Observations on the Isostructure of Currents*

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Model-independent implications of the isostructure of currents for γ-, e-, ν-, and $\bar{\nu}$-induced production reactions are noted. Reactions on hydrogen and deuterium only are considered; separate neutron information is not used. Numerous tests are given for the absence of $I=2$ (or specific higher I values), for the electromagnetic current, and for the weak $\Delta S=0$ current (independent of class). Similar results are derived for the $|\Delta I|=\frac{1}{2}$ rule for $|\Delta S|=1$ currents, where $\bar{\nu}$-d reactions are most favorable. A consequence for K_{l4} decays is mentioned. The expansion of cross sections in terms of the azimuthal angle between the lepton and a hadron plane yields equalities implied by the absence of second-class currents (independent of isospin content) in T-conserving situations. Here the ν and $\bar{\nu}$ disintegrations of the deuteron are of special interest.

I. INTRODUCTION

The purpose of this paper is twofold. First, relations will be given which bear on the isospin content of electromagnetic and of weak currents. Secondly, for the weak $\Delta S=0$ currents, relations will be given which test the presence of second-class currents.

Use will be made of inelastic high-energy reactions: photoproduction and electroproduction in the electromagnetic case, ν- and $\bar{\nu}$-induced reactions in the weak case. Exclusively, such relations will be considered which are model-independent. In particular, no assumptions will be made on dominance of particular states (resonances) at some specific energy. Neutron targets will not be considered in order to be free of any subtraction problematics.

Both problems have roots in nuclear physics. Thus the rule $I=0,1$ for electromagnetic currents was shown to lead to model-independent mass relations within isomultiplets of sufficiently large isospin.[1] Likewise, the distinction made by Weinberg[2] between first- and second-class currents originated in β-decay studies. Here, too, model-independent statements can be made. Most recently, these have been reviewed and extended by Bég and Bernstein[3] who have introduced a further more refined classification of second-class currents. We also refer to this paper for further references to recent contributions on this subject.

All the results just mentioned have reference to the leading electromagnetic (and weak) order of the effects concerned. Similarly, in what follows, only this leading order will be considered. Effects will therefore be novel only if with confidence they can be said to exceed the order of electromagnetic (and weak) corrections. Such higher-order contributions will be dropped from here on.

In recent years, the simple theoretical views – no isotensor electromagnetic currents, no second-class weak currents – have come under renewed scrutiny. It was noted by Grishin, Lyuboshitz, Ogievetskii, and Podgoretskii[4] as well as by Dombey and Kabir[5] that the evidence for the absence of isotensor electromagnetic currents is scanty. A number of tests have been proposed by these and other authors, where the main reliance is on resonance production or on effects at threshold.[6] An exception to this is the suggestion[7] to study asymmetries in $e^+e^-\to\pi^+\pi^-\pi^0$ via one-quantum annihilation.[8] The interest in second-class currents has recently been rekindled by the experimental investigations of Wilkinson and Alburger.[9] The existence of such exotic currents of one kind or another would complicate in many ways the current theoretical picture. For this reason, some amount of reservation partly motivated the present work.

The plan of the paper is as follows: In Sec. II, photoproduction and electroproduction off deuterons and protons are discussed. (Throughout, only spin-averaged cross sections are considered.) In the course of treating the first example, single-pion production off deuterium, a distinction (hardly new) between "configuration" and "channel" is made which is central to the present argument. These two separate notions are essential in any application of isospin to states which contain more than one particle belonging to some given isomultiplet. It is then shown, first for the example at hand and then for many other cases, that there exist linear relations between configurational amplitudes which lead to "configurational inequalities" between differential cross sections. These inequalities test in fact the absence of currents with $I>1$. Considering the assemblage of final states in photoproduction or in electroproduction off d or p, one quickly realizes that there are infinitely many such inequalities. In spite of the inherent weakness of inequalities as compared to equalities, their multitude and their validity for all energies may hopefully make them useful.

While these tests are the strongest ones presented here, they are also the most demanding ones experimentally. However, it is found that one can very often (not always) derive "channel inequalities" from configurational inequalities. These channel relations are relatively easier to handle experimentally. Further integration of channel inequalities leads eventually to rate inequalities. This progression of inequalities becomes transparent only by an appropriate choice of dynamical variables in reactions γ + target $\rightarrow$ more than two particles. For example, in the case of $\gamma + d \rightarrow N + N + \pi$ it is advantageous to choose the final dinucleon invariant mass as a variable. A number of examples involving strange-particle production will also be given.

Two-pion production off d enables one, at least in principle, to distinguish between the presence of an $I=2$ current, but the absence of an $I=3$ current (Sec. II D). More generally, information is obtainable on the possible presence of isomultipole moments of any order in the electromagnetic current (Secs. II G and II K).

Deuterium reactions are treated first in honor of the fact that relations can be obtained which involve only a single π^0. Even for the simplest case of production on a proton target (see Sec. II H), one already deals with channels with two π^0's.

The discussion of weak processes is begun in Sec. III, in the first part of which the methods of Sec. II are applied to the weak $\Delta S = 0$ currents. The absence of $I>1$ currents is known to imply an inequality between single-pion production in ν-nucleon collisions.[10,11] However, neutron targets are involved in this case and, again, we shall only use proton and deuterium information here. Likewise, tests for the absence of $I>1$ currents by means of resonance production have been suggested.[12] Also for the weak processes we shall continue to insist on model-independent statements. These again take the form of inequalities. As for the electromagnetic case, one can in principle locate the presence of currents with any fixed value of I.

Since at no point are comparisons made between rates of neutrino and antineutrino processes, the distinction between first- and second-class currents, either with $I=1$ or with $I>1$, remains dormant. Thus all statements in Sec. III A are independent of class.

Since in this paper we only aim at the limited objective of rate comparisons, the question whether $I>1$ currents, if they exist at all, are to be V or A (or otherwise) does not enter. Nor is the validity of T invariance or any other discrete invariance at stake.

In Sec. III B we turn to $|\Delta S| = 1$ weak currents. The question here is whether or not there exist $I>\frac{1}{2}$ currents. Of course, the existence of $I>\frac{1}{2}$ currents is implied if any of the $\Delta S = -\Delta Q$ reactions,

$$\begin{aligned} &\nu + (p \text{ or } d) \rightarrow (\Lambda \text{ or } \Sigma \text{ or } \overline{K}^0 \text{ or } K^-) + X + \mu^-, \\ &\overline{\nu} + (p \text{ or } d) \rightarrow (K^+ \text{ or } K^0) + X + \mu^+, \end{aligned} \tag{1.1}$$

were ever found, where X is a $S=0$ hadronic complex with appropriate charge and baryon number. About such reactions we shall say nothing more.

