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Nonexistence of Baryon Number for Static Black Holes*

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Wheeler has conjectured that black holes should have no well-defined baryon number, and that as a result the law of conservation of baryons should be transcended in black-hole physics. We show here that a static black hole cannot have any exterior classical scalar or massive vector fields. We consider the modifications that would arise from a quantum-theoretical treatment, and we conclude that such a black hole cannot interact with the exterior world via virtual mesons such as the π and ρ. Because of this we find no way for external measurements to assign unambiguously a baryon number to such a black hole in agreement with Wheeler's prediction.

I. INTRODUCTION

General relativity has led to the inescapable conclusion that massive stars must collapse catastrophically at the end of their thermonuclear evolution, possibly to form black holes.[1] Briefly, a black hole is a region of space separated from its exterior by an horizon or one-way membrane which prevents any interior particles or light rays from ever escaping to the exterior. It does not follow that an exterior observer can learn nothing about a black-hole interior because any exterior field associated with the black hole can yield information about the interior. For example, the electromagnetic field associated with a charged black hole would allow such an observer to determine the charge of the black hole.

For simplicity one often focuses attention on exteriors of stationary, bare black holes, i.e., those which are time-independent and devoid of foreign material such as dust or radiation. With an exception, the most general exact solution of Einstein's equations known which represents such black-hole exteriors is the Kerr-Newman solution.[2] It describes a family of black holes parametrized only by mass, charge, and angular momentum. It is currently believed (Israel-Carter conjecture) that all exteriors of stationary, bare, single black holes are of the Kerr-Newman type; they have associated with them no independent properties other than mass, charge, and angular momentum.[3] Considerable progress has already been made toward the proof of the conjecture, primarily as a result of the work of Israel,[4] Carter,[5] Hawking,[6] and Wald.[7] Also important are the results of Price,[8] Chase,[9] Hartle,[10] and Teitelboim.[11]

In the present paper we shall be concerned with one prediction of the conjecture, namely, that stationary bare black holes have no such properties as baryon or strangeness numbers. That this should be the case has repeatedly been emphasized by Wheeler.[12] It is known that scalar mesons such as the π meson and vector mesons such as the ρ meson are responsible for mediating the strong interactions. One could argue that a black hole formed out of strongly interacting material, i.e., nuclei of stellar material, should have associated with it exterior meson fields just as a charged black hole has associated with it an exterior electromagnetic field. Meson fields associated with a black hole would provide, at least in principle, a way to define the baryon and strangeness numbers of the black hole uniquely. This would clearly violate the conjecture.

In the present paper we show that the above procedure for violating the conjecture will fail for the case of static bare black holes.[13] In Secs. III–V we give the proof of the following result: *A static bare black hole can be endowed with no exterior classical massive or massless scalar fields, nor with exterior classical massive vector fields.* This result applies to charged as well as to neutral fields; it holds for single or for multiple black holes, and it is valid in Brans-Dicke theory as well as in Einstein's theory. In Sec. VI we give a short argument which suggests that the full quantum treatment of the problem would show that static bare black holes cannot interact with the exterior world via scalar or vector virtual mesons. This strongly implies that such black holes have no well-defined baryon or strangeness numbers. We also show in Sec. VI that Israel's results may be generalized to show that in the presence of the strong and electromagnetic interactions, all exteriors of static, bare black holes are of the Reissner-Nordström type.

II. ASSUMPTIONS

In general a forming black hole will be changing in time. However, we may expect that damping resulting from gravitational and other radiations will result in the black hole's exterior reaching a stationary state once all available matter and fields have been either ejected or radiated to large distances, or have been absorbed by the black hole. Only those fields intrinsically associated with the black hole will remain in the exterior, but they will become stationary. We may also expect that no naked singularities will appear, i.e., that the horizon and its exterior will be nonsingular. This expectation is based on another current conjecture which states that "all singularities are hidden in black holes." Since a black hole is a localized object, it is reasonable to assume that the corresponding space-time will be asymptotically flat.

In order that the horizon isolate the interior of the black hole from its exterior, it must be closed; all curves connecting a point in the interior with one in the exterior must intersect the horizon in at least one point. The fact that the horizon is a null hypersurface (by definition), and hence a one-way membrane, then insures that the interior of the black hole cannot communicate with the exterior. We will leave the topology of the horizon unspecified. It may be topologically spherical or toroidal, or the black hole may consist of several disconnected pieces. We shall not enter here into the question of which of these topologies are actually realized.

