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Symmetry of Leptonic Weak Interactions and Lepton Mass Spectrum*

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The invariance principle that excludes the neutral leptonic currents and uniquely determines the leptonic weak interactions is discussed. Based on symmetry considerations and the existence of two different neutrinos, we suggest that the neutrino masses are not strictly zero and there are new heavy charged leptons and their neutrinos. A possible lepton mass spectrum is obtained.

We know many facts about leptons and their interactions yet we are still unable to relate them to a symmetry principle. For example, the weak-interaction Lagrangian L_I does not contain the neutral leptonic currents and the weak leptonic current does not explicitly contain the momentum operator. The conservation laws of the lepton number and the muon number are inadequate to forbid the unobserved neutral leptonic decay modes. So, the experimental absence of these decay modes suggests there is a new selection rule forbidding them. Furthermore, we observe the weak leptonic current $i\bar{l}(x)\gamma_\lambda(1+\gamma_5)\nu(x)$ is not a conserved quantity unless $l(x)$ and $\nu(x)$ are massless fields. This suggests $l(x)$ and $\nu(x)$ carry a new kind of quantum number which is not conserved and is probably related to the nonvanishing lepton masses and their differences. This is not a pure speculation, since the connection between the nonconservation of hypercharge and the K_L^0-K_S^0 mass difference is a realistic example.

We shall define an imaginary quantum number iI for the leptons. It is related to a one-dimensional nonunitary transformation and is not conserved. However, it is violated in a very definite and orderly way, and provides a new selection rule to forbid the unobserved neutral leptonic processes. We emphasize that the very violation of the quantum number as well as the very existence of the conserved quantum number is likely to provide clues to the mysterious dynamics of lepton interactions.[1]

Suppose the interaction Lagrangian density $L_I(x)$ takes the form

$$L_I(x)=gl_\lambda(x)W_\lambda(x)+\cdots, \tag{1}$$

where $l_\lambda(x)$ is the lepton current and $W_\lambda(x)$ may be a combination of vector fields. [If L_I is of current-current type, then our following symmetry requirement should apply to the current $l_\lambda(x)$.]

In addition to the usual lepton number L and muon number U, let us introduce an "imaginary lepton quantum number" iI for the leptons. They are defined for a lepton $\psi(U,L,I)$ as follows:

$$\begin{aligned}&\mu^-(1,1,2), \quad \mu^+(-1,-1,2),\\ &e^-(0,1,1), \quad e^+(0,-1,1),\\ &\nu_e(0,1,-1), \quad \bar{\nu}_e(0,-1,-1),\\ &\nu_\mu(1,1,-2), \quad \bar{\nu}_\mu(-1,-1,-2).\end{aligned} \tag{2}$$

The nonleptons are assigned to have $U=L=I=0$. Consider the transformation of the type $\psi(x) \to e^{-iQ\Delta}\psi(x)$, where Q is the quantum number under consideration and Δ is real. For $Q=U$ or L, it is the ordinary gauge transformation. The space-time-independent gauge function Δ is real and arbitrary, and the quantum numbers U and L are conserved. Nevertheless, for $Q=iI$ the transformation is not unitary and therefore I is not conserved. In general, the parameter Λ' in $e^{-i(iI)\Lambda'}$ might be different for different lepton fields, and the quantum number I must have the same magnitude and the same sign for a lepton and its antilepton.

Consider the following transformation for $\psi=e,\mu,\ldots$:

$$\begin{aligned}&\psi(x)\to e^{-iU_\psi\Delta}e^{-iL_\psi\Lambda}e^{I_\psi\Lambda'_\psi}\gamma_5\lambda^2\psi(\lambda x),\\ &m\to -m,\end{aligned} \tag{3}$$

where $\Lambda'_\psi=\Lambda'_{\psi'}$ *if* $|I_\psi|=|I_{\psi'}|$. The transformation (3) is made of the gauge transformation, the scale transformation, and Sakurai's "mass-reversal transformation",[2] and we shall term it the "chord transformation." If we require $\int L_I(x)d^4x$ to be invariant under the chord transformation (3), the $V-A$ structure of the lepton currents can be uniquely determined without assuming that the lepton current does not explicitly contain the momentum operator. Moreover, the neutral leptonic currents such as $(\bar{\mu}\mu)$, $(\bar{\nu}_\mu\nu_\mu)$, etc. are excluded from $L_I(x)$ and we have a unique $L_I(x)$:

$$L_I(x)=ig[\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e+\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu]W_\lambda+\text{H.c.} \quad (4)$$

Note that the imaginary lepton number iI is violated in a definite way, namely, $\Delta I=$even.

It is obvious that the total Lagrangian L_t for the charged lepton fields $\psi=e,\mu$,

$$L_t=\int[L_f(x)+L_I(x)]d^4x, \quad (5)$$

$$L_f(x)=\sum_{\psi=e,\mu}[-\bar{\psi}(x)\gamma_\mu\partial_\mu\psi(x)-m_\psi\bar{\psi}(x)\psi(x)], \quad (6)$$

is not invariant under the chord transformation (3) because of the presence of $L_f(x)$. If we require the canonical commutation rules for the charged leptons to be invariant under the scale transformation $\psi(x)\to e^{I_\psi\Lambda'_\psi}\lambda^2\psi(\lambda x)$, which is a part of the chord transformation, we have

$$\lambda=e^{-2I_\psi\Lambda'_\psi} \quad (\psi=e,\mu). \quad (7)$$

Now, under the chord transformation (3), we have

$$L_t\to L_t'=\int d^4x'\left\{\sum_{\psi=e,\mu}[-\bar{\psi}(x')\gamma_\mu\partial'_\mu\psi(x') -m_\psi e^{2I_\psi\Lambda'_\psi}\bar{\psi}(x')\psi(x')]+L_I(x')\right\}. \quad (8)$$

We see that, aside from the mass term, the total Lagrangian L_t for the charged leptons is symmetric under the chord transformation (3) with $\lambda=e^{-2I_\psi\Lambda'_\psi}$ $(\psi=e,\mu)$. What does this symmetry mean? Why is the symmetry breaking due to the mass term related to the imaginary lepton number in such a specific way?