For $\Delta S = \Delta Q$ reactions, one test of $|\Delta I| = \frac{1}{2}$ is well known[10,12]:

$$\frac{\sigma(\overline{\nu} + n \rightarrow \Sigma^- + l^+)}{\sigma(\overline{\nu} + p \rightarrow \Sigma^0 + l^+)} = 2\,. \tag{1.2}$$

Further n-p comparisons leading to equalities and inequalities have also been noted.[13]

Fortunately, continued insistence to stay away from neutron information does not deprive us of numerous implications of the semileptonic $|\Delta I| = \frac{1}{2}$ rule. There is no question that, among these, the equalities obtained for Y production in $\overline{\nu}$-d reactions are potentially the most useful ones. Y production in ν-p reactions as well as K- and $\overline{K}$-production reactions yield information as well, but it is not as strong. In Sec. III C, a brief comment is made on K_{l4} decay.

In Sec. IV the question is taken up of second-class-current effects in inelastic neutrino reactions. Since one deals here exclusively with transitions between distinct hadronic isomultiplets, the space-time properties of the weak-current matrix elements are not sufficient to make class distinctions. This was shown quite generally for β-decay transitions[14] and the reasoning applies equally well to ν and $\overline{\nu}$ reactions. In this respect the reactions at hand are distinct from analog transitions in β decay; and from "elastic" neutrino reactions, where a single process like $\nu + p \rightarrow n + \mu^+$ does contain information (hard to come by) on second-class effects, especially if polarization information were available.[15]

Thus for inelastic processes, information on the possible existence of second-class currents can come only from comparisons between ν and $\overline{\nu}$ reactions.[16] In regard to such comparisons, another essential distinction with β decays should be recalled. Let us for the moment assume that there is no T violation. Then for β transitions, a difference in rate (or better, *ft* value) between one decay and its mirror process is *prima facie* evidence for the existence of second-class currents[17] (we repeat, excluding electromagnetic effects). The closest analog to such rates are total cross sections, in the present case. However, even for conventional first-class currents, cross sections are generally distinct for the mirror processes

$$\nu + n \to l^- + X^+, \tag{1.3a}$$

$$\bar{\nu} + p \to l^+ + X^0, \tag{1.3b}$$

where $X^{+,0}$ are $S=0$ hadronic complexes belonging to the same multiplet (or superposition of multiplets). As is well known, integrating over all variables of X except its invariant mass, $d^2\sigma/dq^2d\nu$ is parametrized by functions W_i for the reaction (1.3a) and W_i^m for (1.3b), $i=1,2,3$. Absence of second-class currents gives[18] the testable relations $W_i = W_i^m$. Assuming these relations to hold, one has[19]

$$\frac{d^2\sigma(X^0)}{dq^2d\nu} - \frac{d^2\sigma(X^+)}{dq^2d\nu} = \frac{G^2}{4\pi M\epsilon^2}[q^2(\epsilon+\epsilon') - m^2(\epsilon-\epsilon')]W_3. \tag{1.4}$$

W_3 arises from V-A interference. The distinction just mentioned is that such interference effects vanish for total decay rates, but not for total cross sections.

The discussion of second-class-current effects in Sec. IV is along a different route which is perhaps better tuned to the study of individual processes. It allows for integrations over variables like q^2 and ν, something which is not possible when using $d^2\sigma/dq^2d\nu$ in the pursuit of second-class currents. Instead of the quasi-two-body description used in Eq. (1.3) we shall use a three-body (or quasi-three-body) description. Let us exemplify the reasoning by considering the reactions

$$\nu(\bar{\nu}) + N \to N' + \pi + l(\bar{l}). \tag{1.5}$$

For given energy, there are four variables.[20] One of these is the azimuth ϕ (with a sign convention) between the lepton plane and the hadron plane, in the laboratory system. Assume that the lepton current couples locally. Then

$$\frac{d\sigma}{d\phi} = \sigma_0 \cos 2\phi + \sigma_1 \sin 2\phi + \sigma_2 \cos\phi + \sigma_3 \sin\phi + \sigma_4, \tag{1.6}$$

with[19] $\sigma_i = \sigma_i(\epsilon)$. The terms σ_2, σ_3, σ_4 are known to receive contributions of the types (VV, AA, VA). σ_1 is a pure VA-interference term, while σ_0 is a pure (VV, AA) term. It is then readily seen that absence of second-class currents implies that

$$\sigma_0 = \sigma_0^m, \quad \sigma_1 = -\sigma_1^m. \tag{1.7}$$

Here, as in Sec. IV, the superscript m will always denote the mirror transition. See further Sec. IV where it is noted that this class test is independent of isospin content.

So far, T invariance has been assumed. If the possibility of T violation is admitted, a more refined simultaneous specification in terms of class and of T properties is necessary.[21] In this more general situation the relation (1.7) does not fully separate first from second class, since, in this respect, second-class T-violating currents behave much as first-class T-conserving ones. (The same is true in regard to the equality of the W_i and the W_i^m, mentioned above.) This is spelled out in Sec. IV.

What has been said for Eq. (1.5) holds true equally well for similar reactions with the right-hand side $A+B+l(\bar{l})$, where A and B are either hadrons or (one or both) hadron complexes. Sufficient integration always makes it possible to arrive at Eq. (1.6). This means, furthermore, that Eq. (1.7) can be extended to hold at an inclusive level.

The specific reactions $\nu(\bar{\nu}) + d \to N + N + l(\bar{l})$ are of special interest. Since higher isospin components of the current are dormant in this case (if they exist at all), one may hope to obtain direct information, this way, on the class properties of iso*vector* currents.

It is a serious practical limitation on all second-class-current tests mentioned here that they demand ν and $\bar{\nu}$ comparisons at the same energy (or for the same energy spectrum).

To recapitulate the situation for the $\Delta S=0$ weak currents, there are three questions: (1) isospin content, (2) T invariance, and (3) existence of second-class currents. As said, (1) can be tested independently of (2) and (3). Likewise (2) can be analyzed independently of (1) and (3); (3) can be treated independently of (1), but not of (2). Moreover, in the discussion of (3) presented here, the local action of lepton currents has to be assumed.

As said, if current views on what constitutes simplicity are any guide, one would anticipate that all will be normal in regard to these three questions. The present considerations may perhaps be of some use to settle the issues more firmly.

