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Short-Distance Scale Invariance and the Structure of Weak Transitions*

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The effects of short-distance interactions between hadronic weak currents on the structure of weak transitions are investigated within Wilson's scheme of operator-product expansions and broken scale invariance. It is pointed out that a consistent understanding of these effects can be attained with the hypothesis that the dimensions of nonexotic fields are near-canonical while those of exotic fields are anomalously high. The lack of structure-dependent effects in the $\Delta I=\frac{1}{2}$ part of ordinary nonleptonic decay, the suppression of the $\Delta I=\frac{3}{2}$ part in the same, and the smallness of the K_1^0-K_2^0 mass difference are first explained following Wilson's arguments. The structure of higher-order semileptonic decays disobeying the $\Delta S=\Delta Q$ role is then discussed. Finally, the important role played by short-distance effects in radiative nonleptonic decays of hadrons is elucidated and a mechanism for the large asymmetry parameter observed in the reaction $\Sigma^+\to p+\gamma$ is obtained.

I. INTRODUCTION

At present the local current$\times$current Hamiltonian density of the Cabibbo theory is generally regarded as providing a highly successful theoretical framework for the description of the observed weak interactions of hadrons. Combined with SU(3) symmetry or current algebra, it can explain an impressive number of different results.[1] There are, however, some problems connected with the short-distance behavior of products of hadronic weak currents that cannot be properly treated within the above scheme. These involve possible effects of the structure of weak interactions which may appear when the configuration-space distances between the points of interaction of these currents have to be integrated over. In the picture of weak interactions being mediated by spin-one bosons, these complications arise whenever there is a loop integration over the four-momentum of the W boson. Ordinary nonleptonic decays, the $K^0\leftrightarrow\bar{K}^0$ transition contributing to the K_1^0-K_2^0 mass difference, higher-order weak decays involving W loops, radiative nonleptonic decays – all of these belong to this category of theoretically problematic weak processes.

Over the last few years, attempts have been made to handle the problem of the quadratic divergence[2] of single W loops in the weak intermediate-boson theory. Thus, for example, Bouchiat, Iliopoulos, and Prentki[3] have shown that if the SU(3)$\times$SU(3) symmetry of the strong-interaction Hamiltonian is broken only by terms belonging to the $(3,3^*)+(3^*,3)$ representation of that group, the $\Delta S=1$ and the $\Delta S=0$, parity-violating, ordinary weak nonleptonic transitions (these are the only ones observable) are free from quadratic divergences in the lowest-order diagrams. More recently, with the same assumption, Geshkenbein and Ioffe[4] have obtained an identical result for nonleptonic weak processes that emit one and two real photons. However, these arguments cannot rule out the appearance of structure-dependent effects through the presence of enhancement or suppression factors depending on the mass M of the weak boson which should be taken to be large compared to the typical hadron mass $m_H\sim 1$ GeV. Such factors may originate from the loop integration even if the quadratic divergence is absent and these are not taken into account by the simple local current$\times$current Hamiltonian density.

The purpose of the present paper is to study qualitatively but systematically the origin and the behavior of such structure-generated factors. Given Wilson's[5] postulates of operator-product expansions and scale invariance[6] at short distances, this study can be made in configuration space without depending too much on the details of the mechanism that is taken to mediate the weak interactions so long as it is not wildly unconventional. Thus we do not need to use any specific form for the W-boson propagator as in Refs. 3 and 4. All that is necessary is that an effective propagator be written in configuration space as $M^2W_{\mu\nu}(xM)$,[7] where

$$\int d^4x W_{\mu\nu}(xM)=g_{\mu\nu}/M^4\,.$$

Here M is the mass typifying the structure of weak interactions and will be assumed to be $\gg m_H$. The structure-dependent effects that we want to study will appear in the amplitudes of interest through powers of M/m_H so long as we assume that no cutoff Λ is needed for weak interactions once the effective propagator is introduced. Whenever this last assumption is incorrect, some powers of M

will have to be replaced by the corresponding powers of Λ in conformity with conventional usage. With this propagator, the effective Lagrangian density for ordinary nonleptonic weak transitions can be written as

$$\mathcal{L}_W(0)=\frac{GM^4}{2\sqrt{2}}\int d^4x\, W^{\mu\nu}(xM)\times[TJ_\mu^{C\dagger}(x)J_\nu^C(0)+TJ_\mu^C(x)J_\nu^{C\dagger}(0)], \quad (1)$$

where G is the Fermi constant and J_μ^C the Cabibbo current. Since $W_{\mu\nu}(xM)$ goes to zero exponentially[8] when x is large, one can appreciate the importance of the small-x contribution to integrals such as the one in Eq. (1). It is for estimating this kind of contribution that the technique of scale-invariant operator-product expansions at short distances is most useful. Employing this method, the power of M originating from this type of integral can be obtained in terms of the scale dimensions of the hadronic currents and that of the leading field contributing to the operator-product expansion. Sometimes we may encounter the situation (see Sec. VI below) where the dominant scale-invariant terms in an operator-product expansion happen to vanish owing to external constraints in the problem (such as electromagnetic gauge invariance in radiative weak decays). In that case we shall be interested in the leading contribution from the lowest-order scale-breaking term. The effect of such a term on the power of M that one is trying to seek can be evaluated using the spurion technique[5] outlined by Wilson (see Secs. II and V below).

It should be abundantly clear from the above discussion that in order for our statement on the power of M as an enhancement or suppression factor to be meaningful, we would need to have some idea about the dimensions[9] of the relevant operators. We know that the hadronic currents J_μ^i and the stress-energy tensor $\Theta_{\mu\nu}$ must have dimensions 3 and 4, respectively, as predicted canonically by a free-quark model, because of the nonlinear constraints of current algebra and of the commutation relations of the Lorentz group. However, as emphasized by Wilson, the dimensions of operators, not subject to such symmetry constraints, may be changed[10] from their canonical values by renormalization. These changes are expected to be different for different fields. There is no *a priori* model-independent way of telling whether the dimension of a renormalized field is slightly or markedly different from its canonical value. We shall, therefore, make a simple hypothesis which can, in principle, be rejected or substantiated by future experiment. This assumption,[11] to be considered on its *a posteriori* merits, will enable us to develop a coherent picture of structure-dependent effects in weak transitions that is consistent with present knowledge. The hypothesis can be stated in two parts in the following way: First, if an operator is nonexotic [i.e., it belongs to the representations $\underline{1}$, $\underline{3}$, $\underline{3}^*$, $\underline{8}$, $\underline{10}$, $\underline{10}^*$ of SU(3)], then its dimension is not[12] significantly changed from the canonical free-quark-model value by renormalization. Second, if an operator is exotic [i.e., it belongs to a higher representation of SU(3)], then its dimension is markedly increased from the canonical value as a result of renormalization. This hypothesis will not be needed in our discussions of higher-order weak leptonic decays with $\Delta Q=0$, $|\Delta S|=1$. However, the first part of it will be required in understanding the absence and presence respectively of structure-dependent effects in the $\Delta I=\frac{1}{2}$ part of ordinary nonleptonic decays and in radiative nonleptonic decays. *In particular, we obtain the new result that a large violation of SU(3) symmetry should be expected in the latter.* On the other hand, the second part of our hypothesis will be needed to explain the suppression of the $\Delta I=\frac{3}{2}$ part in ordinary nonleptonic decay and the absence of any significant structure-dependent effects in the K_1^0-K_2^0 mass difference as well as to predict the suppression of short-distance effects in $\Delta Q=-\Delta S=\pm 1$ and $\Delta S=2$ semileptonic decays.

The plan of the paper is as follows: A summary of the set of assumptions underlying our considerations is given in Sec. II. Section III contains a discussion of the argument that structure effects leave the $\Delta I=\frac{1}{2}$ part of ordinary nonleptonic decay unaffected but strongly suppress the $\Delta I=\frac{3}{2}$ part. Section IV is on the K_1^0-K_2^0 mass difference and explains why it cannot impose a severe bound on M. In Sec. V we consider three kinds of higher-order structure-dependent weak semileptonic decays: (a) those of the type $\alpha\to\beta l\bar{l}$, where α,β are hadronic states (including vacuum) differing by one unit of strangeness and l is a lepton, (b) those with $\Delta S=-\Delta Q=\pm 1$, and (c) those with $\Delta S=2$. Section VI presents our considerations on radiative nonleptonic weak decays of the type $A\to B+\gamma$ where A, B are hadrons and shows why short-distance effects enhance SU(3)-symmetry-breaking terms. The final section, VII, summarizes our conclusions.

