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Absolute Measurement of the Vertical Cosmic-Ray Muon Intensity near 1 GeV/c at 12°N

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The absolute vertical cosmic-ray muon intensity at sea level at 12° N has been measured with a range spectrometer similar to that used by Allkofer *et al.* and by Jokisch. The integral intensity amounts to 6.86×10^{-3} cm^{-2} sr^{-1} sec^{-1} at 0.954 GeV/c which is 10.3% lower than that of Allkofer *et al.* and 10.6% higher than that of Rossi, generally used as the normalization point for muon spectra. Our results are in favor of experimental findings of Allkofer *et al.* The decrease in intensity may be explained in terms of the geomagnetic latitude effect.

I. INTRODUCTION

In a recent paper Allkofer *et al.*[1] and also Jokisch[2] reported an absolute measurement of muon intensity at the momentum of 1 GeV/c with a range spectrometer. The result obtained from this experiment is about 25% higher than the Rossi[3] value at the same latitude. Supporting evidence of higher vertical muon intensities near 1 GeV/c comes from the recent work of Ayre *et al.*,[4] Bateman *et al.*,[5] Crookes and Rastin,[6] and the preliminary report of the present authors.[7] Ayre *et al.* with their large spectrograph MARS at Durham obtained integral intensities in the range 3–6 GeV/c significantly higher (~7%) than those previously reported by Aurela and Wolfendale.[8] The Aurela and Wolfendale intensities are again 3–4% higher than those reported by Hayman and Wolfendale[9] in the same momentum region. Crookes and Rastin obtained a vertical integral muon intensity 9.13×10^{-3} cm^{-2} sr^{-1} sec^{-1} at 184.7 g cm^{-2} of lead (momentum 0.35 GeV/c). Bateman *et al.* with a magnetic spectrograph obtained absolute muon intensities in the range 3–50 GeV/c which are 12% higher than those of Hayman and Wolfendale.[9] The latter workers[9] normalized their data to the Rossi intensity, viz., 2.45×10^{-3} cm^{-2} sr^{-1} sec^{-1} (GeV/c)$^{-1}$ at 1 GeV/c.

A review of the above works raises doubts as to the exact value of the normalization point itself though the Rossi intensity has been used for normalizing the muon spectra by many authors.[8–15] The muon intensities near and above 1 GeV/c are somewhat higher than the previously accepted values in this momentum interval.

Allkofer *et al.*[1] concluded that in case the differential and integral muon spectra at sea level are normalized they should be enhanced by a factor 1.26. The discrepancies as explained by Allkofer *et al.*[1] are that the integral Rossi spectrum is normalized at 0.3 GeV/c to the Greisen[16] intensity 8.3×10^{-3} cm^{-2} sr^{-1} sec^{-1} in which the zigzag nature of the path of the particle inside the absorber due to multiple Coulomb scattering has not been considered. This is essential when a range spectrum is converted to a momentum spectrum. The conclusion is also in agreement with that of Kraushaar[17] who following Koenig[18] concluded that the vertical thickness of the absorber due to multiple Coulomb scattering inside the absorber should be increased by 11% in the range 50–180 g cm^{-2} of lead. This causes a shift of momentum from 0.3 to 0.33 GeV/c and hence the Rossi intensity at a particular momentum should be increased. This conclusion is true, but the change in the penetrating muon flux for this effect is only 0.7% at the momentum 0.3 GeV/c. The work of York[19] must also be mentioned in this connection. York investigated the differential range spectrum of cosmic muons in the region from 18–76 g cm^{-2} of air-equivalent absorber using a counter-controlled cloud chamber. The results obtained in this region

