TABLE II. Resonance fraction for five-body final states.

Reaction	$K^*(890)$	$\rho(760)$	$\omega(783)$
$\bar{p}p \to K^+K^-\pi^+\pi^-\pi^0$	40 ± 5	0	30 ± 5
$\to K_1^0K_1^0\pi^+\pi^-\pi^0$ [4]	43 ± 15	20 ± 10	10 ± 5
$\to K^\pm K_1^0\pi^\mp\pi^+\pi^-$ [4]	60 ± 15	11 ± 7	$\cdots$

In conclusion, the $K^*(890)$, $\rho(760)$ and $K^*(890)$, $\omega(783)$ productions are the dominant feature of reactions (1) and (2), respectively. There is no indication of $D(1285)$ or $K_A(1250)$ either. Quasi-two-body processes are almost completely absent, while three-body processes occur to some extent. The final state $K^+K^-\omega$ amounts to about 30% of reaction (2).

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Measurement of Λ^0 Mass*

L. G. Hyman, K. O. Bunnell,† M. Derrick, T. Fields, P. Katz,‡ and G. Keyes§
Argonne National Laboratory, Argonne, Illinois 60439

and

J. G. Fetkovich, J. McKenzie,|| and I-T. Wang**
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

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Using Λ hyperons produced by negative kaons in a helium bubble chamber, we have measured the Λ^0 mass to be $M_{\Lambda^0} = 1115.59\pm0.08$ MeV/c^2. This result is based mainly on curvature and angle measurements of the proton and π^- tracks from the Λ decay.

I. INTRODUCTION

Both emulsion and bubble-chamber experiments have provided useful data on hyperon masses.[1,2] The two techniques are somewhat complementary, with emulsions offering high spatial precision and a well-studied range-energy relation, whereas bubble-chamber data are usually characterized by small multiple Coulomb scattering, precise magnetic curvature information ($\Delta p/p \sim 1\%$ on a single track), and the availability of kinematic information from the production reaction.

In the present experiment we have used helium-bubble-chamber pictures to obtain a measurement of the Λ^0 mass. Considerable effort was made to accurately establish the range-energy relation for helium,[3,4] as well as to achieve high spatial precision. In addition, the magnetic field (provided by a superconducting magnet) was measured[3] to an accuracy of ±0.1%. Thus, the experiment possesses some of the advantages of both emulsion and bubble-chamber techniques.

II. DESCRIPTION OF EXPERIMENT

A. General Procedure

To measure the Λ^0 mass we studied reactions of the types

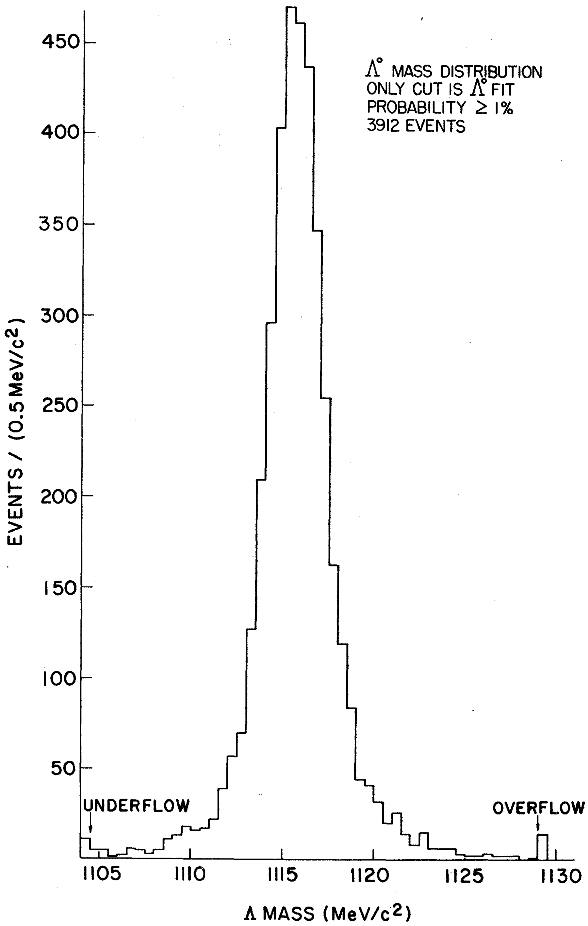

FIG. 1. Histogram of Λ mass for 3912 Λ events making a two-constraint decay fit with probability greater than 1%.

