the whole complex t plane cut along the real axis from t_0 to ∞, the boundary region of Σ being formed by the upper and lower borders of the cut.

In the derivation of the subsequent bounds, in addition to the maximum-modulus theorem for holomorphic functions, we shall make use of the fact that $\Delta(t)$ has no zeros inside the domain Σ because of the positivity of the spectral function ρ $[\rho(t') \geq 0]$.

II. BOUNDS ON $d(t)$ AND $f_+(0)$

First of all we shall consider the analytic function constructed with the aid of $g(t')$ defined in Eq. (3):

$$G(t) \equiv \exp\left(\frac{1}{\pi}(t_0 - t)^{1/2} \int_{t_0}^{\infty} \frac{\ln g(t')dt'}{(t'-t)(t'-t_0)^{1/2}}\right). \quad (5)$$

As can immediately be seen, the function $G(t)$ is holomorphic, has no zeros in the domain Σ, and is of modulus $g(t')$ for $t' \geq t_0$ along the cut. Now considering the function

$$M(t) \equiv d(t)/m_K{}^2 G(t), \quad (6)$$