threshold and 1.4 GeV ($s = 100$). The same should be true to an even greater extent for the absorptive parts of unitary amplitudes.

[8]The derivative with respect to s of this pole term in $A^{(0)2}$ is given by

$$\frac{1}{\pi}\frac{\gamma_3^{(0)2}}{(s_3^{(0)2}-s)^2},$$

which equals 6.31×10^{-5} at threshold.

[9]Each S wave is subtracted at $s_0 = \frac{4}{3}$, and the value predicted by Weinberg from current algebra is imposed there. The P wave is subtracted at threshold, where it is assumed to vanish. These values are imposed in a way which does not constrain the singularities.

[10]Since the ρ resonance of KL is too broad (Ref. 5), the $\text{Im}A^{(l)I}$ displayed in Fig. 1 are not in good quantitative agreement with those which occur in nature. However, the left-hand cuts of KL disagree even more with the $\text{Im}A^{(l)I}$ which occur in nature than they do with the $\text{Im}A^{(l)I}$ displayed in Fig. 1.

[11]There are also $I = 2$ Regge sum rules for low-energy parameters, but these converge only because of extreme cancellations, and cannot be evaluated with sufficient reliability at present to be of any practical use. Cf. E. P. Tryon, Phys. Rev. Letters 22, 110 (1969).

[12]Cf. M. G. Olsson, Phys. Rev. 162, 1338 (1967).

[13]Our S-wave scattering lengths are defined with opposite signs from those of KL.

[14]For example, in the single-term Veneziano model for $\pi\pi$ amplitudes [C. Lovelace, Phys. Letters 28B, 264 (1968)], the contribution to the right-hand side of Eq. (4) from partial waves with $l \geq 2$ is 0.21, and the corresponding contribution to the right-hand side of Eq. (5) is 0.012, assuming one uses Lovelace's values for the Regge parameters and sets $\Gamma(\rho\to\pi\pi) = 125$ MeV.

[15]If one expands the S and P waves as power series in $(s-2)$ over the interval $0 \leq s \leq 4$, then the integrals in two of the five conditions of Roskies are independent of constant and linear terms, and the integrals in one of these conditions are independent of quadratic terms as well. It is also straightforward to verify that if the S waves were linear in s over the interval $0 \leq s \leq 4$, and satisfied the two conditions of Roskies which involve only S waves, then the Martin inequalities would all be marginally satisfied as equalities.

[16]E. P. Tryon, Phys. Rev. D 4, 1216 (1971).

[17]R. T. Park and B. R. Desai, Phys. Rev. D 2, 786 (1970).

[18]E. P. Tryon, Phys. Letters B (to be published).

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Factorization of Multi-Regge Amplitudes. II*

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Some general comments are made on the relationship between the factorization of multi-Regge amplitudes and the singularity structure of multiparticle amplitudes. A close relationship is found to exist between the validity of factorization and the absence of simultaneous discontinuities in overlapping variables suggested by the Steinmann relations.

Multi-Regge factorization of the full amplitude and its total-energy discontinuity have recently been proven in several models: the dual-resonance model,[1] ladder graphs in ϕ^3 perturbation theory,[2,3] and Gribov's hybrid model.[3,4] The procedure of taking into proper account the cuts due to singularities in variables which are dependent owing to nonlinear Gram-determinant constraints was found to play a crucial role in obtaining factorization.[1,2]

Here we make some general, model-independent, comments on the relationship between multi-Regge factorization and the singularity structure of multiparticle amplitudes. In particular, we find that the singularity structure suggested by the Steinmann relations[5] has a very intimate relationship to factorization.