II. PHOTOPRODUCTION AND ELECTROPRODUCTION

What will be said for the photoproduction reactions $\gamma + A \to B$ holds equally well for $e^- + A \to B + e^-$; see the remark at the end of Sec. II A, below. For brevity, we refer below to both reactions as $\gamma + A \to B$. Note that the (γp) system may be considered as a coherent superposition of an $I=\frac{3}{2}$ and a single

$I=\frac{1}{2}$ state, in the absence of currents with $I>1$. Since proton-neutron comparisons are not made here, the distinction between the $I=\frac{1}{2}$ components of (γp) arising from the isovector current on the one hand, and from the isoscalar current on the other remains dormant.

A. $\gamma+d\rightarrow N+N+\pi$

Let us denote by "channel" a specification of a final state by means of the intrinsic quantum numbers only of the final individual particles. Thus in the present case there are three channels: $pp\pi^-$, $pn\pi^0$, $nn\pi^+$ (recall that only spin-averaged cross sections are considered here). Let us define "configuration" as a fuller specification where in addition the individual momenta are exhibited. Thus there are four configurations:

$$p(\vec{p}_1)p(\vec{p}_2)\pi^-(\vec{p}_3),$$
$$p(\vec{p}_1)n(\vec{p}_2)\pi^0(\vec{p}_3),$$
$$n(\vec{p}_1)p(\vec{p}_2)\pi^0(\vec{p}_3),$$

and

$$n(\vec{p}_1)n(\vec{p}_2)\pi^+(\vec{p}_3).$$

For brevity, configurations will hereafter be written as follows:

$$\begin{aligned} pp\pi^-&: A_1,\\ pn\pi^0&: A_2,\\ np\pi^0&: A_3,\\ nn\pi^+&: A_4, \end{aligned} \tag{2.1}$$

with the understanding that particles which appear in a given column have a common momentum. The A symbols refer to the respective amplitudes. Their dependence on an *ordered* set of momenta is understood.

Isospin decomposition of amplitudes may be applied at every point $(\vec{p}_1,\vec{p}_2,\vec{p}_3)$ of momentum space. For $\vec{p}_1\neq\vec{p}_2$ the second and third configuration are of course physically distinguishable. If the electromagnetic current has the usual $I=0,1$ components only, then the four configuration amplitudes can be expressed in terms of three reduced amplitudes, so that there is one relation between the A's:

$$A_1+(A_2+A_3)\sqrt{2}+A_4=0. \tag{2.2}$$

Thus there are four quadrangle "configurational inequalities." A representative is

$$[d^4\sigma(pp\pi^-)]^{1/2}\leq\sqrt{2}\{[d^4\sigma(pn\pi^0)]^{1/2}+[d^4\sigma(np\pi^0)]^{1/2}\} +[d^4\sigma(nn\pi^+)]^{1/2}. \tag{2.3}$$

Remarks. (1) Here and in what follows we write out explicitly only one representative for each set of polygon inequalities. The others are of course obtained by bringing the single quantity on the left to the right and any one of the quantities on the right to the left.

(2) If $a,b,c,d,\ldots$ are complex numbers and if $a+b+c+d+\cdots=0$, then there are also inequalities like $||a|-|b||\leq|c|+|d|+\cdots$. These can be discussed by similar means as inequalities of the type Eq. (2.3). We will not do this explicitly.

(3) Equation (2.2) is easily shown not to hold in the presence of an $I=2$ component in the electric current. Thus, as usual, if Eq. (2.3) is satisfied, this does not necessarily prove anything, but violations of Eq. (2.3) would constitute evidence for the presence of an $I=2$ part. Similar remarks apply to all inequalities to be recorded below.

For given energy, the differential cross sections in Eq. (2.3) are fourfold. Choose as the four variables: the invariant dinucleon mass; the angle of the emerging pion relative to the beam (in a coordinate system of one's choice); and two intrinsic dinucleon variables, such as (i) in the dinucleon rest frame, the angle between the "first" nucleon and the line of flight of the dinucleon, and (ii) the azimuth (with a sign convention) between the dinucleon plane and the (γ,π) plane (in a coordinate system of one's choice). Integrate over the last two variables. Then Eq. (2.3) yields

$$[d^2\sigma(\langle pp\rangle\pi^-)]^{1/2}\leq[d^2\sigma(\langle pn\rangle\pi^0)]^{1/2}+[d^2\sigma(\langle nn\rangle\pi^+)]^{1/2}. \tag{2.4}$$

The notation $\langle\,\rangle$ means that the particle system inside these brackets is specified only by its invariant mass, so that there is *no longer any distinction* between (pn) and (np). Thus we may properly refer to (2.4) as a differential "channel inequality," holding here for each pair of values of the invariant dinucleon mass and the pion angle. Once this far, one can of course integrate further over partial or full domains of the remaining variables. In particular,

$$[\sigma(\langle pp\rangle\pi^-)]^{1/2}\leq 2[\sigma(\langle pn\rangle\pi^0)]^{1/2}+[\sigma(\langle nn\rangle\pi^+)]^{1/2}. \tag{2.5}$$

Note that one gets three more quadrangle inequalities by bringing either A_2 or A_3 or A_4 "to one side." However, in the first two of these three instances, the corresponding channel inequalities are clearly trivial. Thus the number of nontrivial channel inequalities (two in the present case) is in general smaller than the number of configurational inequalities.

Clearly, the above can likewise be applied to electroproduction. The initial inequality (2.3) now refers to a sevenfold differential cross section, but additional integrations can be made with impunity to arrive again at Eqs. (2.4) and (2.5).

B. $\gamma+d\to K+N+\Sigma$

In the first column of Eq. (2.1) substitute $p\to K^+$, $n\to K^0$. In the third column put $\pi\to\Sigma$, for each charge. Retain the symbols $A_1,\ldots,A_4$. Then Eq. (2.2) is again valid, and so is Eq. (2.3) with the same particle substitutions. This latter modified equation is now a channel inequality, of course. It can be integrated as much as desired, up to total cross-section relations. A representative is

$$[\sigma(K^+p\Sigma^-)]^{1/2}\leq\sqrt{2}\{[\sigma(K^+n\Sigma^0)]^{1/2}+[\sigma(K^0p\Sigma^0)]^{1/2}\}+[\sigma(K^0n\Sigma^+)]^{1/2}. \quad (2.6)$$

Needless to say, the collapsed equations (2.4) and (2.5) have no analog here.

C. $\gamma+d\to K+N+\pi+\Lambda$

In the first column of Eq. (2.1) substitute again $p\to K^+$, $n\to K^0$. Add a fourth column Λ to each of the four channels. Equation (2.3) is again valid (with $d^4\sigma\to d^7\sigma$). Integrations can be made up to and including the representative

$$[\sigma(K^+p\pi^-\Lambda)]^{1/2}\leq\sqrt{2}\{[\sigma(K^+n\pi^0\Lambda)]^{1/2}+[\sigma(K^0p\pi^0\Lambda)]^{1/2}\}+[\sigma(K^0n\pi^+\Lambda)]^{1/2}. \quad (2.7)$$

D. $\gamma+d\to\pi+\pi+N+N$

This case is treated mainly to bring out a few points that could not yet be in evidence in the foregoing.