Tennakone and Pakvasa[3] have considered the consequence of a discrete scale transformation on a *free Lagrangian* for a Dirac field. They obtain a "similar" result, which leads them to speculate on the existence of a series of leptons. Unfortunately, their scale transformation and the associated integer n have nothing to do with the weak interactions. One might ask why should these fermions with masses $m\rho^n$ (n integral) be related to the leptons and the weak interactions?

Here, our chord transformation (3) uniquely and completely determines the weak interactions of the leptons; and we consider the transformation of both the free Lagrangian of the charged leptons and their interactions. So, it is more interesting to speculate on the possible connection between the symmetry breaking in (8) and the existence of e and μ whose only difference is in mass.

Now, we postulate that the symmetry breaking in (8) should be interpreted as follows: In the world of charged leptons, where all of them have the same spin, the same weak interactions, etc., the only "observable difference" is in their masses. The mass of a charged lepton is related to the mass of another charged lepton by the chord transformation (3) with $\lambda=e^{-2I_\psi\Lambda'_\psi}$ (ψ may be any charged lepton). The result (8) shows that the difference of their masses is related to the difference in their imaginary lepton numbers.

It is, therefore, natural to identify $m_e e^{2\Lambda'_e}$ as m_μ. We find $\Lambda'_e=2.66$. With the help of (7), we find the next heavy lepton "h" with mass greater than the muon with a mass

$$m_h=m_\mu e^{2\Lambda'_e}\approx 22\text{ GeV}. \quad (9)$$

According to our interpretation, h^- would have $I=3$, $L=1$, $U=2$. In general, we would have a mass spectrum for the charged lepton $l(U,L,I)$:

$$m_{l(U,L,I)}=m_e e^{2\Lambda'_e(I-1)} \quad (L=\pm1,\ I=1,2,3,\dots). \quad (10)$$

We emphasize that these new charged leptons cannot be realized in nature within the domain of the weak interactions alone. Because of the symmetry of the weak interactions, the new charged leptons must have their corresponding new neutrinos to couple with. The existence of two different neutrinos and the speculation on the connection between the symmetry breaking and the existence of a series of leptons would imply that the masses of ν_e and ν_μ are not strictly zero. On the other hand, if m_{ν_e} and/or m_{ν_μ} are strictly zero, then we feel that our interpretation and speculation on the symmetry breaking in (8) are probably wrong because the result (8) with $\psi=\nu_e$ does not shed any light on the existence of neutrinos other than ν_e.

Consider the total Lagrangian of the neutral leptons. If $m_{\nu_e}\neq0$, then by similar argument from (5) to (10) we would have a series of new neutrinos with very small but nonvanishing masses:

$$m_{\nu(U,L,I)}=m_{\nu_e}e^{2\Lambda'_{\nu e}(I+1)} \quad (\nu e\equiv\nu_e;\ L=\pm1;\ I=-1,-2,\dots). \quad (11)$$

Note that our argument based on symmetry and symmetry breaking does not give any relation between m_e and m_{ν_e}.

We have an empirical formula relating the charge q of the leptons to the real and the imaginary lepton number

$$q=-\tfrac{1}{2}L(1+I/|I|), \quad (12)$$

which is easily seen from (2). We naturally expect (12) to be held for the new leptons. Because of symmetry, the new charged leptons l with $I=3,4,5,\dots$ and the new neutrino ν with $I=-3$,

$-4, \ldots$ can only be coupled in the same way as e (μ) is coupled to ν_e (ν_μ) with the same coupling strength.

The decays of h have recently been considered in detail by, for example, Thacker and Sakurai.[4] But, since the mass of the heavy lepton h considered here is so large ($m_h \approx 22$ GeV), it would be very hard to produce and detect the new lepton in the laboratory or even in a cosmic-ray experiment. Nevertheless, if m_{ν_e} or m_{ν_μ} is measured to be nonzero, then our speculation would be more interesting.

Although Tennakone and Pakvasa have predicted a "heavy electron" e^* with $m_{e^*} \approx 22$ GeV, it is not the same as our heavy lepton h with $m_h \approx 22$ GeV. For example e^* is electronlike and could decay through $e^* \to e\gamma$; while h is neither electronlike nor muonlike and it cannot decay through $h \to e\gamma$ or $h \to \mu\gamma$. Nevertheless, most of their discussions of the possible connections between e^* and the cosmic-ray experiment can also be applied to h.

Our interpretation of the symmetry breaking in (8) due to the masses is naturally connected with the most striking symmetry between (μ, ν_μ) and (e, ν_e).

To conclude, we note that if the neutrino mass is small but not zero, then the helicity of ν_e or ν_μ could deviate from -1. Such small deviation cannot be ruled out by the existing accuracy in the present helicity measurement. We may also remark that the $|\Delta I| = \frac{1}{2}$ rule can now be understood within the schizon scheme of Lee and Yang of the intermediate bosons[5] without our worrying about the coupling between the neutral intermediate boson and the neutral leptonic currents. The above symmetry consideration and the interpretation of symmetry breaking imply that, within the domain of the weak interactions, the mass difference of the electron and the muon has its origin in the "symmetry" rather than in the dynamics. Therefore, the ratio m_μ/m_e should be regarded as fundamental as the weak coupling constant.

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