$$V(^3S-^3D)=(-10.463e^{-x}+102.012e^{-2x}-2915e^{-4x}+7800e^{-6x})/x. \quad (57)$$

For particles with antiparallel spin and isospin, $T_z=0$ and $S_z=0$, the potential is an average of $V(^3A)$ $V(^1P_1)$, $V(^3S-^3D)$, and $V(^1S)$ or $V(^1D)$.

$$V(^1P_1)=(31.389e^{-x}-634.39e^{-2x}+2163.4e^{-3x})/x. \quad (58)$$

Owing to the fact that for a crystal system $4\approx k_F a$, the $V(^1D)$ potentials were used for the crystal calculations, while the $V(^1S)$ potentials were used with plane waves. This was not found to affect significantly the results, those with $V(^1D)$ being only about 10% higher than those with $V(^1S)$.

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Gravitation Theory and Oscillating Universe

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The field equations of a noncovariant theory of gravitation, based on the existence of a preferred frame of reference in the universe, are applied to the homogeneous isotropic cosmological model. One is naturally led to a particular value of the previously undetermined constant present in the equations. The second-order equation determining the radius of the universe can be integrated to give a first-order equation similar to that of general relativity, but with an additional term that can lead to oscillations without any singular state. One obtains a conservation law for the total energy, which is found to be positive definite.

I. INTRODUCTION

Recently a noncovariant theory of gravitation was proposed by one of the authors,[1,2] based on the idea that there exists a preferred frame of reference in the universe, determined by the distribution of matter and energy. The field equations chosen were associated with a variational integral which depended on three constants. By considering the case of a weak, static field and comparing the field equations with those of Newton and Einstein one could fix two of the constants, while the third remained undetermined. However, it turned out that, in the case of a static, spherically symmetric field in empty space, the theory agreed with the general relativity theory to the accuracy required for comparison with the well-known crucial tests, for arbitrary values of this constant.

In principle, the remaining constant could be determined from observations on gravitational waves. However, it would be desirable to fix this constant by means of theoretical considerations. The purpose of the present paper is to attempt to do this by applying the field equations to a model of the universe. Aside from the question of the value of the constant, such an application is of interest since, after all, the theory was arrived at from cosmological considerations.

II. APPLICATION TO COSMOLOGY

In the paper referred to[1] it is assumed that, in the preferred frame of reference and with a suitable choice of coordinate system, one can write the line element in the form

$$ds^2 = \Phi^2 dt^2 - \Psi^2(dx^2 + dy^2 + dz^2)\,. \tag{1}$$

The functions Φ and Ψ are determined by two field equations, involving these functions and their first and second derivatives and also the quantities

$$\rho = T_0^{\ 0}, \quad p = -\tfrac{1}{3}T_k^{\ k}, \tag{2}$$

where the tensor $T^{\mu\nu}$ is assumed to satisfy the relation

$$T^{\mu\nu}{}_{;\nu} = 0\,. \tag{3}$$

To begin with, the field equations contain three constants, α, β, γ. However, by comparing the equations with those of Newton and Einstein in the case of a weak static field one arrives at the relations

$$\beta = \alpha + 2\,, \quad \gamma = 2\alpha\,,$$

so that only the constant α is left in the field equations.

Let us now consider the case of a homogeneous, isotropic universe, the preferred system being the comoving system. In the line element of (1) one has

$$\begin{aligned}&\Phi = \Phi(t)\,,\\ &\Psi = R/R_0(1 + kr^2/4R_0^{\ 2})\,, \quad R = R(t)\end{aligned} \tag{4}$$

where k, describing the curvature, can be ±1 or 0, and R might be referred to as the radius of the universe. If one writes down the field equations[1] and makes use of (4), one gets relations which can be combined to give the following equations:

$$\frac{\ddot{\Phi}}{\Phi} + \alpha\frac{\dot{\Phi}^2}{\Phi^2} + (3 + 2\alpha)\frac{\dot{\Phi}\dot{R}}{\Phi R} + \alpha\frac{\dot{R}^2}{R^2} = -4\pi\Phi^2(p + 3p)\,, \tag{5}$$

