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$$\frac{g_{\rho\pi^+\pi^-}{}^2}{4\pi}=\frac{3}{2}\,\frac{\Gamma_\rho m_\rho{}^2}{[(\frac{1}{2}m_\rho)^2-m_\pi{}^2]^{3/2}}\approx 2\,.$$

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# Hadronic Corrections to Goldberger-Treiman Relations for Strangeness-Carrying Currents*

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Hadronic corrections to Goldberger-Treiman relations for vector and axial-vector strangeness-carrying currents are calculated. In both cases, the corrections are found to be less than 10%. A bound for $G_{\Lambda pK}$ is set.

In the dispersion-theoretic version, the PCAC (partial conservation of axial-vector current) hypothesis assumes that the matrix element of the divergence of an axial-vector current satisfies an unsubtracted dispersion relation dominated by the lowest pseudoscalar meson. The Goldberger-Treiman relation (GTR) derived from this hypothesis provides a direct experimental test of this hypothesis itself. For a strangeness-conserving axial-vector current, it is believed that the pion dominance of the divergence of this current is a good approximation, for the pion pole is far below thresholds of other hadronic states. In fact, it is well known that many soft-pion results derived from PCAC and current-algebra assumptions are in very good agreement with experiment. However, the GTR is experimentally found to have a 10% discrepancy. Pagels[1] tried to understand this discrepancy in terms of hadronic continuum corrections but failed. Later, he[2] proposed a once-subtracted dispersion-relation version of PCAC as a remedy.

In this paper, we extend Pagels's calculation to the study of strangeness-carrying vector and axial-vector currents. There are several reasons of interest for such an investigation. First of all, we want to see how good the GTR's for strangeness-carrying currents are and how large the hadronic corrections are in cases where the lowest poles are rather close to the thresholds of the next higher states. Second, such calculations can give us some information about the values on bounds of the form factors $g_\Lambda^A$ and $g_\Lambda^V$ of weak hadronic currents. Such information is useful in checking the Cabibbo theory in strangeness-changing leptonic decays.

In the calculations, we have used experimental information on the coupling constants as far as possible. Where no information is presently available, we have used SU(3) estimates. Our results show that in both cases of vector and axial-vector currents, the corrections to GTR's are less than 10%. A bound for $G_{\Lambda pK}$ is also estimated.

## I. STRANGENESS-CARRYING AXIAL-VECTOR CURRENT

The matrix elements of the strangeness-carrying axial-vector current $A_\mu^{4+i5}$ and its divergence between the proton and $\Lambda$ states are specified by Lorentz invariance as

$$\langle p(p')|A_\mu^{4+i5}(0)|\Lambda(p)\rangle=\bar{u}_p(p')[\gamma_\mu\gamma_5F_1(t)+q_\mu\gamma_5F_2(t)+q_\nu\sigma_{\mu\nu}F_3(t)]u_\Lambda(p)\,, \tag{1}$$

$$\langle p(p')|\partial_\mu A_\mu^{4+i5}(0)|\Lambda(p)\rangle=i\bar{u}_p(p')\gamma_5u_\Lambda(p)D(t)\,, \tag{2}$$

where $q_\mu=(p-p')_\mu$, $t=q^2$, and $F_i(t)$, $i=1,2,3$ are the usual form factors. From Eqs. (1) and (2), it is obvious that

$$D(t)=(m_\Lambda+m_p)F_1(t)-tF_2(t)\,. \tag{3}$$

PCAC assumes that $D(t)$ satisfies an unsubtracted dispersion relation dominated by the $K$ pole, i.e.,

$$D(t)=\frac{f_Km_K{}^2G_{\Lambda pK}}{m_K{}^2-t}+\frac{1}{\pi}\int_{t_0}^\infty\frac{\mathrm{Im}D(t')}{t'-t}dt' \tag{4}$$

$$\simeq\frac{f_Km_K{}^2G_{\Lambda pK}}{m_K{}^2-t}\,, \tag{5}$$

where $t_0$ is the lowest threshold, $f_K$ is the usual kaon decay constant, and $G_{\Lambda pK}$ is the strong $\Lambda pK$ coupling constant. At $t=0$, $F_1(0)\simeq g_\Lambda^A$, the GTR follows from Eqs. (3) and (5):

$$(m_\Lambda+m_p)g_\Lambda^A\simeq f_K G_{\Lambda pK}. \tag{6}$$

Experimentally, $g_\Lambda^A$ and $G_{\Lambda pK}$ have not been accurately determined; a decisive check of Eq. (6) at present is impossible. However, in order to give some ideas about the validity of the relation, we would like to quote some experimental data available. The value[3] of $G_{\Lambda pK}{}^2/4\pi\simeq 13.5\pm2.5$ has recently been obtained using the multichannel effective-range-approximation model, which is consistent with the SU(3) result with the $D$-$F$ mixing parameter $f$ approximately given by $f\simeq 0.35$. We then take $G_{\Lambda pK}\simeq -13$, where the sign is fixed by SU(3) considerations. As for $g_\Lambda^A$ there are several possible values. The SU(3) relation $g_\Lambda^A=-g_A(1+2f')/\sqrt{6}$, with the experimental value $g_A=1.18$ and the best-fit $D$-$F$ mixing parameter $f'\simeq 0.33$,[4] gives the value $g_\Lambda^A\simeq -0.8$. From these estimates, one obtains the deviation from GTR

