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Exotic Currents in Pion Photoproduction

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A detailed analysis is made of the pion photoproduction and pion radiative-capture data in the first resonance region in terms of a simple model incorporating C-violating and isotensor effects. An excellent fit is obtained, and the data can only be understood in the presence of the exotic terms. The stability of the parameters representing these effects is investigated against detailed variations in the model and against possible ambiguities in the proton data. The parameters specifying the ratio of isotensor to isovector resonance excitation (t) and the T-violating phase on neutrons (ϕ_n) are found to be stable, and have the values $t \sim -0.28$ and $\phi_n \sim -11°$. The consequences of this on other processes are briefly summarized, and found to be compatible with the present information. A discussion of the model-independent aspects of these questions is also given, including a critical review of other work known to the authors. Conventional models, i.e., those without exotic terms, are demonstrated to be incapable of describing the present data without violating more fundamental requirements. The particular exotic terms needed depend to some extent on the model; however, this dependence can be eliminated by improved data.

I. INTRODUCTION

In 1967 it was pointed out by one of us[1] that the best way to investigate the existence of an $I=2$ term in the electromagnetic current, suggested as a possibility immediately before by Grishin *et al.*[2] and by Dombey and Kabir,[3] was to study the photoexcitation of the $\Delta(1232)$ resonance in the photoproduction of single pions in this first resonance region. Two experimental tests were specifically suggested:

(A) comparison of the reactions

$$\gamma + n \to \pi^0 + n, \tag{1a}$$

$$\gamma + p \to \pi^0 + p, \tag{1b}$$

both carried out on deuterium, and

(B) measurement of the radiative-capture reaction

$$\pi^- + p \to \gamma + n \tag{2}$$

to compare with the data on the forward reaction

$$\gamma + p \to \pi^+ + n. \tag{3}$$

The object in both cases is to compare the radiative widths of the Δ^+, Δ^0 charge states of the resonance. If the parameter x defined by

$$\Gamma(\Delta^0 \to n\gamma) = \Gamma(\Delta^+ \to p\gamma)(1+x)^2 \tag{4}$$

is not equal to zero, an isotensor term must be present. The advantage of reactions (A), which we will return to later, is that in this case the nonresonant backgrounds are expected to be small. In the case of (2) and (3) the backgrounds are expected to be significant, however, and to be dominated by a large, real, s-wave contribution. In this context, Shaw[1] noted that the normal dispersion-relation approach gave a rather good prediction of the experimentally known background in reaction (3), and might be expected to be equally successful in the dynamically closely related reaction (2). He therefore suggested a philosophy in which the initial data would be analyzed using theory to understand the background, in order to estimate the amount of $I=2$ present, which requirement could be relaxed as more data accumulated.

This philosophy was pursued in part of their work by Sanda and Shaw[4,5] who showed how to incorporate $I=2$ terms into a dispersion-theory model without spoiling the understanding of the proton data, and they examined the rather sparse data then available. However, in an additional and separate point they also showed how to use charged-pion production data to test for the presence of isotensor terms in a rather model-independent way. The idea here is to look for a dip (or peak) in the energy dependence of the difference of total cross sections σ_t for $\pi^\pm$ production in the first resonance region, i.e., in

$$\Delta'(W) = \frac{k}{q}\left[\sigma_t(\pi^-) - \sigma_t(\pi^+)\right], \tag{5}$$

where k/q is just a phase-space factor which removes at least the s-wave threshold dependence. For convenience we shall refer to this subsequently as the dip test. For the detailed argument we refer to the above authors[4,5] but the idea is, roughly speaking, to remove interference terms between the resonant and other partial waves by looking at the total cross sections, and then to study a quantity to which, if there is no isotensor term, the resonance cannot contribute. Thus if there is no isotensor term the resonance cannot contribute to Δ', and it will be given by the slowly varying nonresonant background: If the resonance contributes, i.e., there is a dip or peak of the appropriate position and width, then an $I=2$ term is present.

Let us now consider what sort of dip might be expected. In the above-mentioned dispersion-theory model, predictions were given as a function of the single parameter x defined in Eq. (4), so that $x=0$ means no isotensor. We stress that these results were predictions, and not the result of data fitting. In fact at the time of first publication[4] the data[6] were not in good agreement with the model. A subsequent revision of the data in question[7] together with a further experimental result[8] led, however, to good agreement with the predictions.[5] The conforming of this newer data to the curves previously predicted is a significant success of the model, although admittedly the data are very imprecise. The other obvious point is that the data are based on deuterium experiments analyzed using the spectator model, and so may be subject to errors due to this. It is important to note that this problem can in the future be largely avoided by the measurement of π^-/π^+ ratios on deuterium, and that the few measurements on this which presently exist give some check on the deuterium analysis. This will be discussed in Sec. IV, and a more detailed discussion of the dip question will be given in Sec. VI.

Radiative capture and T violation. Recently, an interesting and important experiment has been reported[9] by a UCLA-LRL group – the first measurement of an angular distribution for the radiative-capture reaction (2) in this region (at $E_\gamma = 350$ MeV) – in other words, test (B) suggested above. This is the first experiment which was performed explicitly to investigate these questions, the deuterium data mentioned above[6-8] having come from general survey experiments. The results indicated a low total cross section at the energy in question as suggested by the above considerations, and the authors reported that while their data were compatible with the above Sanda-Shaw model, they were incompatible with all published accounts based on the $|\Delta I| \leq 1$ rule (i.e., no $I=2$ term).

However, although the total cross section suggested by this experiment is roughly compatible with the deuteron data on the forward reaction

$$\gamma + n \to \pi^- + p\,, \tag{6}$$

the differential cross-section measurements at backward angles are definitely lower. If this is a real effect then it is unambiguous evidence for T violation in electromagnetic interactions, and this experiment had in fact also been suggested in this context by Christ and Lee.[10] Accepting the data at face value (but with a cautionary note and plea for better information), the implications were discussed immediately in two papers. In the first[11] the experimental authors themselves carried out a number of multipole fits to the two reactions (5) and (6). They were able to obtain possible fits with T-violating phases inserted into any of the four isospin amplitudes of the M_{1+} multipole, but in all cases a nonzero isotensor term was required. This latter result is not surprising. Since T violation enters via phases, and since in the total cross sections most (but not quite all) interference terms integrate out, the effect of T violation on the total cross sections is much smaller than its effects on differential cross sections, and it is on the former that the arguments for an isotensor term were initially based.

In the other paper[12] Sanda and Shaw showed the (very restricted) way in which T-violating terms can be introduced into their model. This can only be done at the $\Delta N\gamma$ vertex, so that only isovector and isotensor T-violating terms are possible. Further noting that there was no evidence for T violation in these processes at 520 MeV, where the effects of the "second resonance" (coupling to isoscalar and isovector photons) are beginning to be felt, they suggested that this could be easily understood if the T violation were confined to the $I=2$ term. Although no fits were carried out, the authors were able to show that a rough understanding of all the photoproduction processes ($\pi^0, \pi^\pm$) at this energy (350 MeV) was possible. They also showed on turning to other processes that a theory in which the $I=0,1$ parts of the current had conventional $C=-1$ properties, whereas the $I=2$ part was C (T) violating, was able to provide a rather clear understanding of various CP- (or T-) violating effects in electromagnetic interactions. Subsequently, a rather full account of the theoretical background of this hypothesis, and of its phenomenological implications for a wide variety of processes, has been given,[13] and in particular it has been stressed that the introduction of such "exotic" terms would lead to changes in only those parts of current algebra – SU(3) ⊗ SU(3) predictions for weak and electromagnetic processes – which meet with difficulties, leaving the highly successful predictions of $SU(2)\otimes SU(2)$ unchanged.

In this paper we wish, in the main, to leave aside the question of other processes and to turn again to this central process of pion photoproduction. It is our aim both to provide a complete quantitative account of photoproduction in this region – to provide a more detailed development of the model, to examine all the data, and to fit the (rather few) arbitrary coupling parameters in the model to the data – and also to provide a summary and critical review of all the work on this topic. The paper is laid out as follows:

In Sec. II we summarize for completeness the general formalism necessary to introduce isotensor and T-violating terms into the dispersion-theory discussion.

In Sec. III a description of the model, of which we have provided a more detailed development, is given.

Section IV contains a summary and discussion of the data, and stresses yet again the importance of ratio measurements in removing deuteron corrections.

All the experimental information on possible exotic terms has resulted from experiments on the reactions $\gamma+n \rightleftharpoons \pi^-+p$. In Sec. V we give a brief summary of attempts to perform multipole analyses on the proton reactions $\gamma+p\rightarrow\pi^++n,\ \pi^0+p$ only, without exotic terms. This will be relevant in judging the success of our own account of these reactions.

In Sec. VI we go on to discuss, in some detail, the results of fitting the parameters of the model to the data for all the reactions including a detailed comparison between the results and the data. A short account of some of these results, in the case of $\pi^\pm$ production only, has already been published.[14] In particular, we consider which parts of the results are insensitive to reasonable changes in the model.

So far the work represents a development in the viewpoint running through Refs. 1, 4, 5, 11–14. In Sec. VII we go on to give a review of other work bearing on this subject, some of which has attempted to find alternatives to the above approach.

Finally, in Sec. VIII we summarize the results and indicate the size of effects implied in other related processes. We also give the experimental advances necessary to draw definitive conclusions about the exotic currents.

II. NOTATION AND GENERAL FORMALISM

We consider the process

$$\gamma(K)+N_1(P_1)\rightleftharpoons \pi_\alpha(Q)+N_2(P_2)\,, \tag{7}$$

where α is the isospin index of the pion, and we introduce the usual variables

$$s=-(P_1+K)^2\,,$$
$$t=-(K-Q)^2\,,$$
$$u=-(K-P_2)^2\,,$$

with

$$s+t+u=2M^2+m_\pi{}^2\,.$$

In the center-of-momentum system we define $K=(k,\vec{k}),\ P_1=(E_1,-\vec{k}),\ Q=(\omega_q,\vec{q}),\ P_2=(E_2,-\vec{q})$, and the variables $W=\sqrt{s},\ \omega=W-M$ are often used. Now, in addition to considering the contributions to (7) arising from the conventional electromagnetic current J_μ, satisfying

$$CJ_\mu C^{-1}=-J_\mu\,, \tag{8}$$

we wish to consider possible additional terms arising from a C-violating electromagnetic current[15] K_μ, satisfying

$$CK_\mu C^{-1}=+K_\mu\,. \tag{9}$$

We split the T matrix for the process into two pieces arising from these two terms, i.e.,

$$\begin{aligned}\langle\pi N_2|S|\gamma N_1\rangle &= -i\,\frac{(2\pi)^{5/2}}{(2k)^{1/2}}\,\delta^4(P_2+Q-P_1-K)\epsilon_\mu\\ &\quad\times\langle\pi N_2\mathrm{out}\,\|[J_\mu(0)+K_\mu(0)]\|\,N_1\mathrm{in}\rangle\\ &= -i(2\pi)^{-2}\delta^4(P_2+Q-P_1-K)\,\frac{m}{2(kw_qE_1E_2)^{1/2}}\\ &\quad\times\langle\pi N_2|(T_e+T_o)|N_1\rangle\,,\end{aligned} \tag{10}$$

where the T_e,T_o terms arise from the C-conserving and C-violating terms, respectively. We assume P, CPT invariance throughout so that these also correspond to T-conserving and T-violating terms, respectively. Thus we have for the photoproduction process

$$\langle\pi N_2|T|\gamma N_1\rangle=\langle\pi N_2|(T_e+T_o)|\gamma N_1\rangle\,,$$

and for the time-reversed, radiative-capture process $\pi N\rightarrow\gamma N$

$$\langle\gamma N_1|T|\pi N_2\rangle=\langle\pi N_2|(T_e-T_o)|\gamma N_1\rangle\,. \tag{11}$$

The dispersion formalism for the $I=0,1$ T-conserving terms was first given by Chew, Goldberger, Low, and Nambu[16] (referred to throughout as CGLN) and the extension to include $I=2$ terms is given in Ref. 5. We wish here to introduce a convenient notation for the T-violating terms also, briefly summarizing the basic relations required, together with those for the conventional terms for completeness.

The first step in setting up the dispersion formalism is of course to decompose the T matrix into invariants. For the T-conserving terms the most general form consistent with Lorentz invariance, parity conservation, and gauge invariance was first given by CGLN, and subsequently discussed by Ball.[17] The same decomposition also holds for the C-violating terms, and we write

$$\langle \pi N_2 | T | \gamma N_1 \rangle = \sum_{i=1}^{4} [A_i(s,t,u) + i\overline{A}_i(s,t,u)] \times \overline{u}(P_2) M_i u(P_1), \tag{12}$$

where

$$\begin{aligned} M_1 &= i\gamma_5 \gamma\cdot\epsilon\gamma\cdot K, \\ M_2 &= 2i\gamma_5(P\cdot\epsilon Q\cdot K - P\cdot KQ\cdot\epsilon), \\ M_3 &= \gamma_5(\gamma\cdot\epsilon Q\cdot K - \gamma\cdot KQ\cdot\epsilon), \\ M_4 &= 2\gamma_5(\gamma\cdot\epsilon P\cdot K - \gamma\cdot KP\cdot\epsilon - iM\gamma\cdot K\gamma\cdot\epsilon), \end{aligned} \tag{13}$$

and

$$P = \tfrac{1}{2}(P_1 + P_2). \tag{14}$$

For the capture process

$$\langle \gamma N_1 | T | \pi N_2 \rangle = \sum_{i=1}^{4} [A_i(s,t,u) - i\overline{A}_i(s,t,u)] \times \overline{u}(P_2) M_i u(P_1), \tag{15}$$

so that the total amplitudes are

$$\tilde{A}_i(s,t,u) = A_i(s,t,u) \pm i\overline{A}_i(s,t,u), \tag{16}$$

where the plus (minus) sign is appropriate for photoproduction (capture). Thus the A_i are precisely the amplitudes of CGLN, the $\overline{A}_i$ are the corresponding T-violating amplitudes, and the factor i is introduced for reasons which will become clear later.