There are four channels and 10 configurations. The latter are denoted as follows:

$$\begin{aligned}&\pi^+\pi^0nn:\ A_1, \quad \pi^0\pi^0np:\ A_6,\\&\pi^0\pi^+nn:\ A_2, \quad \pi^+\pi^-np:\ A_7,\\&\pi^+\pi^-pn:\ A_3, \quad \pi^-\pi^+np:\ A_8,\\&\pi^-\pi^+pn:\ A_4, \quad \pi^-\pi^0pp:\ A_9,\\&\pi^0\pi^0pn:\ A_5, \quad \pi^0\pi^-pp:\ A_{10}.\end{aligned} \quad (2.8)$$

If the current has only $I=0,1$, four relations are found, namely,

$$A_1+A_2-A_9-A_{10}=0, \quad (2.9)$$

$$\sqrt{2}(A_1+A_2)+A_3+A_4+2A_5=0, \quad (2.10)$$

$$\sqrt{2}(A_1+A_2)+2A_6+A_7+A_8=0, \quad (2.11)$$

$$A_1-A_2+A_9-A_{10}+\sqrt{3}(A_3-A_4+A_7-A_8)=0. \quad (2.12)$$

However, if the current has $I=0,1,2$ but not an (accessible) $I=3$ component, a single relation still remains:

$$(A_1+A_2+A_9+A_{10})\sqrt{2}+(A_3+A_4+A_7+A_8)+2(A_5+A_6)=0, \quad (2.13)$$

which is of course subsumed in the relations (2.9)–(2.12). Thus the reactions at hand yield one relation, Eq. (2.13), which tests the absence of $I=3$ currents, and three relations which test the absence of $I=3$ *and* $I=2$ currents. For these last three we may choose Eqs. (2.10), (2.11), and (2.12).

Equations (2.10) and (2.11) each yield five pentagon inequalities for configurational cross sections. However, it is easily seen that Eq. (2.10) leads to only a single nontrivial channel inequality:

$$[2d^3\sigma(\langle pn\rangle,\langle 2\pi^0\rangle)]^{1/2}\leq 2[d^3\sigma(\langle nn\rangle,\langle\pi^+\pi^0\rangle)]^{1/2}+[d^3\sigma(\langle pn\rangle,\langle\pi^+\pi^-\rangle)]^{1/2}, \quad (2.14)$$

while Eq. (2.11) only yields the same Eq. (2.14) once again. Equation (2.12) does not give any nontrivial channel inequality at all.

Direct tests of Eq. (2.13) are very hard since none of the 10 configurational inequalities following from it yield a nontrivial channel inequality.

E. $\gamma+d\to\Sigma+\pi+N+K$

Substitute in Eq. (2.8) as follows: $\pi\to\Sigma$ in the first column, $n\to K^0$, $p\to K^+$ in the fourth one. Equations (2.9)–(2.13) again obtain. All relations are now integrable to channel relations. All inequalities trivial in the case of $\pi\pi NN$ now become nontrivial, such as

$$[\sigma(\Sigma^+\pi^0nK^0)]^{1/2}\leq[\sigma(\Sigma^0\pi^+nK^0)]^{1/2}+[\sigma(\Sigma^+\pi^-pK^0)]^{1/2}+[\sigma(\Sigma^-\pi^+pK^0)]^{1/2}, \quad (2.15)$$

which is the representative of four inequalities for total cross sections following from Eq. (2.9) and which tests the absence of $I>1$. For $\pi\pi NN$ we saw that the consequences of Eq. (2.10) coincided with Eq. (2.11) on the channel level of integration. This is not the case here.

The applications of Eq. (2.13) remain difficult, but at least they are now integrable to cross-section relations in a nontrivial way.

F. $\gamma+d\to\pi+\pi+N+K+\Lambda$

Substitute in Eq. (2.8) $n\to K^0\Lambda$, $p\to K^+\Lambda$ in the fourth column. Once again Eqs. (2.9)–(2.13) hold true. The integration problem is intermediate between what happened in Secs. II D and II E. Thus Eqs. (2.9) and (2.12) are trivial on the channel level for $\pi\pi NN$ and are equally so in the present case though this was not so for $\Sigma\pi NK$. On the other hand, Eqs. (2.10) and (2.11) collapsed for

$\pi\pi NN$ at the channel level, but did not do so for $\Sigma\pi NK$ and neither for the present case, where they yield

$$[2\sigma(\langle 2\pi^0\rangle p\Lambda K^0)]^{1/2} \leq [2\sigma(\langle\pi^+\pi^0\rangle n\Lambda K^0)]^{1/2} + [\sigma(\langle\pi^+\pi^-\rangle p\Lambda K^0)]^{1/2}, \quad (2.16)$$

$$[2\sigma(\langle 2\pi^0\rangle n\Lambda K^+)]^{1/2} \leq [2\sigma(\langle\pi^+\pi^0\rangle n\Lambda K^0)]^{1/2} + [\sigma(\langle\pi^+\pi^-\rangle n\Lambda K^+)]^{1/2}. \quad (2.17)$$

Also for the discussion of Eq. (2.13), the present case is intermediate to the previous two. For $\pi\pi NN$ no nontrivial integration is possible, for $\Sigma\pi NK$ one gets total cross-section inequalities by bringing any one of the A's "to one side." In the present case there are two total cross-section inequalities, namely, by bringing either A_5 or A_6 to one side.

G. $\gamma+d\to+N+N+m\pi$

The total number of configurations equals the total number of reduced isospin amplitudes of the $(2N, m\pi)$ system. There is one amplitude for $I=m+1$. Therefore there is one configurational test for the absence of $I=m+1$ in the electromagnetic current. For $I\leq m$ the relations fall into groups which follow from the absence of successively lower isospins. Example: For $m=3$ the number n_I of relations which follow from the absence of isospin values I is $n_4=1$, $n_3=5$, $n_2=13$. The relation for $I<4$ is subsumed in the relations for $I<3$. Likewise for the relations for $I<3$ in regard to $I<2$, etc. Note that for sufficiently large m, equalities begin to develop. This starts with $m=4$, where one finds 11 equalities and 21 inequalities.

