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Comparison of On- and Off-Mass-Shell $\pi^- p$ Elastic and Inelastic Scattering*

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One-pion-exchange (OPE) dominance is examined in the reactions $\pi^- p \to \rho^0\pi^- p$ and $\pi^- p \to \rho^0\pi^-\pi^0 p$ at 6 and 8 GeV/c. On-mass-shell and off-mass-shell scattering angular distributions for $\pi^- p \to \pi^- p$ and $\pi^- p \to \pi^-\pi^0 p$ are compared. The data substantiate the use of the OPE model in reactions such as $\pi^- p \to \rho^0\pi^- p$ and demonstrate that the model can be extended to reactions such as $\pi^- p \to \rho^0\pi^-\pi^0 p$.

Many reactions involving the production of quasi-two-body final states have long been assumed to be dominated by the exchange of a single pion (OPE model). More recently, the model has been extended[1,2] to reactions where resonance production occurs at only one vertex, with "virtual" elastic or charge-exchange scattering between the off-mass-shell ($t<0$) exchanged pion and the incident particle at the other vertex. For instance, the reactions

$$\pi^+ p \to \pi^+\pi^-\Delta^{++} \quad \text{and} \quad K^+ p \to K^+\pi^-\Delta^{++}$$

are assumed to be dominated by the diagrams of Figs. 1(a) and 1(b). With this assumption one is able to study virtual $\pi\pi$ and $K\pi$ scattering, and, through some extrapolation method, infer the behavior of on-shell ($t=m_\pi^2$) $\pi\pi$ and $K\pi$ scattering. As a check on the validity of these results, the reaction

$$K^+ p \to (K^+\pi^-) + (\pi^+ p)$$

has been treated in the same way assuming the diagram in Fig. 1(c). The $\pi^+ p$ scattering angular distributions thus measured are found to be in good agreement with the results from actual $\pi^+ p$ scattering experiments.[3]

In this note we consider the question of OPE dominance in the reactions

$$\pi^- p \to \rho^0\pi^- p \qquad (1)$$

and

$$\pi^- p \to \rho^0\pi^-\pi^0 p\,, \qquad (2)$$

for which the diagrams in Figs. 1(d) and 1(e), respectively, apply. We point out that Fig. 1(e) represents a further extension of the OPE model to inelastic scattering at the nonresonant vertex. This extension has previously been made[4] to study the reactions

$$pp \to \Delta^{++} n\pi^+\pi^- \quad \text{and} \quad pp \to \Delta^{++} p\pi^-\pi^0\,,$$

where the upper vertex represents $p \to \Delta^{++}\pi$ dissociation rather than $\pi^- \to \rho^0\pi^-$ dissociation as in the present case.

The data in this report come from a study of the reactions

$$\pi^- p \to \pi^-\pi^+(\pi^- p) \qquad (3)$$

and

$$\pi^- p \to \pi^-\pi^+(\pi^-\pi^0 p) \qquad (4)$$

at 6-GeV/c incident momentum in the Argonne National Laboratory 30-in. bubble chamber and at 8 GeV/c in the Brookhaven National Laboratory 80-in. bubble chamber. These data have been discussed in previous reports.[5,6] The cross sections and μb equivalents are given in Table I.

Figures 2(a) and 2(b) show the $\pi^+\pi^-$ mass distributions from these reactions after requiring the square of the momentum transfer $t' = |t - t_{\min}|$ between the incident π^- and the $\pi^+\pi^-$ system to be less than 0.3 (GeV/c)^2 and after removing ω^0 events [$0.74 < M(\pi^+\pi^-\pi^0) < 0.84$ GeV] from reaction (4).[7] If both $\pi^+\pi^-$ combinations have $t' < 0.3$, we choose[8] the combination with the smaller value of t'. A very strong ρ^0 signal is seen in both cases. The $\pi^- p$ spectrum for events in reaction (3) satisfying the ρ [$0.65 < M(\pi^+\pi^-) < 0.85$ GeV] and t' cuts is exhibited in Fig. 2(c). The $\pi^-\pi^0 p$ spectrum in reaction (4) with the same cuts appears in Fig. 2(d).