We thus arrive at the definition: The exterior of a stationary, bare black hole is a stationary, asymptotically flat region of space-time which is devoid of matter, foreign fields (radiation, or fields with sources at spatial infinity), or singularities, and which is bounded by a nonsingular horizon.

We shall work in units for which $G=c=\hbar=1$; the only dimension left in these units is length (measured in units of the Planck length 1.6×10^{-33} cm). Since we have excluded matter from the exterior, all the fields in question will be sourceless. We will also exclude any interactions among the various fields except for the electromagnetic coupling of charged fields with one another and with the black hole's exterior electromagnetic field. To take into account this coupling we will make use of the appropriate covariant form of the minimal-coupling rule.

We will be interested in pseudoscalar and pseudovector fields. In flat space one describes these by means of the Klein-Gordon and Maxwell-Proca equations, respectively. In passing to curved space we will use the comma-goes-to-semicolon rule with no direct couplings to the curvature (which violate the strong equivalence principle) to obtain the generally covariant form of these equations.

III. FORMALISM

For our stationary and asymptotically flat geometry we may choose a coordinate chart x^μ to cover the black-hole exterior, and a metric tensor $g_{\mu\nu}$ which is independent of the time x^0, and which asymptotically reduces to the metric tensor of flat space-time in spherical polar coordinates.[14] The horizon, being stationary, is described by an equation of the form $F(x^i)=0$, where F is some function. We see that its normal n_μ given by $n_\mu = F_{,\mu}$ as well as its surface element dS_μ (which is proportional to n_μ) both have vanishing time components. The fact that the horizon is a null hypersurface may be expressed as $dS_\mu dS^\mu=0$.[15]

We now consider a set of local fields Φ_k (labeled by k) which are associated with the black-hole exterior. We choose to describe them with a Lagrangian density $\mathcal{L}$. The variational principle $\delta\int\mathcal{L}(-g)^{1/2}d^4x=0$ then yields as field equations

$$(-g)^{-1/2}[(-g)^{1/2}\partial\mathcal{L}/\partial\Phi_{k,\mu}]_{,\mu}-\partial\mathcal{L}/\partial\Phi_k=0\,. \qquad (1)$$

Multiplying each of these by $\Phi_k(-g)^{1/2}d^4x$, integrating over the black-hole exterior, converting one term to a surface integral, and summing over the fields of interest gives

$$-\int b^\mu dS_\mu + \sum_k \int (\Phi_{k,\mu}\partial\mathcal{L}/\partial\Phi_{k,\mu} + \Phi_k\partial\mathcal{L}/\partial\Phi_k)(-g)^{1/2}d^4x = 0 . \quad (2)$$

Here dS_μ signifies the element of hypersurface bounding the domain in question (the horizon, spatial infinity, and the future and past timelike infinities), and

$$b^\mu = \sum_k \Phi_k \partial\mathcal{L}/\partial\Phi_{k,\mu} . \quad (3)$$

In this paper we confine our attention to static, that is, nonrotating black holes. (We have also generalized all the following results to the case of stationary black holes.[13]) Then we can always arrange for g_{0i} and g^{0i} to vanish. The fact that the horizon is null is then expressed as[15]

$$g_{ij}dS^i dS^j = 0 \text{ on horizon}, \quad (4)$$

and we can also write

$$b^\mu dS_\mu = g_{ij} b^i dS^j . \quad (5)$$

It will be shown in the Appendix that the spatial metric g_{ij} is positive definite in the black-hole exterior and positive semidefinite on the horizon. From the resulting Schwarz inequality also proven there it then follows that, provided the b^i are real,

$$(g_{ij}dS^i b^j)^2 \leqslant (g_{ij}dS^i dS^j)(g_{nm}b^n b^m) . \quad (6)$$

In all the cases to be studied it will turn out that $b^\mu b_\mu$ is bounded on the horizon and that $b^0 = 0$. Then $g_{ij}b^i b^j$ is bounded, and it follows from Eqs. (4), (5), and (6) that $b^\mu dS_\mu = 0$ on the horizon.

It may easily be verified from the field equations to be given later (with the use of the asymptotic form of the metric) that for all physically relevant fields the b^μ vanish asymptotically as $1/r^3$ for massless fields, and exponentially for massive fields. Thus there are no contributions from spatial infinity to the boundary integral in (2). At timelike infinity the boundary has $n_i = dS_i = 0$ so that $b^\mu dS_\mu = 0$ (since we shall show that $b^0 = 0$ always) on it. All these results indicate that

$$\int b^\mu dS_\mu = 0 \quad (7)$$

and thus that the second integral in Eq. (2) vanishes by itself. This is the central result of this paper.