$$\delta_{\exp}=1-\frac{(m_\Lambda+m_p)g_\Lambda^A}{f_K G_{\Lambda pK}}\simeq 0.044. \tag{7}$$

However, the recent compilation[5] of experimental data on $e$-$\nu$ correlation measurements in $\Lambda\to pe\nu$ decay gave the value $g_\Lambda^A/g_\Lambda^V\simeq 0.77^{+0.13}_{-0.09}$. If we take the SU(3) ratio for vector-current form factors, $g_\Lambda^V/g_n^V=-(\frac{3}{2})^{1/2}$, then we get $g_\Lambda^A\simeq -0.94$ and $\delta_{\exp}\simeq -0.12$, while the recent result[5] of the up-down asymmetry experiment in $\Lambda$ decay, $g_\Lambda^A/g_\Lambda^V\simeq 0.40^{+0.17}_{-0.13}$, leads to $\delta_{\exp}\simeq 0.42$.

The main interest of this paper is to evaluate the possible correction to the GTR due to hadronic continuum contributions. As seen from Eq. (4), this correction is equal to

$$\delta=\frac{-1}{\pi f_K G_{\Lambda pK}}\int_{t_0}^{\infty}\frac{\mathrm{Im}D(t)}{t}dt. \tag{8}$$

Using the PCAC relation as an operator equation in the field-theoretic version, $\partial_\mu A_\mu^{4-i5}(x)=f_K m_K{}^2\phi_K(x)$, we can relate $D(t)$ to the vertex function $K(t)$ defined by

$$\langle p(p')|j_K(0)|\Lambda(p)\rangle=i\bar{u}_p\gamma_5 u_\Lambda K(t) \tag{9}$$

in the following way:

$$D(t)=\frac{f_K m_K{}^2 K(t)}{m_K{}^2-t}. \tag{10}$$

Then $\delta$ can be rewritten as

$$\delta=\frac{1}{\pi}\frac{m_K{}^2}{G_{\Lambda pK}}\int_{t_0}^{\infty}\frac{\mathrm{Im}K(t)}{t(t-m_K{}^2)}dt =\delta_L+\delta_H, \tag{11}$$

with $\delta_L$, $\delta_H$ defined by

$$\delta_L=\frac{1}{\pi}\frac{m_K{}^2}{G_{\Lambda pK}}\int_{t_0}^{(m_\Lambda+m_p)^2}\frac{\mathrm{Im}K(t)}{t(t-m_K{}^2)}dt,\qquad \delta_H=\frac{1}{\pi}\frac{m_K{}^2}{G_{\Lambda pK}}\int_{(m_\Lambda+m_p)^2}^{\infty}\frac{\mathrm{Im}K(t)}{t(t-m_K{}^2)}dt. \tag{12}$$

Now we discuss the low- and high-energy contributions separately.

*(1) High-energy contribution.* From Eq. (9), the imaginary part of $K(t)$ is found to be

$$\mathrm{Im}K(t)=\frac{\frac{1}{2}(2\pi)^4\sum_n\delta^4(p_n-q)\langle 0|j_K(0)|n\rangle\bar{u}_p\langle n|j_p(0)|\Lambda(p)\rangle}{i\bar{u}_p\gamma_5 u_\Lambda}.$$

The application of the Schwarz inequality to the sum on states gives the inequality

$$|\mathrm{Im}K(t)|^2\leq\tfrac{1}{4}AB, \tag{13}$$

with

$$A=(2\pi)^4\sum_n\delta^4(p_n-q)|\langle 0|j_K(0)|n\rangle|^2,$$
$$B=(2\pi)^4\sum_n\delta^4(p_n-q)|\bar{u}_p\langle 0|j_p(0)|n\rangle/\bar{u}_p\gamma_5u_\Lambda|^2.$$