H. $\gamma+p\to\pi+\pi+N$

The five configurations are

$$\begin{aligned} &\pi^+\pi^0 n:\ A_1,\\ &\pi^+\pi^- p:\ A_2,\\ &\pi^0\pi^0 p:\ A_3,\\ &\pi^0\pi^+ n:\ A_4,\\ &\pi^-\pi^+ p:\ A_5. \end{aligned} \quad (2.18)$$

If there are no $I>1$ currents,

$$2A_3+\sqrt{2}(A_1+A_4)+(A_2+A_5)=0. \quad (2.19)$$

Only one channel inequality results,

$$[2d^2\sigma(\langle\pi^0\pi^0\rangle p)]^{1/2} \leq [2d^2\sigma(\langle\pi^+\pi^0\rangle n)]^{1/2} + [d^2\sigma(\langle\pi^+\pi^-\rangle p)]^{1/2}. \quad (2.20)$$

I. $\gamma+p\to\pi+\pi+K+\Lambda$

Put $n\to K^0\Lambda$, $p\to K^+\Lambda$ in the third column of Eq. (2.18). Equation (2.19) again holds true, as does Eq. (2.20) with the same substitutions. (Of course there are more variables to integrate over.)

J. $\gamma+p\to\Sigma+\pi+K$

Put $\pi\to\Sigma$ in the first column of Eq. (2.18) and $n\to K^0$, $p\to K^+$ in the third one. Equation (2.19) results once more, leading at once to five channel inequalities, with representative

$$2[d^4\sigma(\Sigma^0\pi^0K^+)]^{1/2} \leq [2d^4\sigma(\Sigma^+\pi^0K^0)]^{1/2} + [2d^4\sigma(\Sigma^0\pi^+K^0)]^{1/2} + [d^4\sigma(\Sigma^+\pi^-K^+)]^{1/2} + [d^4\sigma(\Sigma^-\pi^+K^+)]^{1/2}. \quad (2.21)$$

These inequalities may be integrated *ad libitum*.

K. $\gamma+p\to N+m\pi$

The total number of configurations equals the total number of reduced isospin amplitudes of the $(N, m\pi)$ system. There is one amplitude for $I=m+\frac{1}{2}$. Therefore there is one configurational relation implied by the absence of $I=m$ in the electromagnetic current. Example: For $m=4$, the number of n_I of relations following from the absence of isospin values greater than I is $n_4=1$, $n_3=5$, $n_2=14$. Subsumptions take place just as explained at the end of Sec. II G. Equalities develop for $m\geq 6$.

III. ISOSPIN CONTENT OF WEAK CURRENTS

A. $\Delta S=0$. Implications Independent of Class

As stated in Sec. I, one can ask questions about the isospin content of weak currents which do not touch on whether second-class currents are present or not. We give a few examples. The line of argument is as in Sec. II.

1. *Reactions off Deuterium*

Consider the reactions

$$\nu+d\to N+N+\pi+\mu^-. \quad (3.1)$$

There are three configurations. Absence of $I>1$ components in the weak current implies three inequalities with representative

$$[2d^7\sigma(pp\pi^0)]^{1/2} \leq [d^7\sigma(pn\pi^+)]^{1/2} + [d^7\sigma(np\pi^+)]^{1/2}. \quad (3.2)$$

Upon integration,

$$d^2\sigma(\langle pp\rangle\pi^0) \leq d^2\sigma(\langle pn\rangle\pi^+). \quad (3.3)$$

Here one has integrated over all variables but the invariant dinucleon mass and the angle of pion emission. Further integration is possible as usual.

As in Secs. II B and II C one can directly infer corresponding results for some hyperon production reactions. Thus for

$$\nu + d \to K + N + \Sigma + \mu^-, \tag{3.4}$$

absence of $I > 1$ currents yields three triangle relations, with representative

$$[2\sigma(K^+p\Sigma^0)]^{1/2} \leq [\sigma(K^+n\Sigma^+)]^{1/2} + [\sigma(K^0p\Sigma^+)]^{1/2}. \tag{3.5}$$

Application to

$$\nu + d \to K + N + \pi + \Lambda + \mu^- \tag{3.6}$$

also gives three triangle relations, with representative

$$[2\sigma(K^+p\pi^0\Lambda)]^{1/2} \leq [\sigma(K^+n\pi^+\Lambda)]^{1/2} + [\sigma(K^0p\pi^+\Lambda)]^{1/2}. \tag{3.7}$$

All these results have analogs, of course, for the antineutrino reactions

$$\begin{aligned} &\bar\nu + d \to N + N + \pi + \mu^+, \\ &\bar\nu + d \to K + N + \Sigma + \mu^+, \\ &\bar\nu + d \to K + N + \pi + \Lambda + \mu^+. \end{aligned} \tag{3.8}$$

Here one obtains the inequalities corresponding to Eqs. (3.3), (3.5), and (3.7) by replacing in these equations each particle symbol by its charge-symmetric counterpart.

For multipion production, one can prove general theorems analogous to those given in Sec. II G.

2. *Reactions off Hydrogen*

Consider first the three configurations corresponding to

$$\nu + p \to N + \pi + \pi + \mu^-. \tag{3.9}$$

As in Eq. (3.3) one finds a single $d^2\sigma$ relation; or, in fully integrated form,

$$\sigma(p\langle\pi^+\pi^0\rangle) \geq \sigma(n\langle\pi^+\pi^+\rangle). \tag{3.10}$$

By going to $\Lambda K\pi\pi$ one likewise finds a single integrated inequality

$$\sigma(\Lambda K^+\langle\pi^+\pi^0\rangle) \geq \sigma(\Lambda K^0\langle\pi^+\pi^+\rangle). \tag{3.11}$$

$\Sigma K\pi$ yields a triple of triangle inequalities with representative

$$[\sigma(\Sigma^+K^+\pi^0)]^{1/2} + [\sigma(\Sigma^0K^+\pi^+)]^{1/2} \geq [2\sigma(\Sigma^+\pi^+K^0)]^{1/2}. \tag{3.12}$$

For

$$\bar\nu + p \to N + \pi + \pi + \mu^+, \tag{3.13}$$

there are five configurations. There is one pentagon inequality which yields

$$[2\bar\sigma(n\langle 2\pi^0\rangle)]^{1/2} \leq [2\bar\sigma(p\langle\pi^-\pi^0\rangle)]^{1/2} + [\bar\sigma(n\langle\pi^+\pi^-\rangle)]^{1/2}. \tag{3.14}$$

The same inequality holds for $\Lambda K\pi\pi$ by substituting $n \to \Lambda K^0$, $p \to \Lambda K^+$ in Eq. (3.14). For $\Sigma K\pi$ one obtains a set of pentagon inequalities.

For multipion production, there are again theorems similar to what was found in Sec. II K.