The Steinmann relations state that the full amplitude has no simultaneous discontinuity in energy variables of overlapping channels[6] in the physical region. Here we wish to make the assumption that the full amplitude can be expressed in terms of generalized multi-Froissart-Gribov signatured amplitudes (see Eq. (1.1) of Paper I [(I 1.1)]). The Steinmann relation can then be applied to the signatured amplitude itself, since simultaneous discontinuities in overlapping subenergies in the signatured amplitude would lead to corresponding discontinuities in the full amplitude (within a given physical region, the full amplitude has discontinuities coming from only one term in Eq. (1.1) of Paper I [(I 1.1)]). Therefore, whereas the signatured amplitude for the process $p_a + p_b \to p_0 + p_1 + \cdots + p_{n-1}$ in general has discontinuities in the subenergies of all groups of adjacent outgoing particles

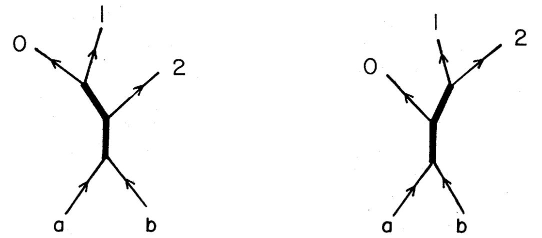

FIG. 1. Allowable simultaneous discontinuities for $A_{2\to 3}$.

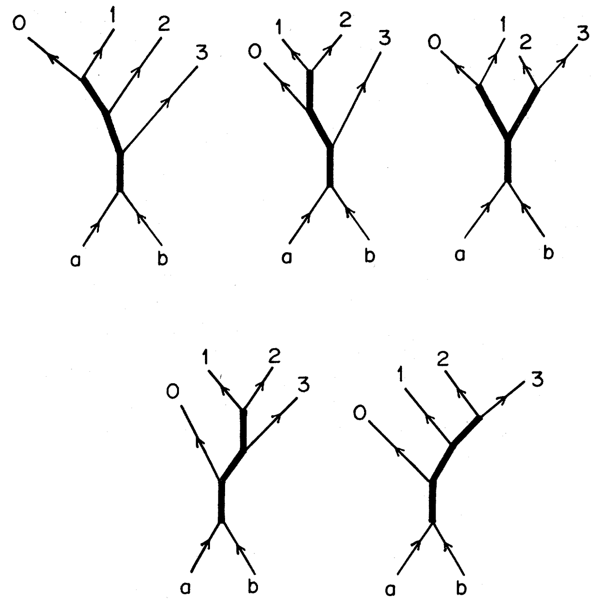

FIG. 2. Allowable simultaneous discontinuities for $A_{2\to 4}$.

[e.g., $s_{01}=(p_0+p_1)^2$, s_{12}, s_{012}, s_{123}, $s_{1,2,\ldots,n-1}$, $s_{0,1,\ldots,n-1}$, etc.], it is assumed to have no simultaneous discontinuities in two subenergies which overlap (e.g., s_{01} and s_{12}, s_{012} and $s_{1,2,\ldots,n-1}$, etc.). We believe this is a very plausible assumption and clearly leads to a full amplitude that satisfies the Steinmann relation, but do not attempt to justify it further here. Both the dual-resonance model and the ϕ^3 ladder model satisfy it, as we shall see in the Appendix.

We first briefly discuss the $2\to 3$ amplitude and then discuss the factorization of the $2\to 4$ amplitude. We expect that the discussion can be straightforwardly generalized to arbitrary production amplitudes $A_{2\to n}$.

The signatured $2\to 3$ amplitude is assumed to have the Regge behavior [see Eq. (I 1.3)]

$$A_{2\to 3}^{\tau_1\tau_2}\sim\beta(t_1)\Gamma(-\alpha_1)(-s_{01})^{\alpha_1}\beta(t_1,t_2;\kappa_1)\times\Gamma(-\alpha_2)(-s_{12})^{\alpha_2}\beta(t_2), \tag{1}$$

where $\kappa_1=s_{01}s_{12}/s_{012}$. As it stands, Eq. (1) appears to exhibit a simultaneous discontinuity in the overlapping variables s_{01} and s_{12}, whereas the only allowed simultaneous discontinuities are those corresponding to the heavy lines in the tree diagrams shown in Fig. 1. Thus consistency with our use of the Steinmann relations demands[2,7] that β have singularities in κ_1,

$$\beta(t_1,t_2;\kappa_1)=(-\kappa_1)^{-\alpha_2}V_2(t_1,t_2;\kappa_1)+(-\kappa_1)^{-\alpha_1}V_1(t_1,t_2;\kappa_1), \tag{2}$$

so that

$$A_{2\to 3}^{\tau_1\tau_2}\sim(-s_{01})^{\alpha_1-\alpha_2}(-s_{012})^{\alpha_2}V_2+(-s_{12})^{\alpha_2-\alpha_1}(-s_{012})^{\alpha_1}V_1. \tag{3}$$