Note that $\langle 0|j_K(0)|n(q)\rangle=(m_K{}^2-t)\langle 0|\phi_K(0)|n\rangle$, and the spectral function $\rho_K$ appearing in the kaon propagator defined by

$$\Delta^K(t)=-i\int d^4x\,e^{iqx}\langle 0|T\phi_K^\dagger(x)\phi_K(0)|0\rangle =\frac{1}{m_K{}^2-t}+\int_{(m_K+2m_\pi)^2}^{\infty}\frac{\rho_K(t')}{t'-t}dt' \tag{14}$$

is equal to

$$\rho_K(t)=(2\pi)^3\sum_n\delta^4(p_n-q)|\langle 0|\phi_K(0)|n\rangle|^2.$$

We then get

$$A=(2\pi)(t-m_K{}^2)^2\rho_K(t).$$

For $t>(m_\Lambda+m_p)^2$, we can show that $B=\sigma_T v$, where $\sigma_T(t)$ is the total $\Lambda\bar{p}$ annihilation cross section in the ${}^1S_0$ state with relative velocity $v$. Thus,

$$|\mathrm{Im}K(t)|^2\leq\tfrac{1}{2}\pi(t-m_K{}^2)^2\rho_K(t)\sigma_T(t)v. \tag{15}$$

$\sigma_T$ is bounded by the unitarity condition

$$\sigma_T(t)\leq 4\pi/k^2,\qquad k^2=(t-m_+{}^2)(t-m_-{}^2)/4t,\quad m_\pm=m_\Lambda\pm m_p, \tag{16}$$

where $k$ is the momentum in the c.m. system of $\Lambda\bar{p}$. Combining Eqs. (14), (15), and (16), we find that

$$|G_{\Lambda pK}\delta_H|\leq 2\sqrt{2}m_K{}^2\int_{(m_\Lambda+m_p)^2}^{\infty}\frac{\rho_K{}^{1/2}(t)}{(t-m_+{}^2)^{1/4}(t-m_-{}^2)^{1/4}(t^2-m_+{}^2m_-{}^2)^{1/2}}dt. \tag{17}$$

Again, by applying the Schwarz inequality to the integral, one obtains the inequality

$$|G_{\Lambda pK}\delta_H|^2 \leqslant 6\frac{m_K^2}{(m_\Lambda+m_p)^2}\int_{m_+^2}^{\infty}\frac{\rho_K(t)}{t}dt. \tag{18}$$

As seen from Eq. (4), we know that

$$\int_{m_+^2}^{\infty}\frac{\rho_K(t)}{t}dt \leqslant \Delta^K(0)-\frac{1}{m_K^2}.$$

Obviously, the value of $\Delta^K(0)$ obtained from $K$-pole dominance will give a null result. Mathur and Okubo[6] have estimated $\Delta^K(0)$ by a method independent of $K$-pole dominance, and found that $\Delta^K(0) = 14.65 f_\pi^2 m_\pi^2/f_K^2 m_K^4$. With the experimental value $G_{\Lambda pK} \simeq -13$, the inequality (18) gives the bound of $\delta_H$,

$$|\delta_H| \leqslant 0.017. \tag{19}$$

It is one order larger than the corresponding correction to the GTR for axial-vector strangeness-conserving current. If we take a less stringent bound for the integral of the spectral function, e.g.,

$$\int_{m_+^2}^{\infty}\frac{\rho_K(t)}{t}dt \leqslant \Delta^K(0),$$

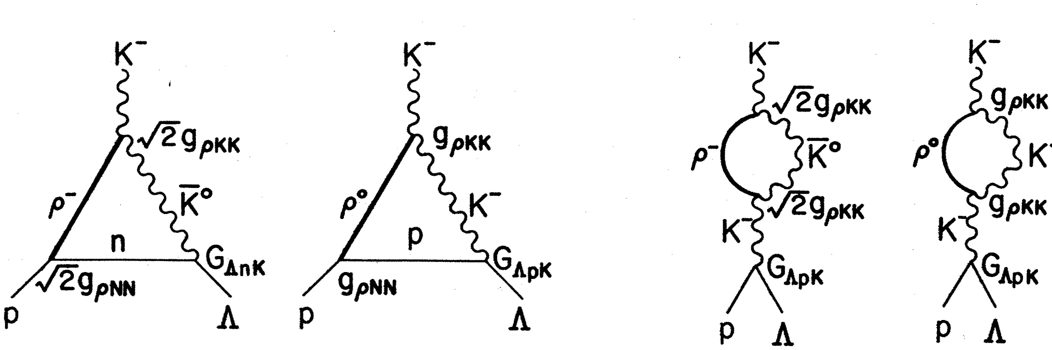
then we get a weaker bound for $\delta_H$, $|\delta_H| \leqslant 0.049$.

*(2) Low-energy contribution* $\delta_L$. For $t < (m_\Lambda+m_p)^2$, there are two-particle states $\rho K$, $\omega K$, $\pi K^*$, $\eta K^*$, $\sigma K$, and $K\pi$, and three-particle states, such as $K\bar{K}K$ and $\pi\pi K$, which may contribute to the correction $\delta_L$. Of the two-particle states, we consider only $\rho K$, $\omega K$, $\pi K^*$, and $\eta K^*$ contributions. We believe that these will give the main contribution, or at least the order of possible low-energy corrections. The contributions of $\sigma K$, $K\pi$, and three-particle states are not discussed because of the lack of experimental data on coupling constants needed, and are neglected. In the case of the strangeness-conserving current, Pagels[1] has made rough estimates and found that the $\sigma\pi$ contribution is at most 2%, and the $3\pi$ contribution is less than 1% because of the smaller available phase space. Similar arguments can apply to our case too.