B. Relations for $|\Delta S| = 1$

1. *Deuterium*

a. Hyperon production. As said in Sec. I, we deal here exclusively with $\bar\nu$ reactions. First, note the following two equalities for which configuration considerations are unnecessary. (In this section, all relations are expressed as much as possible in terms of total cross sections. From the foregoing, it will be obvious where and to what extent they will also be valid differentially.)

For $\bar\nu + d \to \Sigma + N + \mu^+$, $|\Delta I| = \frac{1}{2}$ yields

$$\frac{\bar\sigma(\Sigma^-p)}{\bar\sigma(\Sigma^0 n)} = 2, \tag{3.15}$$

and for $\bar\nu + d \to \Lambda + N + \pi + \mu^+$,

$$\frac{\bar\sigma(\Lambda p\pi^-)}{\bar\sigma(\Lambda n\pi^0)} = 2. \tag{3.16}$$

(All reactions are fully labeled by the hadronic content of the final state.) Equation (3.15) is of some advantage in practice over Eq. (1.2) since it involves a single d experiment.

In the case of the five reactions

$$\bar\nu + d \to \Sigma + N + \pi + \mu^+, \tag{3.17}$$

one finds two equalities and one inequality:

$$\bar\sigma(\Sigma^0 p\pi^-) = \bar\sigma(\Sigma^- p\pi^0), \tag{3.18}$$

$$\bar\sigma(\Sigma^- n\pi^+) + \bar\sigma(\Sigma^+ n\pi^-) = 2\bar\sigma(\Sigma^0 n\pi^0) + \bar\sigma(\Sigma^- p\pi^0), \tag{3.19}$$

$$[\bar\sigma(\Sigma^- n\pi^+)\bar\sigma(\Sigma^+ n\pi^-)]^{1/2} \geq \bar\sigma(\Sigma^0 n\pi^0). \tag{3.20}$$

Note in addition that if the current were to contain $I = \frac{3}{2}$ and $\frac{1}{2}$ but not $I = \frac{5}{2}$, a single inequality still survives. This shows that, hard as it is, information on separate isospin values greater than $\frac{1}{2}$ is in principle available. This holds also for many of the reactions to follow.

For

$$\bar\nu + d \to \Lambda + N + 2\pi + \mu^+, \tag{3.21}$$

the analog to Eq. (3.18) becomes trivial (in integrated form). Here one has

$$2\bar{\sigma}(\Lambda n\langle\pi^+\pi^-\rangle) = 4\bar{\sigma}(\Lambda n\langle 2\pi^0\rangle) + \bar{\sigma}(\Lambda p\langle\pi^-\pi^0\rangle), \quad (3.22)$$

$$\bar{\sigma}(\Lambda n\langle\pi^+\pi^-\rangle) \geq 2\bar{\sigma}(\Lambda n\langle 2\pi^0\rangle). \quad (3.23)$$

Increasing the pion multiplicities yields increasing numbers of relations. Example: $\bar{\nu}+d\rightarrow\Sigma N 3\pi\mu^+$ gives nine relations of which six are equalities.

b. K production. For

$$\nu + d \rightarrow N + N + K + \mu^-, \quad (3.24)$$

one has

$$2\sigma(\langle pn\rangle K^+) \geq \sigma(\langle pp\rangle K^0). \quad (3.25)$$

If in addition a pion is produced, one finds four relations of which two are equalities. The corresponding $\bar{\nu}+d$ results follow by obvious substitution.

2. *Hydrogen*

a. Hyperon production.

$$\bar{\nu} + p \rightarrow \Sigma + \pi + \mu^+ \quad (3.26)$$

yields three inequalities with representative

$$2[\bar{\sigma}(\Sigma^0\pi^0)]^{1/2} \leq [\bar{\sigma}(\Sigma^+\pi^-)]^{1/2} + [\bar{\sigma}(\Sigma^-\pi^+)]^{1/2}. \quad (3.27)$$

For

$$\bar{\nu} + p \rightarrow \Lambda + 2\pi + \mu^+, \quad (3.28)$$

one finds one channel inequality:

$$2\bar{\sigma}(\Lambda\langle 2\pi^0\rangle) \leq \bar{\sigma}(\Lambda\langle\pi^+\pi^-\rangle). \quad (3.29)$$

Additional pion production eventually produces equalities.

b. K production.

$$\nu + p \rightarrow N + \pi + K + \mu^- \quad (3.30)$$

yields three inequalities with representative

$$[\sigma(p\pi^0K^+)]^{1/2} \leq [\sigma(n\pi^+K^+)]^{1/2} + [2\sigma(p\pi^+K^0)]^{1/2}. \quad (3.31)$$

Production of an additional pion yields four relations among which there is one equality.

$$\bar{\nu} + p \rightarrow N + \pi + \bar{K} + \mu^+ \quad (3.32)$$

yields four inequalities with representative

$$[2\bar{\sigma}(n\pi^0K^0)]^{1/2} \leq [\bar{\sigma}(n\pi^+K^-)]^{1/2} + [\bar{\sigma}(p\pi^-\bar{K}^0)]^{1/2} + [2\bar{\sigma}(p\pi^0K^-)]^{1/2}. \quad (3.33)$$

An equality emerges only when two more pions are produced.

3. K_{l4} *Decay*

For K^+ (K^- gives similar results) the decays are

$$K^+ \rightarrow \pi^+ + \pi^- + \bar{l} + \nu_l$$
$$\rightarrow \pi^0 + \pi^0 + \bar{l} + \nu_l. \quad (3.34)$$

There are three configurations. $|\Delta I| = \frac{1}{2}$ implies

$$[d^5R(\pi^+\pi^-)]^{1/2} + [d^5R(\pi^-\pi^+)]^{1/2} \geq [2d^5R(\pi^0\pi^0)]^{1/2}. \quad (3.35)$$

Take as one of the five variables[21] the angle θ_π between the three-momentum of one of the pions (say "the first one" in each configuration) and the line of flight of the dipion in the K rest frame. Integration over that angle yields

$$d^4R\langle\pi^+\pi^-\rangle \geq 2d^4R\langle\pi^0\pi^0\rangle. \quad (3.36)$$

This very detailed implication of $|\Delta I| = \frac{1}{2}$ is written in its full fourfold differential form. The four variables are: the invariant dipion and dilepton masses; the angle between $\bar{l}$ and the dilepton line of flight, in the K rest frame; and the azimuth between the dilepton and dipion planes, in the same frame. Integration over one or more of these variables maintains Eq. (3.36), of course. (Cf., e.g., $K^+\rightarrow 3\pi$ where one gets three configuration relations, leading to an upper and a lower bound for the branching ratio.)