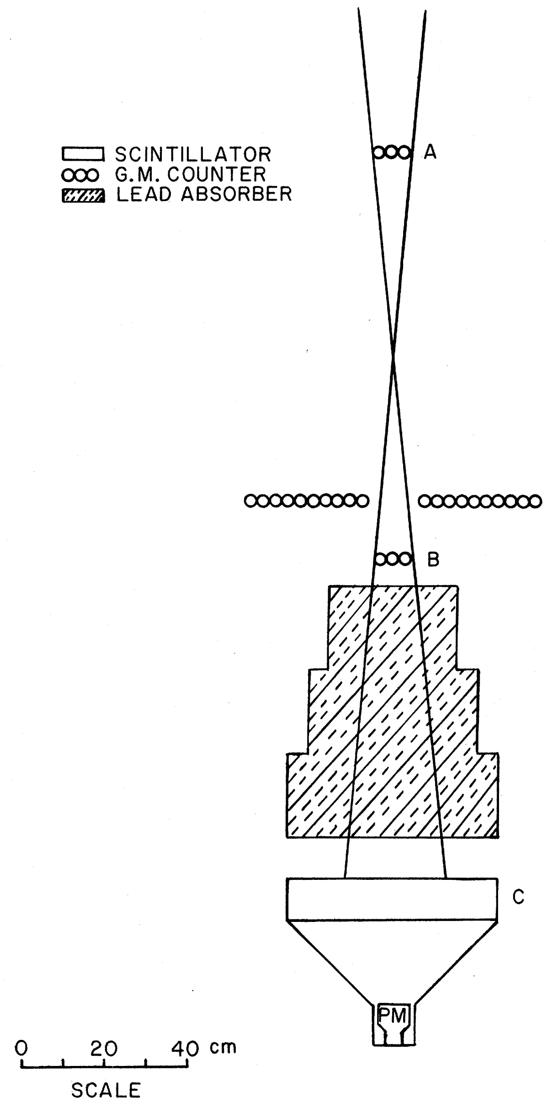

FIG. 1. Front view of the apparatus.

proved to be about 20% higher than predictions based upon similar contemporary measurements[16,17,20] at greater absorber thicknesses. According to him the discrepancy could be attributed to the fact that the Greisen value,[16] as quoted by Rossi,[3] had not been properly corrected for the loss of particles out of the geometry of the apparatus due to multiple Coulomb scattering in the absorber. This point, mentioned earlier by York,[19] has been recently supported by Crookes and Rastin[6] as a major cause for the increase in muon intensity.

We report here the use of an arrangement similar to that used by Allkofer *et al.*[1] for the absolute determination of integral muon intensity near 1 GeV/c at 12° N at sea level (Calcutta) which provides a new datum at the place of observation.

Both the above effects of multiple scattering have been properly estimated.

II. THE EXPERIMENT

Figure 1 shows the geometrical arrangement of the apparatus. Two trays of Geiger-Müller (G.M.) counters A and B, each with a sensitive area 15 $\times 9.2$ cm^2, form a telescope with a geometric acceptance of 2.02 cm^2 sr. The sensitive dimension of each of the beam-defining counters was 15×3.1 cm^2 determined by the method of Greisen and Neresen.[21] It was found that the sensitive length of each counter was, on an average, 7% less than the actual length of the cathode. The efficiency of the counters over the sensitive region determined precisely following Fenyves[22] was found to be 99.8%. Below the counter tray B there is a lead absorber and this together with all the other materials in the apparatus including the thin roof above it provided a total thickness of 663.4 g cm^{-2} of lead. This is the range of a muon with a mean momentum $p_1 = 0.954$ GeV/$c \pm 0.5\%$ considering the zigzag path discussed later. Below this lead absorber is placed a plastic scintillation counter C (50×50 cm^2). The threefold coincidence rate R_{ABC} of ABC coincidence gives the integral rate at the momentum 0.954 GeV/c. As the measured rates include events which are induced by extensive air showers, a ring of four trays of G.M. counters S (each tray of area 30×30 cm^2) has been arranged around the telescope. By simultaneous registration of R_{ABC} and R_{ABCS} counts the actual integral rate can be determined.