The two-Reggeon vertex for the full amplitude is thus (see Fig. 3 of I)

$$R(t_1,t_2;\kappa_1)=\xi_1^{-1}\xi_2^{-1}\big[(e^{-i\pi\alpha_1}+\tau_1+\tau_2e^{-i\pi\alpha_1}e^{i\pi\alpha_2}+\tau_1\tau_2e^{-i\pi\alpha_2})\kappa_1^{\alpha_2}V_2(t_1,t_2;\kappa_1) + (e^{-i\pi\alpha_2}+\tau_1e^{i\pi\alpha_1}e^{-i\pi\alpha_2}+\tau_2+\tau_1\tau_2e^{-i\pi\alpha_1})\kappa_1^{\alpha_1}V_1(t_1,t_2;\kappa_1)\big]. \tag{4}$$

We assume the signatured $2\to 4$ amplitude has the Regge behavior

$$A_{2\to 4}^{\tau_1\tau_2\tau_3}\sim\beta(t_1)\Gamma(-\alpha_1)(-s_{01})^{\alpha_1}\Gamma(-\alpha_2)(-s_{12})^{\alpha_2}\Gamma(-\alpha_3)(-s_{23})^{\alpha_3}\beta(t_3)\beta(t_1,t_2,t_3;\kappa_1,\kappa_2;\phi). \tag{5}$$

The variable $\phi\equiv s_{0123}s_{12}/s_{012}s_{123}$ is required to be unity in the Regge limit, owing to the Gram-determinant constraints. However, we include it as a variable in order to be able to specify where the limit is taken with respect to singularities in the independent variables due to the right-hand cut in s_{0123}.[1] Thus, for example, a change of ϕ by $e^{2\pi i}$ corresponds to taking the asymptotic limit on the opposite side of all cuts in the independent variables due to cuts in s_{0123}.

The dependence on ϕ is also crucial for the satisfaction of the Steinmann relations. Equation (5) appears to have simultaneous discontinuities in s_{01} and s_{12} and s_{12} and s_{23}, whereas the only allowed simultaneous discontinuities are those corresponding to the tree diagrams of Fig. 2. Thus consistency with (5) demands that

$$\beta=(-\kappa_1)^{-\alpha_2}(-\kappa_2)^{-\alpha_3}\phi^{\alpha_3}V^3_{23}+(-\kappa_1)^{-\alpha_1}(-\kappa_2)^{-\alpha_3}\phi^{\alpha_3}V^3_{13}+(-\kappa_1)^{-\alpha_2}(-\kappa_2)^{-\alpha_2}\phi^{\alpha_2}V^2_{22}$$
$$+(-\kappa_1)^{-\alpha_1}(-\kappa_2)^{-\alpha_3}\phi^{\alpha_1}V^1_{13}+(-\kappa_1)^{-\alpha_1}(-\kappa_2)^{-\alpha_2}\phi^{\alpha_1}V^1_{12}, \tag{6}$$

where we have suppressed the arguments, t_1, t_2, t_3, κ_1, and κ_2, so that

$$A_{2\to4}^{\tau_1\tau_2\tau_3}\sim(-s_{01})^{\alpha_1-\alpha_2}(-s_{012})^{\alpha_2-\alpha_3}(-s_{0123})^{\alpha_3}V^3_{23}$$
$$+(-s_{12})^{\alpha_2-\alpha_1}(-s_{012})^{\alpha_3-\alpha_1}(-s_{0123})^{\alpha_3}V^3_{13}+(-s_{01})^{\alpha_1-\alpha_2}(-s_{23})^{\alpha_3-\alpha_2}(-s_{0123})^{\alpha_2}V^2_{22}$$
$$+(-s_{12})^{\alpha_2-\alpha_3}(-s_{123})^{\alpha_1-\alpha_3}(-s_{0123})^{\alpha_1}V^1_{13}+(-s_{23})^{\alpha_3-\alpha_2}(-s_{123})^{\alpha_2-\alpha_1}(-s_{0123})^{\alpha_1}V^1_{12}. \tag{7}$$