IV. SCALAR FIELDS

We first consider here a massive, neutral, and real scalar field ψ of reciprocal Compton wavelength $m > 0$. The customary Lagrangian density used to describe it is[16]

$$\mathcal{L} = -\tfrac{1}{2}(\psi_{,\alpha}\psi_{,}^{\ \alpha} + m^2\psi^2) . \quad (8)$$

We shall use this also in curved space since it is already generally covariant. According to Eq. (1) it leads to

$$\psi_{,\mu;}{}^{\mu} - m^2\psi = 0 , \quad (9)$$

which is the Klein-Gordon equation as generalized to curved space by the comma-goes-to-semicolon rule. The stress-energy tensor corresponding to $\mathcal{L}$ is

$$T_{\mu\nu} = \psi_{,\mu}\psi_{,\nu} - \tfrac{1}{2}g_{\mu\nu}(\psi_{,\alpha}\psi_{,}^{\ \alpha} + m^2\psi^2) . \quad (10)$$

In the static case $T_{\mu\nu}$ will also be static which implies that $\psi_{,0} = 0$. According to (3), $b_\mu = -\psi\psi_{,\mu}$ so that $b^0 = 0$. That $b_\mu b^\mu$ is bounded on the horizon can be seen by obtaining from (10) the relations

$$\psi_{,\mu}\psi_{,}^{\ \mu} = [\tfrac{4}{3}(T_{\mu\nu}T^{\mu\nu}) - \tfrac{1}{3}T^2]^{1/2} \quad (11)$$

and

$$m^2\psi^2 = -\tfrac{1}{2}[\tfrac{4}{3}(T_{\mu\nu}T^{\mu\nu}) - \tfrac{1}{3}T^2]^{1/2} - \tfrac{1}{2}T . \quad (12)$$

As will be pointed out in Sec. VI, physical scalars in general, and T and $T_{\mu\nu}T^{\mu\nu}$ in particular, are bounded on a nonsingular horizon. It then follows from (11) and (12) that $b_\mu b^\mu = \psi^2\psi_{,\mu}\psi_{,}^{\ \mu}$ is bounded on the horizon. Therefore, the formalism of Sec. III informs us that the second integral in Eq. (2) vanishes, or

$$\int (g_{ij}\psi_{,}^{\ i}\psi_{,}^{\ j} + m^2\psi^2)(-g)^{1/2}d^4x = 0 . \quad (13)$$

We will show in the Appendix that g_{ij} is positive definite in the black-hole exterior. Therefore, the only way for the above integral to vanish is for ψ to vanish identically throughout the black-hole exterior.

The case of a massless scalar field is more subtle. We note that Eq. (12) no longer insures that ψ^2 is bounded on the horizon. A related difficulty is that Eq. (9) with $m = 0$ determines ψ only up to an additive constant. If we impose the boundary condition that ψ vanish asymptotically, we may interpret ψ^2 as the invariant probability density for scalar mesons. The full argument for this interpretation will be given in Sec. VI. It then follows that since ψ^2 is a physical scalar, it will be bounded on the horizon. Alternatively, we may *demand* that ψ^2 be bounded as a reasonable condition. Then the same procedure used in the massive case will establish that ψ vanishes identically in the black-hole exterior. The additive constant mentioned need not vanish, but it appears in no physical quantity and thus is totally unobservable. The fact that the asymptotically vanishing part of the massless scalar field vanishes in the exterior of a static black hole has also been established by Chase,[9] who used a different method.

We now consider a charged (complex) scalar field ψ coupled minimally to an electromagnetic field described by a vector potential A_μ and a field tensor $F^{\mu\nu}$. We take as the complete Lagrangian density

$$\mathcal{L} = -(d^\alpha d_\alpha^* + m^2\psi\psi^*) - F^{\mu\nu}F_{\mu\nu}/16\pi\,, \tag{14}$$

where $d_\alpha = \psi_{,\alpha} - ieA_\alpha\psi$, and e is the charge of the field.[16] Associated with ψ itself we have the stress-energy tensor

$$T_{\mu\nu} = d_\mu d_\nu^* + d_\mu^* d_\nu - g_{\mu\nu}(d^\alpha d_\alpha^* + m^2\psi\psi^*)\,. \tag{15}$$