The $\rho K$ contribution in the Born approximation comes from the following diagrams as shown by Fig. 1. Assuming the $\rho$ coupling to mesons and baryons is SU(3)-invariant and universal, then only the charge coupling contributes since the $D$-type coupling is excluded. The contributions from these diagrams[7] are directly calculated to be

$$\begin{aligned}\mathrm{Im}K_a(t) = \frac{3g_{\rho NN}g_{\rho KK}G_{\Lambda pK}}{8\pi}\Bigg\{&\left[\frac{t-m_K^2-m_\rho^2}{m_\rho^2}+(m_\Lambda-m_p)\left(\frac{p_0'(t-m_+^2)}{2t^{1/2}|p|^2 m_p}-\frac{1}{m_p}\right)\right]\frac{p_1}{t^{1/2}}\\ &+\left[2(m_p^2+m_K^2-m_\Lambda^2)+(m_\Lambda-m_p)\left(\frac{(t-m_+^2)(p_0' m_\rho^2+2p_{10}m_p^2)}{2t^{1/2}|p|^2 m_p}-\frac{m_\rho^2}{m_p}\right)\right]\\ &\times\frac{1}{4|p|t^{1/2}}\tan^{-1}\frac{(2p_0'p_{10}+m_\rho^2)4|p|p_1}{(2p_0'p_{10}+m_\rho^2)^2-4|p|^2p_1^2}\Bigg\},\end{aligned} \tag{20}$$

$$\mathrm{Im}K_b(t) = -\frac{3g_{\rho NN}g_{\rho KK}G_{\Lambda pK}}{16\pi m_\rho^2 t(t-m_K^2)}[m_\rho^4-2(t+m_K^2)m_\rho^2+(t-m_K^2)^2][t^2-2(m_\rho^2+m_K^2)t+(m_\rho^2-m_K^2)^2]^{1/2},$$




(a)

FIG. 1. (a) and (b) The $\rho K$ contribution to the divergence of the axial-vector and strangeness-carrying current.

with

$$p_{10}=(t+m_\rho^2-m_K^2)/2t^{1/2}, \quad p_1=(p_{10}^2-m_\rho^2)^{1/2},$$
$$p_0=(t+m_\Lambda^2-m_p^2)/2t^{1/2}, \quad |p|=(m_\Lambda^2-p_0^2)^{1/2},$$
$$p_0'=-(t+m_p^2-m_\Lambda^2)/2t^{1/2}.$$

Assuming that $g_{\rho NN}$, $g_{\rho KK}$, and $G_{\Lambda pK}$ are momentum-independent in $(m_\rho+m_K)^2<t<(m_\Lambda+m_p)^2$, the numerical integration of Eq. (20) leads to

$$\delta_{\rho K}=\frac{g_{\rho NN}g_{\rho KK}}{4\pi}(1.92\times10^{-2}). \tag{21}$$

If we assume SU(3) couplings, and neglect the mass difference between $\omega$ and $\rho$, and $\omega$-$\phi$ mixing, it is simple to see that $\delta_{\rho K}=\delta_{\omega K}$.

Similar calculations for $K^*\pi$ and $K^*\eta$ intermediate states give the results

$$\delta_{\pi K^*}=\frac{g_{\rho NN}g_{\rho KK}}{4\pi}\left(4.8\frac{\sqrt{3}G_{pp\pi}}{G_{\Lambda pK}}+0.56\frac{G_{\Lambda\Sigma^0\pi^0}}{G_{\Lambda pK}}-2.6\right)\times10^{-2},$$
$$\delta_{\eta K^*}=\frac{g_{\rho NN}g_{\rho KK}}{4\pi}\left(4.8\frac{G_{pp\eta}}{G_{\Lambda pK}}+0.56\frac{G_{\Lambda\Lambda\eta}}{G_{\Lambda pK}}-2.6\right)\times10^{-2}, \tag{22}$$

where we have assumed SU(3) couplings for the strong interaction and the universality of vector-meson coupling, i.e., $g_{\rho NN}=g_{\rho KK}=\frac{1}{2}g_{\rho\pi\pi}$. For numerical estimation, we take the average value[5] $G_{\Lambda pK}\simeq-13$ and the $D$-$F$ mixing parameter in the baryon-baryon-meson interaction $f=F/(D+F)\simeq0.35$; while $g_{\rho\pi\pi}^2/4\pi\simeq2.43$ is obtained from $\rho\to2\pi$ decay, taking $\Gamma_\rho=125$ MeV and $m_\rho=765$ MeV. Then we get

$$\delta_{\rho K}\simeq\delta_{\omega K}\simeq0.012,$$
$$\delta_{\pi K^*}\simeq0.033, \tag{23}$$
$$\delta_{\eta K^*}\simeq-0.006,$$

so that

$$\delta_L\simeq\delta_{\rho K}+\delta_{\omega K}+\delta_{\pi K^*}+\delta_{\eta K^*}\simeq0.051. \tag{24}$$