The exact muon rates are given by

$$R_{ABC\bar{S}} = R_{ABC} - K_1(R_{ABCS} - K_2 R_{ABC}). \qquad (1)$$

Here we take $K_1 = 1.3 \pm 0.13$, the value used by Allkofer *et al.*,[1] as the S counters of our experimental arrangement are of exactly the same size as that of Ref. 1. The factor K_1 takes into account that not all shower-induced events have been recorded by the counters S because of the incomplete protection of the counters S. The value of K_1 was experimentally determined by Allkofer *et al.*[2,23] by measuring the rate of incoherent particles (i.e., ABC coincidence produced by different particles, e.g., particles hitting B and C are accompanied by associated particles that hit the tray A) and the fraction of this rate that was detected by the counters S. K_1 is the ratio of these two rates. Then $K_1 \times R_{ABCS}$ gives the rate of incoherent particles which would simulate true R_{ABC} coincidence. The magnitude of this effect is very sensitive to the distribution of materials near the apparatus due to the generation of knock-on electrons and small local showers. The apparatus was placed in

TABLE I. Measured rates and their corrections.

	Time of observation (hours)	No. of events	Random coincidence in %	Noise coincidence in %	Efficiency correction in %	Corrected rates (10^{-4} sec^{-1})
ABC	1507	86 000	0.946 ± 0.05	2.88 ± 0.06	1.75 ± 0.13	155.2 ± 0.59
$ABCS$		9610	...	...	1.95 ± 0.14	18.05 ± 0.18

the middle of a large room where the roof and the walls of the enclosure were such as to contribute little to this effect. The factor K_2 takes into account those single muons which produce knock-on electrons and hence are detected as air showers. This was calculated from the knock-on electron probability given by Bhabha[24] for muon momentum greater than 0.954 GeV/c, energy transfer greater than 5 MeV, and total effective target thickness 3 mm iron between the counter trays A and B and the supporting structure for the counters. All the rates have been corrected for random coincidences, noise coincidences, efficiency of the detectors, scattering and proton contributions. It was possible to determine the noise coincidence rates arising from the chance coincidence of the noise pulses by keeping the whole apparatus at a constant temperature. The rate given in Table I shows the average rate taken for a long time.

The multiple scattering loss correction is done here following the procedure of Sternheimer[25] using the distribution function of Eyges[26] to consider the multiple scattering with energy loss in the absorber. Then we can easily deduce the expression for the probability P that a particle passing through the counters A and B would be found at a distance s from its position without scattering on the counter C. This probability is given by

$$P = \frac{1}{\pi r_0^2} \exp(-s^2/r_0^2), \tag{2}$$

where $r_0 = 2(A_2 + 2A_1 l + A_0 l^2)^{1/2}$, A_0, A_1, A_2 are given in Eyges[26] and l is the distance of the counter C from the lower end of the absorber. This expression is similar to that obtained by Sternheimer [Eq. (1) of Ref. 25] without energy loss. Here the value of r_0 is different. If we disregard the energy loss, the expression (2) reduces to Eq. (1) of Ref. 25.

Then according to the procedure of Ref. 25, the fraction $F(p)$ of the incident beam which passes through C after traversing A and B can be easily found out. The numerical values of $F(p)$ for different values of momentum can be evaluated with the help of Fig. 4 of Ref. 25 by extrapolation. In order to find out the value of scattering loss correction for integral count rates R_{ABC}, another set of integral count is taken with an additional absorber of thickness 10 cm (momentum p_2). The difference of the integral counts will give the differential counts in the intermediate momentum value. The scattering loss correction for the integral count rates R_{ABC} will be K times the differential counts where K will be

$$K = \int_{p_1}^{\infty} [1 - F(p)] N(p) dp \Big/ \int_{p_1}^{p_2} N(p) dp, \tag{3}$$

where $N(p)dp$ is the approximate differential muon intensity. Here K is calculated to be 20.8%.

To find the momentum for a certain length of absorber, Serre's[27] table has been used. The Serre values have been corrected according to the fact that a particle going through an absorber makes a zigzag motion due to multiple scattering. The difference between the geometrical range T(= 663.4 g cm^{-2}) and the effective range R of a muon with momentum p is given by Koenig[18]:

TABLE II. Corrections and final corrected rates.