The entire theory is invariant under the gauge transformation

$$\psi \to \psi\exp(ie\Lambda),\quad A_\mu \to A_\mu + \Lambda_{,\mu}\,, \tag{16}$$

where Λ is an arbitrary real scalar function. A consequence of the gauge invariance is the existence of the conserved electric current

$$j_\mu = ie(\psi d_\mu^* - \psi^* d_\mu)\,. \tag{17}$$

Since we are concerned only with a static situation, we may always make a choice of gauge for which $A_i = 0$ and $A_{0,0} = 0$. In a static situation it is always true that $j^i = 0$ and $j^0{}_{,0} = 0$. It then follows from (17) that $\psi\psi^*$ must be independent of x^0, and that the phase of ψ must be of the form $\omega x^0 + \varphi$, where ω and φ are real constants. By making a gauge transformation with $\Lambda = -(\omega x^0 + \varphi)/e$ we can make ψ real and time-independent without altering our conditions on A_μ. Having done so, we proceed to calculate b^μ by carrying out the sum implied in (3) over ψ and ψ^*. We obtain $b_\mu = -(\psi d_\mu^* + \psi^* d_\mu)$. It is now easy to see that $b^0 = 0$ while the b^i are real.

In order to show that $b^\mu b_\mu$ is bounded on the horizon, we obtain from the stress-energy tensor the expressions

$$T = -2d^\mu d_\mu^* - 4m^2\psi\psi^* \tag{18}$$

and

$$T_{\mu\nu}T^{\mu\nu} = 2\,|d^\mu d_\mu|^2 + \tfrac{3}{4}Td^\mu d_\mu^* + \tfrac{1}{16}T^2 + \tfrac{1}{2}(d^\mu d_\mu^*)^2\,. \tag{19}$$

Both T and $T_{\mu\nu}T^{\mu\nu}$, being physical scalars, must be bounded on the horizon. We see that both of the terms in (19) which are explicitly of fourth order in d_μ are positive. It is therefore necessary that both $d_\mu d^\mu$ and $d^\mu d_\mu^*$ be bounded on the horizon for if either or both became unbounded, then $T_{\mu\nu}T^{\mu\nu}$ would also become unbounded. It then follows from (18) that $\psi\psi^*$ is likewise bounded on the horizon. We recall that in our gauge ψ is real so that $b^\mu b_\mu = \psi\psi^*(d^\mu d_\mu + d^{\mu*}d_\mu^* + 2d^\mu d_\mu^*)$. It is now clear that $b^\mu b_\mu$ is indeed bounded on the horizon.

Having met all the prerequisites for the applicability of Eq. (7), we can now write Eq. (2) in the form

$$\int\{g_{ij}\psi_{,}{}^{i}\psi_{,}{}^{j} + [m^2 + g_{00}(eA^0)^2]\,\psi^2\}(-g)^{1/2}d^4x = 0\,. \tag{20}$$

In the Appendix it will be shown that $g_{00} \leq 0$ and g_{ij} is positive definite in the black-hole exterior. The integrand of Eq. (20) is not positive definite if $A^0 \neq 0$, but we shall now show that A^0 must vanish in our gauge *if* the scalar field is present.

From (17) we see that in our gauge $j^0 = -2e^2A^0\psi^2$. In accordance with the argument to be given in Sec. VI, we will interpret $\psi\psi^* = \psi^2$ as the invariant probability density for charged scalar mesons. That $\psi\psi^*$ is bounded on the horizon supports this physical interpretation. Since $(-j^\mu j_\mu)^{1/2}$ is the invariant charge density, we see that $(-j^\mu j_\mu)^{1/2}/\psi^2$, the specific charge (per meson) of the field, is a physical scalar and must be bounded on the horizon. Since $j^i = 0$, this implies that $g_{00}(A^0)^2$ is bounded. The quantity $F_{\mu\nu}F^{\mu\nu}$ is itself a physical scalar, and therefore in the static case $g_{00}g_{ij}F^{0i}F^{0j}$ will also be bounded.

We now compute b^μ for the electromagnetic field by summing (3) over the A_μ. We obtain $b^\mu = -F^{\mu\nu}A_\nu/4\pi$ so that in our gauge $b^0 = 0$. From the information obtained in the preceding paragraph it follows immediately that $b^\mu b_\mu$ is bounded on the horizon. The formalism of Sec. III then gives for the electromagnetic field

$$\int g_{00}[g_{ij}F^{0i}F^{0j}/4\pi + 2(eA^0)^2\psi^2](-g)^{1/2}d^4x = 0\,. \tag{21}$$

From the properties of the metric it follows that the above integrand is negative definite in the black-hole exterior, so that the only way for the integral to vanish is for A^0 to vanish throughout the exterior. If we now return to Eq. (20) we see that its integrand is now positive definite, so that ψ must vanish identically over the entire black-hole exterior. It then follows from (16) that this conclusion is true in all gauges.