Figure 3 shows the distribution in the final-state $\pi^- p$ rest frame of reaction (1) of the scattering angle between incident and final protons for four different regions of $\pi^- p$ mass. These regions were chosen such that there was no substantial variation in the on-mass-shell distribution across

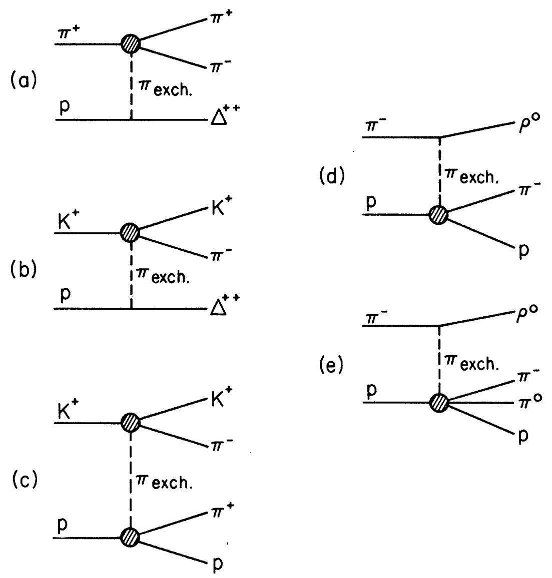

FIG. 1. (a)–(e) Exchange diagrams for investigating one-pion exchange.

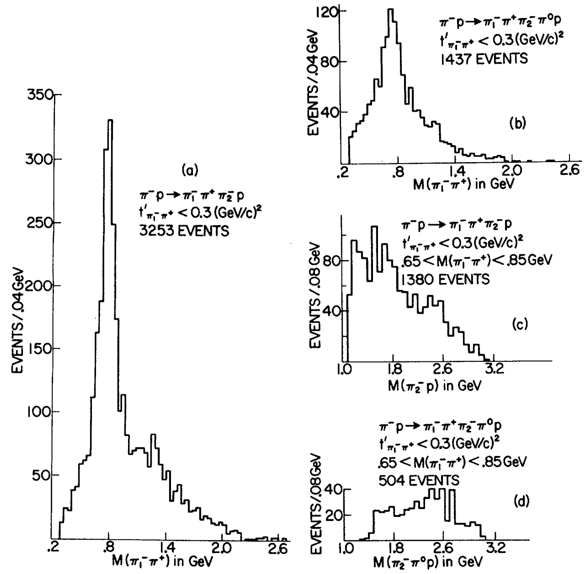

FIG. 2. (a)–(d) Mass spectra of $\pi_1^-\pi^+$, π_2^-p, and $\pi_2^-\pi^0p$ for reactions (3) and (4) after selections discussed in the text.

a given region. The solid curves in each region represent the behavior of $d\sigma/d\cos\theta$ observed in π^-p scattering at a mass near the center of the region (normalized to the data).[9] The good agreement is evidence in favor of the OPE-dominance assumption. Similar results[10] have been used to support the assumptions of the "Deck effect" models explaining the broad, low mass enhancement in the A_1 region of the $\rho\pi$ mass spectrum.

Figure 4 shows the ratio[11] of the cross sections for reactions (1) and (2) as a function of the mass of the system (π^-p or $\pi^-\pi^0p$) recoiling against the ρ^0. The shaded band in Fig. 4 represents the allowed range of the ratio determined from π^-p elastic and inelastic scattering data.[12] The agreement of the ratios for on-mass-shell and off-mass-shell interactions suggests that OPE is dominant for reaction (2) also, and implies that the off-mass-shell corrections for reactions (1) and (2) are the same.

Figure 5 shows the scattering angle between the incident and final protons in the final-state $\pi^-\pi^0p$