II. ASSUMPTIONS OF THE MODEL OF SHORT-DISTANCE SCALE INVARIANCE

It will be convenient to summarize our basic assumptions and rules here. These are based on the set of hypotheses introduced by Wilson[5] and are enumerated as follows:

(1) Products[13] of two or more local operators

near the same space-time point can be expanded as a sum of terms involving local operators at that point. Thus for two local operators A and B, we can write

$$A(x)B(y) \underset{x-y\to 0}{\sim} \sum_n f_n(x-y)O_n(y), \tag{2}$$

where O_n are local operators and f_n are c-number functions involving powers of $x-y$. Similarly,

$$A(x)B(y)C(z) \underset{x-y,x-z\to 0}{\sim} \sum_n f_n(x-y, x-z)O_n(y), \tag{3}$$

and so on. The sums in Eqs. (2) and (3) are finite.

(2) The nature of the short-distance singularities of the functions f_n is determined by the symmetry principle of broken scale invariance. Thus in Eq. (2)

$$f_n(x-y) \underset{x-y\to 0}{\propto} (x-y)^{-d_A-d_B+d_n}, \tag{4}$$

where d_A, d_B, d_n are the dimensions[9] of A, B, and O_n, respectively. Similarly, the short-distance singularities of $f_n(x-y, x-z)$ can be determined up to an arbitrary dependence on the ratio $(x-z)^2/(x-y)^2$ about which we have nothing to say.

(3) Among the local operators of the theory, the unit operator I has dimension zero, the octets of hadronic vector and axial-vector currents V_μ^i and A_μ^i have dimension 3 and the stress-energy tensor $\Theta_{\mu\nu}$ has dimension 4.

(4) The other local operators are all constructed from the free-quark model with the rule that the dimension of a nonexotic operator is left at a nearly canonical value whereas that of an exotic one is increased to a markedly anomalous value by renormalization. Thus the scalar and pseudoscalar densities u^i, v^i ($=\bar{q}\frac{1}{2}\lambda^i q$, $i\bar{q}\frac{1}{2}\lambda^i\gamma_5 q$, respectively, in the free-quark model) have the dimension $\Delta \simeq 3$. Similarly, the antisymmetric tensor current $T_{\alpha\beta}^i$, which has the form $\bar{q}\sigma_{\alpha\beta}\frac{1}{2}\lambda^i q$ in the free-quark model, has the dimension $d_T \simeq 3$, and so on.

(5) The Hamiltonian density can be written as

$$\Theta_{00}(x) = \Theta_{00}^{(0)}(x) + \lambda w(x) + \lambda_0 u_0(x) + \lambda_8 u_8(x),$$

where the λ's are constants. The last two terms in the above equation break SU(3)×SU(3) invariance and λ_0, λ_8 have the ordinary dimension $4-\Delta$ in mass units. If the SU(3)×SU(3)-invariant scale-breaking term $w(x)$ is a c number (as the author would guess), λ has the dimension 4 in mass units. If $w(x)$ is an operator (e.g., it may break the axial-vector baryon number[14]), its dimension Δ_1 is assumed to be <4 so that λ has the ordinary dimension $4-\Delta_1$ in mass units. This assumption is necessary to keep all symmetry-breaking interactions superrenormalizable.[5] However, the lowest dimensional operator in the free-quark model that is SU(3)×SU(3)-invariant has dimension 6. Hence *if* $w(x)$ is a q number, it and the corresponding pseudoscalar operator $w_5(x)$ will have to be treated as exceptions[14] to rule (4).

(6) The limit λ's $\to 0$ takes one to the skeleton theory which is scale- and U(3)×U(3)-invariant. Operators in the skeleton theory will be assigned the superscript (0). Symmetry-breaking effects on these operators will be evaluated by interpreting the λ's as spurions belonging to the U(3)×U(3) conjugate representation of $u_i(x)$ and $w(x)$.

III. ORDINARY NONLEPTONIC WEAK DECAY

We shall now present a review of Wilson's[5,6] consideration of nonleptonic weak decays. The effective Lagrangian density for such decays was introduced in Eq. (1). For the observed $\Delta Q = \Delta S$, $|\Delta S| = 1$ decays this can be further specified as

$$\mathcal{L}_W(0) = \frac{GM^4}{\sqrt{2}}\cos\theta\sin\theta \times \int d^4x\, W^{\mu\nu}(xM)[TJ_\mu^{W\dagger}(x)J_\nu^S(0) + \text{H.c.}]. \tag{5}$$

In Eq. (5) θ is the Cabibbo angle, $J = V - A$, and the superscripts W and S stand for the SU(3) index combinations $1+i2$ and $4+i5$, respectively. To study the small-x contribution to the integral in Eq. (5), we write

$$TJ_\mu^{W\dagger}(x)J_\nu^S(0) \underset{x\to 0}{\sim} \sum_n C_{n\mu\nu}(x)O_n(0), \tag{6}$$

where the subscript n incorporates both possible Lorentz and gauge indices. Substituted in Eq. (5), Eq. (6) yields

$$\mathcal{L}_W(0) = \frac{G}{\sqrt{2}}\cos\theta\sin\theta \sum_n [C_n(M)O_n(0) + C_n^*(M)O_n^\dagger(0)]. \tag{7}$$

In Eq. (7)

$$C_n(M) = M^4\int d^4x\, W^{\mu\nu}(xM)C_{n\mu\nu}(x) \propto M^{6-d_n}$$

and similarly for $C_n^*(M)$, where d_n is the dimension of O_n. From the structure of the above integral we see that C_n (as well as O_n) has to be a Lorentz tensor of even rank, e.g., O_n could be a scalar field. Because of the μ,ν symmetry in Eq. (5) and because $\mathcal{L}_W$ has to alter strangeness, the only SU(3) representations to which O_n can belong are the symmetric octet and the 27-plet. First of all, we note that the O_n's cannot be the scalar and pseudoscalar densities $u^i(x)$, $v^i(x)$ which have the dimension Δ with $1<\Delta<4$. This is because these densities are divergences which vanish between states of equal four-momentum. In the free-quark model, the field of smallest dimension that can contribute can be written as

$$O_0 = :\bar{q}(1-\gamma_5)\tfrac{1}{2}\lambda^{W\dagger}q\bar{q}(1-\gamma_5)\tfrac{1}{2}\lambda^{S}q: . \tag{8}$$

O_0, as given in Eq. (8) in terms of the quark spinor q, has the canonical dimension 6 and contains both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ parts. The former belongs to the octet, is nonexotic and – in accordance with our hypothesis – ought to have an actual dimension nearly equal to 6. On the other hand, the latter belongs to the 27-plet, is exotic, and hence by fiat should have a significantly higher dimension. This means that $C_{3/2}(M)$ is suppressed through weak-structure effects by a power of M^{-1}. However, $C_{1/2}(M)$ is nearly independent of M so that neither enhancement nor suppression occurs in this case; hence the $\Delta I = \frac{1}{2}$ part of $\mathcal{L}_W(0)$ does not depend too much on the structure of the weak interactions. Wilson has, in fact, shown[15] that the above considerations of $\Delta I = \frac{1}{2}$ nonleptonic decay are perfectly compatible with current-algebra treatments of the same with a local current-current weak Lagrangian density.

IV. SMALLNESS OF THE K_1^0-K_2^0 MASS DIFFERENCE

Several authors[16] have argued in the past that both in the four-fermion point coupling model and in the intermediate vector-boson model of the weak interactions, the observed smallness of the K_1^0-K_2^0 mass difference $\Delta m_{K_{1,2}^0}$ imposed a severe upper bound (~5 GeV) on the weak-interaction cutoff Λ. Since presumably the mass M – considered in this paper – is $<\Lambda$, this would suggest that M is not that large compared with the typical hadronic mass of 1 GeV. Such a fact would be inconsistent with the basic approach of our work. Moreover, it would raise the interesting puzzle why the particle mediating weak interactions has not been seen so far. We shall show in this section that within our theoretical framework one can understand why the structure of weak interactions is unable to affect $\Delta m_{K_{1,2}^0}$ significantly, so that its observed smallness cannot be used to generate a strong upper bound on M. The gist of this argument was given by Wilson in the third paper of Ref. 5.

