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There Is a Measurement Problem: A Comment

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A possible difficulty is suggested with Moldauer's approach to the problem of measurement.

I agree with the interpretation adopted by Moldauer in response to the measurement problem, namely, to suggest that pure state $\Psi^{(2)}$ [his Eq. (2)] as well as mixed state $\underline{W}$ [his Eq. (3)] should each be interpreted as representing the situation in which the apparatus observable has some definite value, with the probabilities given by the coefficients of (3). Indeed this is the strategy I suggested in Secs. V and VI of the paper cited by Moldauer as Ref. 1 above.

I would point out, however, that this interpretation is at variance with an almost universally accepted rule for the application of quantum mechanics. It is the rule according to which if the state of a system is pure, and if some observable of the system has a specific but perhaps unknown value, then in fact the state of the system is an eigenstate of that observable. For, state $\Psi^{(2)}$ is not an eigenstate for the apparatus observable, yet that observable is claimed to have a value (that is a specific but perhaps unknown value) in that state.

Thus the strategy adopted by Moldauer is at variance with quantum mechanics as it is usually interpreted. This is just to say that there is indeed a measurement problem and that this approach to its "solution" involves a modification of ordinary quantum mechanics.

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Some Remarks on Symmetry Groups*

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A number of general remarks are given in connection with earlier discussions of the outer automorphisms of internal-symmetry groups.

I. INTRODUCTION

Recently it has been suggested[1,2] that the intrinsic properties of internal-symmetry groups are very important. Indeed, a "bootstrap principle" was proposed: The outer automorphisms of degenerate-symmetry groups are themselves symmetries. These symmetries are in general hidden.[3] From a number of conversations and correspondences that we have had, it appears that a few more, largely pedagogical, remarks may be useful in clarifying the ideas in these considerations.

II. THE MEANING OF THE BOOTSTRAP PRINCIPLE

It should be emphasized, first of all, that the proposition in Sec. I is a physical principle, as opposed to a possible mathematical theorem. In fact, it is not difficult to construct mathematical models violating the principle. As such, the bootstrap principle can only be proved or disproved by comparing with reality.

What, then, in discussing a symmetry group G, motivates our consideration of its outer automorphism $\mathrm{Out}(G)$? It turns out that, in applying G to physical problems, $\mathrm{Out}(G)$ has to be reckoned with.

Let us be explicit. When using G, we are interested in its representation:

$$g \rightarrow \mathfrak{D}(g), \quad g \in G. \tag{1}$$

Now, if G has an outer automorphism of the form

$$g \overset{\mathrm{Out}(G)}{\longleftrightarrow} \bar{g}, \tag{2}$$

it is quite clear that there are two possible representations of G corresponding to the same set of matrices $\mathfrak{D}$:

$$\text{(a) } g \to \mathfrak{D}(g),\quad \bar{g} \to \mathfrak{D}(\bar{g}),$$
or (3)
$$\text{(b) } g \to \mathfrak{D}(\bar{g}),\quad \bar{g} \to \mathfrak{D}(g).$$

Further, $\mathfrak{D}(g)$ and $\mathfrak{D}(\bar{g})$ are in general inequivalent[4]:

$$\mathfrak{D}(g) \not\sim \mathfrak{D}(\bar{g}). \tag{4}$$

[Being inequivalent, the use of $\mathfrak{D}(g)$ or $\mathfrak{D}(\bar{g})$ has different physical implications.] The question is: Which representation, $\mathfrak{D}(g)$ or $\mathfrak{D}(\bar{g})$, should one use for g? The bootstrap principle in Sec. I is merely the following statement[5]: It is physically equivalent to use either $\mathfrak{D}(g)$ or $\mathfrak{D}(\bar{g})$.

Mathematically, this physical equivalence means that Out(G) is a symmetry operator. Note that this statement entails that under Out(G) the Hamiltonian remains invariant and the operators transform among themselves. The action of Out(G) on physical state vectors requires some careful analysis, and will be done in Sec. III.