Factorization of the signatured amplitude for all $s_{ij\cdots k}\to-\infty$ (or all $s_{ij\cdots k}\to+\infty+i\epsilon$), where $\phi=1$, implies [using (1), (2), (5), and (6)]

$$\begin{aligned}V^3_{23}(\kappa_1,\kappa_2)&=V_2(\kappa_1)V_3(\kappa_2),\\ V^2_{22}(\kappa_1,\kappa_2)&=V_2(\kappa_1)V_2(\kappa_2),\\ V^1_{12}(\kappa_1,\kappa_2)&=V_1(\kappa_1)V_2(\kappa_2),\\ V^3_{13}(\kappa_1,\kappa_2)+V^1_{13}(\kappa_1,\kappa_2)&=V_1(\kappa_1)V_3(\kappa_2).\end{aligned} \tag{8}$$

The full amplitude $A_{2\to4}$ is a sum over eight terms of the form (5) (see Fig. 4 of I). Factorization implies that it must have the form

$$A_{2\to4}\sim\beta(t_1)[\Gamma(-\alpha_1)\xi_1 s_{01}{}^{\alpha_1}]R(t_1,t_2;\kappa_1)[\Gamma(-\alpha_2)\xi_2 s_{12}{}^{\alpha_2}]R(t_2,t_3;\kappa_2)[\Gamma(-\alpha_3)\xi_3 s_{23}{}^{\alpha_3}]\beta(t_3). \tag{9}$$

Using (6), (8), and Fig. 4 of I, one finds that $A_{2\to4}$ indeed has the form (9) if one additional condition holds:

$$e^{i\pi\alpha_1}e^{-i\pi\alpha_2}e^{-i\pi\alpha_3}V^3_{13}(\kappa_1,\kappa_2)+e^{-i\pi\alpha_1}e^{-i\pi\alpha_2}e^{i\pi\alpha_3}V^1_{13}(\kappa_1,\kappa_2)$$
$$=\frac{1}{-2i\sin\pi\alpha_2}(e^{i\pi\alpha_1}e^{-2i\pi\alpha_2}e^{i\pi\alpha_3}+e^{-i\pi\alpha_1}e^{-i\pi\alpha_3}-e^{-i\pi\alpha_1}e^{i\pi\alpha_3}-e^{i\pi\alpha_1}e^{-i\pi\alpha_3})V_1(\kappa_1)V_3(\kappa_2). \tag{10}$$

The verification of this requires a considerable amount of tedious algebra, which will not be given here. We only remark that the precise phases due to ϕ from (6) are essential in compensating the mismatches between the phases in κ_1 and κ_2 obtained by applying the $+i\epsilon$ prescription to $A_{2\to4}$ and the phases in κ_1 and κ_2 in the two-Reggeon residue (4).[8] The existence of two terms in (6) contributing to the coefficient of $\kappa_1{}^{-\alpha_1}\kappa_2{}^{-\alpha_3}$ is important for the same reason. The singularity structure (6) suggested by the Steinmann relations is therefore intimately related to the factorization of $A_{2\to4}$.

It is interesting to solve Eqs. (8) and (10) for V^3_{13} and V^1_{13}:

$$\begin{aligned}V^3_{13}(\kappa_1,\kappa_2)&=\frac{\sin\pi(\alpha_2-\alpha_3)\sin\pi\alpha_1}{\sin\pi\alpha_2\sin\pi(\alpha_1-\alpha_3)}V_1(\kappa_1)V_3(\kappa_2),\\ V^1_{13}(\kappa_1,\kappa_2)&=\frac{\sin\pi(\alpha_2-\alpha_1)\sin\pi\alpha_3}{\sin\pi\alpha_2\sin\pi(\alpha_3-\alpha_1)}V_1(\kappa_1)V_3(\kappa_2).\end{aligned} \tag{11}$$