$$\frac{\ddot{R}}{R} - (\tfrac{7}{2} + \alpha + 3/\alpha)\frac{\dot{\Phi}^2}{\Phi^2} - (4 + 2\alpha)\frac{\dot{\Phi}\dot{R}}{\Phi R} + (\tfrac{1}{2} - \alpha)\frac{\dot{R}^2}{R^2} + \tfrac{3}{2}k\frac{\Phi^2}{R^2} = 4\pi\Phi^2[\rho + (3 + 6/\alpha)p]\,. \tag{6}$$

Here dots denote derivatives with respect to t. The density ρ and the pressure p are functions only of t, and Eq. (3) gives the relation

$$\dot{\rho} + 3(\dot{R}/R)(\rho + p) = 0\,. \tag{7}$$

Let us now take advantage of the fact that Φ depends only on t by introducing a new time variable T defined by the relation

$$\frac{dT}{dt} = \Phi\,. \tag{8}$$

Then the line element (1) takes the form

$$ds^2 = dT^2 - [R^2/R_0^{\ 2}(1 + kr^2/4R_0^{\ 2})^2](dx^2 + dy^2 + dz^2)\,, \tag{9}$$

where R is now a function of T.

If one now rewrites Eqs. (5) and (6) in terms of the new variable, using primes to denote derivatives with respect to T, one obtains

$$\frac{\Phi''}{\Phi} + (1 + \alpha)\frac{\Phi'^2}{\Phi^2} + (3 + 2\alpha)\frac{\Phi' R'}{\Phi R} + \alpha\frac{R'^2}{R^2} = -4\pi(\rho + 3p)\,, \tag{10}$$

$$\frac{R''}{R} - (\tfrac{7}{2} + \alpha + 3/\alpha)\frac{\Phi'^2}{\Phi^2} - (3 + 2\alpha)\frac{\Phi' R'}{\Phi R} + (\tfrac{1}{2} - \alpha)\frac{R'^2}{R^2} + \frac{3}{2}\frac{k}{R^2} = 4\pi[\rho + (3 + 6/\alpha)p]\,. \tag{11}$$

In Eq. (7) one can simply replace the dots by primes.

Since in Eq. (9) only the function R occurs, the function Φ having disappeared from the line element, one would expect to determine R by means of a second-order equation not depending on Φ. One readily sees that this will be the case if, and only if,

$$\alpha = -\tfrac{3}{2}\,. \tag{12}$$

It seems reasonable therefore to adopt this value for α. Equation (11) then has the form

$$\frac{R''}{R} + 2\frac{R'^2}{R^2} + \frac{3}{2}\frac{k}{R^2} = 4\pi(\rho - p)\,. \tag{13}$$

Having solved this equation for R, one can go back to Eq. (10), which now has the form

$$\frac{\Phi''}{\Phi} - \frac{1}{2}\frac{\Phi'^2}{\Phi^2} - \frac{3}{2}\frac{R'^2}{R^2} = -4\pi(\rho + 3p)\,, \tag{14}$$

and solve it for Φ. One can raise the question: What is the physical significance of Φ in the coordinate system having the line element (9)? We shall see below that Φ is associated with the gravitational energy density.

It should be remarked that, for $\alpha = -\frac{3}{2}$, the gravitational energy density in the general case is not positive definite. However, the requirement is satisfied that a physical system emitting gravitational radiation must lose energy.[3]

III. OSCILLATING UNIVERSE

Let us go back to Eq. (13). It turns out that it can be integrated. This follows from the fact that Eq. (7) is equivalent to the relation

$$\frac{d(\rho R^6)}{dR}=3(\rho - p)R^5 . \tag{15}$$

If one multiplies Eq. (13) by $2R^5R'$ and makes use of (15) one obtains after integration

$$\frac{R'^2}{R^2}=\frac{8\pi}{3}\rho - \frac{3}{4}\frac{k}{R^2}-\frac{A}{R^6}, \tag{16}$$

where A is a constant of integration.