III. HIDDEN SYMMETRIES

Hidden symmetries were defined[3] as symmetries of the Hamiltonian, but not of the physical states. Their nature was fully understood in solid-state physics, and has been extensively discussed. However, there has been some reluctance in using hidden symmetries in particle physics. We will now argue that, fundamentally, hidden symmetries should be the rule, rather than the exception.

It is generally agreed that[6] symmetries refer to the laws of nature, but not the initial conditions. Quantum mechanically, the laws of nature correspond to operators (including the Hamiltonian), while the initial conditions correspond to the state vectors, or wave functions. As such, a symmetry operator maps the whole set of physical operators into themselves, while leaving the Hamiltonian invariant.

The degenerate symmetries (S) are those for which, from a given state $|\alpha\rangle$, we generate other states

$$S|\alpha\rangle = S_{\alpha\beta}|\beta\rangle, \tag{5}$$

with the condition that $|\beta\rangle$ is also one of the states realizable in the same physical system under consideration. Indeed, this very property, plus the superposition principle, "explains"[6] the fact that symmetry principles are far more powerful in quantum physics as compared with classical physics (since most symmetries are degenerate for quantum systems, but very few are degenerate for classical systems). Nonetheless, it is also unnecessary to restrict ourselves to degenerate symmetries exclusively.

Let us consider an explicit example. The binding force of atoms is electromagnetic, and parity is conserved. In fact, the atomic states are eigenstates of P, so that P is surely a degenerate symmetry. However, let us consider an aggregate of atoms, say our left hand, to be denoted by $|\psi_L\rangle$. From $|\psi_L\rangle$ we may also study its excited states, but parity always remains good:

$$[P,H]=0. \tag{6}$$

On the other hand,

$$P|\psi_L\rangle = |\psi_R\rangle, \tag{7}$$

where $|\psi_R\rangle$ is an identical mirror image of $|\psi_L\rangle$ (the right hand). Further, corresponding to each excited state of $|\psi_L\rangle$ we also have one from $|\psi_R\rangle$. However, the state $|\psi_R\rangle$ may or may not exist. Its existence is also unimportant for the description of the behavior of $|\psi_L\rangle$. Therefore, for the aggregate of atoms $|\psi_L\rangle$, P becomes a hidden symmetry. As such, P is not very useful. However, even here it does restrict the possible forms of H by $[P,H]=0$. Further, if $|\psi_L\rangle$ together with its excited states behave in a certain way, then, provided $|\psi_R\rangle$ might be prepared, it must behave identically.

The difference between P in particle physics (where P is supposedly degenerate) and in classical physics (where it is hidden) lies in the use of the superposition principle.[6] Indeed, we could have defined states

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\psi_L\rangle \pm |\psi_R\rangle), \tag{8}$$

which are eigenstates of P. The trouble is that, physically, it is very difficult to prepare the states $|\psi_\pm\rangle$. Alternatively, we may say that the transition probability between $|\psi_L\rangle$ and $|\psi_R\rangle$ is very small, or, there is a selection rule forbidding the transition.[7] This selection rule does not have anything in common with the familiar selection rules in quantum mechanics (such as energy conservation, charge conservation, etc.) It may be called a "macroscopy selection rule." It is due primarily to the complexity of the states[8] $|\psi_L\rangle$ and $|\psi_R\rangle$.

But are there also complicated states in particle physics? Perhaps a prototype is the vacuum state. Because of displacement invariance

$$e^{ip_\mu a_\mu}|0\rangle = |0\rangle, \tag{9}$$

we may say that the vacuum $|0\rangle$ is actually a macroscopic state. A localized "particle" state may then be regarded as a "local excitation" of the vacuum. It is perhaps not surprising that there may exist symmetries G for which $|0\rangle$ is not an eigenstate:

$$G|0\rangle = |\bar{0}\rangle \neq |0\rangle. \tag{10}$$

Indeed, it is not meaningful to superimpose

$$|0\rangle_{\pm}=\frac{1}{\sqrt{2}}\,(|0\rangle \pm |\bar{0}\rangle), \tag{11}$$

since $|0\rangle_{\pm}$ are very difficult to prepare, physically. Even though the "physical vacuum" $|0\rangle$ is not an eigenstate of G (so that G is hidden), much can still be learned from the existence of G. Thus, the possible forms of the Hamiltonian is restrained:

$$[H,G]=0. \tag{12}$$

Further, a nontrivial restriction is placed on the set of physical operators in that they must transform among themselves[9] under G.