The angular momentum decomposition is also given by CGLN, expanding the A_i into electric and magnetic multipole amplitudes $E_{l\pm}, M_{l\pm}$, leading to final πN states of angular momentum $J = l \pm \frac{1}{2}$. The equations are given by CGLN, and a precisely similar set define multipole amplitudes for the T-violating parts $\overline{E}_{l\pm}, \overline{M}_{l\pm}$, so that the complete multipole amplitudes for photoproduction and capture are

$$\begin{aligned} \tilde{E}_{l\pm} &= E_{l\pm} + i\overline{E}_{l\pm}, E_{l\pm} - i\overline{E}_{l\pm}, \\ \tilde{M}_{l\pm} &= M_{l\pm} + i\overline{M}_{l\pm}, M_{l\pm} - i\overline{M}_{l\pm}, \end{aligned} \tag{17}$$

respectively.

In addition to the usual isospin amplitudes for photoproduction,[18] an isoscalar[19] amplitude A^0, leading to the $I = \frac{1}{2}$ final πN state and isovector amplitudes A^1, A^3 leading to the $I = \frac{1}{2}, \frac{3}{2}$ πN final states, we have also the isotensor amplitude A^2 leading to the $I = \frac{3}{2}$ πN final state.[4] The amplitudes for the observable processes are

$$(3/\sqrt{2})A(\gamma p \to \pi^+ n) = 3A^0 + A^1 + (\tfrac{3}{5})^{1/2}A^2 - A^3, \tag{18a}$$

$$(3/\sqrt{2})A(\gamma n \to \pi^- p) = 3A^0 - A^1 + (\tfrac{3}{5})^{1/2}A^2 + A^3, \tag{18b}$$

$$3A(\gamma p \to \pi^0 p) = 3A^0 + A^1 - 2(\tfrac{3}{5})^{1/2}A^2 + 2A^3, \tag{18c}$$

$$3A(\gamma n \to \pi^0 n) = -3A^0 + A^1 + 2(\tfrac{3}{5})^{1/2}A^2 + 2A^3. \tag{18d}$$

It has also been convenient to introduce[4] amplitudes leading to the $I = \frac{1}{2}, \frac{3}{2}$ πN states on protons, neutrons, respectively, i.e.,

$${}_pA^3(W) = (\tfrac{2}{3})^{1/2}[A^3(W) - (\tfrac{3}{5})^{1/2}A^2(W)], \tag{19a}$$

$${}_nA^3(W) = (\tfrac{2}{3})^{1/2}[A^3(W) + (\tfrac{3}{5})^{1/2}A^2(W)], \tag{19b}$$

$${}_pA^1(W) = -(\tfrac{1}{3})^{1/2}[A^1(W) + 3A^0(W)], \tag{19c}$$

$${}_nA^1(W) = (\tfrac{1}{3})^{1/2}[A^1(W) - 3A^0(W)]. \tag{19d}$$

Finally, for purposes of discussing crossing symmetry it is useful to write the isospin amplitude between nucleon isospinors for the emission of a pion of isospin index α in the form[16,18]

$$A = A_i^+ g_\alpha^+ + A_i^- g_\alpha^- + A_i^0 g_\alpha^0 + A_i^2 g_\alpha^2, \tag{20}$$

where

$$\begin{aligned} g_\alpha^+ &= \delta_{\alpha 3}, \quad g_\alpha^- = \tfrac{1}{2}[\tau_\alpha, \tau_3], \\ g_\alpha^0 &= \tau_3, \quad g_\alpha^2 = (15)^{-1/2}(\delta_{\alpha 1}\tau_1 + \delta_{\alpha 2}\tau_2 - 2\delta_{\alpha 3}\tau_3), \end{aligned} \tag{21}$$

and

$$\begin{aligned} A_i^+ &= A_i^1 + 2A_i^3, \\ A_i^- &= A_i^1 - A_i^3. \end{aligned} \tag{22}$$

It is well known that unitarity and T invariance lead, for the conventional $C = -1$ amplitudes, to the result that the multipole amplitudes $M_{l\pm}^I, E_{l\pm}^I$ have the same phase (modulo π) as the πN scattering amplitude $f_{l\pm}^I$ leading to the same final state. This result, the Watson theorem,[20] holds below the $(\pi\pi N)$ production threshold, to lowest order in the fine-structure constant. For an amplitude odd under T, the same argument leads to an amplitude 90° out of phase with $f_{l\pm}^I$. In our definition of the T-violating multipoles $\overline{E}_{l\pm}^I, \overline{M}_{l\pm}^I$ [see Eqs. (16) and (17)] this factor i has been removed so that the phase of these amplitudes, like the conventional $E_{l\pm}^I, M_{l\pm}^I$, is also given by the πN phase shift (modulo π) in the elastic region.

Thus the complete multipole amplitudes can be written

$$\begin{aligned} \tilde{M}_{l\pm}^I &= M_{l\pm}^I \pm i\overline{M}_{l\pm}^I \\ &= (\{M_{l\pm}^I\} \pm i\{\overline{M}_{l\pm}^I\})e^{i\delta_{l\pm}^I}, \end{aligned} \tag{23}$$

where the plus (minus) sign is appropriate for photoproduction (capture) and $\{\ \}$ means $\pm$ the modulus. It can also be written

$$\tilde{M}^I_{l\pm}=\{\tilde{M}^I_{l\pm}\}e^{i\delta^I_{l\pm}}e^{\pm i\phi^I_{l\pm}}. \tag{24}$$

While this is sometimes convenient as a parametrization, it is not convenient for theoretical discussion since the amplitudes $A, \bar{A}$ have quite different analytic structures, and it is these amplitudes, but not $\tilde{A}$, that have simple crossing properties and satisfy simple dispersion relations. Finally, it is convenient to mention here that in their initial paper on T violation in these processes,[10] Christ and Lee made the approximation

$$|\tilde{M}^I_{l\pm}|=|M^I_{l\pm}|, \quad |\tilde{E}^I_{l\pm}|=|E^I_{l\pm}|, \tag{25}$$

which is sometimes referred to as the Christ and Lee model. This was done in order to test the sensitivity of the data to small T-violating effects, and, as was very clearly stressed by Christ and Lee, it is important to remember that it is only valid if

$$|\bar{M}^I_{l\pm}| \ll |M^I_{l\pm}|, \quad \text{etc.} \tag{26}$$

and cannot be used if this condition is not satisfied.

Crossing relates the processes

$$\gamma+N\to\pi+N,$$
$$\gamma+\bar{N}\to\pi+\bar{N},$$

and so to obtain the crossing-symmetry results it is necessary to use C invariance (or noninvariance) to obtain another relation between them.[16,17] Thus the C-violating amplitudes have opposite crossing properties to the C-conserving amplitudes. Further as can be seen from Eq. (21), the isotensor amplitudes have the same isospin crossing properties as the isoscalar amplitudes.[5] The complete results are

$$\begin{aligned} A_i^{(0,+,2)}(s,t,u)&=\xi_i A_i^{(0,+,2)}(u,t,s),\\ A_i^-(s,t,u)&=-\xi_i A_i^-(u,t,s) \end{aligned} \tag{27}$$

and

$$\begin{aligned} \bar{A}_i^{(0,+,2)}(s,t,u)&=-\xi_i \bar{A}_i^{(0,+,2)}(u,t,s),\\ \bar{A}_i^-(s,t,u)&=\xi_i \bar{A}_i^-(u,t,s), \end{aligned} \tag{28}$$

where

$$\begin{aligned} \xi_i&=+1, \quad i=1,3,4\\ &=-1, \quad i=2. \end{aligned} \tag{29}$$

We can now write down the fixed-t dispersion relations for all the amplitudes $A_i, \bar{A}_i$, which are both defined so as to be real, analytic functions. The necessary crossing relations are given above, and the only other thing to note is that since none of the $I=2$ or C-violating terms can couple to on-shell nucleons or pions, none of the "exotic" amplitudes have Born terms. The results are, for the conventional amplitudes,

$$\begin{aligned} A^{(\pm,0)}(s,t,u)=&A^{(\pm,0)}(s,t,u\text{:Born})\\ &+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\,\mathrm{Im}A^{(\pm,0)}(s',t)\\ &\times\left(\frac{1}{s'-s}\pm\xi_i\frac{1}{s'-u}\right), \end{aligned} \tag{30}$$

and for the "exotic" amplitudes,

$$A_i^2(s,t,u)=\frac{1}{\pi}\int ds'\,\mathrm{Im}A_i^2(s',t)\left(\frac{1}{s'-s}+\xi_i\frac{1}{s'-u}\right), \tag{31}$$

$$\begin{aligned} \bar{A}_i^{(\pm,0,2)}(s,t,u)=&\frac{1}{\pi}\int ds'\,\mathrm{Im}\bar{A}_i^{(\pm,0,2)}(s',t)\\ &\times\left(\frac{1}{s'-s}\mp\xi_i\frac{1}{s'-u}\right), \end{aligned} \tag{32}$$

where the lower sign is appropriate for the $A_i^-, \bar{A}_i^-$ amplitudes. The expressions for the Born terms are given, for example, by Ball.[17] It is sometimes convenient not to work with these relations, but with the coupled set of equations for the multipole amplitudes which can be projected from them. For the conventional case, these are given in some detail in, for example, the paper of Berends, Donnachie, and Weaver.[21] The results for the exotic amplitudes are easily deduced from these by comparing Eqs. (31) and (32) with Eq. (30). Examples of this, in the static model at least, will be given later.

Finally, it is perhaps worth making a few remarks on the range of validity of these relations.[22] The relations have only been proved rigorously for[23] $0>t>-12m_\pi^2$, which covers the whole of the physical region, allowing partial waves to be projected out, up to $s\sim 78m_\pi^2$, $E_\gamma\sim 340$ MeV. However, the Mandelstam representation indicates a much larger domain of applicability, for isovector amplitudes up to $s\sim 88m_\pi^2$, i.e., $E_\gamma\sim 450$ MeV and for isoscalar (and isotensor) amplitudes the larger range $s\leq 100m_\pi^2$, $E_\gamma\leq 570$ MeV. This is quite sufficient for the present purposes. However, it is reasonable to expect the relations to be approximately true to much higher energies. If the t-channel exchanges are small or dominated by narrow resonances,[22] or are dual to the s-channel exchanges,[24] then the limit is set by the s,u double-spectral function to be $s\leq 133m_\pi^2$, $E_\gamma\leq 920$ MeV, the result being the same for all the isospin cases. Thus the use of the partial-wave projected relations over the region we shall consider ($E_\gamma \leq 400$ MeV) would appear to be amply justified.

III. THE MODEL

The model suggested by Sanda and Shaw,[5,13] which we shall take as our point of departure, is based like most others on the fixed-t dispersion relations. The important assumptions are as follows.

(a) The discontinuities for the conventional amplitudes $A_i^{0,1,3}$ are assumed to be dominated by the diagrams of Fig. 1. On the other hand, the $I=2$ and C-violating terms in the current cannot couple to the Born-term poles, since these involve only the on-shell γNN, $\gamma\pi\pi$ couplings. Thus, the discontinuities for these terms are assumed to be dominated by the diagrams 1(b) and 1(c) only. In particular, the $C=+1$ terms couple in only via the $\gamma N\Delta$ vertex and so must be isovector and isotensor, with the result that in this model

$$\bar{A}_i^0=0\,. \tag{33}$$

(b) The resonant excitation ($\Delta N\gamma$) is assumed to be almost entirely magnetic dipole. The electric quadrupole amplitude on protons, ${}_pE_{1+}^3$, is known experimentally to be very small at resonance[25] compared to the magnetic dipole ${}_pM_{1+}^3$ (about -3%) and to show little sign of resonant structure – the absorptive part has no peak at resonance but is slowly varying.[26] The assumption is that this is the case on neutrons also. Although this is no more than a plausible guess based on the above results on protons, and on the fact that dynamical models (see, e.g., Refs. 16, 21, and 26) usually give a very small isovector E_{1+}^3, it should be remembered that, in the absence of isotensor terms,

$${}_pE_{1+}^3={}_nE_{1+}^3,$$

so that it ceases to be an assumption at all in this case.