The effective Lagrangian density for the $K^0 \to \bar{K}^0$ transition causing the K_1^0-K_2^0 mass difference can be written in our notation as[16]

$$\mathcal{L}_{K\bar{K}}(0) = \tfrac{1}{4}G^2M^8\cos^2\theta\,\sin^2\theta \times\int d^4x\int d^4y\int d^4z\, W^{\mu\nu}(M(x-y))\,W^{\lambda\sigma}(Mz) \times TJ_\mu^{W\dagger}(x)J_\nu^{S}(y)J_\lambda^{W\dagger}(z)J_\sigma^{S}(0)\,. \tag{9}$$

As discussed in the third paper of Ref. 5, the dominant part of the integration comes from the region where x, y, z are all of the order of M^{-1}, in other words, when all four currents are at short distances. In the W-boson picture this region is contained in the diagrams with overlapping W loops[17] [e.g., Fig. 1(a)]. Thus the operator-product expansion of present interest is of the type

$$TJ_\mu^{W\dagger}(x)J_\nu^{S}(y)J_\lambda^{W\dagger}(z)J_\sigma^{S}(0) \underset{x,y,z\to 0}{\sim} \sum_l f_{l\mu\nu\lambda\sigma}(x,y,z)O_l(0)\,. \tag{10}$$

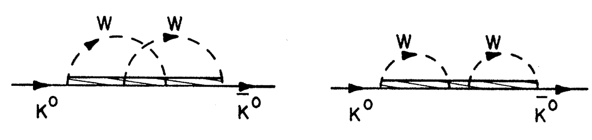
In Eq. (10) O_l has to be an exotic field carrying two units of strangeness. Substituting Eq. (10) in Eq. (9), we have

$$\mathcal{L}_{K\bar{K}}(0) = \tfrac{1}{4}G^2\cos^2\theta\;\sin^2\theta\sum_l f_l(M)O_l(0) + \text{non-short-distance part,} \tag{11}$$

where

$$f_l(M) = M^8\int d^4x\int d^4y\int d^4z\,W^{\mu\nu}(M(x-y)) \times W^{\lambda\sigma}(Mz)f_{l\mu\nu\lambda\sigma}(x,y,z) \propto M^{8-d_l}\,,$$

d_l being the dimension of O_l. Once again, f_l (as well as O_l) has to be an even-ranked Lorentz tensor, e.g., a scalar. In the free-quark model, the fields of lowest dimension that can contribute to O_l in Eq. (10) are $:\bar{q}\frac{1}{2}\lambda^{W\dagger}q\bar{q}\frac{1}{2}\lambda^{W\dagger}q:$, $:\bar{q}\gamma_\mu\frac{1}{2}\lambda^{W\dagger}q\bar{q}\times\gamma^\mu\frac{1}{2}\lambda^{W\dagger}q:$, etc., all of which have the canonical dimension 6. Thus in the free-quark model [or, equivalently, in the approximation of retaining only the vacuum intermediate state in Eq. (9), as made in Ref. 15] the K_1-K_2 mass difference is proportional to $G^2\cos^2\theta\sin^2\theta M^2$; for large M this is too big to fit the known mass difference. However, within our theoretical framework, the leading exotic field O_l is expected to have a dimension significantly higher than 6, thereby weakening the bound on M. We do not know precisely what the leading value of d_l is. However, we note that if it is 7, one can only say $M < 25$ GeV and if it is >8, the K_1^0-K_2^0 mass difference becomes free from any effects due to the structure of the weak interac-



(a) (b)

FIG. 1. Two types of contributions to the $K^0 \to \bar{K}^0$ transition: (a) overlapping W loops; (b) separated W loops.

tions. Thus, within our theoretical framework, the observed K_1^0-K_2^0 mass difference cannot impose a severe upper bound on M.

V. HIGHER-ORDER SEMILEPTONIC DECAYS VIOLATING THE $\Delta S=\Delta Q$ RULE

There are three types of structure-dependent higher-order weak decays of hadrons that are of experimental interest in the immediate future: (a) semileptonic decays with $\Delta Q=0$, $|\Delta S|=1$, (b) those with $\Delta S=-\Delta Q=\pm 1$, and (c) those with $\Delta S=2$. The first kind of reactions are of the form $\alpha\to\beta l\bar{l}$ and have amplitudes of order $G^2\sin\theta\cos\theta$. Here α,β are hadronic states (including the vacuum) that differ by one unit of strangeness and l is a lepton; typical examples are $K_L^0\to\mu^+\mu^-$, $K\to\pi\nu\bar{\nu}$, $K\to\pi e^+e^-$, $\Sigma\to N e^+e^-$, etc. (Electromagnetic effects are ignored here and will be considered in Sec. VI.) The second type of decays have amplitudes of order $G^2\cos^2\theta\sin\theta$ and are exemplified by $K^0\to\pi^+e^-\bar{\nu}_e$, $\bar{K}^0\to\pi^-e^+\nu_e$, $\Sigma^+\to ne^+\nu_e$, $\Xi^0\to\Sigma^-e^+\nu_e$, etc. The third type of decays, with amplitudes of order $G^2\cos\theta\sin^2\theta$, include $\Xi^0\to pe^-\bar{\nu}_e$ and $\Xi^-\to ne^-\bar{\nu}_e$.

(a) $\alpha\to\beta l\bar{l}$.[18] These decays have been considered in Refs. 15 and 19 in specific models of weak interactions. In any model that reduces to the Cabibbo Hamiltonian in the appropriate limit (i.e., does not have neutral weak currents), every diagram contributing to such reactions has to contain a virtual fermion line connecting the two external leptons. This is transparent in the intermediate-vector-boson theory for which the relevant diagrams are shown in Fig. 2. In general, the dominant short-distance contribution can be written in the form of an effective interaction in configuration space as

$$\mathcal{L}_{\alpha\beta}(0)\simeq\tfrac{1}{2}G^2\sin\theta\cos\theta M^8\int d^4x\int d^4y\int d^4z\, W^{\lambda\nu}(M(y-x))W^{\rho\mu}(Mz)[TJ_\nu^{W\dagger}(x)J_\mu^S(0)+W\leftrightarrow S]$$
$$\times\bar{u}(k)\gamma_\lambda(1-\gamma_5)S_F(z-y)\gamma_\rho(1-\gamma_5)v(\bar{k}),\qquad(12)$$

where S_F is the lepton propagator in configuration space which is singular when $z\to y$ as $(\not{z}-\not{y})/(z-y)^4$. This makes $z\to y$ in the dominant contribution to the integrals whereas the weak propagators make $y-x$, $x\sim O(1/M)$ and small. In Eq. (12) we have thrown away certain exponentials whose arguments are in effect very small. The relevant operator-product expansion now is

$$TJ_\nu^{W\dagger}(x)J_\mu^S(0)\underset{x\to 0}{\sim}\sum_m D_{m\mu\nu}(x)O_m(0).\qquad(13)$$

Substituting Eq. (13) in Eq. (12), we obtain

$$\mathcal{L}_{\alpha\beta}(0)\simeq\sum_m\tfrac{1}{2}G^2\sin\theta\cos\theta\,\bar{u}(k)\gamma^\lambda(1-\gamma_5)$$
$$\times D_{m\lambda\rho}(M)\gamma^\rho(1-\gamma_5)v(\bar{k})O_m+\text{H.c.},\qquad(14)$$

where

$$D_{m\lambda\rho}(M)=M^8\int d^4x\int d^4y\int d^4z\, W_{\lambda\nu}(M(y-z))W_{\rho\mu}(Mz)\times S_F(z-y)D_m^{\mu\nu}(x),$$

d_m being the dimension of O_m. Writing

$$S_F(z-y)\underset{z\to y}{\propto}(\not{z}-\not{y})(z-y)^{-4},$$

we then have

$$D_{m\lambda\rho}(M)$$
$$\propto\gamma^\tau M^8\int d^4x\int d^4y\int d^4z\,\frac{W_{\lambda\nu}(M(y-z))W_{\rho\mu}(Mz)}{(z-y)^4}\times(z-y)_\tau D_m^{\mu\nu}(x).\qquad(15)$$

From the structure of the right-hand side in the above equation it is clear that O_m – as introduced in Eq. (14) – has to be a Lorentz tensor of odd rank starting with the vector field O_σ. When m stands