The case of an electrically neutral but complex scalar field may be treated also in the above manner by coupling ψ minimally to a fictitious A_μ which is itself a gradient. This ghost field will make no contributions to any physical quantity, i.e., stress-energy tensor, by virtue of the gauge invariance of the theory, but does allow us to take over our results for the charged scalar field. In conclusion we state that a static, bare black hole can be endowed with no exterior scalar field of any type.

V. VECTOR FIELDS

We will first consider a massive, neutral, real vector field B_μ of reciprocal Compton wavelength

$m>0$. Associated with it is the field tensor

$$H_{\mu\nu} = B_{\nu,\mu} - B_{\mu,\nu} . \quad (22)$$

We shall employ the usual Lagrangian density[16]

$$\mathcal{L} = -H^{\mu\nu}H_{\mu\nu}/16\pi - m^2 B_\mu B^\mu/8\pi \quad (23)$$

which is already in general covariant form. From it follow according to (1) the Proca equations in general covariant form with no couplings to the curvature:

$$H^{\mu\nu}{}_{;\nu} + m^2 B^\mu = 0 . \quad (24)$$

The $H^{\mu\nu}$ is entirely analogous to the electromagnetic $F^{\mu\nu}$. But the Proca field B_μ is not subject to gauge transformations as is the electromagnetic A_μ. In fact, B_μ can be obtained uniquely from $H^{\mu\nu}$ through Eq. (24). Therefore, B_μ, unlike A_μ, is a physical field.

We may learn something about the Proca field by studying its behavior under time reversal. The Proca equation (24) should be invariant under time reversal even if it had a source current on the right-hand side. If we recall that under time reversal $g_{ij} \to g_{ij}$, $g_{00} \to g_{00}$, and $g_{0i} \to -g_{0i}$, we see that Eq. (24) with a source current will be invariant only if $B_0 \to B_0$, $B_i \to -B_i$ and consequently $H^{0i} \to H^{0i}$ and $H^{ij} \to -H^{ij}$. However, time reversal should change no physical quantities in the static case; therefore, B_i and H^{ij} must vanish.

By summing (3) over the B_ν we obtain $b^\mu = -H^{\mu\nu}B_\nu/4\pi$ and we see that $b^0 = 0$ in the static case. Furthermore, $b^\mu b_\mu$ is clearly a physical scalar and therefore it will be bounded on the horizon. We may thus apply the analysis of Sec. III to conclude that

$$\int g_{00}[g_{ij}H^{0i}H^{0j} + m^2(B^0)^2](-g)^{1/2}d^4x = 0 . \quad (25)$$

From the properties of g_{00} and g_{ij} we see that the integrand above is negative definite so that the integral will vanish as required only if H^{0i} and B^0 vanish everywhere. Therefore, for a massive, neutral, real vector field, $B_\mu = 0$ throughout the black-hole exterior.

For a massless field ($m=0$) our proof breaks down because B_μ is then subject to gauge transformations and thus lacks any direct physical meaning. Thus $b^\mu b_\mu$ is not a physical scalar and need not be bounded on the horizon. In fact, it is well known that the Reissner-Nordström solution of Einstein's equations represents a static, spherically symmetric black hole with an exterior electromagnetic field. Clearly, exterior massless vector fields may exist. However, since the only such field known is the electromagnetic field, it alone is an exception to our rule.

In the case of a charged (complex) vector field of charge e coupled minimally to the electromagnetic field, we replace (22) by

$$H_{\mu\nu} = B_{\nu,\mu} - B_{\mu,\nu} - ie(A_\mu B_\nu - A_\nu B_\mu) \quad (26)$$

and take as the full Lagrangian density

$$\mathcal{L} = -H^*_{\mu\nu}H^{\mu\nu}/8\pi - m^2 B^*_\mu B^\mu/4\pi - F_{\mu\nu}F^{\mu\nu}/16\pi . \quad (27)$$