Scattering correction	Proton correction	Final corrected rates (sec^{-1})	Integral intensity at 0.954 GeV/c ± 0.5%
20.8% of the differential count rate (7.575×10^{-4} sec^{-1}), i.e., 1.146% of the $ABC\bar{S}$ counts	0.72% of $ABC\bar{S}$ counts	$(138.6 \pm 0.64) \times 10^{-4}$	6.86×10^{-3} cm^{-2} sr^{-1} sec^{-1} ± 0.46%

TABLE III. Integral intensities.

Muon momentum	Geomagnetic latitude	Reference	Integral intensity ($cm^{-2} sr^{-1} sec^{-1}$)
0.954 GeV/c	12° N	Present experiment	$6.86\times 10^{-3} \pm 0.46\%$
	55° N	Ref. 3	6.20×10^{-3} [a]
	55° N	Ref. 1	$7.65\times 10^{-3} \pm 4\%$
	55° N	Ref. 30	7.35×10^{-3}
	57.5° N	Ref. 9	6.00×10^{-3} [b]

[a] This value is 6.00×10^{-3} if the experimental points of Wilson [J. G. Wilson, Nature 158, 415 (1946)] plotted in Fig. 5 of Rossi (Ref. 3) be taken.

[b] Normalized to the intensity at 1 GeV/c given by Rossi (Ref. 3).

$$R(p)-T(p)=\int_0^p \frac{\theta^2(R)}{2}\,\frac{dR}{dp}\,dp\,. \tag{4}$$

$\theta^2(R)$ is given in Rossi and Greisen.[28] In the present case T is 5% shorter than R. This gives a muon momentum of 0.954 GeV/c corresponding to $R=663.4$ g cm^{-2} of lead +5%.

The different rates and corrections are given in Tables I and II.

III. RESULTS AND DISCUSSION

The integral intensity calculated from our experiment is given by 6.86×10^{-3} cm^{-2} sr^{-1} sec^{-1} at 0.954 GeV/c at 12° N. Rossi's intensity at sea level for a range 663.4 g cm^{-2} of lead (i.e., 417.8 g cm^{-2} of air) is given by 6.2×10^{-3} cm^{-2} sr^{-1} sec^{-1}. As shown in Table III the present value is therefore higher than that of Rossi[3] by 10.6%. It is again less than that of Allkofer *et al.*[1] by 10.3%. From this we may conclude that our result goes in favor of Allkofer *et al.*[1] who got a 23.4% higher muon intensity than did Rossi[3] in this momentum region. According to Olbert,[29] the present measurements at 12° N geomagnetic latitude should give 8.3% lower intensity than that of Allkofer *et al.*[1] at 55° N latitude. Again if we consider the recent survey of Allkofer *et al.*,[30] our result is 7% less than this value at this momentum region. This 10.3% and 7% decrease in muon intensity of the present results as compared to 8.3% expected from Olbert[29] is not in bad agreement, if we consider that the experimental values correspond to two different, though similar, experimental setups.

The only data on low-energy muon spectrum at 12° N are given by Basu and Sinha[31] and Bhattacharyya.[32] Basu and Sinha[31] obtained a decrease in muon intensity at 12° N of (15 ±4)% from that of Brini *et al.*[33] at 45° N in the low-momentum region of 0.3 GeV/c. Our result contradicts that of Bhattacharyya[32] who recently reported an integral muon intensity 7.3×10^{-3} cm^{-2} sr^{-1} sec^{-1} at 320 MeV/c which is in agreement with Rossi's value at 55° N.

The increase in the muon intensity obtained in the present experiment is in good agreement with the values of the magnetic spectrograph measurement of Allkofer and Clausen[34] and Allkofer *et al.*[30] and the absolute intensity measurement of Crookes and Rastin.[6] Crookes and Rastin[6] obtained a vertical intensity of 9.13×10^{-3} cm^{-2} sr^{-1} sec^{-1} at 0.35 GeV/c which is ~11% higher than the Greisen intensity. Allkofer *et al.*[30] presented an absolute spectrum in the range 0.2–1000 GeV/c based on four different measurements carried out at Kiel. The integral intensity at 0.35 GeV/c amounts to 9.35×10^{-3} cm^{-2} sr^{-1} sec^{-1} and is in agreement with that of Crookes and Rastin.[6]

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