(c) High-energy photoproduction cross sections decrease rapidly ($\alpha_{\text{eff}}\approx 0$) so that there is no need to consider more than one subtraction. With the above model for the absorptive parts, partial conservation of axial-vector current (PCAC) and gauge invariance ensure that the subtraction constants vanish to terms of order of the pion mass squared.[27] These constants are therefore neglected. The empirical status of this will be returned to below.

(d) Allowance must be made for the effects of the highly inelastic Roper resonance on the background amplitudes $M_{1-}^{0,1}$. This is of little importance for total cross-section considerations, but can be crucial if differential cross sections or polarizations are discussed.

(e) The contributions of the crossed-Δ-exchange diagram 1(b) are small compared with those of the Born term and direct-channel resonance contributions. Thus it is a quite good first approximation to calculate these particular terms in the conventional model, neglecting $I=2$ and $C=+1$ terms, which in turn means that to a good approximation these effects will be confined to the resonant M_{1+} multipole, and to the energy region of the resonance itself.

(f) The conventional model is on the whole successful in describing the data on protons, although there is evidence for some discrepancies in detail.[28,29] However, since the amplitudes are dominated by the Born terms and ${}_pM_{1+}^3$, if this latter term is kept reasonably close to its usual value, this success can be easily retained. Equally easily, it can be seen from Eqs. (19a) and (19b) that this imposes no restriction at all on the isotensor contribution, to determine which it is clearly necessary to examine neutron data also.

This describes the qualitative features of the model, which are the same here as in earlier work. However, we have not restricted ourselves to $I=2$ T violation, as in Refs. 12 and 13, but have allowed an arbitrary mixture of $I=1,2$ T violation, and have also made a number of quantitative improvements. We now go on to describe the actual details, and define the parameters of the model used.

For the resonance shape, we have taken the same form for both isovector and isotensor amplitudes, namely,

$$M(W)=5.5\times 10^{-2}\,\frac{k}{q^2}\,e^{i\delta_{33}}\sin(\delta_{33}+q^3x_1/k^3)\,, \tag{34}$$

where we are using units with $\hbar=m_\pi=c=1$. This reduces to the usual resonance form if the phase x_1 is zero. The δ_{33} phase shift of Bugg *et al.*[30] is used, and the numerical factor is merely to give a convenient scale. We allow T-violating phases in both the isovector and isotensor parts as noted above, the multipoles being given by

$$\tilde{M}_{1+}^3=M(W)x_2e^{ix_4}\,, \tag{35}$$

$$\tilde{M}_{1+}^2=M(W)x_3e^{ix_5}\,, \tag{36}$$

the T-violating phases x_4, x_5 changing sign under T reversal. Strictly speaking, x_4 cannot be exactly constant, since

$$M_{1+}^3=M(W)x_2\cos x_4$$

must contain the Born term poles whereas

$$\bar{M}_{1+}^3=M(W)x_2\sin x_4$$

must not. However, to the extent that the amplitude is dominated by the second-sheet resonance poles, x_4 is constant so that in this case the approximation should be a reasonably good one. It is also convenient to define corresponding parameters for ${}_p\tilde{M}_{1+}^3$, ${}_n\tilde{M}_{1+}^3$, i.e.,

$$ {}_p\tilde{M}^3_{1+} = \left(\frac{2}{3}\right)^{1/2} M(W)\left[x_2 e^{ix_4} - \left(\frac{3}{5}\right)^{1/2} x_3 e^{ix_5}\right] $$
$$ = \left(\frac{2}{3}\right)^{1/2} M(W) x_p e^{i\phi_p} , \qquad (37) $$
$$ {}_n\tilde{M}^3_{1+} = \left(\frac{2}{3}\right)^{1/2} M(W)\left[x_2 e^{ix_4} + \left(\frac{3}{5}\right)^{1/2} x_3 e^{ix_5}\right] $$
$$ = \left(\frac{2}{3}\right)^{1/2} M(W) x_n e^{i\phi_n} . \qquad (38) $$

The problem with the $M^{0,1}_{1-}$ multipoles is of course the impossibility of theoretically calculating the contribution of the P_{11} resonance to photoproduction, and its effects in this region. We treat this by adding to the theoretical values, approximated by the formulas given below, contributions from the mass region of this resonance, i.e.,

$$ M^0_{1-} = -1.1\times 10^{-3}\,\frac{q}{k}\left(1 + \frac{x_6}{10.72 - W}\right) \qquad (39) $$

and

$$ M^1_{1-} = 4.9\times 10^{-3}\,\frac{q}{k}\left(\frac{M}{W} + \frac{x_7}{10.72 - W}\right) . \qquad (40) $$

The "other" partial waves, up to and including f waves, are then given by the model, using conventional values to estimate the contribution of Fig. 1(c). All the higher partial waves, previously neglected,[5,13] have now been included in the Born approximation. The numerical values have in fact been taken from the evaluation of Berends, Donnachie, and Weaver,[21] which in addition to the above contributions include some small corrections from "rescattering" terms roughly represented in Fig. 1(d). The only slight exception to this is in E^3_{1+}, where we have changed the δ_{33} phase used to its more recent value (Ref. 30), but kept the same modulus as these authors.

This completes the description of the model under assumption (e), in which the crossed-Δ contribution is calculated in the conventional model, and this is the model used in most of our fits. However, in order to check that the results are not sensitive to this approximation, the corrections to it have been evaluated in the static model, and included in some of the fits. As we shall see, the inclusion of these extra terms does not change any of the conclusions drawn, although the details of the fits alter somewhat. The same result will also be found to apply when we add to our multipoles some small corrections from the second resonance region calculated by Berends and Weaver.[28]

We thus see that our model attempts a description of the process in terms of only seven parameters x_1–x_7, the four interesting ones being the isovector, isotensor resonance coupling strengths x_2, x_3, and the isovector, isotensor T-violating phases x_4, x_5. In addition there is a resonance shape parameter x_1, and the two parameters of the $M^{0,1}_{1-}$ waves x_6, x_7.

IV. THE DATA

We have performed fits over the photon laboratory energy range 200–400 MeV. The data used are as follows:

(a) $\gamma + p \to \pi^+ + n$. The differential cross section (d.c.s.) has been extensively studied, and we used the accurate recent results of Fischer *et al.*[31] and Betourni *et al.*[32] In both cases systematic and statistical errors have been combined, as is the case throughout for differential cross-section measurements. The polarized-photon asymmetry and recoil-neutron polarization data were also included.[33–36]

(b) $\gamma + p \to \pi^0 + p$. The recent data of Fischer *et al.*[37] and Hilger *et al.*[38] have been used, and those of Morand *et al.*,[39] although not included in the actual fits, have been compared with the results afterwards. As can be seen the bulk of the data are from Ref. 37 and it is these data which determines the fits, the other data being relatively unimportant by comparison. However, it is important to notice that there are definite disagreements between this and the other two experiments which, though not large, are greater than the quoted errors. Thus conclusions based on fitting the details of the above experiment[37] should be treated as yet with some caution.

The polarized-photon asymmetry data of Drickey *et al.*,[40] Barbiellini *et al.*,[41] and Antufyev *et al.*[42] were included and the recoil-proton polarization data were from Althoff *et al.*[43]

(c) Other d.c.s. data on protons. There are of course many other earlier d.c.s. data on protons,

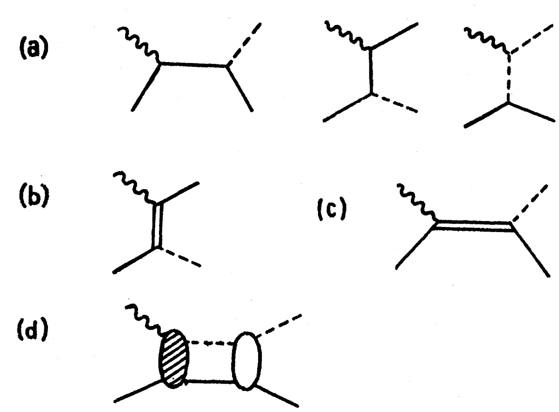

FIG. 1. Born terms and contributions to the low-energy absorptive parts for pion photoproduction.

as given for example in the data compilation of Beale, Ecklund, and Walker.[44] These earlier experiments contain severe inconsistencies among themselves, and we have preferred to retain here only the more modern work. However, we wish to record here that a fit including all the old data has been carried out to check that this results in no significant changes in the parameters. This expectation is in fact borne out.

(d) $\pi^- + p \to \gamma + n$. In addition to the differential cross-section result quoted earlier[9] there is also an excitation curve at 30° measured over the resonance region.[45] In both cases, systematic and statistical errors are again included.

(e) $\gamma + n \to \pi^- + p$. For the differential cross sections there are the two extensive bubble-chamber experiments on deuterium, using the spectator model for corrections.[7,8] In ABBHHM[7] a cut has been made on the exchange-nucleon mass to ensure that it is close to the mass shell before applying the spectator model. We use the total error which is quoted. In the PRFN data[8] the normalization error has been combined with the quoted statistical error.

There is an important experiment at backward angles from Fujii *et al.*[46] since this should be to a large extent independent of deuterium corrections. They measure both $\pi^\pm$ production on deuterium and π^+ production on protons, so that the ratio method can be applied. The backward π^+ data were not included in our fits, which were made to the data listed in (a) of this section. Their results are in good agreement with this. The π^+ results on deuterium and protons are the same within errors with no corrections, and the authors note that this is expected since the known corrections are small in the backward direction. Thus the π^- results on neutrons are the same as those on deuterium by the ratio argument mentioned below. Again we have combined statistical and systematic errors. There are also a few polarized-photon asymmetry points.[47]

Finally there are two experiments which were not included in the fit, but to which we have compared the results to show that they are consistent. The first of these are the total cross-section measurements of White *et al.*[48] and the second,[49] three differential cross-section measurements at 275 MeV obtained by detecting both the π^- and proton in coincidence, and performing a modified Chew-Low extrapolation.

(f) Ratio measurements. The use of measurements of charge ratios on deuterium to eliminate much of the uncertainty due to the deuteron corrections in obtaining neutron cross sections has been advocated for a long time[50]: We wish to stress its value yet again. The point is simply that most of the corrections to the free-nucleon charge ratio in the measured ratio on deuterium,

$$R_d = \frac{d\sigma/d\Omega(\gamma + d \to \pi^- + p + p)}{d\sigma/d\Omega(\gamma + d \to \pi^+ + n + n)}, \tag{41}$$

cancel out. They are the same for both reactions. The final states are charge-symmetric, so that the final-state corrections cancel out, and in the initial state, photons of the energy concerned (about 300 MeV) do not shadow. Further, if the ratio is not rapidly varying, which it is not, then the deuteron wave-function corrections will cancel out also. Thus the above ratio should be a good approximation to the free-nucleon ratio

$$R = \frac{d\sigma/d\Omega(\gamma + n \to \pi^- + p)}{d\sigma/d\Omega(\gamma + p \to \pi^+ + n)}. \tag{42}$$

The same remark applies to the production of neutral pions on deuterium,

$$R_d^0 = \frac{d\sigma/d\Omega(\gamma + d \to \pi^0 + n + p_s)}{d\sigma/d\Omega(\gamma + d \to \pi^0 + p + n_s)}, \tag{43}$$

except that the neutron, proton reactions need to be distinguished by kinematic identification of the spectator.

Corrections which obviously do not cancel out are the Coulomb corrections for the $\pi^- pp$ state, which have been studied by Baldin.[51] These should of course be applied. Some numerical results are given for example by Bazin and Pine[52] over the energy range $165 \leq E_\gamma \leq 200$ MeV. The corrections given at the angles measured are up to 4% near threshold decreasing to about 1% at 200 MeV. They would thus not seem to be a serious source of uncertainty but should be applied if data reach this level of precision.

The upshot of the above remarks is that by the combination of ratio measurements on deuterium with measurement of those reactions accessible on protons, the deuteron corrections to be applied are effectively measured in the experiment. Of course the discussion neglects components in the deuterium ground-state wave function of higher nucleonic resonances, and interactions of the incoming photon with both nucleons simultaneously. All these effects are small in the physics of light nuclei, especially in a very open structure like deuterium, and since the deuterium corrections are not large themselves, to neglect these small modifications to the corrections is presumably rather safe.

Thus the use of ratio measurements is an important check on the data obtained in (e) of this section using deuterium corrections. There are so far no π^0 ratios, which is rather unfortunate considering their sensitivity to the presence of isotensor terms. There are some measurements of π^-/π^+ ratios however. The only ones included in the fits are those of Fujii *et al.*[46] mentioned above, which were not actually direct measurements of ratios, but, since both $\pi^{\pm}$ were measured (separately), can be regarded as such. The only other reasonably recent data in the resonance region are those of Hogg[53] which at 145° extend to 290 MeV, and at lower energies the data of Bazin and Pine[52] and Burq and Walker.[54] These three experiments were not included in the fit, but the results of the fit will be compared with them, and in particular they will bear on the question of possible s-wave subtractions. There are also a number of much older measurements summarized by Hogg,[53] especially in the threshold region. Unfortunately their consistency is far from satisfactory – a notorious if understandable feature of early photoproduction experiments – although Hogg concludes that only the very early data of Sands *et al.*[55] are in clear disagreement with the other experiments. For this reason we confine ourselves to the most recent and presumably reliable experiments mentioned above.