The field equations obtained by varying B^*_ν according to (1) are

$$H^{\mu\nu}{}_{;\nu} - ieH^{\mu\nu}A_\nu + m^2 B^\mu = 0 . \quad (28)$$

Associated with the vector field is the stress-energy tensor

$$T_{\mu\nu} = \mathrm{Re}\,[H^*_{\mu\alpha}H_\nu{}^\alpha + m^2 B^*_\mu B_\nu - \tfrac{1}{4}g_{\mu\nu}(H^*_{\alpha\beta}H^{\alpha\beta} + 2m^2 B^*_\alpha B^\alpha)]/2\pi \quad (29)$$

which is easiest obtained by the method of Landau and Lifshitz.[17] The above theory is invariant under the gauge transformation

$$B_\mu \to B_\mu \exp(ie\Lambda), \quad A_\mu \to A_\mu + \Lambda_{,\mu} \quad (30)$$

and this gauge invariance is reflected in the existence of a conserved gauge-invariant electric current

$$j_\mu = ie(H^*_{\mu\alpha}B^\alpha - H_\mu{}^\alpha B^*_\alpha) . \quad (31)$$

Because of the gauge freedom the absolute phases of the various fields lack any physical significance. It is only the absolute values and *relative* phases of the fields $H^{\mu\nu}$ and B_μ that are physical quantities. Only these gauge-invariant quantities appear in j^μ and $T_{\mu\nu}$; the common phase of the fields cancels out.

One may always choose A_μ in such a way that under time reversal $A_i \to -A_i$ and $A_0 \to A_0$. The time-reversal invariance of Eq. (28) requires that the fields transform as

$$B_0 \to B^*_0, \quad H_{0i} \to H^*_{0i}, \quad B_i \to -B^*_i, \quad \text{and} \quad H_{ij} \to -H^*_{ij} \quad (32)$$

up to a common phase factor corresponding to the gauge freedom. In the static case one can take $A_i = 0$ and $A_{0,0} = 0$. In order for the physical relative phases of the fields to be unchanged under time reversal, it is necessary according to (32) that up to a common phase the H^{0i} and B_0 be real and the H^{ij} and B_i be pure imaginary. In the static case the absolute values of the fields cannot depend on time. The most immediate consequence of the above results is that $j^i = 0$ and $j^0{}_{,0} = 0$ in the static case as expected.

If one now looks at Eq. (28), one sees that the common phases of the fields must be of the form

$\omega x^0+\varphi$ with ω and φ real constants. Any other dependence on x^μ would result in the breakdown of the results obtained above in the static case. By a simple gauge transformation we may set the common phase to zero without disturbing the conditions we imposed on A_μ. In this new gauge the H^{0i} and B_0 are pure real and the H^{ij} and B_i pure imaginary.

We may learn more about the fields by considering a test particle which can interact directly with the vector field. It can be represented as a source current J^μ on the right-hand side of Eq. (28). One can show that the expression for the stress-energy tensor (29) is still valid in the presence of the source. The divergence of this $T_{\mu\nu}$ represents the force density exerted by the vector field on the particle. If we use (28) with the source J^μ and the fact that $H_{[\alpha\beta;\gamma]}=0$ we have

$$T_\mu{}^\nu{}_{;\nu}=\mathrm{Re}\{-H^*_{\mu\alpha}(J^\alpha+ieH^{\alpha\nu}A_\nu)+[J^\alpha{}_{;\alpha}+ie(\tfrac{1}{2}H^{\alpha\beta}F_{\alpha\beta}+m^2B^\alpha A_\alpha-J^\alpha A_\alpha)]B^*_\mu\}/2\pi\,. \quad (33)$$

We notice immediately that some of the terms in (33) are independent of J^α; they are the same for all test particles. These unphysical terms are expected to cancel out in a physical situation. In particular, in our gauge we see that in the static case these terms will disappear from the force for arbitrary symmetries only if the B_i and consequently the H^{ij} vanish. Therefore, in the static case the charged vector field is no different in principle from the real, neutral one. We may thus apply our previous results to conclude the B_μ vanishes everywhere in the black-hole exterior. From (30) we see that this conclusion is gauge-independent. The case of an electrically neutral but complex vector field may be treated as above with the same results by using the fictitious A_μ mentioned in Sec. IV.