The results of Eqs. (19) and (24) show that hadronic corrections are a few percent in magnitude, one order larger than those of the strangeness-conserving current as discussed by Pagels. We find that disregarding some variation in $\delta$ caused by the experimental uncertainty on strong coupling constants, the correction is of the same order as $\delta_{\text{exp}}$ given by Eq. (7), and so at the present level of experimental information, the kaon PCAC hypothesis can indeed be understood through an unsubtracted dispersion relation.

## II. STRANGENESS-CARRYING VECTOR CURRENT

By Lorentz invariance, the matrix elements of vector strangeness-carrying current and its divergence between the $\Lambda$ and proton states can be written as

$$\langle p(p')|V_\mu^{4+i5}(0)|\Lambda(p)\rangle=i\bar{u}_p(p')(f_1\gamma_\mu+f_2q_\mu+f_3\sigma_{\mu\nu}q_\nu)u_\Lambda(p), \tag{25}$$

$$\langle p(p')|\partial_\mu V_\mu^{4+i5}(0)|\Lambda(p)\rangle=D(t)\bar{u}_p(p')u_\Lambda(p), \tag{26}$$

where $q_\mu=(p-p')_\mu$ and $t=-q^2$. The form factors $f_{1,2,3}$ and $D$ are dependent on $t$ only. It follows from Eqs. (25) and (26) that

$$D(t)=(m_\Lambda-m_p)f_1(t)+tf_2(t),$$

so that

$$D(0)=(m_\Lambda-m_p)g_\Lambda^V, \tag{27}$$

where $g_\Lambda^V=f_1(0)$ is the $\Lambda$-decay constant of the vector current.

Assume $D(t)$ satisfies an unsubtracted dispersion relation, i.e.,

$$D(t)=\frac{1}{\pi}\int_0^\infty\frac{\text{Im}D(t')}{t'-t}dt',$$

then we may write

$$D(0)=(m_\Lambda-m_p)g_\Lambda^V=\frac{1}{\pi}\int_0^\infty\frac{\text{Im}D(t)}{t}dt \tag{28}$$
$$=D_L+D_H, \tag{29}$$

with

$$D_L=\frac{1}{\pi}\int_0^{(m_\Lambda+m_p)^2}\frac{\text{Im}D(t)}{t}dt,$$
$$D_H=\frac{1}{\pi}\int_{(m_\Lambda+m_p)^2}^\infty\frac{\text{Im}D(t)}{t}dt. \tag{30}$$

From Eq. (28), it is obvious that in the SU(3) limit, $D(0)=0$. The Goldberger-Treiman relation follows from the PCVC (partial conservation of vector current) assumption that the $\kappa$ pole dominates the dispersion integral,

$$(m_\Lambda-m_p)g_\Lambda^V\simeq f_\kappa G_{\Lambda p\kappa}, \tag{31}$$

where $f_\kappa$ is the $\kappa$-decay constant and $G_{\Lambda p\kappa}$ is the strong coupling constant.

*(1) High-energy contribution.* Similar to Sec. I, the Schwarz inequality implies that

$$|\text{Im}D(t)|^2\le\tfrac{1}{2}\pi\rho^V(t)\sigma_T^V(t)v, \tag{32}$$

where $\rho^V(t)$ is the spectral function appearing in the following vector-current propagator:

$$\Delta^V(t)=-i\int d^4x\,e^{iqx}\langle0|T(\partial_\mu V_\mu^{4+i5}(x)\partial_\nu V_\nu^{4-i5}(0))|0\rangle=\int_0^\infty\frac{\rho^V(t')}{t'-t}dt, \tag{33}$$