$$K^- + He^4 \rightarrow \pi^- + \Lambda^0/\Sigma^0 + He^3 \quad (1)$$

$$\rightarrow \pi^- + \Lambda^0/\Sigma^0 + p + d \quad (2)$$

$$\rightarrow \pi^- + \Lambda^0/\Sigma^0 + p + p + n\,. \quad (3)$$

The events were measured on Hermes film-plane digitizers and processed through Argonne versions of the geometric reconstruction program TVGP and the kinematic fitting program GRIND. Kinematic fitting of the production reactions, Eqs. (1)–(3), was not used.

All events in which the Λ^0 gave a three-constraint (3C) (connecting) decay fit were then refitted, taking the Λ^0 mass as a variable. For each event the Λ^0 mass which minimized the χ^2 fit for Λ^0 decay was chosen. The mass error was assigned in the standard GRIND way by propagating the fitted errors on p, λ, ϕ for the proton and pion.

Figure 1 is a mass distribution for 3912 events with χ^2 probabilities $\geq 1\%$. The weighted mean mass obtained from events with $1109 \leq M_{\Lambda^0} \leq 1122$ MeV/c^2 (3786 events) was $M_{\Lambda^0} = 1115.59 \pm 0.03$ MeV/c^2. In Fig. 2 we show ideograms of the resolution function and the events. The resolution function has a full width at half-maximum (FWHM) of 3.9 MeV/c^2, and the events have a FWHM of 5.3 MeV/c^2. The ratio of the two FWHM is then 1.33

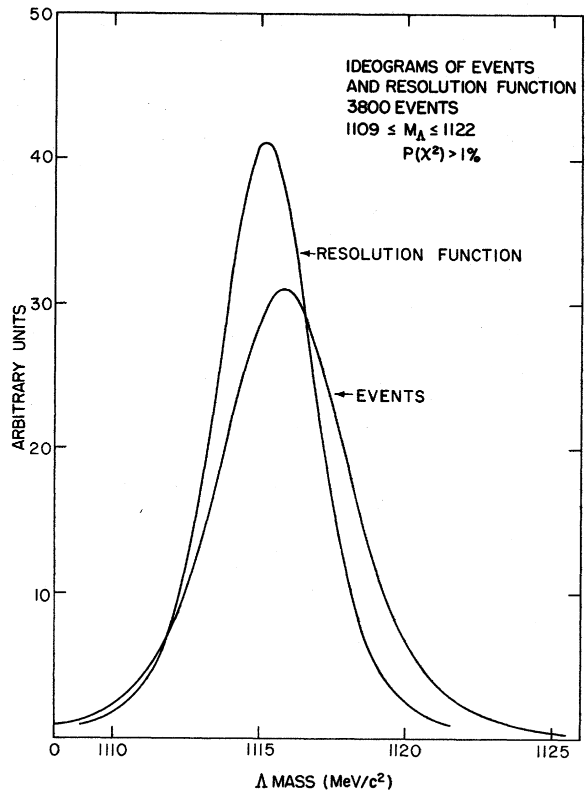

FIG. 2. Ideogram of Λ mass using 3800 events and ideogram of resolution function for those events. The center of the resolution function is arbitrary and has no physical significance.