V. MULTIPOLE ANALYSES: THE PROTON DATA

As stressed in (e) of Sec. III, in the absence of T violation, for the proton reactions

$$\gamma + p \rightarrow \pi^+ + n, \tag{44}$$

$$\gamma + p \rightarrow \pi^0 + p, \tag{45}$$

the model reduces precisely to a conventional dispersion-theory model, except that the coupling and shape of M_{1+}^3 and $M_{1-}^{0,1}$ are determined phenomenologically. If the T-violating phase $\phi_p \neq 0$, there will of course be some changes, but if ϕ_p is not too big, the predictions should not be changed very much. It is therefore obviously of interest to ask if there is any phenomenological evidence for deviations from the dispersion theory results in the nonresonant multipoles.

Recently there have been two detailed energy-independent multipole analyses of the reactions (44) and (45) over this energy region, by Noelle, Pfeil, and Schwela[29] and by Berends and Weaver.[28] The results of these analyses do in general agree with the dispersion-relation predictions; however, there are discrepancies in detail, especially in M_{1-}^3, but also to some extent in other multipoles. The effect in M_{1-}^3 is the most serious. Berends and Weaver[28] have calculated the corrections in this region arising from the second and third resonance contributions, using the coupling parameters estimated by Walker.[56] Whereas these corrections move things in the right direction, in the case of M_{1-}^3 they are only half as big as is necessary to remove the discrepancy. We shall in fact investigate the effect of including these calculated second-resonance corrections on our results (it is negligible) but this leaves some discrepancy still not understood. Berends and Weaver also show that this discrepancy results essentially from fitting the π^0 asymmetry data with polarized photons. We will pay particular attention to this data. However we will not allow M_{1-}^3 to deviate in general from its predicted value.

What about this residual discrepancy (apart from the second-resonance effects) in M_{1-}^3, and the small discrepancies in other waves, e.g., E_{1+}^3 above resonance? These are certainly present in the results of these analyses, but how firmly does this establish them for the physical amplitudes? This obviously depends on the quality of the data. To split the $I=\frac{1}{2}, \frac{3}{2}$ πN states information on both reactions (44) and (45) is required, and in view of the discrepancies mentioned in Sec. III between the data of Fischer *et al.*,[37] which by their statistical weight determine the fit results, and the other recent experiments,[38,39] suspicion falls on the $\pi^0 p$ data. To investigate this, it is obviously more instructive to plot the multipole analysis results as a function of energy, not for the eigenstates of isospin as is done by the above authors,[28,29] but for the π^0, π^+ channels themselves for which the data are given.

This is done for the s-wave amplitude $E_{0+}^{\pi^0}$ in Fig. 2. We also show the present dispersion-relation result for this wave. Little change results from adding the higher-resonance correction of Berends and Weaver.[28] It is important to realize that the ambiguities of the dispersion relations are associated only with slowly varying terms; the absence of resonances in photoproduction which are not present in scattering is a result of unitarity alone. In contrast the curve which fits the results of Noelle *et al.*[29] so well is the real part of the p-wave $\Delta(1232)$ resonance multiplied by a constant. The actual numbers are taken from Berends *et al.*,[21]

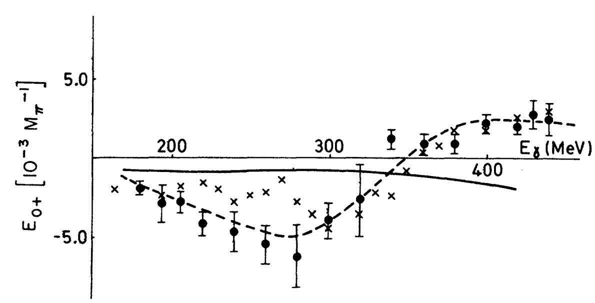

FIG. 2. The electric dipole contribution E_{0+} to π^0 photoproduction. Closed circles, with error bars, are the results of the fit by Noelle, Pfeil, and Schwela (Ref. 29) and crosses the results of the fit by Berends and Weaver (Ref. 28). The solid line is the conventional dispersion-relation solution (i.e., the present model) and the broken line $-0.2\,\mathrm{Re}M_{1+}^3$.

and the coupling is quite large (−30% of the contribution of the resonance to π^0 in M_{1+}). Above 280 MeV, the results of Berends and Weaver[28] also follow this line very closely. We note that the first full angular distribution of Fischer *et al.*[37] is at 260 MeV.

Obviously this result is nonsensical. It means that the results of the analysis for π^0 are unreliable, and clearly something associated with ${}_pM_{1+}^3$ has gone wrong. Two possibilities spring to mind. It could have resulted because of neglect of the possibility of T violation. Or, alternatively, and perhaps more plausibly, it could be the result of some systematic error in the data. In either case it is clear that claims for detailed violations of the dispersion relations based on these analyses should be treated with skepticism. And in the second case it is clear that attempts to fit the π^0, π^+ data simultaneously within any reasonable theoretical framework may have difficulty in detail. However, as we will explicitly show, these detailed questions will have no effect on our main conclusions.

VI. THE FITS

The object of performing fits to the data is to determine the resonance-excitation parameters defined in Eqs. (37) and (38), namely, the parameters x_p, x_n, ϕ_p, ϕ_n. x_p, x_n are the coupling strengths, and ϕ_p, ϕ_n are the T-violating phases on protons, neutrons, respectively. From values of these the isovector and isotensor couplings can be deduced, and we shall sometimes use the subsidiary parameters

$$x = (x_n - x_p)/x_p, \tag{46}$$

$$t = x_2/x_3. \tag{47}$$

In the absence of an isotensor term, $x_p - x_n = x = t = 0$.

Obviously the results for these can only be reliable to the extent that they are not sensitive to the details of the model, and of course that the input data are correct. How will these parameters be determined by the data?

(a) x_p, x_n will be fixed essentially by the total cross sections, or general magnitude of the data.

(b) ϕ_n is determined by the fact that it changes sign between the reactions $\gamma n \to \pi^- p$, $\pi^- p \to \gamma n$. There is no interpretation other than T violation for such a behavior. The magnitude of the effect is governed by the interference term between the T-violating part of the resonant amplitude, and the background, and since for the π^- reaction the background is largely given by the Born term, the magnitude of ϕ_n will be reasonably independent of the details of the model.

(c) In contrast to this, the value obtained for ϕ_p will be sensitive to details of the model. This arises since, for example, in the absence of data on the reaction

$$\pi^+ + n \to \gamma + p \tag{48}$$

for comparison with that on the corresponding forward reaction, there is no model-independent way to demonstrate the existence of T violation on the proton data. The actual value of ϕ_p obtained will result from adjusting the interference terms between ${}_pM_{1+}^3$ and the background for the forward reactions only. This is particularly serious for the $\pi^0 p$ reactions, since the background here is small and difficult to calculate to high accuracy. Since the interference terms in question are proportional to this background, they cannot be reliably trusted to give ϕ_p, especially in view of the inconsistencies in the $\pi^0 p$ d.c.s. data already mentioned.

We thus see that if the data are reliable we can expect to obtain reasonable estimates of x_p, x_n, ϕ_n, but not of ϕ_p. Hence we cannot tell from the fit to this energy region alone the isospin nature of any T violation that might occur. Further, all the information which we have at present that bears on the question of exotic terms in a model-independent way – the comparison of $\pi^\pm$ total cross sections in the case of isotensor terms, and of $\gamma n \rightleftharpoons \pi^- p$ differential cross sections in the case of T violation – occurs in the charged reactions, where the background is given largely by the Born terms. We have therefore found it convenient to divide our account of the many fits carried out into three parts, first discussing the fits to the $\pi^\pm$ data alone, then those incorporating the π^0 data also, and finally a number of fits in which modifications are made to the details of the model in order to test the sensitivity of the results to these.

A. Fits to $\pi^\pm$ Production

The fits to the data achieved in our model are shown in Figs. 3–7, and as can be seen are very satisfactory. The large amount of rather accurate π^+ d.c.s. data (with typical errors of about 7%) is very well accounted for (Fig. 3) as are both sets of radiative capture data (Fig. 4). The fits to the π^- photoproduction cross sections are shown in Fig. 5, and to the π^+ asymmetry data with polarized photons in Fig. 6. As can be seen, the fit is as good as any smooth set of curves will allow, the somewhat higher χ^2 values, which are by no means high, being due to the slightly uneven nature of the data. Finally we would in particular draw attention to the good fit to the 180° $\gamma n \to \pi^- p$ data of Fujii *et al.*[46] shown in Fig. 7. The polarized-photon asymmetry measurements for π^- at 90° (Ref. 47) are well reproduced by the fit.

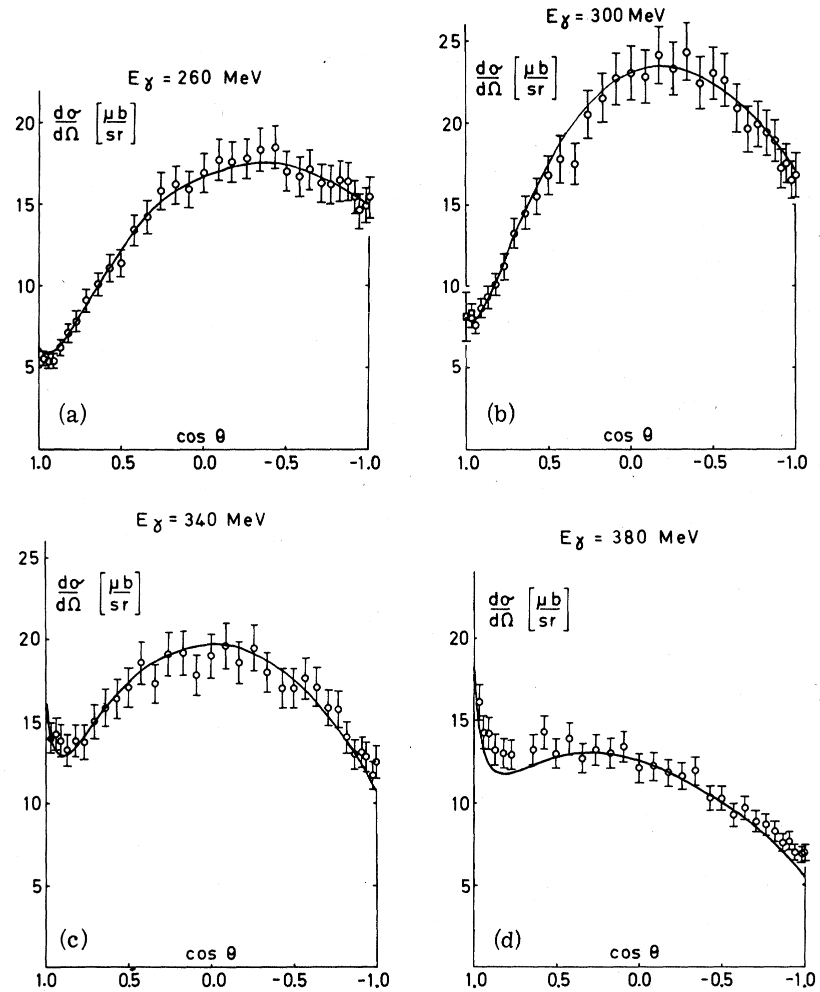

FIG. 3. (a)–(d) Fit of the present model to the π^+ photoproduction differential cross sections. The data are from Fischer *et al.* (Ref. 31) (open circles) and Betourni *et al.* (Ref. 32) (open squares).

We have also shown the comparison with the π^-/π^+ ratio data summarized in (f) of Sec. IV, which was not included in the actual fit, in Fig. 8. The only data which extend into the resonance peak are those of Hogg (to 290 MeV) and they are in good agreement with our results. These also receive some confirmation in this region (275 MeV) from the data of Garelick and Cooperstein [see (e) of Sec. IV] which also agree with our results, but not with the results of the conventional model fits. However, perhaps the most useful information in these extra data is the angular distribution of the π^-/π^+ ratio at 180 MeV, which tells us two things. First, if a subtraction is to be inserted into the π^- s wave,

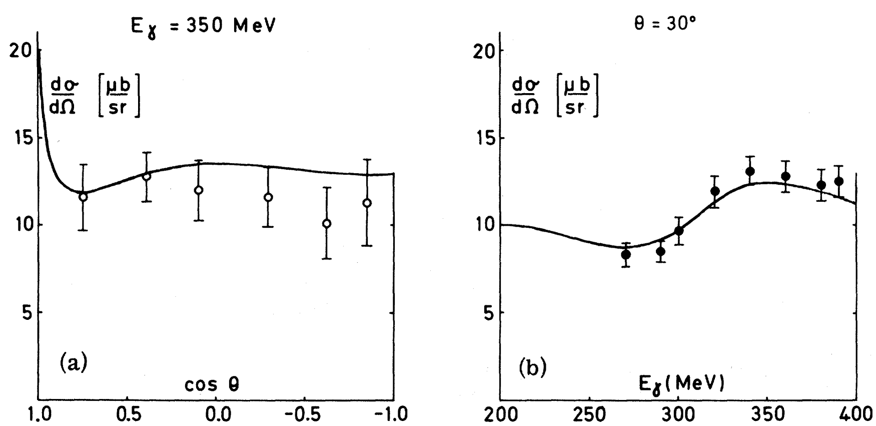

FIG. 4. (a) and (b) Fit of the present model to the π^- radiative-capture data of Berardo *et al.* (Ref. 9) [Fig. 4(a)] and of Schinzel (Ref. 45) [Fig. 4(b)].

it must be rather small and such as to increase this ratio. Second, it allows a good estimate of the total cross section at 180 MeV to be made. Both of these points will be referred to in the discussion of the total cross sections given below.