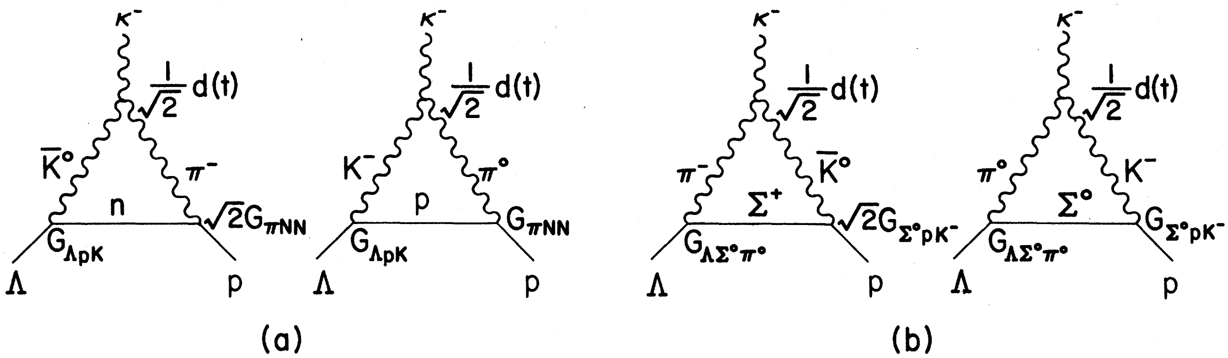



FIG. 2. (a) and (b) The $K\pi$ contribution to the divergence of the vector and strangeness-carrying current.

where

$$\rho^V(t)=(2\pi)^3\sum_n \delta^4(p_n-q)\left|\langle 0|\,\partial_\mu V_\mu^{4+i5}(0)\,|n\rangle\right|^2 . \qquad (34)$$

$\sigma_T^V(t)$ is now the total $\bar{\Lambda}p$ annihilation cross section in the $I=1$, $J=0$ state, and so is bounded by the unitarity condition

$$\sigma_T^V(t)\le 12\pi/k^2 . \qquad (35)$$

From Eqs. (32), (33), and (35), applying the Schwarz inequality to the dispersion integral $D_H$, one obtains

$$|D_H|\le(\tfrac{3}{4})^{1/2}(m_\Lambda+m_p)^{-1}[\Delta^V(0)-f_K{}^2m_K{}^2]^{1/2} \qquad (36a)$$

$$\le(\tfrac{3}{4})^{1/2}(m_\Lambda+m_p)^{-1}[\Delta^V(0)]^{1/2} . \qquad (36b)$$

*(2) Low-energy contribution.* For $t<(m_\Lambda+m_p)^2$, there are many intermediate states, such as $\kappa$, $K\pi$, $KA$, $K_A\pi$ states, etc., which may contribute to the dispersion integral $D_L$. However, we believe that the lowest states $K\pi$ or $\kappa$ dominate the low-energy contribution, or at least this contribution will give the order of $D_L$. Furthermore, since experiments indicate that $m_\kappa$ is higher than the $K\pi$ threshold, in order to avoid double counting, we can only take into account either the $\kappa$-pole or $K\pi$ contribution.

If we dominate $D_L$ by the $\kappa$-pole contribution we simply have the relation

$$D(0)=(m_\Lambda-m_p)g_\Lambda^V\simeq f_K G_{\Lambda pK}+D_H, \qquad (37)$$

so that the correction $\delta$ to GTR is given by

$$\delta=1-\frac{f_K G_{\Lambda pK}}{(m_\Lambda-m_p)g_\Lambda^V} =\frac{D_H}{(m_\Lambda-m_p)g_\Lambda^V} . \qquad (38)$$

If we dominate $D_L$ by the $K\pi$ contribution and use the Born approximation as indicated by the diagrams shown in Fig. 2, direct calculation[7] gives

$$\begin{aligned}
\mathrm{Im}D_a(t)=\frac{-(2+\sqrt{2})G_{\Lambda pK}G_{\pi NN}\,d(t)}{16\pi}\Bigg(&\frac{p_1}{tp^2}\,[m_\Lambda t^{1/2}-(m_\Lambda-m_p)p_0]\\
&+\frac{1}{4|p|\,t^{1/2}}\tan^{-1}\frac{4|p|\,p_1(-2p_{10}p_0+m_K{}^2+m_\Lambda{}^2-m_p{}^2)}{(-2p_{10}p_0+m_K{}^2+m_\Lambda{}^2-m_p{}^2)^2-4|p|^2p_1{}^2}\\
&\times\{(m_p-m_\Lambda)+(m_\Lambda-m_p)(p_{10}/t^{1/2})\\
&-\tfrac{1}{2}[m_\Lambda t^{1/2}-(m_\Lambda-m_p)p_0](-2p_{10}p_0+m_K{}^2+m_\Lambda{}^2-m_p{}^2)p^{-2}t^{-1/2}\}\Bigg),\\
\mathrm{Im}D_b(t)=\frac{-(2+\sqrt{2})G_{\Lambda\Sigma^0\pi^0}G_{\Sigma pK}\,d(t)}{16\pi}\Bigg(&\frac{p_1}{tp^2}\,[m_pt^{1/2}-(m_\Lambda-m_p)p_0']\\
&+\frac{1}{4|p|t^{1/2}}\tan^{-1}\frac{(2p_{10}p_0'+m_K{}^2+m_p{}^2-m_\Sigma{}^2)4|p|\,p_1}{(2p_{10}p_0'+m_K{}^2+m_p{}^2-m_\Sigma{}^2)^2-4|p|^2p_1{}^2}\\
&\times\{(m_\Sigma-m_p)-(m_\Lambda-m_p)p_{10}t^{-1/2}\\
&-\tfrac{1}{2}[m_pt^{1/2}-(m_\Lambda-m_p)p_0'](2p_{10}p_0'+m_K{}^2+m_p{}^2-m_\Sigma{}^2)p^{-2}t^{-1/2}\}\Bigg),
\end{aligned} \qquad (39)$$