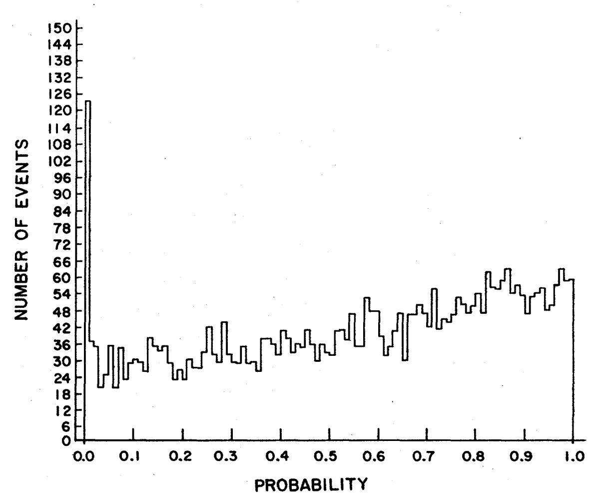

FIG. 3. Probability distribution for χ^2 as calculated in GRIND for two-constraint fit to Λ.

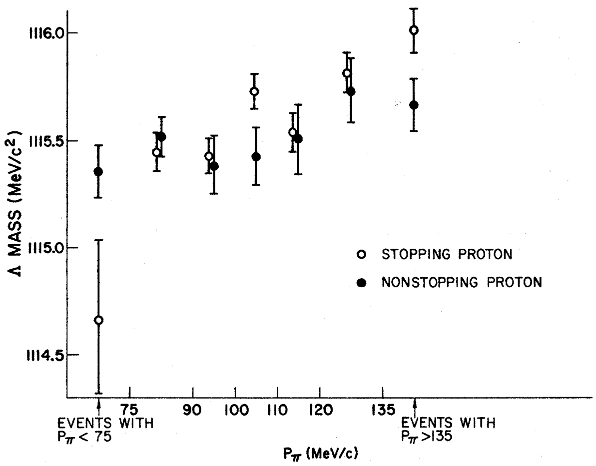

FIG. 4. Λ mass as a function of pion momentum for events with and without stopping protons. The two outside points are for $P_\pi \le 75$ MeV/c and $P_\pi \ge 135$ MeV/c.

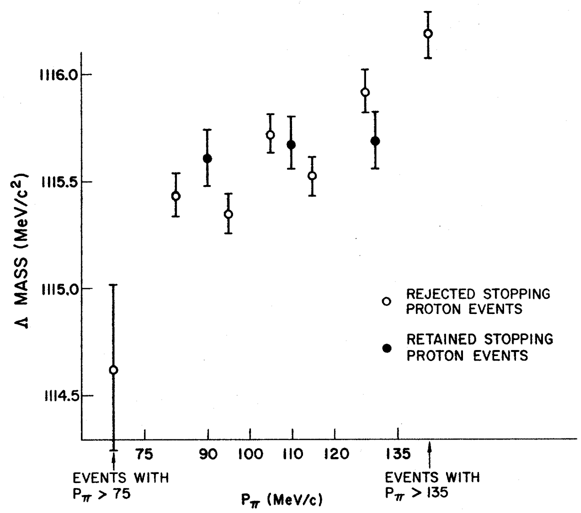

FIG. 5. Λ mass as a function of pion momentum for events with stopping protons. Events with an acceptable stopping-proton track have been condensed from six bins to three bins because of the small number of events.

where one expects $\sqrt{2}$ for a Gaussian distribution.

The probability distribution as calculated in GRIND for 2C fits (i.e., variable Λ mass) to these events is shown in Fig. 3. About 80 of the 100 excess events clustered at small probabilities are caused by events where it is difficult to measure the stopping proton. This problem is discussed in the following section. Events with probability less than 10% were not used in the final sample. From the over-all probability distribution, we conclude that the error assignment on kinematic variables is reasonable, with actual errors slightly smaller than those used in calculating χ^2.

B. Consistency Checks and Cuts

Numerous consistency checks were made on the sample. In these tests, the sample was subdivided according to the value of some parameter, and the Λ mass was determined separately for each subdivision. When testing on a parameter with a continuous range, the sample was divided into four or five bins with approximately equal numbers of events. The total sample was divided according to whether or not the proton from the Λ^0 decay came to rest in the chamber, and the two classes were tested separately. For completeness, the parameters tested are listed in Table I.