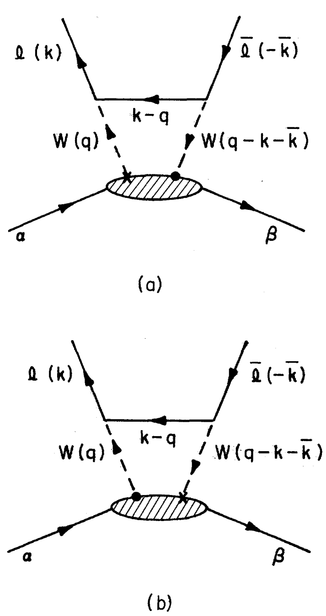


FIG. 2. Diagrams for the reaction $\alpha\to\beta+l+\bar{l}$ in the intermediate-vector-boson model.

for the Lorentz index σ, the integral in Eq. (15) can be made up of the tensors $g_{\lambda\rho}g_{\sigma\tau}$, $g_{\lambda\sigma}g_{\rho\tau}$, $g_{\lambda\tau}g_{\rho\sigma}$, and $\epsilon_{\lambda\sigma\rho\tau}$. Thus the leading contribution to Eq. (14) comes when O_σ is the strangeness-changing neutral current of dimension 3, i.e., $O_\sigma = J_\sigma^{6+i7}$. The maintenance of the form V-A [because O_σ is formed out of the product $(8,1)\oplus(8,1)$ of $\mathrm{SU}(3)\times\mathrm{SU}(3)$ representations] is noteworthy. We can now rewrite Eqs. (14) and (15) as

$$\mathfrak{L}_{\alpha\beta}(0) \simeq \tfrac{1}{2}G^2 \sin\theta \cos\theta\, \bar{u}(k)\gamma_\lambda(1-\gamma_5) \times D^{\sigma\lambda\rho}(M)\gamma_\rho(1-\gamma_5)v(\bar{k})J_\sigma^{6+i7} + \text{H.c.}, \tag{16}$$

$$D^{\sigma\lambda\rho}(M) = M^2(a g^{\lambda\rho}\gamma^\sigma + b g^{\lambda\sigma}\gamma^\rho + c g^{\rho\sigma}\gamma^\lambda + d\epsilon^{\lambda\sigma\rho\tau}\gamma_\tau),$$

where a, b, c, and d are dimensionless constants.

According to Eqs. (16), the amplitudes for decays of the type $\alpha \to \beta l \bar{l}$ are proportional to $G^2M^2\sin\theta\cos\theta$ as a consequence of the weak hadronic current having dimension 3. This means that for these decays the formulas obtained in Refs. 15 and 19 relating the corresponding rates to the fourth power of the weak-interaction cutoff Λ in specific models are compatible with our considerations. Using the W-boson model formula of Ioffe and Shabalin [Eq. (16) of the second paper of Ref. 16], we note that the present experimental upper limit[18] of 1.9×10^{-9} for the branching ratio of the decay $K_L^0 \to \mu^+\mu^-$ leads to the bound $M \leq \Lambda \leq 20$ GeV.

(b) $\Delta S = -\Delta Q$ semileptonic decays.[18] In the intermediate-boson picture a diagram for this kind of transitions involves two virtual W's – one of them in a loop – as shown in Fig. 3. An additional diagram involving a triple-W vertex may be present. However, this type of vertex does not seem to be necessary on any fundamental grounds and it needlessly complicates our considerations. Hence such a diagram will be ignored in the subsequent discussion. Now the effective Lagrangian density for the $\Delta S = -\Delta Q = \pm1$ leptonic decays can be written as

$$\mathfrak{L}_{\Delta S=-\Delta Q}(0) = \tfrac{1}{2}G^2M^6 \sin\theta\cos^2\theta\, L_\nu \Delta^{\nu\mu}(q)\int d^4x \int d^4z\, W^{\lambda\rho}(M(z-x))[TJ_\lambda^{W\dagger}(x)J_\rho^{S}(z)J_\mu^{W\dagger}(0) + \text{permutations} + \text{H.c.}]\,. \tag{17}$$

In Eq. (17),

$$\Delta_{\nu\mu}(q) = \int d^4y\, e^{iq\cdot y}M^2 W_{\nu\mu}(My),$$

q is the four-momentum carried by the final lepton pair, and $L_\nu = \bar{u}\gamma_\nu(1-\gamma_5)v$ is the current associated with the pair. The dominant short-distance contribution to the integral in Eq. (17) comes from the region where $x, z \sim O(1/M)$, i.e., when all three currents act at nearly the same point.[20] Hence the relevant operator-product expansion is

$$TJ_\lambda^{W\dagger}(x)J_\rho^{S}(z)J_\mu^{W\dagger}(0) \underset{x,z\to0}{\sim} \sum_k G_{k\lambda\rho\mu}(x,z)O_k(0). \tag{18}$$

Equation (18), used in Eq. (17), leads to the result

$$\mathfrak{L}_{\Delta S=-\Delta Q}(0) = \tfrac{1}{2}G^2\sin\theta\cos^2\theta\, L_\nu\Delta^{\nu\mu}(q)G_\mu(M) + \text{non-short-distance term}. \tag{19}$$

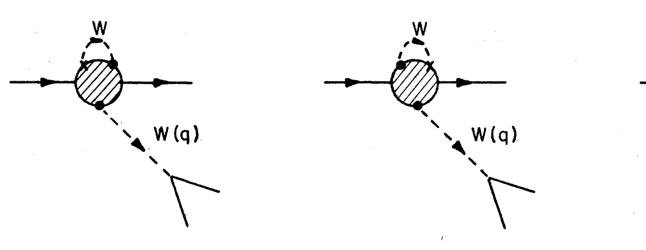
In Eq. (19)

$$G_\mu(M) = M^6\sum_k \int d^4x \int d^4z\, W^{\lambda\rho}(M(z-x)) \times G_{k\lambda\rho\mu}(x,z)O_k(0) + \text{perm.} + \text{H.c.} \tag{20}$$

From the structure of the integral

$$\int d^4x \int d^4z\, W^{\lambda\rho}(M(z-x))G_{k\lambda\rho\mu}(x,z),$$

it is clear that k has to stand for an odd number of Lorentz indices so that O_k has to be a Lorentz tensor of odd rank such as a vector field O_σ. Since it has to carry quantum numbers with $\Delta S = -\Delta Q$, it cannot be the weak hadronic current J_σ as in case (a) but has to be some higher-dimensional field of dimension d_0, say. Thus Eq. (20) may be rewritten



x = Strangeness – Changing Current
● = Strangeness – Conserving Current

FIG. 3. W-boson-model diagrams for $\Delta S = -\Delta Q = \pm1$ leptonic decays.

as

$$G_\mu(M)\simeq M^6\int d^4x\int d^4z\, W_{\lambda\rho}(M(z-x))\times[G^{\sigma\lambda\rho\mu}(x,z)O_\sigma^{\Delta S=-\Delta Q}(0)+\text{H.c.}]$$
$$\propto M^{7-d_0}O_\mu^{\Delta S=-\Delta Q}+\text{H.c.} \qquad (21)$$

Substituting Eq. (21) in Eq. (17) and writing $\Delta^{\nu\mu}(q)\simeq g^{\nu\mu}/M^2$ for $M^2\gg q^2$, we have

$$\mathcal{L}_{\Delta S=-\Delta Q}(0)\simeq\tfrac{1}{2}G^2M^{5-d_0}\sin\theta\cos^2\theta\,(L^\mu O_\mu^{\Delta S=-\Delta Q}+\text{H.c.})$$
$$+\text{non-short-distance term.} \qquad (22)$$

In the free-quark model, the lowest-dimensional field contributing to $O_\mu^{\Delta S=-\Delta Q}$ is

$$:\bar{q}\overleftrightarrow{\partial}_\mu(1-\gamma_5)\tfrac{1}{2}\lambda^{6+i7}q\bar{q}(1-\gamma_5)\tfrac{1}{2}\lambda^{W\dagger}q:$$

which has dimension 7. Thus, with naive canonical dimensions, we would have $\mathcal{L}_{\Delta S=-\Delta Q}\propto G^2\cos^2\theta\times\sin\theta\, M^{-2}$ which is suppressed by the factor M^{-2} since $O_\mu^{\Delta S=-\Delta Q}$ is an exotic field, in accordance with our basic hypothesis, we would expect $d_0\gg 7$, i.e., an even stronger suppression of $\mathcal{L}_{\Delta S=-\Delta Q}$. Thus our prediction is that because of the strong suppression factor M^{5-d_0} in the short-distance contribution, the $\Delta S=-\Delta Q$ semileptonic decays will not be observed in the immediate future unless there is radically new physics at the level of second-order weak interactions.