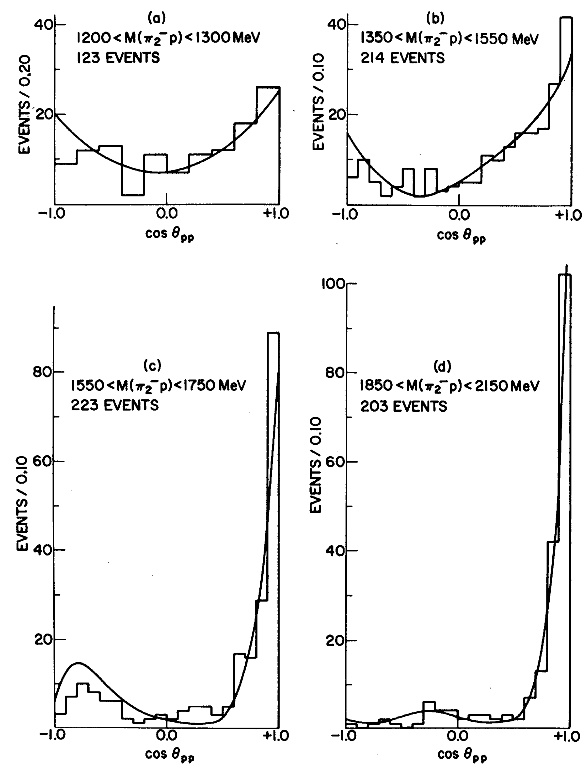

FIG. 3. Distribution of the scattering angle between incident and final protons in the π_2^-p rest frame for reaction (1). (a) 1200 $<M(\pi_2^-p)<$ 1300 MeV. (b) 1350 $<M(\pi_2^-p)<$ 1550 MeV. (c) 1550 $<M(\pi_2^-p)<$ 1750 MeV. (d) 1850 $<M(\pi_2^-p)<$ 2150 MeV. The solid curves represent the distribution as measured in on-mass-shell π^-p elastic scattering data.

TABLE I. Summary of cross sections at 6 and 8 GeV/c.

Momentum (GeV/c)	Final state	Cross section (mb)	
6	$\pi^-\pi^+\pi^-p$	1.39 ± 0.15	0.61 μb/event
8	•••	1.27 ± 0.07	0.54 μb/event
6	$\pi^-\pi^+\pi^-\pi^0p$	1.52 ± 0.13	0.62 μb/event
8	•••	1.39 ± 0.09	0.71 μb/event

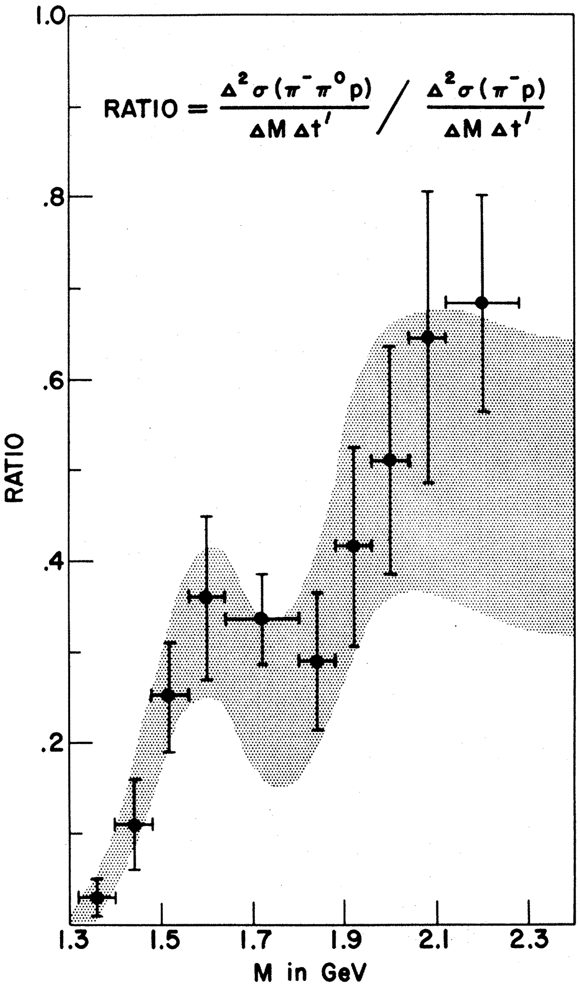

FIG. 4. Ratio of the cross sections for $(\pi^- p \to \rho^0\pi^-\pi^0 p)/(\pi^- p \to \rho^0 \pi^- p)$ as a function of the mass recoiling against the ρ^0 for the selections discussed in the text. The shaded band represents a summary of the ratios of the known experimental cross sections $(\pi^- p \to \pi^-\pi^0 p)/(\pi^- p \to \pi^- p)$.