We have thus shown that a static bare black hole cannot be endowed with exterior classical fields that would correspond to the following mesons: π^0 (real pseudoscalar), $\pi^\pm$ (complex charged pseudoscalar), $K^0\overline{K}^0$ (complex neutral pseudoscalar), ρ^0 (complex neutral pseudovector), ρ^- (complex charged pseudovector), and others connected with the strong interactions.[18]

VI. CONCLUSIONS

If we assume that the electromagnetic field is the only massless vector field in nature, we may conclude that static bare black holes cannot have exterior scalar or vector fields other than the electromagnetic field. The key physical assumption made is that physical scalars are bounded on a nonsingular horizon. To justify this assumption we consider a freely falling observer who carries an orthonormal frame with him. On crossing the horizon the observer should find in his frame that physical quantities such as $T^{\mu\nu}$, j^μ, and $F^{\mu\nu}$ and others remain bounded unless there is something singular in the physics at the horizon. We exclude this possibility with the statement that the horizon is nonsingular, by which we mean that local physics is nonsingular. Any scalar constructed out of physical quantities will clearly be bounded also.

As a result of the above broad definition of a nonsingular horizon, we were able to dispense with the use of the gravitational field equations. Therefore, the proof given is valid in both Einstein's theory and in the Brans-Dicke theory. A moment's thought will convince one that the proof is also valid if fields other than the ones considered are present provided that no interactions other than the electromagnetic one are in operation. Thus the scalar and massive vector fields must vanish even in the presence of a neutrino field.

The proof given applies only to the classical fields. What changes can one expect from a full quantum treatment? Such a treatment should be possible along the lines of Hartle's study of the question of whether a black hole has weak-interaction properties.[10] However, we shall be content here with giving a simple argument which suggests the results to be expected from a full quantum treatment.

Consider first the real scalar field. In the quantum theory it would be represented by an Hermitian operator $\psi_{\mathrm{op}}(x^\mu)$. We assume that there exists a complete set of one-meson eigenfunctions $U_m(x^\mu)$; we may expand

$$\psi_{\mathrm{op}}(x^\mu)=(2)^{-1/2}\sum_m(a_mU_m+a_m^\dagger U_m^*)\,, \quad (34)$$

where $a_m^\dagger$ and a_m are the creation and annihilation operators for a meson in the state labeled by m. With respect to a many-meson state $|i_m\rangle$ labeled by the occupation numbers i_m, the classical quantity corresponding to $\psi_{\mathrm{op}}{}^2$ is

$$\psi^2=\langle i_m|{:}\psi_{\mathrm{op}}{}^2{:}|i_m\rangle, \quad (35)$$

where the normal ordering (:) ensures that ψ^2 vanishes in the vacuum state for which all $i_m=0$. We thus have

$$\psi^2=\sum_m i_mU_m^*U_m \quad (36)$$

and similarly

$$b_\mu=-\tfrac{1}{2}(\psi^2)_{,\mu}=-\tfrac{1}{2}\sum_m i_m(U_m^*U_m)_{,\mu}\,. \quad (37)$$

If the U_m are properly normalized, the space integral of ψ^2 in (36) will be the number of mesons in all states. Thus it is consistent to interpret ψ^2 as the invariant (nonconserved) probability density for scalar mesons. (In the massless case the U_m and consequently the ψ will have to be chosen asymptotically vanishing in order for normalization of the U_m to be feasible.) In the complex-scalar-field case the ψ_{op} would be non-Hermitian; this would require the introduction of creation operators for the particles and antiparticles, but the interpretation of $\psi\psi^*$ would be similar.

Going back to the real scalar field, we suppose that for a state m the U_m vanishes asymptotically at least as rapidly as $1/r$, $(U_m^* U_m)_{,0}=0$, and $(U_m^* U_m)_{,\mu}$ contracted with itself is bounded on the horizon. Since U_m satisfies the equation for a complex field with no charge, we may immediately apply the results of Sec. IV to show that it vanishes identically in the black-hole exterior. Such a state is missing from the expansion for ψ_{op} in the exterior. If any of the above conditions is not satisfied the proof will fail. But we now show that in this case $i_m=0$.

We note first that since we are dealing with bosons the occupation numbers are unrestricted. If U_m does not vanish asymptotically for some m, then (36) shows that in general the classical field will not be localized near the black hole. Also if the contraction of $(U_m^* U_m)_{,\mu}$ with itself is unbounded on the horizon, (37) shows that $b^\mu b_\mu$ is unbounded in general. The only way to exclude the above improper behavior of the classical quantities is to set $i_m=0$ for the appropriate m's. We are left with states m for which $U_m^* U_m$ depends on time. Clearly, any classical quantity will now depend on time in general, and since we cannot allow this we set $i_m=0$ for these states also. We are left with the conclusion that in the exterior of the black hole only the meson vacuum state is possible. Similar arguments could no doubt be given for the other types of mesons considered here.