It is interesting to compare this equation with that given by the general relativity theory[4] for the case of the line element (9) if we take the cosmological constant Λ to vanish,

$$\frac{R'^2}{R^2}=\frac{8\pi}{3}\rho - \frac{k}{R^2}. \tag{17}$$

One sees that, except for the slightly different coefficient of the second term, the essential difference between the equations is in the presence of the third term on the right-hand side of Eq. (16).

Now, if one takes $k=1$ and $A>0$, this term can lead to oscillations of the universe. This will be the case if there is a domain of values of R for which ρ is sufficiently large so that the right-hand member of (16) is positive. Since, with any reasonable equation of state, for large values of R ρ decreases with increasing R more rapidly than $1/R^2$, and for small values of R ρ increases with decreasing R more slowly than $1/R^6$, one sees from (16) that there will be two finite turning points of R as a function of T. Oscillations will take place between these turning points, so that the universe never goes through a singular state.

It might be remarked that in the past one of the authors,[5] in seeking a way to obtain an oscillatory behavior of the universe without a singular state, assumed the existence of a "cosmic field" described by a scalar and thus obtained an equation for R essentially of the same form as (16). We see that, in the framework of the present theory, such an assumption is not necessary.

IV. ENERGY CONSERVATION

It has been shown[1] that one can define the gravitational energy-momentum density $\theta_{\mu\nu}$ so that a conservation law holds:

$$[(-g)^{1/2}(T_\mu{}^\nu+\theta_\mu{}^\nu)]_{,\nu}=0 . \tag{18}$$

Using the expression given previously,[1] one finds that with t as the time coordinate

$$16\pi\theta_{00}=\frac{1}{2}\frac{\dot\Phi^2}{\Phi^2}-\frac{3}{2}\frac{\dot R^2}{R^2}-\frac{3\dot\Phi\dot R}{\Phi R}-\frac{3}{8}\frac{k^2r^2\Phi^2}{R_0{}^2R^2}, \tag{19}$$

$$16\pi\theta_{0j}=\frac{\frac{3}{2}k(\dot\Phi/\Phi+\dot R/R)x^j}{R_0{}^2(1+kr^2/4R_0{}^2)}. \tag{20}$$

Equation (18) then gives for $\mu=0$

$$\left(32\pi\Phi R^3\rho+\frac{R^3\dot\Phi^2}{\Phi^3}-\frac{3R\dot R^2}{\Phi}-\frac{6R^2\dot\Phi\dot R}{\Phi^2}-9k\Phi R\right)_{,0}=0 . \tag{21}$$

If one goes over from t to T, one can write

$$\Phi R^3\left(32\pi\rho+\frac{\Phi'^2}{\Phi^2}-\frac{3R'^2}{R^2}-\frac{6\Phi'R'}{\Phi R}-\frac{9k}{R^2}\right)=\text{const} . \tag{22}$$

The left-hand member is proportional to the total energy. If one eliminates ρ by using Eq. (16), one obtains

$$\Phi R^3\left[\left(\frac{\Phi'}{\Phi}-\frac{3R'}{R}\right)^2+\frac{12A}{R^6}\right]=\text{const} . \tag{23}$$

We see that the energy (with $A>0$) is positive definite.

One can integrate the conservation relation (23) to obtain the solution for Φ. If one denotes the right-hand constant by $2B^2$, one finds in the general case

$$\Phi=(B^2/3A)R^3\sin^2\left[(3A)^{1/2}\int dT/R^3+C\right], \tag{24}$$

where C is an integration constant.

For $A=0$ the solution is given by

$$\Phi=R^3\left(B\int dT/R^3+D\right)^2, \tag{25}$$

where D is a constant of integration. For $B=0$, Φ is proportional to R^3.

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