The sines in (11) have a very natural interpretation. From (3) we note that only V_1 contributes to the simultaneous discontinuity in s_{12} and s_{012}. Since there is no reason *a priori* to expect this discontinuity to vanish for $\alpha_2-\alpha_1$ or α_3 integral, or to be singular for α_2 integral, we expect [note the $\Gamma(-\alpha_1)$ and $\Gamma(-\alpha_2)$ in (1)]

$$V_1(\kappa_1)=\frac{\sin\pi\alpha_2}{\sin\pi(\alpha_2-\alpha_1)}\tilde V_1(\kappa_1),$$

where $\tilde V_1(\kappa_1)$ need not have such zeros or poles. Similar reasoning suggests

$$V_3(\kappa_2)=\frac{\sin\pi\alpha_2}{\sin\pi(\alpha_3-\alpha_2)}\tilde V_3(\kappa_2),$$

and, comparing (5) and (7), we have

$$V^3_{13}(\kappa_1\kappa_2)=\frac{\sin\pi\alpha_1\sin\pi\alpha_2}{\sin\pi(\alpha_2-\alpha_1)\sin\pi(\alpha_3-\alpha_1)}\tilde V^3_{13}(\kappa_1,\kappa_2),$$

$$V^1_{13}(\kappa_1,\kappa_2)=\frac{\sin\pi\alpha_2\sin\pi\alpha_3}{\sin\pi(\alpha_2-\alpha_3)\sin\pi(\alpha_1-\alpha_3)}\tilde V^1_{13}(\kappa_1,\kappa_2).$$

Equations (11) are just what is needed to allow consistency between these "natural" zeros and poles.

Using (6), (8), and (11), it is straightforward to show that the total-energy discontinuity of $A_{2\to4}$ also factorizes. The singularity structure (6) thus plays a vital role in giving factorization for both the amplitude and its total-energy discontinuity.

Finally, we recall that in I we noted that factorization follows if the signatured amplitude has the form [see Eq. (I3.5)]

$$A_{2\to4}^{\tau_1\tau_2\tau_3} \sim (-s_{01})^{\alpha_1}(-s_{12})^{\alpha_2}(-s_{23})^{\alpha_3} \int_0^\infty \int_0^\infty \int_0^\infty dz_1\, dz_2\, dz_3\, g(t_1) f(z_1; t_1, t_2) f(z_2; t_2, t_3) g(t_3)$$
$$\times \int_0^\infty \int_0^\infty \int_0^\infty dy_1\, dy_2\, dy_3\, y_1^{-\alpha_1-1} y_2^{-\alpha_2-1} y_3^{-\alpha_3-1}$$
$$\times \exp\left[-y_1 - y_2 - y_3 + \left(\frac{z_1}{\kappa_1}\right) y_1 y_2 + \left(\frac{z_2}{\kappa_2}\right) y_2 y_3 - \left(\frac{z_1 z_2 \phi}{\kappa_1 \kappa_2}\right) y_1 y_2 y_3\right], \tag{12}$$

if the integrations over z_j introduce no further singularities in κ_j. The integral over the y_i in (12) is precisely the dual-resonance amplitude with $\kappa_j \to \kappa_j / z_j$. Since we show in the Appendix that the dual-resonance model satisfies (6), (8), and (11), the form (12) will also. The form (12) is therefore a very general form for amplitudes which incorporates satisfaction of the Steinmann relations [as well as the factorization conditions (8) and (11)].[9]

APPENDIX

We show that the dual-resonance model (DRM) is consistent with our assumption about the singularity structure of the signatured amplitudes, in particular its consequence (6).[10] In the DRM, we have [see Eq. (I 2.9)]

$$\Gamma(-\alpha_1)\Gamma(-\alpha_2)\Gamma(-\alpha_3)\beta(t_1, t_2, t_3; \kappa_1\kappa_2; \phi) = \int_0^\infty \int_0^\infty \int_0^\infty dy_1\, dy_2\, dy_3\, y_1^{-\alpha_1-1} y_2^{-\alpha_2-1} y_3^{-\alpha_3-1}$$
$$\times \exp\left(-y_1 - y_2 - y_3 + \frac{y_1 y_2}{\kappa_1} + \frac{y_2 y_3}{\kappa_2} - \frac{\phi y_1 y_2 y_3}{\kappa_1 \kappa_2}\right). \tag{A1}$$