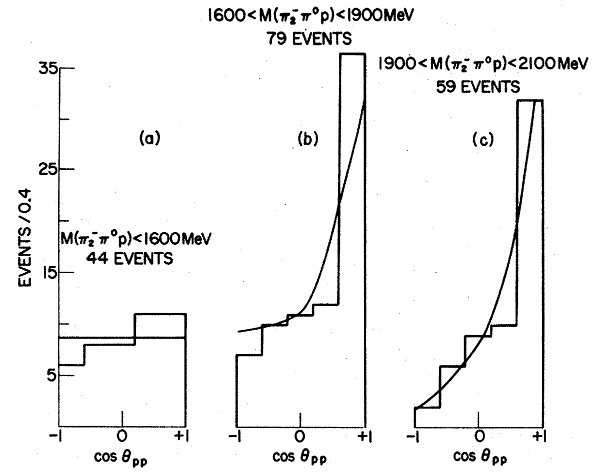

FIG. 5. Distributions of the scattering angle between the incident and final protons in the $\pi_2^-\pi^0 p$ rest frame of reaction (2). (a) $M(\pi_2^-\pi^0 p) < 1600$ MeV. (b) $1600 < M(\pi_2^-\pi^0 p) < 1900$ MeV. (c) $1900 < M(\pi_2^-\pi^0 p) < 2100$ MeV. The solid curves represent the production angle of the proton measured in the reaction $\pi^- p \to \pi^-\pi^0 p$. (a) $M(\pi^-\pi^0 p) = 1520$ MeV. (b) $M(\pi^-\pi^0 p) = 1720$ MeV. (c) $M(\pi^-\pi^0 p) = 2000$ MeV.

rest frame of reaction (2) for three regions of $\pi^-\pi^0 p$ mass. These represent the production angular distributions of protons in the off-mass-shell inelastic scattering process. The solid curves represent the behavior of $d\sigma/d\cos\theta$ observed in the reaction on the mass shell at energies near the centers of the regions chosen,[13] and agree well with the off-mass-shell angular distributions. This agreement is additional evidence for OPE dominance in reaction (2).

All distributions have been examined separately for the 6-GeV/c and 8-GeV/c data and are consistent with each other as would be expected from the OPE hypothesis. The agreement we have observed between on-shell and off-shell scattering represents further substantiation of the OPE model in reactions such as (1) and suggests, as has been pointed out previously,[4] that the model can be extended to a wide range of "inelastic" reactions such as (2).

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[7]The t' distributions for the $\pi^-\pi^+$ in the ρ^0 region are approximately exponential in shape. When the events of reactions (3) and (4) are plotted, we observe a break in the t' distribution at $t' = 0.3$ (GeV/c)^2. We have chosen $t' < 0.3$ (GeV/c)^2 to maximize the OPE contribution.

[8]This selection results in less than a 10% loss of ρ^0's in either sample.

[9]For a compilation of πp elastic scattering data, see D. M. Alvaredo and E. Urvater, University of Colorado Internal Report No. UA-3, 1967 (unpublished).

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[11]In calculating this ratio, the differences in yields between the 6-GeV/c and 8-GeV/c data as well as between reactions (3) and (4) were accounted for (see Table I).

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Formula for the Baryon Mass Spectra*

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It is observed that the central and isospin-multiplet masses in the baryon octets and decuplets are given by the formula $M = M_P(1+L+\frac{2}{5}S)^{1/2}f$, where $M_+ = 1014$ MeV for positive-parity states, $M_- = 1135$ MeV for negative-parity states, L and S denote the total orbital and total spin angular momentum of the spin-$\frac{1}{2}$ baryon constituents, and f is the hypercharge-isospin-splitting function. A simple modification of the semitheoretical formula yields the mass values observed for the unitary-singlet Λ's. The only experimentally established baryonic isospin multiplet with a mass not given by the formula (to within present experimental accuracy and the likely magnitude of electromagnetic shifts and splittings) is the $N(1470)$ Roper resonance, believed to be a radial excitation of the nucleon with a mass that depends on a radial quantum number.

It has been noted that the fractional hypercharge-isospin splittings in the $L=0$ baryon octet and decuplet with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ are equal,[1] and a semitheoretical formula for the masses of the isospin-multiplet members of these $L=0$ states has been reported.[2] The following simple extension of the $L=0$ mass formula,

$$M = M_P(1+L+\tfrac{2}{5}S)^{1/2}f, \quad (1)$$

accounts for the central and isospin-multiplet masses of all isospin-multiplet members in the baryon octets and decuplets. In the semitheoretical formula (1), the prefactor constant has the empirical value $M_+ = 1014$ MeV for positive-parity

TABLE I. Numerical coefficients a, b in the hypercharge-isospin-splitting function (2) for the main series.

$J^P = (L+S)^+$	a	b
$\frac{1}{2}^+, \frac{3}{2}^+$	2	$\frac{4}{3}$
$\frac{5}{2}^+, \frac{7}{2}^+$	1	1
$\frac{9}{2}^+, \frac{11}{2}^+$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{13}{2}^+, \frac{15}{2}^+$	$\frac{1}{3}$	$\frac{1}{2}$
$\frac{17}{2}^+, \frac{19}{2}^+$	$\frac{2}{9}$	$\frac{1}{2}$