All of the tests listed in Table I gave acceptable results except test (j) for events with stopping protons. The results of this test are plotted in Fig. 4 for both stopping protons and nonstopping protons. The decays with a stopping proton exhibit a clear systematic trend of increasing Λ mass with increasing pion momentum. The stopping-proton events give $M_{\Lambda^0} = 1115.64 \pm 0.04$ MeV/c^2 with a χ^2 probability $\ll 1\%$. The events in which the proton does not stop give $M_{\Lambda^0} = 1115.51 \pm 0.05$ MeV/c^2 with a χ^2 probability of 28%.

TABLE I. Parameters for consistency checks.

(a)	Probability of χ_{min}^2
(b)	Number of prongs at production vertex
(c)	Λ proton-track stopping/proton-track nonstopping
(d)	Length of proton track
(e)	Length of π^- track
(f)	Center-of-mass decay angle
(g)	Laboratory opening angle
(h)	Λ momentum
(i)	Proton momentum
(j)	Pion momentum
(k)	Λ dip angle
(l)	Proton dip angle
(m)	π^- dip angle
(n)	Λ length
(o)	Spatial location of production vertex and decay vertex

TABLE II. Final cuts.

(a)	Stopping-proton tracks pass visual inspection
(b)	$1109 \leq M_0 \leq 1122$ MeV/c^2
(c)	Probability $(\chi_{\min}{}^2) \geq 10\%$
(d)	Proton-track length (proton stops) ≥ 0.5 cm
(e)	Proton-track length (proton does not stop) ≥ 7.0 cm
(f)	Pion-track length ≥ 7.0 cm
(g)	Λ momentum ≤ 515 MeV/c
(h)	Dip: $\lvert\sin\lambda\rvert \leq 0.8$ for both π and p
(i)	Lab opening angle $\theta_{\pi p}$: $\lvert\cos\theta_{\pi p}\rvert \leq 0.9$
(j)	$\Lambda°$ decays within three mean lives

The error on the Λ mass may be written as the sum of three terms:

$$\delta M_{\Lambda^0} = \left[\frac{P_\pi E_p}{E_\pi} - P_p \cos\theta\right]\frac{\delta P_\pi}{M_{\Lambda^0}} + \left[\frac{P_p E_\pi}{E_p} - P_\pi \cos\theta\right]\frac{\delta P_p}{M_{\Lambda^0}} - \frac{P_\pi P_p\, \delta(\cos\theta)}{M_{\Lambda^0}}, \tag{4}$$

where P_π is the pion lab momentum, E_π the total pion lab energy, P_p and E_p are the corresponding proton quantities, and θ is the lab angle between the proton and pion. Qualitatively, the second term exhibits the observed behavior as a function of P_π. For small P_π, $\cos\theta$ is negative, and for large P_π, $\cos\theta$ is positive. Thus, if the proton momentum were systematically underestimated, we would expect M_{Λ^0} to have a slope when plotted as a function of P_π, but the average shift in M_{Λ^0} for all events would be small.

To minimize the chance of systematic measuring error on the stopping-proton tracks, we inspected all events with a stopping proton and selected only those events in which the proton track was clear and readily measured in all three views. There were allowed to be no δ rays or tracks crossing near either end of the proton track in any view. This cut was probably far more severe than necessary, and resulted in our rejecting 75% of the events with stopping protons. The events retained showed no systematic trend of M_{Λ^0} with P_π while the rejected sample showed the effect more strongly. This is shown in Fig. 5.

To obtain our final answer we imposed the set of cuts listed in Table II. These cuts lead to the following results:

(1) Nonstopping proton,

$M_{\Lambda^0} = 1115.55 \pm 0.063$ MeV/c^2, 682 events.

(2) Stopping proton,

$M_{\Lambda^0} = 1115.66 \pm 0.088$ MeV/c^2, 253 events.

The combined answer (based on 935 events) is 1115.59 ± 0.051 MeV/c^2, where the standard deviation is statistical only.