The strong interactions are known to be mediated by pseudoscalar mesons such as the π and the K, and by pseudovector mesons such as the ρ. Our argument strongly suggests that a static bare black hole does not interact strongly with the exterior world because it cannot exchange virtual mesons with an exterior object. Since both baryon and strangeness numbers go hand in hand with the strong interactions, we cannot see any way to determine these numbers for a static, bare black hole by external measurements alone.

It could be argued that such black holes have zero baryon number, but this choice would be inconsistent if the black hole in question was formed out of baryonic matter. Again one could say that the baryon number of such a black hole is just the number of baryons that went into its making. But this argument is circular; it uses the law of conservation of baryons which has content only after the baryon number of each object present has been determined by some independent method, not by the use of the law itself. Therefore, it appears that static bare black holes have no well-defined baryon number. As a consequence, the law of conservation of baryon number ceases to be meaningful in black-hole physics in a semiglobal scale, though for all we know it may still hold locally. Similar conclusions apply to strangeness number and its conservation. Wheeler, who has often remarked that black holes should have no baryon number, expresses the above by saying that the law of conservation of baryon number is transcended in black-hole physics.[12]

Another important consequence of our result follows immediately. Israel has proven that in Einstein's theory the most general exterior of a single static black hole with only an electromagnetic field present is of the Reissner-Nordström type.[4] *A priori*, the "only electromagnetic fields" restriction excludes the possibility of the black hole containing strongly interacting material. But our results show that even if this is the case the stress-energy tensor in the exterior will contain no contributions from the strong interactions (mesons). Therefore, Israel's theorem is valid even in this more general case.

We hasten to point out that we have shown only that exterior scalar and massive vector fields are incompatible with a nonsingular horizon. We cannot exclude the possibility that the gravitational collapse of a star could lead to the formation of a naked singularity with exterior fields (not a black hole in the usual usage of the word). However, the work of Price,[8] treating the almost spherical collapse of a star coupled to various massless fields, has shown that a black hole actually forms while the fields are radiated away or absorbed by the black hole.

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APPENDIX

In the coordinates chosen, $g_{00}=-1$ asymptotically. The g_{00} is the square of that Killing vector which generates time translations in our static geometry. If no singularities are present we expect the invariant g_{00} to be continuous in the black-hole exterior. Can g_{00} ever be positive in the exterior? No. For suppose there existed a surface on which g_{00} changed from negative to positive in the exterior. We could trace out this $g_{00}=0$ surface. It cannot extend to infinity (where $g_{00}=-1$) so it will either close on itself surrounding a region exterior to the horizon, or it will intersect the horizon and close on itself outside of it, or it will close about the horizon. It cannot remain open because g_{00} has opposite signs on either side.

Vishveshwara has shown that a $g_{00}=0$ surface of a static metric is always null.[19] Therefore our surface is null. In addition, it is nonsingular by hypothesis so it is the horizon or at least part of it. It is thus clear that $g_{00}<0$ in the exterior except for a possible isolated surface which has g_{00} negative on either side and zero on it (so that it could be open). In any case $g_{00}\leq 0$ on and outside the horizon.

When $g_{00}\leq 0$ a static coordinate system such as ours is realizable with test particles or photons,[17] and so the spatial distance dl between two points separated by the coordinate interval dx^i is well defined:

$$dl^2=g_{ij}\,dx^i dx^j\,.$$

Clearly the g_{ij} is a positive definite matrix except on the horizon where in view of Eq. (4) it is positive semidefinite only. So for an arbitrary real number λ and real triplets b^i and dS^i,

$$g_{ij}(dS^i+\lambda b^i)(dS^j+\lambda b^j)\geq 0\,.$$

If we choose $\lambda=-g_{ij}\,dS^i b^j/g_{ij}b^i b^j$ (providing the denominator does not vanish), we immediately obtain the Schwarz inequality (6).

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[15]Write $dS_\mu=\sigma n_\mu$, where σ is an invariant. Recall the definition of dS_μ in terms of three coordinate intervals. In Kruskal-like coordinates (whose existence is guaranteed by the assumed nonsingular character of the horizon) the dS_μ are clearly nonsingular, so that σ is bounded at the horizon. But σ is an invariant. Therefore, in any coordinates $n_\mu n^\mu=0$ implies $dS_\mu dS^\mu=0$.

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