(c) $\Delta S=2$ leptonic decays.[18] The W-boson diagrams for these decays are identical to those for case (b) (Fig. 2) except that two of the three hadronic currents now change strangeness. The theoretical considerations for these decays proceed exactly as in case (b) and, in analogy with Eq. (22), we have

$$\mathcal{L}_{\Delta S=2}(0)=\tfrac{1}{2}G^2M^{5-d_0}\sin^2\theta\cos\theta\, L^\mu O_\mu^{\Delta S=2}$$
$$+\text{non-short-distance term.} \qquad (23)$$

The lowest-dimensional field contributing to Eq. (23) in the free-quark model is

$$O_\mu=:\bar{q}\overleftrightarrow{\partial}_\mu(1-\gamma_5)\tfrac{1}{2}\lambda^{6+i7}q\bar{q}(1-\gamma_5)\tfrac{1}{2}\lambda^{4+i5}q:$$

with canonical dimension 7. However, once again, the renormalized dimension of this exotic field is expected to be $\gg 7$. Thus the short-distance contributions here should be strongly suppressed and these decays are not expected to be observed in the near future.

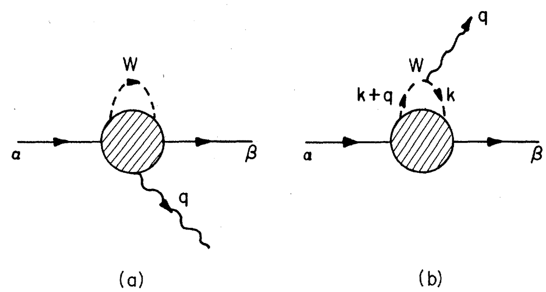


FIG. 4. Radiative nonleptonic decay in the weak-boson picture: (a) photon coupling to a hadron; (b) photon coupling to the W.

VI. RADIATIVE NONLEPTONIC WEAK DECAY

We shall consider decays of the type $\alpha\to\beta+\gamma$, where α and β are hadronic states and γ is a real photon.[1] Our discussion will be extended to include the case where the photon is virtual and decays electromagnetically into a lepton pair. In principle, the same approach can also be applied to reactions with two or more photons (e.g., $K_L^0\to 2\gamma$) or to those with an internal photon loop (e.g., the electromagnetic contribution to $K^+\to\pi^0\pi^+$); the question whether weak structure effects become enhanced in those reactions[21] is an intriguing one. However, the treatment of such problems is beyond the scope of the present work and we shall confine ourselves to processes where only one photon – real or virtual – emerges from the weak nonleptonic decay of a hadron.

First consider the decay $\alpha\to\beta+\gamma$. There are two diagrams for this reaction in the W-boson picture as illustrated in Fig. 4. Both have to be considered to preserve gauge invariance. The part corresponding to Fig. 4(b) may appear to depend strongly on the specific mechanism chosen to mediate weak interactions. However, Geshkenbein and Ioffe[4] have shown that the dominant contribution in this case can be calculated by ignoring the difference in the four-momenta carried by the W before and after the electromagnetic interaction and by using the Ward identity. Then [converting Eqs. (1) and (18) of Ref. 4 into integrals in configuration space] one is able to write[22] a less model-dependent effective interaction for this decay in the form

$$\mathcal{L}_{\alpha\to\beta\gamma}(0)=\frac{G\cos\theta\sin\theta\, e\mathcal{A}^\lambda}{\sqrt{2}}M^4\int d^4x\left\{\int d^4y\, W^{\mu\nu}(M(y-x))TJ_\mu^{W\dagger}(x)J_\nu^S(y)V_\lambda^{\text{em}}(0)+x_\lambda W^{\mu\nu}(-Mx)TJ_\mu^{W\dagger}(x)J_\nu^S(0)+\text{H.c.}\right\}. \qquad (24)$$

In Eq. (24) $\mathcal{A}^\lambda$ stands for the electromagnetic field and V_λ^{em} is the hadronic electromagnetic current. The appropriate operator-product expansions needed to evaluate the short-distance contributions to the right-hand side of Eq. (23) are the following:

$$TJ_\mu^{W\dagger}(x)J_\nu^S(y)V_\lambda^{\text{em}}(0) \underset{x,y\to 0}{\sim} \sum_a G_{\mu\nu\lambda a}(x,y)O_a(0),$$
$$TJ_\mu^{W\dagger}(x)J_\nu^S(0) \underset{x\to 0}{\sim} \sum_b C_{\mu\nu b}(x)O_b(0)\,. \tag{25}$$

Equations (24) and (25) lead to the following expression for the effective Lagrangian density:

$$\mathfrak{L}_{\alpha\to\beta\gamma}(0) = \frac{Ge\mathfrak{A}^\lambda M^4\cos\theta\sin\theta}{\sqrt{2}}\int d^4x\Big(\sum_a\int d^4y\, W^{\mu\nu}(M(y-x))G_{\mu\nu\lambda a}(x,y)\,O_a(0)$$
$$+\sum_b x_\lambda W^{\mu\nu}(-Mx)C_{\mu\nu b}(x)O_b(0)\Big)+\text{H.c.}+\text{non-short-distance term.} \tag{26}$$

From the integrals

$$\int d^4x\int d^4y\, W^{\mu\nu}(M(y-x))G_{\mu\nu\lambda a}(x,y)$$

and

$$\int d^4x\int d^4y\, x_\lambda W^{\mu\nu}(-Mx)C_{\mu\nu b}(x)$$

of Eq. (26), we can say that a and b must stand for odd numbers of Lorentz indices, i.e., O_a, O_b must be Lorentz tensors of odd rank. The first candidate for the leading member is (in analogy with the cases considered in Sec. V) the weak hadronic current, i.e., $O_\lambda = V_\lambda^{6+i7}$ or A_λ^{6+i7} (or any combination[23] of these). However, this leading contribution has to be gauge-invariant by itself since the nonleading contributions cannot compete with it. But the relevant currents required here (i.e., strangeness-changing neutral vector and axial-vector currents) are not conserved; this means that the corresponding contribution to the amplitude for the decay are not gauge-invariant since $\partial^\lambda V_\lambda^{6+i7}\neq 0$ and $\partial^\lambda A_\lambda^{6+i7}\neq 0$. Thus the contributions from $O_\alpha = V_\alpha^{6+i7}$ or A_α^{6+i7} to the two terms in the right-hand side of Eq. (23) must cancel out because of gauge invariance. Since such contributions, if present, would have made the amplitude for the decay $\alpha\to\beta+\gamma$ proportional to GeM^2 (V_λ, A_λ having dimension 3) we can see in a simple way why the quadratic divergence in the amplitude cancels out (see Ref. 4). Gauge-invariant operators can, of course, be made out of V_λ^{6+i7}, A_λ^{6+i7} by applying derivatives on them, i.e., one could choose $O_\lambda^{6+i7} = \Box V_\lambda^{6+i7} - \partial_\lambda\partial^\nu V_\nu^{6+i7}$ or $\Box A_\lambda^{6+i7} - \partial_\lambda\partial^\nu A_\nu^{6+i7}$ or any combination of these. In this case, $\partial^\lambda O_\lambda^{6+i7}$ vanishes and we have gauge invariance. However, the contributions of such terms to the transition amplitude are proportional to

$$\epsilon^\lambda\langle\beta|(q^2V_\lambda^{6+i7} - q_\lambda q^\nu V_\nu^{6+i7})|\alpha\rangle \quad\text{or}\quad \epsilon^\lambda\langle\beta|(q^2A_\lambda^{6+i7} - q_\lambda q^\nu A_\nu^{6+i7})|\alpha\rangle\,,$$

where ϵ is the photon polarization; these vanish since $q^2 = 0 = \epsilon\cdot q$.