An alternative way of presenting the results of the fit is given in Table I, where χ^2-per-data-point values for the various types of data are given. The large number of data points and the small number of parameters should be remembered. In this table we have also shown for comparison the results obtained without isotensor and/or T-violating terms. In comparing the various π^- d.c.s. fits it should be borne in mind that the effect is centered on the resonance peak, so that in this region the improvement on introducing an $I=2$ term is much greater than the average shown in the table, which includes the extremes of the energy range where isotensor effects are small. It is clear from the table that within the model, neither the π^- photoproduction data nor the π^- radiative-capture data can be accounted for without an isotensor effect, and that both can only be accounted for simultaneously in the presence of nonzero T-violating phases. The amount of isotensor term is measured by the parameter values $x=-0.28$, $t=-0.24$, and the T-violating phase on neutrons is $\phi_n=-8.5°$. For comparison the rough estimates of Sanda and Shaw[12,13] based on the data at 360 MeV (without the Fujii *et al.* data[46]) were $x\sim-0.30$, $t\sim-0.35$, $\phi_n\sim-20°$. This larger ϕ_n value is preferred by the UCLA-Berkeley π^- capture data,[9] the data of Favier *et al.*[45] preferring the smaller value. The simultaneous fit to both is however good.

Consideration of some qualitative aspects of the data allows us to see what is giving rise to these

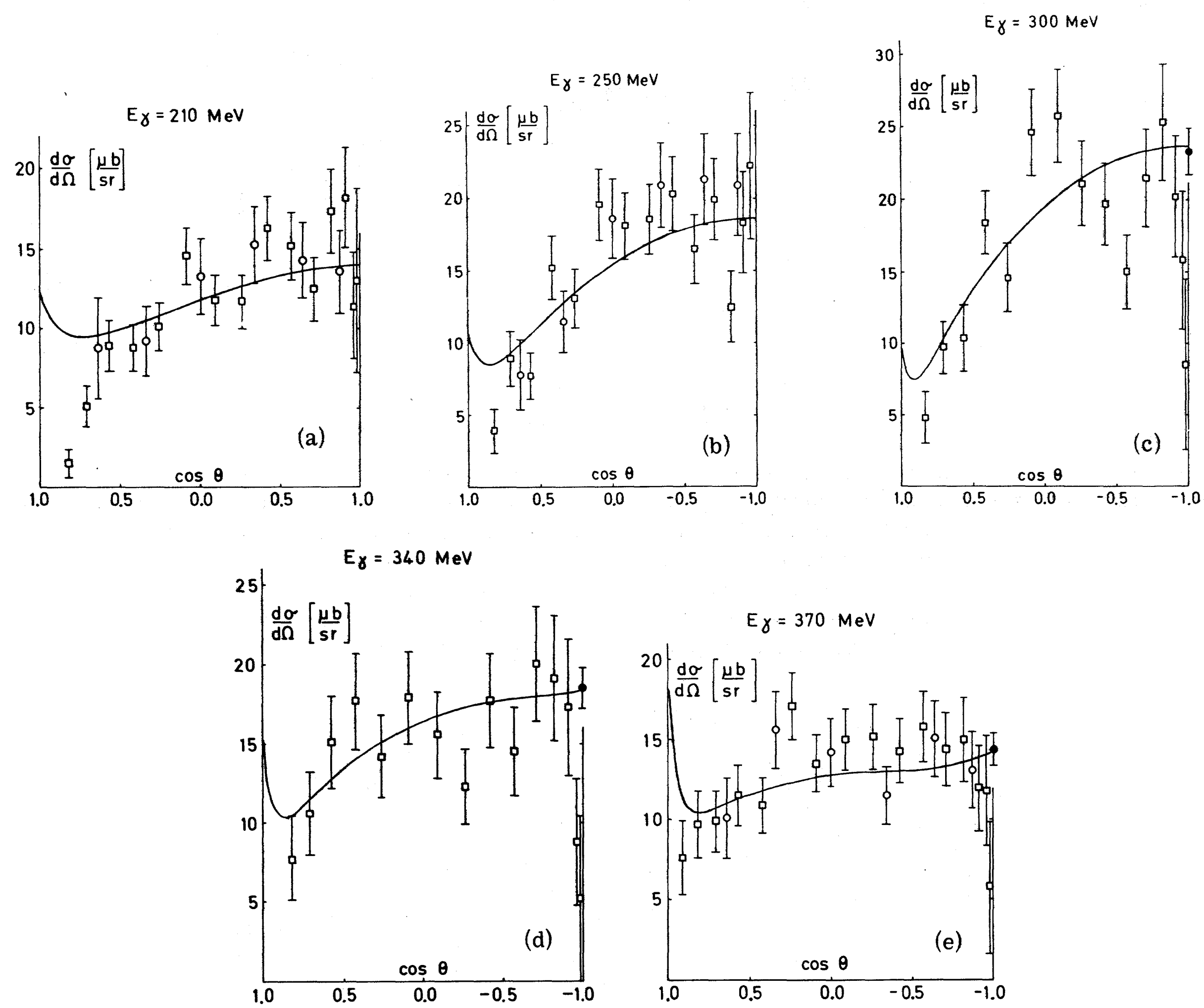

FIG. 5. (a)–(e) Fit of the present model to the π^- photoproduction differential cross sections. The data are from the ABBHHM Collaboration (Ref. 7) (open circles), the PRFN Collaboration (Ref. 8) (open squares), and Fujii *et al.* (Ref. 46) (closed circles).

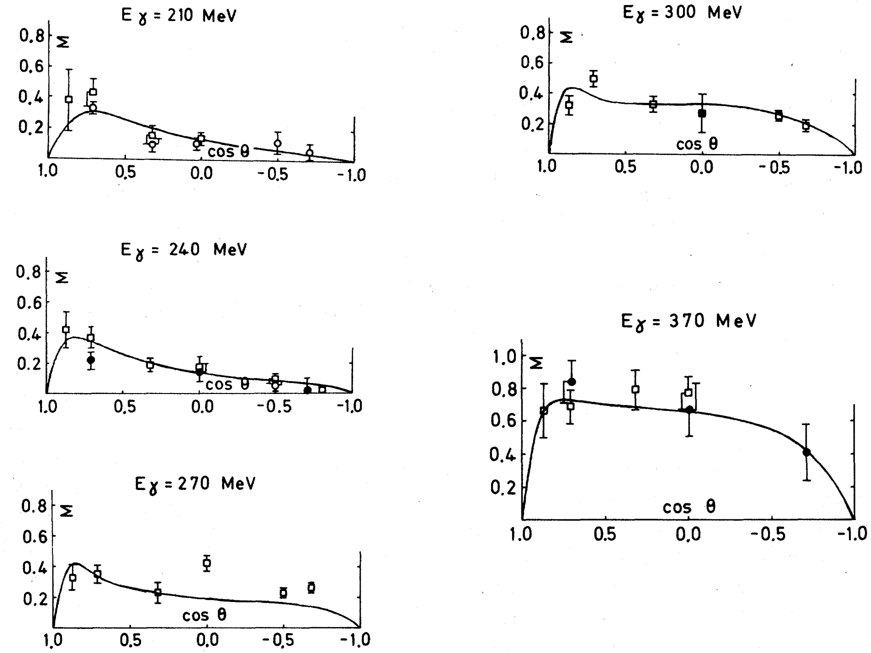

FIG. 6. Fit of the present model to the polarized-photon asymmetry in π^+ photoproduction. The data are from Taylor *et al.* (Ref. 33) (closed squares), Smith *et al.* (Ref. 34) (closed circles), and Grilli *et al.* (Ref. 35) (open squares and open circles).

definite results. First consider the extrapolation of the radiative capture data shown in Fig. 4(a) to the backward direction. The forward peak in this cross section is well known to be due to one-pion exchange, and no appreciable structure, even of this width, is allowed in the backward direction. Any reasonable extrapolation yields a value at 180° of 10 ± 2 μb. On the other hand the corresponding photoproduction reaction at 180° is given in a way that should be independent of deuteron corrections by the data of Fujii *et al.*[46] and is 16 ± 1 μb. Clearly if both these experiments are correct, the existence of some T violation is established; the fit is only to estimate the magnitude and express it as a phase. This also serves to extrapolate the effect to other energies.

The need for the presence of isotensor terms is perhaps most easily seen in the behavior of the total cross sections. The results of our fit for $\sigma_t(\pi^+)$ are in good agreement with those of Noelle *et al.*[29] and Berends and Weaver.[28] Using these values for $\sigma_t(\pi^+)$ and the data for $\sigma_t(\pi^-)$, the quantity $\Delta'(W)$ of Eq. (1) is plotted in Fig. 9. In addition to the bubble-chamber data of the ABBHHM[7] and PRFN[8] groups we have shown also the data of White *et al.*[48] not included in the fit, and the value at 180 MeV resulting from the π^-/π^+ ratio data mentioned above [Fig. 8(a)] and also not included in the fit. The result of the fit is shown, and is clearly satisfactory. The result for the conventional model is also shown, and we note that no published results which obey the fixed-t dispersion relations have produced any noticeable change in the shape of these curves (see, for example, Schwela[57] and Devenish, Lyth, and Rankin[58]). Further, the possibility of lowering the conventional curve (but leaving the shape unchanged) by a subtraction is excluded by the low-energy ratio data, especially that of Fig. 8(a).

The result for the analogous quantity involving the capture reaction

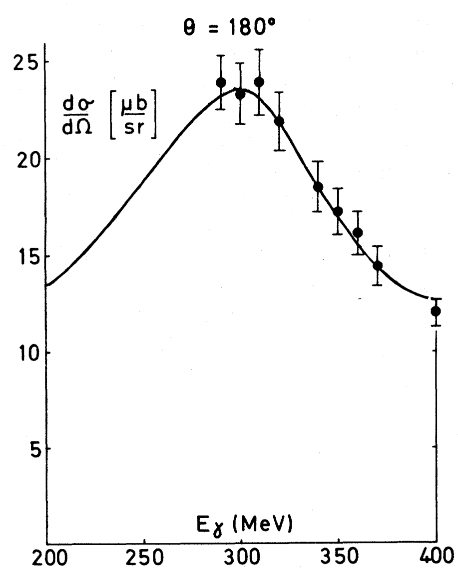

FIG. 7. Fit of the present model to the backward π^- photoproduction excitation curve. The data are from Fujii *et al.* (Ref. 46).

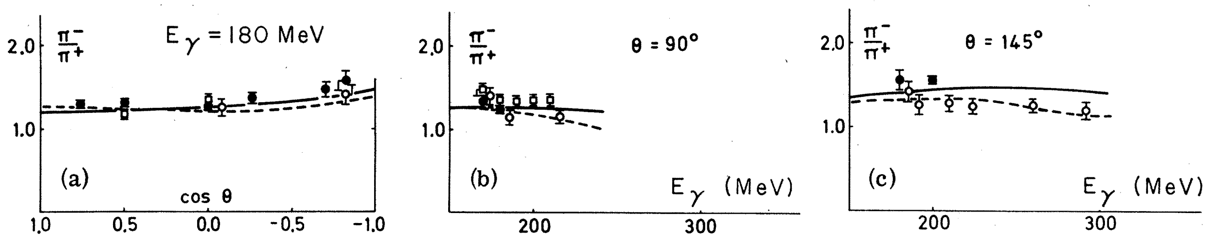

Fig. 8. (a)–(c) Comparison of the π^-/π^+ ratio at low energies with the conventional dispersion relation calculation (solid line) and the present model (broken line). The data are from Hogg (Ref. 53) (open circles), Bazin and Pine (Ref. 52) (closed circles), and Burq and Walker (Ref. 54) (open squares).

$$\delta = \frac{k}{q}\left[\sigma_t(\pi^- p \to \gamma n) - \sigma_t(\pi^+)\right] \tag{49}$$

is shown also. The value for the fit to the capture data at 350 MeV shown in Fig. 4(a) can be read off from this curve, so that it is clear why these data also require an $I=2$ term. Further, Sanda and Shaw[13] showed that in the case of $I=2$ T violation, to first order (i.e., keeping only those terms involving the largest of the resonant amplitudes, M_{1+}^3) $\Delta = \delta$ so that the effects of T violation on the total cross section would be small compared with those of a T-conserving $I=2$ term. This feature also occurs in the results of the fit, i.e., the effect of the T-violating term on the total cross sections is small, and the conclusions about the existence of an isotensor term are unaffected by its presence.