with

$$p_{10}=(t+m_K{}^2-m_\pi{}^2)/2t^{1/2},\qquad p_1=(p_{10}{}^2-m_K{}^2)^{1/2},\qquad p_0=(t+m_\Lambda{}^2-m_p{}^2)/2t^{1/2},$$

$$p=(p_0{}^2-m_\Lambda{}^2)^{1/2},\qquad p_0'=-(t+m_p{}^2-m_\Lambda{}^2)/2t^{1/2} .$$

The form factor $d(t)$ is defined by

$$\langle 0|\partial_\mu V_\mu^{4+i5}(0)|K^-\pi^0\rangle=(1/\sqrt{2})d(t)$$

which is bound by an inequality given by Li and Pagels[8]:

$$|d(t)|\leq\frac{8\pi}{3}\frac{[t\rho^V(t)]^{1/2}}{[(t+m_K^2-m_\pi^2)^2-4m_K^2t]^{1/4}}. \quad (40)$$

From Eqs. (30), (39), and (40), applying the Schwarz inequality again, we obtain

$$|D_L|^2\leq\Delta_L^V(0)\frac{64\pi^2}{9}\times\int_{(m_K+m_\pi)^2}^{(m_\Lambda+m_p)^2}\frac{|\mathrm{Im}D_a+\mathrm{Im}D_b|^2}{[(t+m_K^2-m_\pi^2)^2-4m_K^2t]^{1/2}}dt, \quad (41)$$

where

$$\Delta_L^V(0)=\int_{(m_K+m_\pi)^2}^{(m_\Lambda+m_p)^2}\frac{\rho^V}{t}dt\simeq f_\kappa^2m_\kappa^2 \leq\Delta^V(0).$$

The integral in Eq. (41) can be evaluated numerically, and we obtain

$$|D_L|\leq(\Delta_L^V)^{1/2}G_{\pi NN}^2(0.2\ \mathrm{BeV}^{-1}) \quad (42a)$$

$$\leq(\Delta^V)^{1/2}G_{\pi NN}^2(0.2\ \mathrm{BeV}^{-1}), \quad (42b)$$

where again we use SU(3) coupling ratios, i.e., $G_{\Lambda pK}/G_{\pi NN}=-(1+2f)/\sqrt{3}$ and $G_{\Lambda\Sigma^0\pi^0}G_{\Sigma^0pK^-}/G_{\pi NN}^2=2(1-f)(1-2f)/\sqrt{3}$, and take the $D$-$F$ mixing parameter $f\simeq0.35$.

Now we have Eqs. (37) and (38), and inequalities (36a), (36b), (42a), and (42b). Inequalities (36a) and (42a) give better bounds, while (36b) and (42b) are more reliable because they are independent of the experimental uncertainty in $f_\kappa$. One can also obtain a bound for $G_{\Lambda pK}$ from Eq. (37),

$$\frac{(m_\Lambda-m_p)|g_\Lambda^V|-D_H^0}{|f_\kappa|}\leq|G_{\Lambda pK}|\leq\frac{(m_\Lambda-m_p)|g_\Lambda^V|+D_H^0}{|f_\kappa|}, \quad (43)$$

where $D_H^0$ is the bound of $|D_H|$ as given by (36a) and (36b).

The factor $\Delta^V(0)$ is calculated by Mathur and Okubo[6] as

$$\Delta^V(0)=2K_{44}=\tfrac{9}{2}\gamma ab, \quad (44)$$

with

$$a\simeq-0.89,$$

$$b=\frac{1-x^2}{\frac{1}{2}+x^2},\quad x=\frac{f_K}{f_\pi},$$

$$\gamma=\frac{f_\pi^2m_\pi^2}{2(1+a)(1+b)}.$$

The factor $f_\kappa$ may be estimated in two ways; (A) The sum rule given by Glashow and Weinberg,[9] $2f_+(0)f_Kf_\pi=f_\pi^2+f_K^2-f_\kappa^2$, with the experimental value $f_K/f_\pi f_+(0)=1.28$, leads to

$$f_\kappa^2/f_\pi^2=1-0.5625x^2. \quad (A)$$

Note that $\Delta_H^V\geq0$ implies $|x|\geq1.25$. (B) We may obtain from the simple sum rule,[9,10] $f_\pi=f_K+f_\kappa$, that

$$f_\kappa/f_\pi=1-x. \quad (B)$$

$\Delta_H^V\geq0$ implies $1\leq|x|\leq2.52$. If we accept the result of Okubo[11] and Li and Pagels[8] that $f_+(0)<1$, then the experimental value of $f_K/f_\pi f_+(0)=1.28$ implies $1<x<1.28$. For $x$ in this range, as $x$ increases, $|f_\kappa|$ will increase in case (A) and decrease in case (B).