Following the prescription given in rule number (4) of Sec. II, we see that the leading operator in the right-hand side of Eqs. (25) that can make a nontrivial gauge-invariant contribution to the decay is the divergence of the antisymmetric octet tensor current $T^i_{\alpha\beta}$. We can have both tensor and pseudotensor contributions in general, i.e., $O^i_\alpha = \partial^\beta T^i_{\alpha\beta}$ or $\epsilon_{\alpha\beta\gamma\delta}\partial^\beta T^{i\gamma\delta}$ or any combination. However, these operators belong[24] to the representation $(3,3^*)+(3^*,3)$ of SU(3)×SU(3). On the other hand, those appearing in the left-hand side of Eqs. (25) belong to (8, 1) or (8, 1) + (1, 8) and it is not possible to construct operators of the representation $(3,3^*)+(3^*,3)$ out of these products. This means that the contributions of interest to the operator-product expansions in Eqs. (25) come not from the dominant scale- [and SU(3)×SU(3)-] invariant piece but from the lowest-order scale- [and SU(3)×SU(3)-] breaking terms.[25] For the purpose of investigating the symmetry properties of these operator-product expansions, we can resort to the spurion analysis. In accordance with rule number (6) of Sec. II, we can assert that the presence of $\partial^\beta T^i_{\alpha\beta}$ or $\epsilon_{\alpha\beta\gamma\delta}\partial^\beta T^{i\gamma\delta}$ in the two operator-product expansions of Eq. (25) arises from the products $TJ_\mu^{W\dagger(0)}J_\nu^{S(0)}V_\lambda^{\text{em}(0)}\mathfrak{L}_I$ and $TJ_\mu^{W\dagger(0)}J_\nu^{S(0)}\mathfrak{L}_I$, respectively. Here $\mathfrak{L}_I$, carrying dimension 4 and associated with the scale-breaking hadron mass m_H, is $\lambda_0u_0+\lambda_8u_8+\lambda w$, but the SU(3)×SU(3)-invariant w cannot contribute in generating operators of the representation $(3,3^*)+(3^*,3)$. Since the u^0 and u^8 fields carry the dimension Δ, we can rewrite Eq. (25) as

$$TJ_\mu^{W\dagger}(x)J_\nu^S(y)V_\lambda^{\text{em}}(0) \underset{x,y\to 0}{\sim} [m_H(x-y)]^{4-\Delta}[G_{\mu\nu\lambda\alpha}(x,y)g_{\beta\gamma}+\bar{G}_{\mu\nu}(x,y)\epsilon_{\lambda\gamma\alpha\beta}]\partial^\gamma T^{6+i7,\alpha\beta}\,,$$
$$TJ_\mu^{W\dagger}(x)J_\nu^S(0) \underset{x\to 0}{\sim} [m_H(x-y)]^{4-\Delta}[C_{\mu\nu\alpha}(x)g_{\beta\gamma}+\bar{C}_{\mu\nu\delta}(x)\epsilon^{\delta\gamma\alpha\beta}]\partial_\gamma T^{6+i7}_{\alpha\beta}, \tag{27}$$

and similarly for the products of the Hermitian conjugate currents. Substituting Eq. (27) and the corresponding Hermitian conjugate relations in Eq. (26), we obtain

$$\mathcal{L}_{\alpha\to\beta\gamma}(0)=\frac{G\cos\theta\,\sin\theta\, e\,\mathfrak{A}_\lambda}{\sqrt{2}}\,S^{\lambda\gamma\alpha\beta}(M)\partial_\gamma T^{6+i7}_{\alpha\beta}+\text{H.c.}+\text{non-short-distance term.} \tag{28}$$

In Eq. (28), the structure factor $S_{\lambda\gamma\alpha\beta}(M)$ is given by

$$S_{\lambda\gamma\alpha\beta}(M)=M^2 m_H{}^{4-\Delta}\int d^4x\left(\int d^4y\, W^{\mu\nu}(M(y-x))\,\{[(x-y)^2]^{1/2}\}^{4-\Delta}[G_{\mu\nu\lambda\alpha}(x,y)g_{\beta\gamma}+\bar{G}_{\mu\nu}(x,y)\epsilon_{\lambda\gamma\alpha\beta}]\right.$$
$$\left.+(\sqrt{x^2}\,)^{4-\Delta}x_\lambda W^{\mu\nu}(-Mx)[C_{\mu\nu\alpha}(x)g_{\beta\gamma}+\bar{C}^{\mu\nu\delta}(x)\epsilon_{\delta\gamma\alpha\beta}]\right), \tag{29}$$

or

$$S_{\lambda\gamma\alpha\beta}(M)=S_1(M)g_{\lambda\alpha}g_{\beta\gamma}+S_2(M)\epsilon_{\lambda\gamma\alpha\beta}, \tag{30}$$

where $S_{1,2}(M)\propto M^{\Delta-d_T}$, d_T being the dimension of the octet of antisymmetric tensor fields $T^i_{\alpha\beta}$.

There are two important points concerning Eq. (28) that we shall comment on:

(1) According to Eqs. (28) and (30), $\mathcal{L}_{\alpha\to\beta\gamma}(0)$ is proportional to $M^{\Delta-d_T}$. The canonical free-quark-model value for both Δ and d_T is 3. The associated fields u^i and $T^i_{\alpha\beta}$ are nonexotic[26]; by fiat their dimensions are near-canonical. Hence we see that structure-dependent short-distance effects in nonleptonic radiative weak decays of the kind $\alpha\to\beta+\gamma$ are neither enhanced nor suppressed by a power of M. In fact they have a logarithmic dependence on M which arises from the region with $x-y\sim O(1/M)$, $O(1/M)\ll x,y\ll O(1/m_H)$ in the integrals of Eq. (29). In general, these effects would make finite contributions to the transition amplitude resulting in the failure of model calculations (e.g., the pole-model or the inner-bremsstrahlung hypothesis as discussed in Ref. 1, p. 660 *et seq.*) that ignore short-distance effects.

(2) Both the tensor and the pseudotensor terms will in general contribute to $S_{\lambda\gamma\alpha\beta}$. For example, in the free massless quark model, it can be demonstrated via Wick's theorem and Taylor expansions that for the second of Eqs. (27) one has

$$TJ^{i(0)}_\mu(x)J^{j(0)}_\nu(y)u^k(0)\underset{x,y\to 0}{\sim}\frac{f^{ijm}}{2(x-y)^4}\left(x_\alpha\frac{x_\mu(x-y)_\nu+x_\nu(x-y)_\mu-x\cdot(x-y)g_{\mu\nu}}{x^4}-y_\alpha\frac{y_\mu(x-y)_\nu+y_\nu(x-y)_\mu-y\cdot(x-y)g_{\mu\nu}}{y^4}\right)$$
$$\times(d^{mkl}\partial_\beta T^{l\,\alpha\beta}+f^{mkl}\epsilon^{\alpha\beta\gamma\delta}\partial_\beta T^l_{\gamma\delta})+\cdots.$$

Thus with $i=1-i2$, $j=4+i5$, and $k=8$, the combination

$$3i\epsilon^{\alpha\beta\gamma\delta}\partial_\beta T^{6+i7}_{\gamma\delta}-\partial_\beta T^{6+i7,\alpha\beta}$$

contributes to the above right-hand side. Hence, in general, it is quite reasonable to expect comparable contributions from both tensor and pseudotensor terms.

We shall now discuss the application of the above considerations to observed nonleptonic radiative decays.

(A) Two-body radiative hyperon decays[1]: $B\to B'\gamma$. The effective Lagrangian density here has the form[27]

$$\mathcal{L}_{B\to B'\gamma}(0)=\frac{Ge\cos\theta\,\sin\theta\,\mathfrak{A}^\lambda}{\sqrt{2}}\partial^\beta\bar{\psi}_{B'}\sigma_{\lambda\beta}(\lambda+\eta\gamma_5)\psi_B\,. \tag{31}$$

Strict SU(3) implies[28] $\eta=0$ for transition within a U-spin doublet, and the pole model, used in conjunction with SU(3), can predict rates for various possible decays. In this model the electromagnetic current acts outside of and at distances of order $1/m_H$ from the W loop so that $\mathcal{L}\propto M^{6-d_n}\sim M^0$ (see Sec. III). The only reaction of this type which has been studied experimentally so far is $\Sigma^+\to p\gamma$. Although the observed rate is not too different from that predicted by the pole model, a determination[29] of the correlation parameter between the direction of the final baryon and the polarization of the initial hyperon requires[27] η^{Σ^+p} to be $\simeq\lambda^{\Sigma^+p}$. Holstein[27] has made an exhaustive investigation of possible SU(3)-symmetry-breaking effects in the pole model within the chiral-Lagrangian framework and has concluded that such effects are unlikely to make η^{Σ^+p} large enough and usually change it in the wrong direction. There is, of course, the possibility that non-short-distance structure effects contribute significantly. In fact, by relating the amplitudes for $\Sigma^+\to p\pi^0\gamma$ and $\Sigma^+\to p\gamma$ by means of the soft-pion technique and current algebra, Ahmed[30] made the claim that this was actually the case. However, recently some authors[31] have demonstrated that this claim is incorrect having been based on an inappropriate extrapolation procedure; the correct application of this technique again gives too small a value for η^{Σ^+p}. These theories thus cannot account for the large observed value of this parameter. On the other hand, we have demonstrated that weak structure effects in the region where both weak currents and the electromagnetic current act near the same point are likely to cause a considerable enhancement of a symmetry-breaking term in the transition amplitude competitive with the contributions of the pole terms. We are unable to provide a quantitative evaluation of this effect. However, in view of our remark (2) above, a significant pseudotensor term is expected to be present to which the observed large magnitude of $\eta^{\Sigma^+ p}$ can be attributed. Regarding this as a success of our considerations, we make the general prediction that in all other two-body radiative hyperon decays, to be observed in future, the predictions of exact SU(3) symmetry as well as those of the pole model will fail and in general large values will be obtained for the parameter η even for transitions within a U-spin doublet.