B. Fits to $\pi^\pm$ and π^0 Data

We now go on to discuss the effects of including the π^0 data in the data set. This of course involves a large increase in the number of data points from 673 to 1061, whereas the number of parameters in the model remains unchanged at seven.

The quality of the fits to the π^0 data which result are shown in Figs. 10 and 11. The excitation curve for the polarized-photon asymmetry data at 90° is shown in Fig. 10. These were the data primarily responsible for the shift in M_{1-}^3 from the normally accepted values in the multipole analyses discussed in Sec. V. This multipole is fixed at its conventional dispersion-theory value here, but the account of the data is not unreasonable considering the very small errors, except for the two lowest-energy points at 230 MeV and 250 MeV where the fit is much too low.

The fits to the differential cross sections are shown in Fig. 11. The data of Morand *et al.*[39] which are shown here were not included in the minimization. An excellent account is achieved except at the highest three energies, and even at the highest (400 MeV) the fit curve lies within the band given by the different experiments. The recoil proton polarization measurements are also well fitted.

In view of the difficulties of simultaneous fitting of the details of the π^+, π^0 reactions experienced in the multipole analyses described in Sec. V, we regard this as a very satisfactory performance of our model. Of course such difficulties can have nothing to do with the presence or absence of isotensor amplitudes A_l^2 since these occur in a fixed linear combination for the two reactions.

The results are presented in the form of χ^2-per-data-point information in Table II, where we have also included the previous fit to $\pi^\pm$ for comparison. As can be seen the fits to π^+ (with the inclusion of the π^0 data) are not so good as previously. We have also shown the results of a fit to the π^0, π^- data alone. We note in particular that in this case the lowest-energy π^0 asymmetry points are well fitted also, and clearly the π^0, π^+ data are, in a sense, competing – for example, the shape parameter x_1 is $-1°$ in the $\pi^\pm$ fit, 5.6° for the π^0, π^- fit, and 4.3° for the $\pi^\pm, \pi^0$ fit.

Do these details have any effect on the fit to the

TABLE I. χ^2 per data point for the different models and data subsets.

	π^+ d.c.s.	π^+ Asym.	π^+ Pol.	π^- d.c.s.	π^- Asym.	Capture d.c.s.	Total
No. of points	378	86	1	192	2	14	673
Conventional model	1.35	1.91	1.44	3.99	0.49	7.22	2.30
T violation ($I=1$); no $I=2$	1.34	1.96	1.90	3.93	0.61	7.28	2.28
$I=2$; no T violation	1.23	1.80	1.42	2.14	0.08	2.82	1.60
$I=2$; T violation ($I=1,2$)	1.23	1.80	1.50	2.18	0.01	0.66	1.56

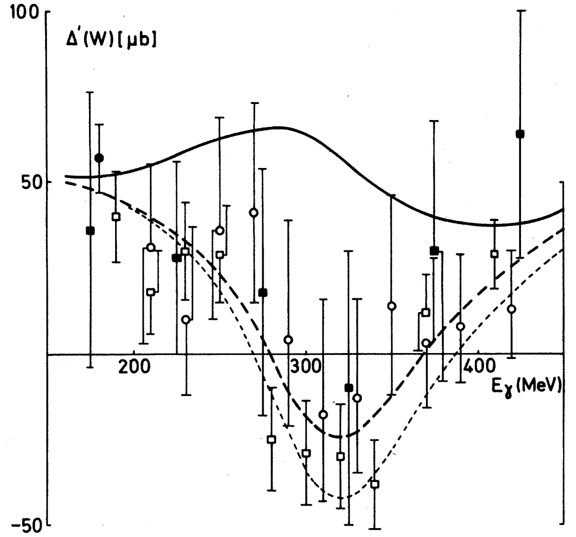

FIG. 9. The function $\Delta'(W)$, defined in Eq. (5), as a function of energy from the fit (dashed line) and the conventional model (solid line). The dotted line shows for comparison the function δ, defined in Eq. (49), as obtained in the fit. The data are from the ABBHHM Collaboration (Ref. 7) (open circles), the PRFN Collaboration (Ref. 8) (open squares), White *et al.* (Ref. 48) (closed squares), and calculated from the π^-/π^+ angular distribution at 180 MeV [see Fig. 8(a)] and the π^+ total cross section there (closed circle).

π^- data and its interpretation? As can also be seen in Table II the quality of the fit to the π^- data is unaffected by these considerations. Further, precisely the same features are found on attempting to fit without isotensor and/or T-violating terms as were found in the $\pi^\pm$ only case – none of the π^- data can be accounted for without an isotensor term, and the capture and photoproduction data can only be described simultaneously with nonzero T violation. The general features giving rise to this are as described for the $\pi^\pm$ fits. Finally, what of the parameters x, t, ϕ_n which are related most directly to these general features? We find the following:

For the $\pi^\pm$ fit,

$$x=-0.28, \quad t=-0.24, \quad \phi_n=-8.5°.$$

For the $\pi^0\pi^-$ fit,

$$x=-0.31, \quad t=-0.29, \quad \phi_n=-10.4°.$$

For the $\pi^\pm\pi^0$ fit,

$$x=-0.30, \quad t=-0.24, \quad \phi_n=-11.1°.$$

It is clear then that these detailed problems with the π^0 data experienced in all analyses so far, and possibly due to the existence of small systematic errors in the π^0 data, are not relevant to the questions we are investigating here.

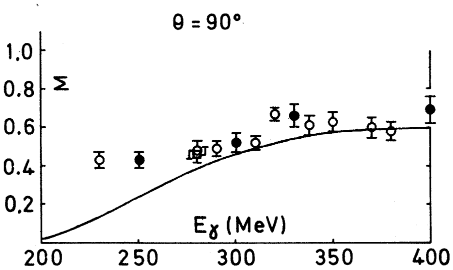

FIG. 10. Fit of the present model to the polarized-photon asymmetry in π^0 photoproduction. The data are from Drickey *et al.* (Ref. 40) (open squares), Barbiellini *et al.* (Ref. 41) (open circles), and Antufyev *et al.* (Ref. 42) (closed circles).

C. Variations on the Model, and the Data Set

The above fits which we have been discussing represent only a small subset of the large variety of hypotheses we have investigated. So far the data sets we have discussed are

(i) All $\pi^\pm, \pi^0$ data.
(ii) All $\pi^\pm$ data.
(iii) All π^-, π^0 data.

In addition we have also performed the following fits:

(iv) All $\pi^\pm$ data, plus the polarized-photon asymmetry data for π^0 production.

(v) All $\pi^\pm, \pi^0$ data, but with the normalization of the π^0 d.c.s. data left free (it changes by about 5% in the fit).

These various data sets have been analyzed with a variety of changes in the model, the object being to investigate also what is, or is not, sensitive to details of the model. The following cases have been investigated:

(a) The "normal model" which we have been discussing so far. This has been applied to all data sets (i)–(v).

(b) The effects of higher-resonance contributions. The effects of the P_{11} resonance on the $M_{1-}^{0,1}$ amplitudes are already allowed for in the normal model. As we have noted in Secs. III and V, Berends and Weaver[28] have calculated some additional corrections from the second and third resonance regions, which are largest in the E_{0+} and M_{1-}^3 multipoles. Fits with these corrections added have been performed to data sets (i) and (ii). This kind of correction does much to improve the fits to the π^0 d.c.s. of Fischer *et al.*[37] at the highest energies.

(c) Calculation of the crossed-Δ terms. In this case, instead of using the crossed-Δ contributions given by the usual model, the corrections caused

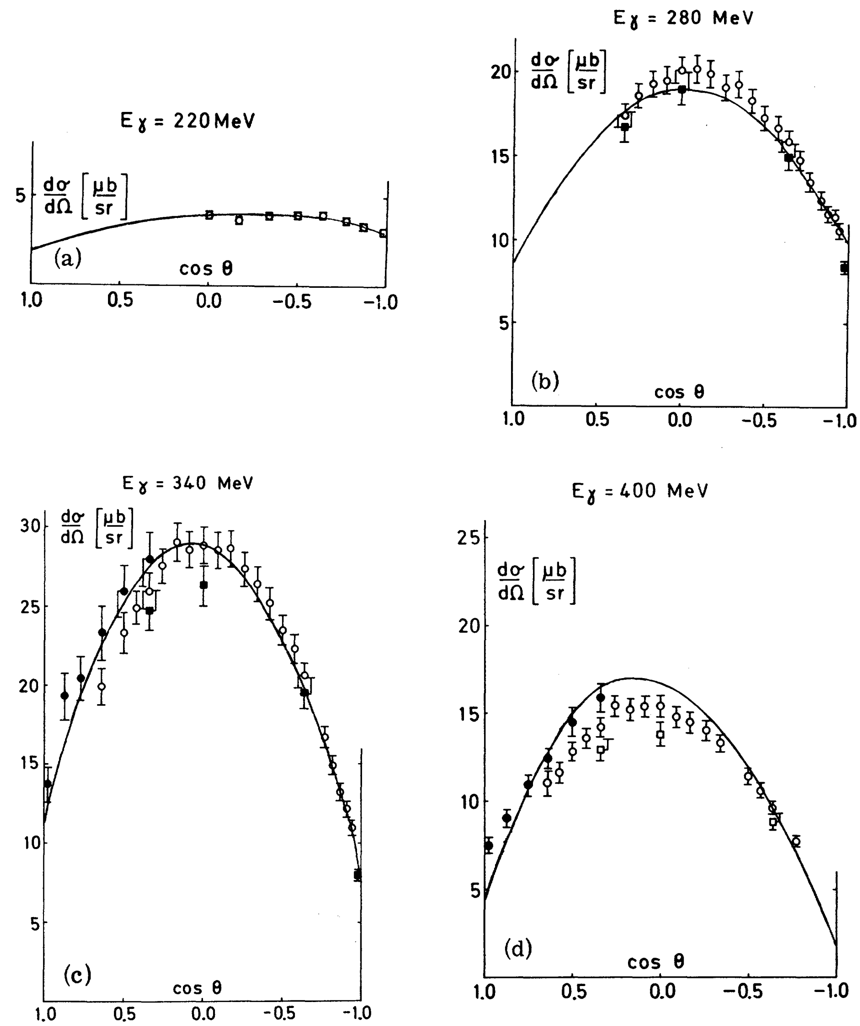

FIG. 11. (a)–(d) Fit of the present model in the π^0 photoproduction differential cross section. The data are from Fischer *et al.* (Ref. 37) (open circles), Hilger *et al.* (Ref. 38) (closed circles), and Morand *et al.* (Ref. 39) (closed squares).

by the introduction of isotensor and T-violating terms have been added using the static model. This cannot be expected to be accurate at the highest energies, but is sufficient to test sensitivity. Fits including these have been made to data sets (i) and (iii). This makes very little difference to the quality of the fit except at the highest energies where, as we have said, the approximations made cannot be trusted, and where it makes the fit somewhat worse.

(d) A fit to the complete data set (i) was carried out, but with M_{1-}^{3} multiplied by an arbitrary parameter to be fitted by the data. The value of M_{1-}^{3} changed from the theoretical value by about 25%, which is much the same size of effect as the second and third resonance corrections of Berends and Weaver[28] to this multipole.

(e) A similar fit to the complete data set (i) was carried out, but with the theoretical contributions to $M_{1-}^{0,1}$ multiplied by variable parameters. No appreciable improvement in the fit occurred.

Thus we see that in particular all data sets (i)–(v) were analyzed using the normal model (a), and all models (a)–(e) were applied to the complete data set (i). We note that in some cases it was necessary to weight the capture data to ensure that it was fitted and not ignored by the computer because of the comparatively vast bulk of other data. The important information which results from this are the ranges of values obtained for the crucial parameters x, t, ϕ_n. They are $-0.23 \geq x \geq -0.36$, $-0.23 \geq t \geq -0.31$, and $-7.9° \geq \phi_n \geq -11.4°$. The extreme constancy of ϕ_n is due not only to its insensitivity to details of the model and the proton data, but also because all the fits to the capture data at 350 MeV are near the upper end of the error bars, as shown in Fig. 4(a). If a fit is made in which it goes through the center of the error bars the value increases to $\phi_n = -18.3°$. In a fit in which it is allowed to lie higher than the error bars (the χ^2 con-

TABLE II. Fits to $\pi^{\pm}$ and π^0 data.

	π^+ d.c.s.	π^+ Asym.	π^+ Pol.	π^0 d.c.s.	π^0 Asym.	π^0 Pol.	π^- d.c.s.	π^- Asym.	Capture d.c.s.	Total
No. of points	378	86	1	361	16	11	192	2	14	1061
Conventional model	1.95	1.69	1.16	3.26	6.82	2.28	5.07	0.58	7.00	3.08
T violation ($I=1$); no $I=2$	1.96	1.77	0.60	2.60	8.88	2.36	5.30	0.34	8.35	2.95
$I=2$; no T violation	1.96	1.69	1.19	3.22	6.69	2.27	2.21	0.07	2.86	2.50
$I=2$; T violation ($I=1,2$)	2.07	1.80	0.53	2.40	9.19	2.26	2.19	0.00	0.58	2.27
$I=2$; T violation ($I=1,2$) $\pi^-\pi^0$ only				1.12	2.42	5.04	2.18	0.00	0.52	1.55
$I=2$; T violation ($I=1,2$) $\pi^-\pi^+$ only	1.23	1.80	1.50				2.18	0.01	0.66	1.56

tribution of all the capture data doubling) it decreases to $-5.8°$. The x, t values in both these cases still lie in the above ranges. However, what the above small range does show is how little ambiguity is introduced into the interpretation of a given $\pi^- p \rightarrow \gamma n$ d.c.s. by detailed questions concerning the proton data, and variations in the model used. It is important to recall that in addition to the variations discussed here, the further ambiguity of a general lowering of the π^- data due to an s-wave subtraction has been eliminated by the discussion of the π^-/π^+ ratio data at 180 MeV given earlier in this section.