C. Other Sources of Error

The magnetic field was calibrated using the methods listed in Table III. Results using all of these methods are in agreement and the combined result yields a field uncertainty of ±0.1%.[5] This field uncertainty of ±0.1% introduces an uncertainty in M_{Λ^0} of ±0.05 MeV/c^2.

TABLE III. Field calibration.

Method	Reaction	Accuracy
(1) Momentum of π^+ peak in $K_{\pi 2}$ decays at rest	$K^+ \to \pi^+ + \pi^0$	±0.22%
(2) Momentum of μ^+ peak in $K_{\mu 2}$ decays at rest	$K^+ \to \mu^+ + \nu$	±0.15%
(3) Kinematic fit of τ^+ decays	$K^+ \to \pi^+\pi^+\pi^-$	±0.25%
(4) Kinematic fit of τ^- decays	$K^- \to \pi^-\pi^+\pi^-$	±0.26%
(5) Compare momentum found from range of stopping pion to momentum from curvature		±0.5%
(6) Disassemble bubble chamber and measure absolute central value as well as field shape		±0.2%

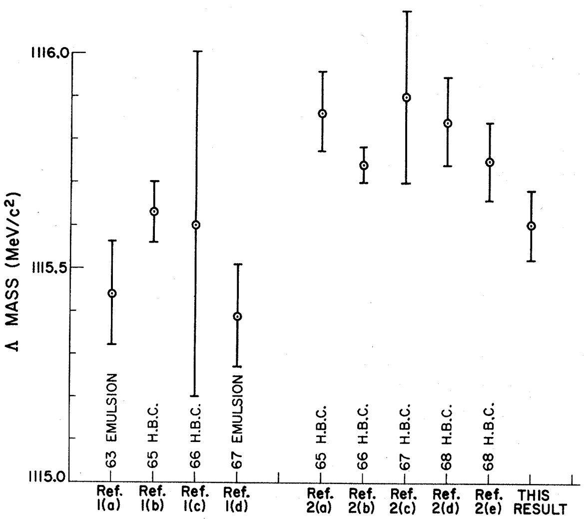

FIG. 6. Summary of measurements of the Λ mass. The first four values are those used to obtain the world average.

We have investigated possible systematic effects in the geometry program TVGP and the kinematics program GRIND by Monte Carlo generation of Λ^0 events with multiple scattering and measuring errors included. This was done using the computer program TRACK [6] which generates ideal particle orbits (with energy loss and multiple Coulomb scattering) in the bubble chamber, projects them through the optics system, and simulates errors in measurement. In these tests, the output Λ^0 mass was shifted from the input value by 0.017 ± 0.041 MeV/c^2. There is thus no evidence for a systematic bias; however, we do allow for the uncertainty of this test by adding ± 0.04 MeV/c^2 in quadrature with the other errors.

The proton and pion masses used in this experiment are[7]

$$M_p = 938.256 \pm 0.005 \text{ MeV}/c^2,$$

$$M_\pi = 139.58 \pm 0.013 \text{ MeV}/c^2.$$

The errors on these masses contribute a negligible error to our result.

The final result of this experiment is $M_{\Lambda^0} = 1115.59 \pm 0.08$ MeV/c^2. This is to be compared with the present world average[7] of $M_{\Lambda^0} = 1115.59 \pm 0.06$ MeV/c^2.

III. DISCUSSION

Figure 6 shows various measurements of M_{Λ^0} including the present result. The results used by the Particle Data Group are the first four plotted. The present world average for the Λ^0 mass is listed[7] with a scale factor of 1.3, which means the individual experimental results are not statistically consistent. Generally, the values of M_{Λ^0} measured in hydrogen bubble chambers have been higher than the emulsion measurements. As pointed out by Bohm *et al.*,[8] the emulsion values may have systematic problems arising from errors in the range-energy relationship for π^- mesons with velocities $>0.6c$.

When a quantity such as the Λ mass has a large scale factor, it is important to do several experiments under different and carefully controlled conditions. We believe after careful scrutiny that the error assigned to our result is realistic.

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†Presently at Stanford Linear Accelerator Center, Stanford, Calif. 94305.