Doing one integration, applying three times the Mellin-Barnes formula[11]

$$\Gamma(-\alpha)(1+z)^\alpha = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dr\, \Gamma(-\alpha + r)\Gamma(-r) z^r,$$

and then doing the remaining two y_i integrations gives

$$\Gamma(-\alpha_1)\Gamma(-\alpha_2)\Gamma(-\alpha_3)\beta = \left(\frac{1}{2\pi i}\right)^3 \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} dp\, dq\, dr\, \Gamma(-\alpha_1 + p)\Gamma(-\alpha_2 + p + q - r)\Gamma(-\alpha_3 + q)$$
$$\times \Gamma(-p + r)\Gamma(-q + r)\Gamma(-r)(-\kappa_1)^{-p}(-\kappa_2)^{-q}\phi^r. \tag{A2}$$

An integral representation of the hypergeometric function[12] can be used to rewrite the right-hand side of (A2) as

$$[(2\pi i)^2 \Gamma(-\alpha_2)]^{-1} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} dp\, dq\, \Gamma(-\alpha_1 + p)\Gamma(-\alpha_2 + p)\Gamma(-p)(-\kappa_1)^{-p}\Gamma(-\alpha_2 + q)\Gamma(-\alpha_3 + q)\Gamma(-q)(-\kappa_2)^{-q}$$
$$\times {}_2F_1(-p, -q; -\alpha_2; 1 - \phi). \tag{A3}$$

Using the usual formula for analytic continuation of the hypergeometric function,[13] we can extract the singularity in ϕ to obtain

$$[(2\pi i)^2 \Gamma(-\alpha_2)]^{-1} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} dp\, dq\, \Gamma(-\alpha_1 + p)\Gamma(-\alpha_2 + p)\Gamma(-p)(-\kappa_1)^{-p}\Gamma(-\alpha_2 + q)\Gamma(-\alpha_3 + q)\Gamma(-q)(-\kappa_2)^{-q}$$
$$\times \left[\frac{\sin\pi(p - \alpha_2)\sin\pi q}{\sin\pi\alpha_2 \sin\pi(p - q)}\phi^p + \frac{\sin\pi(q - \alpha_2)\sin\pi p}{\sin\pi\alpha_2 \sin\pi(q - p)}\phi^q\right], \tag{A4}$$

where the remaining hypergeometric function has been evaluated at unit argument.[14] Finally, we can close the p and q contours in their left-half planes, picking up the singularities in $\Gamma(-\alpha_1 + p)$ and $\Gamma(-\alpha_2 + p)$ and $\Gamma(-\alpha_2 + q)$ and $\Gamma(-\alpha_3 + q)$, respectively. Of the possible eight terms only five are finite, since one of the two factors in the final bracket vanishes if $p = \alpha_2$ or $q = \alpha_2$. These five terms correspond to the five terms in (6).[9] Equations (8) and (11) are also satisfied where

$$V_1(\kappa_1) = [\Gamma(-\alpha_1)\Gamma(-\alpha_2)]^{-1} \sum_{i=0}^\infty \Gamma(-\alpha_1 + i)\Gamma(\alpha_1 - \alpha_2 - i)\frac{\kappa_1^{\,i}}{i!},$$

etc.

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[1]J. H. Weis, Phys. Rev. D 4, 1777 (1971), hereafter referred to as I.

[2]I. G. Halliday, Nucl. Phys. B33, 285 (1971).

[3]I. G. Halliday and G. W. Parry, Nucl. Phys. B36, 162 (1972).

[4]D. K. Campbell, Phys. Rev. 188, 2471 (1969).

[5]O. Steinmann, Helv. Phys. Acta 33, 257 (1960); 33, 347 (1960); H. Araki, J. Math. Phys. 2, 163 (1960); H. P. Stapp, Phys. Rev. D 3, 3177 (1971).

[6]Overlapping channels are those having some particles in common but neither is a subchannel of the other.