VII. OTHER MODELS AND CALCULATIONS

One of the most interesting topics discussed in work other than that mentioned in the introduction is the question of the total cross sections for π^+ and π^- production, and in particular the difference between them – the dip test. While the relation between a dip in $\Delta'(W)$ and an isotensor resonance excitation has not been challenged, it is nonetheless important to ask how much the predictions of the present model can be changed, within a dispersion-relations framework. In particular, how much can the prediction in the absence of exotic terms be changed? In this latter case there are only two obvious ways of changing the predictions of $\Delta'(W)$ – the introduction of nonzero s-wave subtraction constants, and the consideration of higher-energy, and in particular higher-resonance, contributions to the dispersion integrals. The first of these is rather simple, and one would expect that while the curves could be moved up or down in magnitude by such terms, it could only be by a very slowly varying amount – the shape of the curves with energy would be essentially unchanged. This is explicitly borne out by the model of Schwela[57] where subtractions are effectively allowed in the s waves, and where this is precisely what happens. The second possible effect, that of higher resonances, has been dealt with by Devenish, Lyth, and Rankin.[58] Using their own extensive phenomenological analysis[59] of the higher resonance regions, these authors have explicitly calculated the contributions of the second, third, and fourth resonance "bands" to the region of the first resonance, including the contribution of the F_{15} resonance which Berends and Weaver[60] have speculated might change the result. The effects found are very small compared with the isotensor effects, and to the errors on the π^- total cross-section data. In fact, on comparing with the total cross-section data[7,8] they find an isotensor term of the same magnitude as was found earlier[5] from this data without taking into account higher-resonance effects (and as is found here). Thus quite apart from general statements concerning the presence or absence of a dip in $\Delta'(W)$ previously stressed, this work would also seem to suggest that the actual shapes of the curves predicted for $\Delta'(W)$, $\sigma_t(\pi^{\pm})$ are also rather independent of the particular model used.

A second contribution of these authors[58] has been to the discussion of the shape of the resonance. If the conventional CGLN coupling is used for ${}_pM^3_{1+}$, ${}_nM^3_{1+}$, then the conventional Breit-Wigner shape for the radiative width given by the CGLN form is a rather good solution to the dispersion relation. When the coupling strength is changed, a slight modification to the shape will be required so that the dispersion relation will remain satisfied. The above authors have proposed a simple parametric form which ensures that this will be so to a good approximation. The resultant changes are not very large, but clearly the new form is theoretically more satisfactory.

We now turn to a very different approach which has proved rather instructive – an attempt to fit the π^- data without isotensor terms, ignoring the restrictions of dispersion relations.[61] In the absence of isotensor terms the amplitudes for π^- production can be written

$$-A^{\pi^-} = A^{\pi^+} - 2\sqrt{2}\, A^{(0)} . \qquad (50)$$

The π^+ amplitudes used are taken from the multipole analysis of Noelle *et al.*[29] Thus one need only determine the isoscalar amplitudes $A^{(0)}$, and if the usual dispersion-relation values are used for these amplitudes (namely, the Born terms), the authors find that the predicted curve for $\Delta(W)$ is closely reproduced once more. They then go on to attempt to discard the dispersion-relation predictions for the $A^{(0)}$ and to fit. However, before going on to describe this, it is convenient to digress on the work of Berends and Weaver[60] who have attempted to use their multipole analysis[28] as the springboard for a similar investigation. However, these authors have used (in the absence of $I=2$ terms)

$$(3/\sqrt{2})A^{\pi^-} = 3A^0 - A^1 + A^3 \tag{51}$$

and used their multipole analysis for A^3 only, using dispersion predictions for A^0, A^1. Since they have assigned 10% errors to the predictions of both A^0 and A^1, and since the A^1 are much larger than the A^0, it is clear that a good deal of imprecision has been unnecessarily introduced by the use of (51) rather than (50). In addition, the A^3 amplitudes used as input are less reliable than the A^{π^+} amplitudes, since the latter obviously do not depend on fits to the π^0 data, and are thus not open to the criticisms of Sec. V. Further, they have examined a much smaller data set than Noelle and Pfeil,[61] and it is not surprising then that their conclusions are less specific.

We now return to the work of Noelle and Pfeil,[61] therefore, who as we have said use the multipole-analysis results for the A^{π^+} amplitudes and fit the isoscalar multipoles to the data. The latter amplitudes are represented by polynomials in the variable $(\omega - 1)$, and deviations from the Born terms are found for $M_{1-}^{(0)}$, and especially for $E_{0+}^{(0)}$ for which a cubic in the variable $(\omega - 1)$ results, i.e.,

$$\mathrm{Re}E_{0+}^{(0)} = (-2.04 \pm 0.13) + (3.64 \pm 0.37)(\omega - 1) - (1.20 \pm 0.18)(\omega - 1)^3. \tag{52}$$

The units are $10^{-3} m_\pi^{-1}$. As the authors note, they do not fit well the CERN π^- capture data[45] or the 180° π^- data,[46] and also the results are in bad agreement with the UCLA-LRL capture data, which were not included. Again as the authors note, they do not fit well the π^- data near the resonant peak – although the values of $\Delta'(W)$ are generally lowered, they do not obtain a real dip structure near resonance, despite the restrictions of dispersion relations being ignored.

It is this last point which seems to us particularly significant, rather than any possible defects in the fit to the data. As we have noted in Sec. II the fixed-t dispersion relations are well founded theoretically, and there is no problem with the partial-wave projections at this energy, so that an amplitude that is not compatible with these relations cannot reasonably be considered as a physical amplitude – unless of course one is willing to abandon much more fundamental properties of the S matrix than those in question here. So one may ask whether the above suggested amplitude is compatible with the dispersion relations? Now in all published evaluations of $E_{0+}^{(0)}$ this amplitude is dominated by the Born terms, which are almost constant over the energy region in question. A single subtraction, which is the most that can reasonably be reconciled with the known high-energy behavior of the photoproduction cross sections, can change the magnitude of the result, but clearly leaves the energy dependence unchanged, and in general none of the allowed modifications to the Born-term result – namely, subtractions, s-wave rescattering corrections, or contributions from higher resonances – can do more than change the result by a slowly varying amount, as can be seen in Fig. 12. In other words the above dispersion relation dictates a slowly varying $E_{0+}^{(0)}$ amplitude over this region. As can also be seen in Fig. 12, the above result is in flagrant disagreement with this.

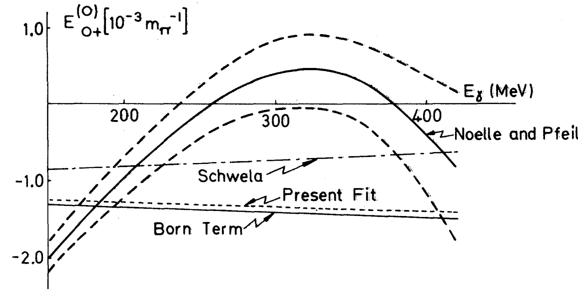

FIG. 12. The isoscalar electric dipole term $E_{0+}^{(0)}$ according to Noelle and Pfeil (Ref. 61) (solid line, the dashed lines indicating upper and lower limits), Schwela (Ref. 57) (dot-dash line), the Born term (lower solid line), and the present fit (dotted line).

In our opinion this result, that an attempt to fit without isotensor terms leads to a clear violation of rather general theoretical requirements, is not evidence for indeterminancy as suggested by the authors themselves,[61] but is further compelling evidence that an isotensor term must be introduced, provided only of course that the input data are correct.

Going back to the question of the fit, we noted above that the UCLA-LRL experiment was not included. In a subsequent paper, Pfeil and Schwela[62] have included these data, working at fixed energy. However, the authors have taken the results of the analysis of Noelle and Pfeil as input, including their $E_{0+}^{(0)}$ amplitude, and then added T-violating isotensor terms in order to improve the fit. In view of our earlier remarks concerning the $E_{0+}^{(0)}$ amplitude used, we do not think conclusions drawn in this way can be reliable. In any case, in general, if one wishes to consider the effects of exotic terms, the results of analyses made assuming their absence should not be used as input, unless of course arguments can be given to show that the results are really independent of this assumption. The exotic terms should be incorporated from the beginning.

Finally, for completeness, we turn to an early claim of Gittelmann and Schmidt[63] that the amount of isotensor resonance excitation could not exceed more than 2% or 3% of the isovector excitation. Two arguments were given. The first of these was that the conventional dispersion-theory prediction for the resonance excitation on protons, assuming $|\Delta I| \leq 1$, is close to experiment, so that only a very small isotensor term can be added. This conclusion obviously rests on the assumption that the solution is unique. However, it is the solution of an inhomogenous, linear, singular integral equation for which the boundary condition is uncertain. Mathematically the solution of such an equation is not unique. In particular Sanda and Shaw[5] have indicated how to construct solutions of these integral equations in which an isotensor term does occur without appreciably changing the resonance excitation on protons, and this has subsequently been done in other discussions also.[57,58] It should be mentioned here that Gittelmann and Schmidt themselves stress that this argument is model-dependent and should not be trusted. A second argument consisted in comparing $\pi^\pm$ cross-section data at $90°$ with the results of a calculation of Engels *et al.*[64] However, at fixed angle there are large interferences with the background amplitude in both $\pi^\pm$ production, and there are excellent reasons (the existence of inelasticity and the Roper resonance in the P_{11} wave for example) to believe that any calculation of the M_{1-} multipole is likely to be wrong. Thus this argument is also model-dependent, and the model used[64] is, like all published conventional models, in serious disagreement with the neutron data studied here.

VIII. CONCLUSIONS

In this paper we have been able to account for all of the π^- photoproduction and radiative-capture data in the energy region of the $\Delta(1232)$ resonance in terms of a simple dynamical model of the type initially suggested by Sanda and Shaw.[4,5,12,13] At present this model is the only candidate with this property which does not violate the rigorously proved fixed-t dispersion relations. The main feature of the model, which accounts for this success, is the introduction of contributions from exotic currents – namely, isotensor and T-violating terms – into the radiative couplings of the $\Delta(1232)$ resonance. This results in the introduction of three extra parameters – which can be taken to be ϕ_p, ϕ_n, and t, defined in Eqs. (37), (38), and (47) – and the model reduces to a conventional one in the limit $\phi_p = \phi_n = t = 0$.

Clearly, in order to complete a model-independent demonstration of the presence of exotic terms (always assuming of course that the data are reliable), it is necessary only to show that the effects of the exotic terms indicated in this model – in particular, the dip in $\Delta'(W)$ and the breakdown of reciprocity in the $\gamma n \rightleftharpoons \pi^- p$ reactions – cannot be produced in a conventional theory in any model. (It is a requirement of course that such models give a reasonable description of the proton data and are not incompatible with the fixed-t dispersion relations.) Hence the many variations on the initial model which were discussed in Sec. VI were all variations on the conventional terms, and not on the exotic terms themselves. We think that it is rather clear from both this and the other discussions cited in Secs. I and VII that neither of those effects – the dip or the breakdown of reciprocity – can be produced in any conventional theory, so that exotic terms of some type must be present provided only that the data are reliable, as we have said.

This still leaves the question of what can be concluded in a model-independent way about the nature of these exotic terms. We would summarize the results on this as follows. First, the breakdown of reciprocity can obviously only be due to T violation, so that this is unambiguously indicated by the data. We note however that this rests largely on the results of a single experiment[9] so far. Second, in the absence of T violation the dip effect would unambiguously indicate an $I = 2$ term. This conclusion survives the presence of T violation if the effects of this on the total cross sections are small compared to the size of the dip effect. This is so in the present fit, but again the results are based on a single angular distribution in the $\pi^- p \to \gamma n$ case. Third and last, measurement of the $\pi^0 n/\pi^0 p$ ratio on deuterium in this region constitutes a completely model-independent test for $I = 2$ terms of the presently suggested size, irrespective of the presence or absence of T violation.

Thus the measurement of the $\pi^0 n/\pi^0 p$ ratio on deuterium, the R_d^0 of Eq. (43) is an extremely crucial test of the present results. As noted in the Introduction [test (A)], this was the first of the tests for $I = 2$ terms to be suggested,[1] and is extremely sensitive to T-conserving $I = 2$ terms since the π^0 reactions are almost completely dominated by the resonant M_{1+} term, and this also makes the extraction of the ratio of magnitudes of the neutron and proton radiative widths from such data rather trivial. If isotensor terms are present in the magnitude estimated by the present fit, then for example the total cross-section ratio at 340 MeV should be 0.51, compared with the conventional model result 1.02. The predictions for the angular and energy dependence of the ratio are available on request – we only note here that it is not rapid.