(B) Three-body radiative hyperon decays[1]: $B \to B'\pi\gamma$. The equivalent of the pole model in this case is the inner-bremsstrahlung hypothesis that only considers diagrams where the electromagnetic current acts outside the W loop and which are proportional to M^{6-d_n}. Only the decays $\Sigma^\pm \to n + \pi^\pm + \gamma$ have been experimentally studied until now. The inner-bremsstrahlung predictions are difficult to test accurately because they involve the magnetic moment of the decaying hyperon and at present there is only qualitative agreement with experiment. Our prediction is that, with detailed experimental investigations, significant deviations from those predictions will show up because of short-distance structure effects of the sort we have considered.

(C) Nonleptonic radiative K decays.[1] There are several decays of this type which have been or are on the verge of being experimentally studied. Rudimentary data existing on[1] $K^+ \to (2\pi^+)\pi^-\gamma$ and on[2] $K_S^0 \to \pi^+\pi^-\gamma$ agree qualitatively with crude estimates based on the inner-bremsstrahlung hypothesis; however, we predict that more detailed investigations will uncover deviations due to short-distance structure effects. For the decay $K_L^0 \to \pi^+\pi^-\gamma$, the inner-bremsstrahlung contribution is related to the CP-violating decay $K_L^0 \to \pi^+\pi^-$ and is small. The non-short-distance structure-dependent contribution has been estimated[1] and found to be 1 order of magnitude smaller than the present experimental upper limit. However, short-distance effects should alter the magnitude of this prediction. Finally, we note that the success of the inner-bremsstrahlung hypothesis in explaining[1] the observed smallness of the $K^+ \to \pi^0\pi^+\gamma$ decay rate (by relating it to the $\Delta I = \frac{1}{2}$ violating decay $K^+ \to \pi^0\pi^+$) is due mostly to the kinematic enhancement of the bremsstrahlung term over most of the phase space. Gaillard[26] has analyzed the data on the $K^+ \to \pi^0\pi^+\gamma$ decay and has found that only very weak upper limits can be established on the non-bremsstrahlung terms.

Before concluding this section, we wish to remark on weak decays of the type $\alpha \to \beta l^+ l^-$ where α, β are hadronic states and the charged lepton pair $l^\pm$ emerge from a single virtual photon.[22] This process is similar to the decay $\alpha \to \beta\gamma$ except that the photon is now virtual. Because of this difference, the choice $O_\lambda^i = \Box V_\lambda^i - \partial_\lambda \partial^\nu V_\nu^i$ or $\Box A_\lambda^i - \partial_\lambda \partial^\nu A_\nu^i$ or any combination[23] of these is now permitted in the operator-product expansions of Eqs. (25). Since these operators carry the dimension 5, their contribution to the effective Lagrangian density is independent of M. Hence for these decays, we can rewrite Eq. (28) as

$$\mathfrak{L}_{\alpha\to\beta+l^++l^-} = \frac{G\cos\theta\,\sin\theta\, e^2 L^\lambda}{\sqrt{2}} [S_\lambda^{\gamma\alpha\beta}(M)\partial_\gamma T_{\alpha\beta}^{6+i7} + C_V(\Box V_\lambda^{6+i7} - \partial_\lambda \partial^\nu V_\nu^{6+i7}) + C_A(\Box A_\lambda^{6+i7} - \partial_\lambda\partial^\nu A_\nu^{6+i7}) + \text{H.c.}]$$
$$+\text{non-short-distance term}, \tag{32}$$

where L_λ is the lepton current and $C_{V,A}$ are unknown constants. Since $S_{\lambda\gamma\alpha\beta}(M)$ is expected to be only logarithmically dependent on M, the vector and tensor current contributions are competitive and both should produce observable structure-dependent effects. Of course, the above electromagnetic mechanism cannot cause any lepton nonlocality the study[19] of which can cull information on the (presumably smaller) purely weak contribution of the type discussed in article (a) of Sec. V.

VII. CONCLUSIONS

With the hypothesis that nonexotic and exotic fields have near-canonical and anomalously high dimensions, respectively, we have obtained the following results and predictions within Wilson's[5] scheme of operator-product expansions and broken-scale invariance:

(1) The $\Delta I = \frac{1}{2}$ part of the effective interaction for ordinary nonleptonic weak decays is not af-

fected by the short-distance structure of weak interactions whereas the $\Delta I = \frac{3}{2}$ part is strongly suppressed.

(2) The observed smallness of the K_1^0-K_2^0 mass difference cannot impose a severe bound on the mass M typifying the short-distance structure of weak interactions.

(3) The amplitudes for reactions of the type $K_L^0 \to \mu^+ + \mu^-$, $K \to \pi + \nu + \bar{\nu}$ are proportional to $G^2 \cos\theta \sin\theta M^2$ and are enhanced; those for the $\Delta S = -\Delta Q = \pm 1$ and the $\Delta S = 2$ leptonic decays are proportional to $G^2 \cos^2\theta \sin\theta$ and $G^2 \cos\theta \sin^2\theta$, respectively, and are not enhanced.

(4) Pole-model or inner-bremsstrahlung-hypothesis calculations as well as exact SU(3) considerations for nonleptonic radiative decays should fail in general. In particular, in radiative hyperon decays the SU(3)-violating asymmetry parameter ought to be large as observed in $\Sigma^+ \to p + \gamma$.

It is hoped that these considerations will stimulate a more vigorous experimental pursuit of structure-dependent effects in weak interactions with the more intense kaon and hyperon beams that will become available shortly.

Note added. After completing this work we have come across a paper by de Alwis[34] where the hypothesis of near-canonical and anomalously high dimensions for nonexotic and exotic fields, respectively, has been independently proposed.

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[1]A review will be found in the book by R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).

[2]It is not possible to dispose of these divergences by generating cutoffs due to strong interactions [see M. Halpern and G. Segrè, Phys. Letters 19, 611 (1967); 19, 1000(E) (1967)].

[3]C. Bouchiat, J. Iliopoulos, and J. Prentki, Nuovo Cimento 56A, 1150 (1968).

[4]B. V. Geshkenbein and B. L. Ioffe, Yadern. Fiz. 12, 1011 (1970) [Soviet J. Nucl. Phys. 12, 552 (1971)].

[5]K. Wilson, Phys. Rev. 179, 1499 (1969); SLAC Report No. SLAC-PUB-737 (unpublished); in *Broken Scale Invariance and the Light Cone,* 1971 Coral Gables Conference on Fundamental Interactions at High Energy, Vol. 2 (Gordon and Breach, New York, 1971).

[6]The idea of scale invariance is originally due to Kastrup and Mack [H. A. Kastrup, Phys. Rev. 150, 1183 (1966); G. Mack, Nucl. Phys. B5, 499 (1968)].

[7]$M^2 W_{\mu\nu}(xM) = \int [d^4q/(2\pi)^4] e^{-iq\cdot x} \Delta_{\mu\nu}(q,M)$, where, in a specific spin-one W-boson theory, $\Delta_{\mu\nu}(q,M) = (q^2 - M^2)^{-1} \times (-g_{\mu\nu} + M^{-2} q_\mu q_\nu)$.