IV. SECOND-CLASS CURRENTS

A. General Considerations

Consider the $\Delta S = 0$ transitions (T = target)

$$\nu + T \rightarrow A + B + \mu^-, \quad (4.1)$$

$$\bar{\nu} + T^m \rightarrow A^m + B^m + \mu^+. \quad (4.2)$$

A and B are hadrons, or complexes thereof; m is the mirror. Thus A^m is obtained from A by a 180° isospin rotation around the 2 axis. $(A+B)$ may be in a variety of configurations, labeled by (c). Call $\mathfrak{M}_\mu^{(c)}$ the hadronic current matrix element for Eq. (4.1) in a given configuration. Likewise $\mathfrak{M}_\mu^{(c)m}$ refers to Eq. (4.2).

The respective spin-averaged cross sections are proportional to[22]

$$W_{\mu\nu}^{(c)}\tau_{\mu\nu}^+, \quad W_{\mu\nu}^{(c)m}\tau_{\mu\nu}^-,$$
$$W_{\mu\nu}^{(c)} = (\mathfrak{M}_\mu^{(c)*}\mathfrak{M}_\nu^{(c)})_{\text{av}}, \quad (4.3)$$
$$W_{\mu\nu}^{(c)m} = (\mathfrak{M}_\mu^{(c)m*}\mathfrak{M}_\nu^{(c)m})_{\text{av}},$$

where $(\;)_{\text{av}}$ denotes spin average and where

$$\tau_{\mu\nu}^{\pm} = n_\mu n_\nu - q_\mu q_\nu + \delta_{\mu\nu}(q^2+m^2) \mp \epsilon_{\mu\nu\alpha\beta}n_\alpha q_\beta, \quad (4.4)$$

with[19] $n = q_1 + q_2$, $q = q_1 - q_2$. The ± sign in Eq. (4.4) leads to the cross-section differences independent of class, discussed after Eq. (1.3).

We have

$$\mathfrak{M}_\mu^{(c)} = \langle A, B; c|\mathcal{J}_\mu|T\rangle,$$
$$\mathfrak{M}_\mu^{(c)m} = \langle A^m, B^m; c|\mathcal{J}_\mu^\dagger|T^m\rangle, \quad (4.5)$$

where $\mathcal{J}_\mu$ is the charge-raising weak $\Delta S = 0$ current and, with suitable conventions, $\mathcal{J}_\mu^\dagger$ is its Hermitian conjugate. The latter is a charge-lowering current. The question of class arises when one asks for the relation between $\mathcal{J}_\mu^\dagger$ and $\mathcal{J}_\mu^m$ which is also a charge-lowering current.

Let

$$\mathcal{J}_\mu = V_\mu + A_\mu \tag{4.6}$$

be the decomposition in vector and axial-vector parts. Decompose each current further into its isospin components labeled by j:

$$\begin{aligned} V_\mu &= \sum_j c_V(j) V_\mu(j), \\ A_\mu &= \sum_j c_A(j) A_\mu(j), \end{aligned} \tag{4.7}$$

with the convention that the $c(j)$ be real numbers ($j=1$ only corresponds to the conventional picture). Treat the adjoint and the mirror likewise.

Definitions.[23] If

$$\begin{aligned} V_\mu(j)^\dagger &= (-1)^{j-1}\eta_T(j) V_\mu^m(j), \\ A_\mu(j)^\dagger &= (-1)^{j-1}\eta_T(j) A_\mu^m(j), \end{aligned} \tag{4.8}$$

then the respective currents are pure first-class. If

$$\begin{aligned} V_\mu(j)^\dagger &= (-1)^{j}\eta_T(j) V_\mu^m(j), \\ A_\mu(j)^\dagger &= (-1)^{j}\eta_T(j) A_\mu^m(j), \end{aligned} \tag{4.9}$$

then they are pure second-class. If $\eta_T=1$, then they are T-conserving; for $\eta_T=-1$ they are T-violating. Of course, T conservation or violation is a question of relative phases. We choose $\eta_T(1)=1$ to refer to T-conserving iso*vector* currents by convention. Then T conservation corresponds to $\eta_T(j)=1$, all j.

The system (AB) can be expanded in isospin eigenstates with total isospin labeled by i. If there is more than one state for given i, additional labels may be necessary, collectively denoted by n. The expansion of $(A^m B^m)^{(c)}$ is then fixed without further ado. Expand Eq. (4.5) as follows:

$$\mathfrak{M}_\mu^{(c)} = \sum_{i,n,j} \mathfrak{M}_\mu^{(c)}(i,n,j), \tag{4.10}$$

$$\mathfrak{M}_\mu^{(c)m} = \sum_{i,n,j} \mathfrak{M}_\mu^{(c)m}(i,n,j). \tag{4.11}$$

j again refers to the isospin content of the current. Then the relation between the reduced matrix elements $\mathfrak{M}_\mu^{(c)}(i,n,j)$ and $\mathfrak{M}_\mu^{(c)m}(i,n,j)$ is completely determined once we specify the class properties and the T properties of the currents in terms of Eqs. (4.8) and (4.9). With reference to Eq. (4.3) one now readily sees that

$$W_{\mu\nu}^{(c)} = W_{\mu\nu}^{(c)m}, \tag{4.12}$$

for any superposition of first-class currents with $\eta_T=1$ and second-class currents with $\eta_T=-1$. For brevity I shall refer to this situation as mirror symmetry. Thus mirror symmetry ≡ charge symmetry if T is conserved.

It remains to find experimental consequences which can uniquely be attributed to mirror symmetry. To this end consider first the case that A and B are single hadrons. Choose as variables[19] ϵ, q^2, ν; ϕ defined in Sec. I; and Δ^2, the invariant momentum transfer between T and A. Then

$$W_{\mu\nu}^{(c)}\tau_{\mu\nu}^{+} = \sum_{A=1}^{9} F_A^{(c)}(q^2,\nu,\Delta^2) X_A(\epsilon,\phi,q^2,\nu), \tag{4.13}$$

where the F_A depend on the detailed dynamics, but where the X_A are explicitly known as a consequence of local action:

$$\begin{aligned} &X_1=1,\quad X_2=z^2,\quad X_3=(2x-1)z\cos\phi,\quad X_4=z^2\cos 2\phi, \\ &X_5=z\sin\phi,\quad X_6=(2x-1),\quad X_7=z\cos\phi, \\ &X_8=(2x-1)z\sin\phi,\quad X_9=z^2\sin 2\phi, \end{aligned} \tag{4.14}$$

with

$$x=\epsilon/\nu,\quad y=q^2/4\nu^2,\quad z=[x(x-1-y/x)]^{1/2}. \tag{4.15}$$

The terms $A=1,\ldots,5$ are pure (VV, AA) terms, while $A=6,\ldots,9$ arise from VA interference. The corresponding expansion for $W_{\mu\nu}^{(c)m}\tau_{\mu\nu}^{-}$ is obtained by substituting

$$F_A^{(c)} \to F_A^{(c)m},\quad X_A \to \eta_A X_A,$$
$$\eta_A = \begin{cases} 1 & (A=1,\ldots,5) \\ -1 & (A=6,\ldots,9). \end{cases} \tag{4.16}$$

The validity of (4.12) implies that

$$F_A^{(c)} = F_A^{(c)m}. \tag{4.17}$$

Since the dependence on ϵ and on ϕ is explicitly known, one can in principle translate Eq. (4.12) into nine equalities. However, it is vastly more economical to integrate over all variables except ϕ. This then leads to Eq. (1.6) and Eq. (1.7) is once again a consequence of Eq. (4.12).