By $\kappa$-pole dominance of $D_L$, Eqs. (37)–(38) and inequalities (36a), (36b), and (43) set bounds for $|D_H|$, $|G_{\Lambda pK}|$, and $|\delta|$, the correction to GTR. If we approximate $g_\Lambda^V$ by its SU(3) value, $g_\Lambda^V=-(\frac{3}{2})^{1/2}$,[12] and take $f_\pi=131.7$ MeV, $G_{\pi NN}=13.66$, $m_\kappa=1050$ MeV, we obtain the result listed in Table I, so that we reach two conclusions:

(1) The correction to GTR is less than 10%, a bound independent of the uncertainty of $f_\kappa$. It may be a few percent or much less, depending on various possible values of $f_\kappa$. For more precise com-

TABLE I. The bounds for $|\delta|$ and $|G_{\Lambda pK}|$ using the $K\pi$ dominance.

| | | | With (36a) | | | | With (36b) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $x$ | $\lvert f_\kappa/f_\pi\rvert$ | $\lvert D_H\rvert\leq$ (MeV) | $\lvert\delta\rvert\leq$ | $\leq\lvert G_{\Lambda pK}\rvert\leq$ | | $\lvert D_H\rvert\leq$ (MeV) | $\lvert\delta\rvert\leq$ | $\leq\lvert G_{\Lambda pK}\rvert\leq$ | |
| (A) | 1.28 | 0.28 | 14.2 | 0.065 | 5.51 | 6.275 | 21.64 | 0.10 | 5.3 | 6.45 |
| | 1.25 | 0.35 | 0.95 | 0.004 | 4.72 | 4.76 | 20.31 | 0.094 | 4.3 | 5.18 |
| (B) | 1.25 | 0.25 | 14.15 | 0.065 | 6.17 | 7.03 | 20.31 | 0.094 | 5.98 | 7.21 |
| | 1.20 | 0.2 | 13.67 | 0.063 | 7.33 | 8.77 | 17.97 | 0.083 | 7.56 | 8.93 |
| | 1.08 | 0.08 | 10.02 | 0.046 | 19.66 | 21.58 | 11.05 | 0.051 | 19.57 | 21.66 |
| | 1.04 | 0.04 | 7.24 | 0.033 | 39.86 | 42.62 | 7.61 | 0.035 | 39.79 | 42.68 |

TABLE II. The bound for $|g_\Lambda^V|$.

| | $x$ | With (36a) and (42a) $|D_L| \leq$ (MeV) | $|g_\Lambda^V| \leq$ | With (36b) and (42b) $|D_L| \leq$ (MeV) | $|g_\Lambda^V| \leq$ |
|---|---|---|---|---|---|
| (A) | 1.28 | 1427.70 | 8.09 | 1892.16 | 10.74 |
| | 1.25 | 1471.52 | 9.95 | 1776.33 | 10.08 |
| (B) | 1.25 | 1274.78 | 7.23 | 1776.33 | 10.08 |
| | 1.2 | 1057.99 | 5.8 | 1571.04 | 8.92 |
| | 1.08 | 407.91 | 2.35 | 965.80 | 5.49 |
| | 1.04 | 203.96 | 1.19 | 665.40 | 3.78 |

parison, we need more accurate experimental values for $f_K$, $x$, $g_\Lambda^V$, and the strong coupling constants $G_{\Lambda pK}$, $G_{\Lambda\Sigma^0\pi^0}$, and $G_{\Sigma^0 pK^-}$.

(2) $G_{\Lambda pK}$ has opposite sign to $f_K$, as seen from Eq. (37) and the bound of $|D_H|$ calculated in Table I. Its value varies with $f_K$. However, the most reasonable value would be around $|G_{\Lambda pK}|=5$ as given in case (A).

On the other hand, using the $K\pi$ dominance of $D_L$, the relation follows from Eqs. (28)–(29),

$$|g_\Lambda^V| \leq (m_\Lambda - m_p)^{-1}(|D_L|+|D_H|). \tag{45}$$

One can estimate the value of $|g_\Lambda^V|$ if one can determine $|D_L|$ and $|D_H|$. By using inequalities (36a), (36b), (42a), and (42b), we list the results in Table II. Unfortunately, the bound for $|g_\Lambda^V|$ is much bigger than the SU(3) value, $|g_\Lambda^V| \simeq 1.225$, except when $f_K$ becomes very small, which seems unlikely. This is due to the fact that the inequality (42) is not very stringent, $|D_L|$ is proportional to $f_K$ and $|D_L| \gg |D_H|$ in the acceptable range of $x$.

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