[7]C. E. DeTar and J. H. Weis, Phys. Rev. D 4, 3141 (1971); see Appendix B.

[8]These mismatches are what led to the apparent nonfactorization pointed out by J. W. Dash, Phys. Rev. D 3, 1016 (1971).

[9]Using the results of the Appendix and Eq. (2.7) of Ref. 2, it is easy to see that the ϕ^3 ladder model satisfies Eq. (12) and thus our Steinmann assumption.

[10]This is no surprise, of course. The cuts in the asymptotic variables are asymptotic representations of poles and we know the dual model has no simultaneous poles in overlapping variables.

[11]*Higher Transcendental Functions* (Bateman Manuscript Project), edited by A. Erdélyi (McGraw-Hill, New York, 1953), Vol. 1, p. 256.

[12]E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge Univ. Press, Cambridge, England, 1927), p. 290.

[13]Reference 11, Vol. 1, p. 109.

[14]Reference 11, Vol. 1, p. 104.

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Eikonal Approximation and the Light Cone*

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This comment considers the high-energy behavior of a particle of arbitrary spin interacting with an external potential $A_{\mu_1 \cdots \mu_n}(x)$ by means of a $J^{\mu_1 \cdots \mu_n} A_{\mu_1 \cdots \mu_n}(x)$ interaction, where $J^{\mu_1 \cdots \mu_n}$ is a function of the fields and their derivatives. It is shown that in the eikonal approximation, "exponentiation" occurs when the light-cone commutation relations satisfy

$$\int_{-\infty}^{\infty} dx^- \int_{-\infty}^{\infty} dy^- [J^{+\cdots+}(x),\, J^{+\cdots+}(y)]_{x^+=y^+=0} = 0.$$

It has become apparent that the knowledge of current commutators on the light cone provides useful theoretical and experimental information about the high-energy behavior. The structure of light-cone commutators of currents in various model field theories has been exposed by Cornwall and Jackiw,[1] Fritzsch and Gell-Mann,[2] and Gross and Treiman,[3] and many applications of the theory have already been given.[4-6]

In this comment we wish to indicate a new area of applicability of light-cone techniques – the study of the eikonal approximation. This approximation has become a useful method in calculating high-energy scattering amplitudes in various model theories.[7,8]

That there should be a connection between the light-cone approach and the eikonal approximation can be seen in various ways. For example, the natural variables in the eikonal methods are the + and – components familiar from the light cone. Also, the work of Gross and Treiman[3] abounds in eikonal-type exponentials in their light-cone formulas. More specifically, Lee[9] and Chang[10] have related "exponentiation" of the eikonal approximation in quantum electrodynamics to the properties of current commutators on the light cone.

Recently, Weinberg[11] has shown for the scattering of a fast particle of arbitrary spin in an external electromagnetic field that exponentiation occurs provided that the anomalous magnetic moment obeys a generalized Drell-Hearn sum rule.[12] The Drell-Hearn sum rule can be related to the light-cone commutation relations.[5] Thus there is a general relation between the exponentiation of the eikonal approximation and light-cone commutators. In this comment that relationship is directly obtained. It is found that for a particle of arbitrary spin [field $\phi(x)$] interacting with an arbitrary external potential $A^{\alpha_1 \cdots \alpha_m}_{\mu_1 \cdots \mu_n}(x)$ by the interaction $J^{\mu_1 \cdots \mu_n}_{\alpha_1 \cdots \alpha_m} A^{\alpha_1 \cdots \alpha_m}_{\mu_1 \cdots \mu_n}(x)$, exponentiation of the eikonal approximation to the wave function $\Phi(x; p, \lambda, \alpha; A) \equiv \langle 0\, \mathrm{out} | \phi(x) | p, \lambda, \alpha\, \mathrm{in} \rangle$ is valid provided that

$$\int_{-\infty}^{\infty} dx^- \int_{-\infty}^{\infty} dy^- [J^{+\cdots+}_{\alpha_1 \cdots \alpha_m}(0, \vec{x}_\perp, x^-),\, J^{+\cdots+}_{\beta_1 \cdots \beta_m}(0, \vec{y}_\perp, y^-)] = 0\,.$$