The next most urgent experimental priority is to extend the data on the radiative-capture reaction. At present only one angular distribution[9] and one excitation curve[45] are available; it is of the greatest importance to extend these results to other angles and energies in this region.

Finally, it is also important to make more extensive measurements of the π^-/π^+ ratio in order to deduce π^- angular distributions, as discussed in (f) of Sec. IV. We note that information from this measurement is greatly improved if a tagged photon beam is used, since if only one pion is detected and a bremsstrahlung beam is used, the reaction is kinematically badly underdetermined, and Land[64] has found that in the region of 290 MeV, measurements made in this way are systematically higher than those using a monochromatic photon beam. These latter, lower, results, which agree with the results used in our fit, have also been confirmed in a somewhat different way by the experiment of Garelick and Cooperstein[49] where both π^- and p were detected in coincidence, again improving the kinematic determination of the events.

This concludes our discussion of the photoproduction process itself – we feel that these above experiments will completely settle the question of exotic terms in a model-independent way, and further that our model affords a good basis for analyzing such data in detail to determine their quantitative nature. The present data, as we have noted, clearly indicate the presence of both $I=2$ and T-violating terms, and a rather extensive amount of the data will have to be wrong for both effects to disappear.

This leaves only one thing to be dealt with before ending, and that is the implications of the results of our fit for other closely related processes in which the $\Delta N\gamma$ vertex plays a prominent role. As we have seen, the results of two of the parameters related to this vertex, namely t and ϕ_n, have turned out to be reasonably stable in our fits, and for definiteness we will take fixed values for these within the allowed range, namely, $t=-0.28$ and $\phi_n=-11°$. Unfortunately the T-violating phase on protons ϕ_p, and hence the isovector and isotensor T-violating phases ϕ_1 and ϕ_2, are essentially undetermined. We will consider three alternative models for these.

Model (i)

Pure isotensor T violation. This alternative has been advocated by Sanda and Shaw[12,13] in order to correlate information on a large number of processes. With the above values $t=-0.28$, $\phi_n=-11°$, it leads to $\phi_p=7.6°$, $\phi_1=0°$, $\phi_2=51°$, $x=-0.31$.

Model (ii)

Pure isovector T violation. This case leads to $\phi_p=-7.1°$, $\phi_1=-8.6°$, $\phi_2=0°$, $x=-0.35$.

Model (iii)

We finally consider a model in which instead of $\phi_1=0$ or $\phi_2=0$, we set $\phi_p=0°$. This gives $\phi_1=-4.4°$, $\phi_2=20.8°$, $x=-0.32$.

What do other processes tell us about these phases? We consider these in turn.

(a) $n+p\rightleftarrows\gamma+d$. In the model of Barshay[65] T violation can occur in this process via the $\Delta N\gamma$ vertex. Only the isovector phase is involved, and the latest experiment[66] leads to a value of $\phi_1 = 4\pm 10°$.

(b) Neutron dipole moment. This is related to the photoproduction multipoles by the model of Barton and White[67] and Broadhurst.[68] Only the phase enters, and the values resulting from models *(i)*, *(ii)*, and *(iii)* are $\beta=(5,-5,0)\times 10^{-24}\ e\,\text{cm}$, respectively. The current upper limit is $5\times 10^{-23}\ e\,\text{cm}$.[69]

(c) Electroproduction on a polarized target $e^-p\to e^-\Gamma$. In a recent experiment[70] a possible asymmetry of $(4.5\pm1.4)\%$ has been observed in the first resonance region at a k^2 value of 0.6 $(\text{GeV}/c)^2$. In a simple resonance dominance picture, this measures the phase difference between the transverse and scalar resonance excitations on protons which is zero if T is conserved. Using the recently measured ratio value $|S_{1+}/M_{1+}|\sim 0.1$,[71] the above result corresponds to a phase difference of about $20°\pm 6°$.

We thus see that our results are perfectly compatible with the present information on these processes, which do little in themselves to restrict the isospin nature of the T violation, at least at the present level of precision. In general, the best way to test the conjecture $K_\mu^{0,1}=0$, so that the T violation is pure $I=2$,[12] is to test the selection rules it leads to over a variety of processes, in the way attempted in Refs. 12 and 13. In particular, the process $\eta\to\pi^0e^+e^-$ is forbidden, and T violation in the second and third resonance regions should be much smaller than in the first resonance region. It is obviously important to investigate the question of T violation in the region of these resonances further. In the first resonance region itself, apart from the reactions $(a)-(c)$ above, the isospin nature of the T violation could only clearly be sorted out if the reaction

$$\pi^+ + d \to p + p + \gamma$$

could be used to make a comparison between

$$\gamma + p \rightleftharpoons \pi^+ + n$$

in analogy with that for the π^- reaction.

Note added. Since submission of this work, a new version of the π^- photoproduction cross section has been published by the PRFN Collaboration[72] and a preliminary version from a new analysis by the ABBHHM Collaboration was presented at the Cornell conference.[73] There are no substantial changes in the PRFN total cross sections in the first resonance region, and consequently our conclusions remain unaltered as far as these results are concerned. However, the preliminary cross sections from the ABBHHM Collaboration are significantly changed, showing an increase over their previous values by about 25–30% and being in considerable disagreement with those of the PRFN Collaboration. The ABBHHM data alone would apparently require no isotensor current and are in reasonable accord with the "no-exotics" curve of Fig. 9. However, when taken in conjunction with the data of Berardo *et al.*,[9] they imply a greatly increased T violation. Under these circumstances, the "dip test" breaks down as indicated in Sec. VIII, and no conclusion on the presence of $I=2$ is possible without measurements on the $\pi^0 n$ channel.

It is clear that the emergence of these alternatives is due not to ambiguities in the methods of analysis we have proposed here, but is due to inconsistencies in the data. Thus it is of even more importance than before to improve the data in the ways indicated: In particular, in the case of photoproduction on neutrons, the uncertainty due to the use of deuterium, which may be the cause of present discrepancies, can be greatly reduced by the methods suggested.

[1] G. Shaw, Nucl. Phys. B3, 338 (1967).

[2] V. G. Grishin *et al.*, Yadern. Fiz. 4, 126 (1966) [Soviet J. Nucl. Phys. 4, 90 (1967)].

[3] N. Dombey and P. K. Kabir, Phys. Rev. Letters 17, 730 (1966).

[4] A. I. Sanda and G. Shaw, Phys. Rev. Letters 24, 1310 (1970).

[5] A. I. Sanda and G. Shaw, Phys. Rev. D 3, 243 (1971).

[6] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München (ABBHHM) Collaboration, Nucl. Phys. B8, 535 (1968).

[7] ABBHHM Collaboration, reported in H. Butenschon, DESY Report No. DESY RI-70/1, 1970 (unpublished).

[8] Pavia-Frascati-Rome-Napoli (PRFN) Collaboration, Lett. Nuovo Cimento 3, 697 (1970).

[9] P. A. Berardo *et al.*, Phys. Rev. Letters 26, 201 (1971).

[10] N. Christ and T. D. Lee, Phys. Rev. 148, 1520 (1966).

[11] P. A. Berardo *et al.*, Phys. Rev. Letters 26, 205 (1971).

[12] A. I. Sanda and G. Shaw, Phys. Rev. Letters 26, 1057 (1971).

[13] A. I. Sanda and G. Shaw, Columbia University Report No. NYO-1932(2)-197, 1971 (unpublished).

[14] A. Donnachie and G. Shaw, Phys. Letters 35B, 419 (1971).

[15] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965); S. Barshay, Phys. Letters 17, 78 (1965).

[16] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1395 (1957).

[17] J. S. Ball, Phys. Rev. 124, 2014 (1961).

[18] K. M. Watson, Phys. Rev. 95, 228 (1954).

[19] It would be more consistent with the rest of the notation, based on the Clebsch-Gordan coefficient conventions of the Particle Data Group, Rev. Mod. Phys. 41, 1 (1971), to use $(-\sqrt{3}\,A^0)$, but long-established practice forbids.

[20] K. M. Watson, Phys. Rev. 85, 852 (1952).

[21] F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. B4, 54 (1968).

[22] The following numbers are taken from G. Shaw, Nuovo Cimento 44, 1276 (1966).

[23] R. Oehme and J. G. Taylor, Phys. Rev. 113, 371 (1959).

[24] We would like to thank D. Lyth for drawing our attention to this point.

[25] See, e.g., A. Donnachie and G. Shaw, Nucl. Phys. 87, 556 (1967).

[26] A comparative plot is given in G. Shaw, Phys. Rev. 158, 1479 (1967).

[27] G. W. Gaffney, Phys. Rev. 161, 1599 (1967).

[28] F. Berends and D. L. Weaver, Nucl. Phys. B30, 575 (1971).

[29] P. Noelle, W. Pfeil, and D. Schwela, Nucl. Phys. B26, 461 (1971).

[30] D. V. Bugg *et al.*, Nucl. Phys. B26, 588 (1970).

[31] G. Fischer *et al.*, Nucl. Phys. B16, 119 (1970).

[32] C. Betourni *et al.*, Phys. Rev. 172, 1343 (1968).

[33] R. E. Taylor *et al.*, Phys. Rev. 117, 835 (1960).

[34] R. C. Smith *et al.*, Phys. Rev. 130, 2429 (1963).

[35] M. Grilli *et al.*, Nuovo Cimento 54A, 877 (1968).

[36] K. H. Althoff *et al.*, Phys. Letters 26B, 640 (1968).

[37] G. Fischer *et al.*, Nucl. Phys. B16, 93 (1970).

[38] E. Hilger *et al.*, Bonn report, 1970 (unpublished).

[39] R. Morand *et al.*, Phys. Rev. 180, 1299 (1969).

[40] D. J. Drickey *et al.*, Phys. Rev. 136, B543 (1964).

[41] G. Barbiellini *et al.*, Phys. Rev. 184, 1402 (1969).

[42] Yu. P. Antufyev *et al.*, in *Proceedings of the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R., 1970* (Atomizdat, Moscow, 1971).

[43] K. H. Althoff *et al.*, Phys. Letters 26B, 677 (1968).

[44] J. T. Beale, S. D. Ecklund, and R. L. Walker, Cal-

tech Report No. CALT-68-108 (unpublished).
[45]D. Schinzel (private communications). We note that this supercedes the earlier published result of the same group, J. Favier *et al.*, Phys. Letters 1, 60 (1970).
[46]T. Fujii *et al.*, Phys. Rev. Letters 26, 1672 (1971).
[47]T. Nishikawa *et al.*, Phys. Rev. Letters 21, 1288 (1968).
[48]D. H. White *et al.*, Phys. Rev. 120, 614 (1960).
[49]D. Garelick and G. Cooperstein, Phys. Rev. 136, B201 (1964).
[50]G. Neugebauer, W. Wales, and R. L. Walker, Phys. Rev. 119, 1726 (1960).
[51]A. Baldin, Nuovo Cimento 8, 569 (1968).
[52]M. Bazin and J. Pine, Phys. Rev. 132, 2735 (1963).
[53]W. R. Hogg, Proc. Phys. Soc. (London) 80, 729 (1962).
[54]J. P. Burq and R. L. Walker, Phys. Rev. 132, 447 (1963).
[55]M. Sands, J. G. Teasdale, and R. L. Walker, Phys. Rev. 95, 592 (1954).
[56]R. L. Walker, Phys. Rev. 182, 1729 (1969).
[57]D. Schwela, Nucl. Phys. B26, 525 (1971).
[58]R. C. E. Devenish, D. H. Lyth, and A. Rankin, University of Lancaster report, 1971 (unpublished).
[59]R. C. E. Devenish, D. H. Lyth, and A. Rankin (unpublished).
[60]F. A. Berends and D. L. Weaver, Phys. Rev. D 4, 1997 (1971).
[61]P. Noelle and W. Pfeil, Bonn Report No. Pl 2-87, 1970 (unpublished).
[62]W. Pfeil and D. Schwela, Nucl. Phys. B31, 1 (1971).
[63]B. Gittelman and W. Schmidt, Phys. Rev. 175, 1998 (1968).
[64]R. H. Land, Phys. Rev. 113, 1141 (1959).
[65]S. Barshay, Phys. Rev. Letters 17, 49 (1966).
[66]B. L. Schrock *et al.*, Phys. Rev. Letters 26, 1659 (1971).
[67]G. Barton and E. D. White, Phys. Rev. 184, 1660 (1969).
[68]D. J. Broadhurst, Nucl. Phys. B20, 603 (1970).
[69]J. K. Baird *et al.*, Phys. Rev. 179, 1285 (1969).
[70]S. Rock *et al.*, Phys. Rev. Letters 24, 748 (1970).
[71]W. Albrecht *et al.*, Nucl. Phys. B25, 1 (1971); R. D. Hellings *et al.*, Daresbury Report No. DNPL/P65, 1971 (unpublished).
[72]Pavia-Frascati-Roma-Napoli (PRFN) Collaboration, Frascati Report No. LNF-71/44 (unpublished).
[73]A. Donnachie, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Cornell, 1971 (unpublished).