[8]$M^2 W_{\mu\nu}(xM) = (g_{\mu\nu} + M^{-2}\partial_\mu\partial_\nu) W(xM)$, where $W(xM)$ is proportional to $\exp[-M((-x^2)^{1/2})]/(M^2x^2)^{3/4}$ for $x \gg O(1/M)$ [e.g., N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Wiley-Interscience, New York, 1959), p. 152]. The region corresponding to large x but small or timelike x^2 does not contribute to integrals such as

$$\sum_n \int d^4x\, W_{\mu\nu}(xM) \langle f|TA(x)|n\rangle \langle n|B(0)|i\rangle$$

(where A, B are local operators) that interest us here. This is because one can perform a Cottingham rotation on the x^0 integration; the integrand on the contour at infinity is proportional to $\exp[\pm i(M + E_n - E_{i,f})x^0]$, E standing for energy, and vanishes so long as $E_{i,f} \ll M$, i.e., there are no W bosons in the initial and final states. The above integral can then be written as

$$i \int d^4\bar{x}\, W_{\mu\nu}(\bar{x}M) \langle f|TA(\bar{x})B(0)|i\rangle ,$$

where $\bar{x}$ is a Euclidean four-vector so that large $\bar{x}$ corresponds to large and negative $\bar{x}^2$ where $W_{\mu\nu}(M\bar{x})$ is very small.

[9]Dimensions are defined in the skeleton theory following Ref. 5. An introductory discussion of canonical and renormalized scale dimensions will be found in the review article on broken scale invariance in particle physics by P. Carruthers, Phys. Reports 1C, 1 (1971).

[10]For the Thirring model, this has been rigorously demonstrated: K. Wilson, Phys. Rev. D 2, 1473 (1971).

[11]We stress the *heuristic* nature of our assumption in view of the theoretical controversy currently raging on whether dimensions have to be canonical or not. See Ref. 10, K. Wilson, Phys. Rev. D 2, 1478 (1970), R. Jackiw, *ibid.* 3, 2005 (1971), and R. Gatto and P. Menotti, *ibid.* (to be published).

[12]There is already some indirect evidence from the SLAC data on deep-inelastic electron-nucleon scattering, as compared with the Mack sum rule, that the symmetric tensor current $S^i_{\alpha\beta}$, which has the form $\bar{q}(\gamma_\alpha \overleftrightarrow{\partial}_\beta + \gamma_\beta \overleftrightarrow{\partial}_\alpha)\frac{1}{2}\lambda^i q$ in the free-quark model, has a dimension nearly equal to its canonical value four. See G. Mack, Phys. Rev. Letters 25, 400 (1970), and R. E. Taylor, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R., 1970.

[13]Here time-ordered products are included. The expectation that such expansions hold for products of any number of currents at short distances is based on perturbation-theoretic studies [W. Zimmermann, Commun. Math. Phys. 6, 161 (1967)] as well as on axiomatic considerations [W. Zimmermann, in *Lectures on Elementary Particles and Quantum Field Theory* (M.I.T. Press, Cam-

bridge, Mass., 1971)].

[14]S. Glashow, *Hadrons and Their Interactions*, edited by A. Zichichi (Academic, New York, 1968), pp. 102–103. In the gluon model $w(x)$ can be proportional to the square of the gluon field.

[15]See the last part of Sec. VII E in the first paper quoted in Ref. 5.

[16]R. N. Mohapatra, J. Subba Rao, and R. E. Marshak, Phys. Rev. Letters 20, 1081 (1968); B. L. Ioffe and E. P. Shabalin, Yadern. Fiz. 6, 832 (1967) [Soviet J. Nucl. Phys. 6, 603 (1968)].

[17]For comparison, consider the case of separated W loops [e.g., Fig. 1(b)], i.e., when $z \to 0$ and $x \to y$ but $\not\to 0$. Now effectively

$$\mathcal{L}_{K\bar{K}}(0) \simeq \tfrac{1}{8} G^2 M^8 \cos^2\theta \sin^2\theta \times \int d^4x \int d^4y \int d^4z \; W^{\mu\nu}(M(x-y)) W^{\lambda\sigma}(Mz) \times \{[TJ_\mu^{W\dagger}(x) J_\nu^S(y)][TJ_\lambda^{W\dagger}(z) J_\sigma^S(0)] + \text{H.c.}\}$$

and its contribution to $\Delta m_{K^0_{1,2}}$ is proportional to

$$\begin{aligned} G^2 \sin^2\theta \cos^2\theta\, M^8 \sum_j \delta^{(4)}(k - p_j) \\ \times \int d^4z \,\langle \bar{K}^0(k) | TJ_\lambda^{W\dagger}(z) J_\sigma^S(0) | j\rangle \\ \times W^{\lambda\sigma}(Mz) \int d^4u \,\langle j | TJ_\mu^{W\dagger}(u) J_\nu^S(0) | K^0(k)\rangle \\ \times W^{\mu\nu}(M(x-y)) + \text{H.c.} \\ \simeq G^2 \sin^2\theta \cos^2\theta \sum_j \delta^{(4)}(k-p_j) \sum_n C_n^{\,2}(M) \\ \times \langle K^0(k) | O_n(M) | j\rangle \langle j | O_n(M) | K^0(k)\rangle + \text{H.c.} \end{aligned}$$

(see Sec. III). Since we know that $C_n(M) \propto M^{6-d_n}$ where $d_n \simeq 6$, we can see that the type of contribution is not enhanced by any power of M.

[18]For an up to date review of the experimental upper limits on such decays see Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).

[19]S. K. Singh and L. Wolfenstein, Nucl. Phys. B24, 77 (1970).

[20]Once again, the region where the hadronic current associated with the lepton pair is outside the loop contributes a factor M^{d_n-6} which neither enhances nor suppresses the amplitude (cf. Ref. 17).

[21]We remark that Geshkenbein and Ioffe (Ref. 4) have considered these transitions in the specific intermediate-vector-boson model using the symmetry-breaking assumptions of Bouchiat, Iliopoulos, and Prentki (Ref. 3). They find that quadratic divergence is absent in processes with two real photons but is present in transitions with an internal photon loop. This leads us to believe that the electromagnetic contribution to the $K^+ \to \pi^0\pi^+$ decay amplitude is proportional to $G\alpha \sin\theta \cos\theta\, (M/m_H)^2$.

[22]Here the contribution from any anomalous magnetic moment of the W boson is ignored. The authors of Ref. 4 have shown that any such complication in the W boson does not affect reactions with a real photon but contributes a quadratic divergence to those with a virtual one. For decays of the type $\alpha \to \beta l^+ l^-$, where one virtual-photon contributions are possible, such an effect can overwhelm the quadratic divergence discussed in Sec. VI.

[23]It is easy to show that in the second of Eqs. (25) (but not necessarily in the first) O must have the form $V-A$.

[24]This can be most easily seen in the free-quark model by going to the massless limit.

[25]As will be clear in the subsequent discussion, this makes the tensor-current divergence come in with an effective dimension nearly equal to 5 so that there is no other operator in the scale- [and SU(3)×SU(3)-] invariant piece that can compete with it.

[26]M. K. Gaillard, private communication. See also M. K. Gaillard and P. Roy, Phys. Letters B (to be published).

[27]See B. R. Holstein, Nuovo Cimento 2A, 561 (1971), for a detailed discussion of these decays. G. Farrar, Phys. Rev. D 4, 212 (1971), has considered non-short-distance structure effects with results on the asymmetry parameter that sensitively depend on the hyperon magnetic moments. For a phenomenological approach see M. K. Gaillard, Nuovo Cimento 6A, 559 (1971).

[28]M. Gourdin, *Unitary Symmetry* (North-Holland, Amsterdam, 1967), p. 137.

[29]L. K. Gershwin *et al.*, Phys. Rev. 188, 2077 (1969).

[30]M. A. Ahmed, Nuovo Cimento 58A, 728 (1968).

[31]L. R. Ram Mohan, Phys. Rev. D 3, 785 (1971); L. Heiko and J. Pestieau, Lett. Nuovo Cimento 1, 347 (1971). See also the first paper in Ref. 27.

[32]E. Bellotti *et al.*, Nuovo Cimento 45A, 737 (1969); B. R. Webber, Ph.D. thesis, LRL Report No. LRL-19226 (unpublished), p. 39.

[33]C. Itzykson, M. Jacob, and G. Mahoux, Nuovo Cimento Suppl. 5, 978 (1967).

[34]S. P. de Alwis, Phys. Letters 36B, 106 (1971).