‡Presently at University of Tel-Aviv, Tel-Aviv, Israel.

§Presently at Carnegie-Mellon University, Pittsburgh, Pa. 15213.

‖Presently at University College, London, England.

**Presently at Argonne National Laboratory, Argonne, Ill. 60439.

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Absolute Measurement of the Vertical Cosmic-Ray Muon Intensity near 1 GeV/c at 12°N

A. K. De, P. Ghosh, S. Mitra, P. C. Bhattacharya, and A. K. Das
Department of Physics, University College of Science, Calcutta-9, India
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The absolute vertical cosmic-ray muon intensity at sea level at 12° N has been measured with a range spectrometer similar to that used by Allkofer *et al.* and by Jokisch. The integral intensity amounts to 6.86×10^{-3} cm^{-2} sr^{-1} sec^{-1} at 0.954 GeV/c which is 10.3% lower than that of Allkofer *et al.* and 10.6% higher than that of Rossi, generally used as the normalization point for muon spectra. Our results are in favor of experimental findings of Allkofer *et al.* The decrease in intensity may be explained in terms of the geomagnetic latitude effect.

I. INTRODUCTION

In a recent paper Allkofer *et al.*[1] and also Jokisch[2] reported an absolute measurement of muon intensity at the momentum of 1 GeV/c with a range spectrometer. The result obtained from this experiment is about 25% higher than the Rossi[3] value at the same latitude. Supporting evidence of higher vertical muon intensities near 1 GeV/c comes from the recent work of Ayre *et al.*,[4] Bateman *et al.*,[5] Crookes and Rastin,[6] and the preliminary report of the present authors.[7] Ayre *et al.* with their large spectrograph MARS at Durham obtained integral intensities in the range 3–6 GeV/c significantly higher (~7%) than those previously reported by Aurela and Wolfendale.[8] The Aurela and Wolfendale intensities are again 3–4% higher than those reported by Hayman and Wolfendale[9] in the same momentum region. Crookes and Rastin obtained a vertical integral muon intensity 9.13×10^{-3} cm^{-2} sr^{-1} sec^{-1} at 184.7 g cm^{-2} of lead (momentum 0.35 GeV/c). Bateman *et al.* with a magnetic spectrograph obtained absolute muon intensities in the range 3–50 GeV/c which are 12% higher than those of Hayman and Wolfendale.[9] The latter workers[9] normalized their data to the Rossi intensity, viz., 2.45×10^{-3} cm^{-2} sr^{-1} sec^{-1} (GeV/c)$^{-1}$ at 1 GeV/c.

A review of the above works raises doubts as to the exact value of the normalization point itself though the Rossi intensity has been used for normalizing the muon spectra by many authors.[8-15] The muon intensities near and above 1 GeV/c are somewhat higher than the previously accepted values in this momentum interval.

Allkofer *et al.*[1] concluded that in case the differential and integral muon spectra at sea level are normalized they should be enhanced by a factor 1.26. The discrepancies as explained by Allkofer *et al.*[1] are that the integral Rossi spectrum is normalized at 0.3 GeV/c to the Greisen[16] intensity 8.3×10^{-3} cm^{-2} sr^{-1} sec^{-1} in which the zigzag nature of the path of the particle inside the absorber due to multiple Coulomb scattering has not been considered. This is essential when a range spectrum is converted to a momentum spectrum. The conclusion is also in agreement with that of Kraushaar[17] who following Koenig[18] concluded that the vertical thickness of the absorber due to multiple Coulomb scattering inside the absorber should be increased by 11% in the range 50–180 g cm^{-2} of lead. This causes a shift of momentum from 0.3 to 0.33 GeV/c and hence the Rossi intensity at a particular momentum should be increased. This conclusion is true, but the change in the penetrating muon flux for this effect is only 0.7% at the momentum 0.3 GeV/c. The work of York[19] must also be mentioned in this connection. York investigated the differential range spectrum of cosmic muons in the region from 18–76 g cm^{-2} of air-equivalent absorber using a counter-controlled cloud chamber. The results obtained in this region