For the case where B is a complex, integrate first over its internal variables, after which one has six residual variables: the five mentioned above and (the invariant mass) s_B. From there on all that was said from Eq. (4.13) continues to hold, except that $F_A^{(c)}$ now depends also on s_B.

B. Applications

Let us first dispose of proton targets. Consider the six reactions

$$\begin{aligned} &\nu+p\to p+\pi^+ +\mu^-:\ \mathfrak{M}_\mu^{(1)},\quad \bar\nu+n\to n+\pi^- +\mu^+:\ \mathfrak{M}_\mu^{(1)m}, \\ &\nu+n\to p+\pi^0 +\mu^-:\ \mathfrak{M}_\mu^{(2)},\quad \bar\nu+p\to n+\pi^0 +\mu^+:\ \mathfrak{M}_\mu^{(2)m}, \\ &\nu+n\to n+\pi^+ +\mu^-:\ \mathfrak{M}_\mu^{(3)},\quad \bar\nu+p\to p+\pi^- +\mu^+:\ \mathfrak{M}_\mu^{(3)m}. \end{aligned} \tag{4.18}$$

Here Eq. (4.12) leads to three pairs of equalities Eq. (1.7). But now, of course, one gets involved with n-p comparisons which we want to avoid. However, there is a result for which neutrons are not necessary; namely, if there is no isotensor current, then there exists an inequality between the $(p\pi^+)$, $(n\pi^0)$, and $(p\pi^-)$ channels implied by mirror symmetry. All three channels are produced on proton targets. However, since equalities do exist as tests, it is most inefficient to study mirror symmetry on proton targets.

Once again we turn to deuterium. The equations (1.7) for

$$\nu + d \rightarrow p + p + \mu^- \tag{4.19}$$

as compared with

$$\overline{\nu} + d \rightarrow n + n + \mu^+ \tag{4.20}$$

are perhaps the simplest case one can have for the study of mirror symmetry, purely for $I = 1$ currents.

If one wishes to compare

$$\begin{aligned} &\nu + d \rightarrow N + N + \pi + \mu^-, \\ &\overline{\nu} + d \rightarrow N^m + N^m + \pi^m + \mu^+, \end{aligned} \tag{4.21}$$

one has options. One can either integrate over internal dinucleon variables and use the (dinucleon, π) plane to define ϕ. Or else one can integrate such that ϕ has reference to the $(N, \langle N\pi\rangle)$ plane. In either case Eq. (1.7) is a consequence of mirror symmetry. Clearly one can also sum over channels. Thus, generally Eq. (1.7) may be applied as well to

$$\begin{aligned} &\nu + d \rightarrow p + X^+ + \mu^-, \\ &\overline{\nu} + d \rightarrow n + X^0 + \mu^+, \end{aligned} \tag{4.22}$$

where $X^{+,0}$ are $\Delta S = 0$ complexes related by charge symmetry. Likewise, instead of singling out (p, n) one may take any pair of charge-symmetric particles and sum over all else, as long as an azimuth remains properly definable.

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[1]S. Treiman and S. Weinberg, Phys. Rev. 116, 464 (1959).

[2]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[3]M. A. B. Bég and J. Bernstein, Phys. Rev. D 5, 714 (1972).

[4]V. Grishin, V. Lyuboshitz, V. Ogievetskii, and M. Podgoretskii, Yadern. Fiz. 4, 126 (1966) [Soviet J. Nucl. Phys. 4, 90 (1967)].

[5]N. Dombey and P. Kabir, Phys. Rev. Letters 17, 730 (1966).

[6]See, e.g., A. Sanda and G. Shaw, Phys. Rev. D 3, 243 (1971), also for references to earlier contributions.

[7]E. Bergmann, Phys. Rev. 172, 1441 (1968); G. Domokos and P. Suranyi, Lett. Nuovo Cimento 4, 307 (1970).

[8]Attempts to distinguish charge conjugation alternatives of isotensor currents do not form part of the present work. On this question see A. Sanda and G. Shaw, Phys. Rev. Letters 26, 1057 (1971).

[9]D. Wilkinson and D. Alburger, Phys. Rev. Letters 24, 1134 (1970).

[10]T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).

[11]C. Llewellyn Smith, Phys. Reports (to be published). In this paper further inequalities are also given involving associated production; see Eq. (2.19).

[12]M. M. Block, Phys. Rev. Letters 12, 262 (1964).

[13]Reference 11, Eqs. (2.15)–(2.17).

[14]Reference 3, Sec. III.

[15]A. Pais, Ann. Phys. (N.Y.) 63, 361 (1971) and erratum 69, 604 (1972).

[16]Barring special mechanisms, see, e.g., N. Dombey, Phys. Rev. 174, 2127 (1968).

[17]S. Weinberg, Phys. Rev. 115, 481 (1959). It is well known (see Ref. 2) that second-class-current effects, if present at all, are kinematically suppressed in select instances.

[18]Reference 11, Eq. (2.20).

[19]See Ref. 15, Eq. (3.12). q^2 is the lepton momentum transfer, ν is the lepton energy transfer in the lab system, ϵ and ϵ' are the initial (final) lepton energies in that system, m is the lepton mass, and q_1 (q_2) is the incoming (outgoing) lepton four-momentum.

[20]For further details see A. Pais and S. Treiman, Phys. Rev. D 1, 907 (1970).

[21]See Refs. 10 and 15, and especially the general discussion in Ref. 3.

[22]More details are given in A. Pais and S. Treiman, Phys. Rev. 168, 1858 (1968).

[23]These definitions follow standard usage, cf. Ref. 3. Equations (4.8) and (4.9) refer to a real metric. In a Minkowski metric one must insert a familiar minus sign for the fourth component.