Let us now go back to our discussion of the bootstrap principle. When we say that the outer automorphism Out(G) is a hidden symmetry, we are restricting the forms of the Hamiltonian,

$$[H,\mathrm{Out}(G)]=0\,, \tag{13}$$

as well as the possible set of physical operators that must be used.[9] On the other hand, starting from a "physical Hilbert space" for G, Out(G) carries the state vectors outside of the original space. Thus, to properly treat Out(G), we must enlarge the original Hilbert space, even though it was quite adequate as long as we restricted ourselves to operators in G.

IV. AN EXAMPLE

Let us return once more to the well-known discussion of Yang and Tiomno,[10] which is a careful analysis of the following familiar, yet somewhat vague, statement: The parity of half-integral-spin states is not absolute.

Consider the rotational symmetry [(SU(2)] combined with parity (Z_2). Yang and Tiomno defined the two types of spinors ψ_A and ψ_B by

$$P(\psi_A,\psi_B)=(+\psi_A,-\psi_B). \tag{14}$$

The important point is that ψ_A and ψ_B are different when they coexist, but, when isolated, they are not intrinsically different. In other words, the two types are physically equivalent. Or, what amounts to the same thing, the exchange operator f, defined by

$$\psi_A \overset{f}{\longleftrightarrow} \psi_B\,, \tag{15}$$

is a symmetry operator.[11] However, there is no reason to expect that ψ_A and ψ_B must coexist in a given system (in "our" world), or that f must be a degenerate-symmetry operator.

Mathematically, we have the symmetry group $\mathrm{SU}(2)\times Z_2$. The center consists of four elements: 1, $(-1)^{2J}$, P, and $P'\equiv(-1)^{2J}P$. They are the "observables" in the theory. Note that

$$P'(\psi_A,\psi_B)=(-\psi_A,+\psi_B), \tag{16}$$

so that f also effects the exchange of the operators P and P',

$$P \overset{f}{\longleftrightarrow} P', \tag{17}$$

while leaving all other operators invariant; i.e., f is an (and the only[1]) outer automorphism of the group $\mathrm{SU}(2)\times Z_2$. The question is: Can we distinguish, in an intrinsic and absolute way, the two observables P and P'? The preceding discussion says that we cannot, and therefore f is a symmetry operator.

Looked upon in this way, the "bootstrap principle" in Sec. I is really a mathematical formulation and a generalization of Yang and Tiomno's theory of types. As in Eq. (3), to represent g by the inequivalent $\mathfrak{D}(g)$ or $\mathfrak{D}(\bar{g})$ amounts to using two different types of state vectors. The intrinsic physical equivalence of the two different types is reflected in the symmetry Out(G). On the other hand, Out(G) is in general hidden since the two types do not necessarily coexist in the physical system under consideration.

V. AMBIGUITIES OF THE PARITY OPERATOR

The physical equivalence of P and $P'=(-1)^{2J}P$ [Eq. (17)] expresses the ambiguity of the parity operator when we combine the rotational symmetry and the parity. The basis of this result may be traced back to the two-to-one correspondence between "mathematical" and "physical" symmetry operators.[12] Now, in the SU(2) group, the element $(-1)^{2J}$, being in the center of SU(2) and hence commuting with any group element, is represented by $\pm I$, where I denotes the identity matrix. However, under the assumption of SU(2) symmetry, the $\pm$ sign cannot be observed physically since there cannot be transitions between half-integral- and integral-spin states. We must then conclude that, even though 1 and $(-1)^{2J}$ are different mathematical operators, physically they both represent the identity operator.

We turn next to the question of combining P with an arbitrary symmetry group G. One must first identify the enlarged group which contains P and G, and, having done that, ask whether P is well defined.

Let us limit ourselves to the case when G is either the gauge group ($e^{i\theta Y}$) or the U(2) group [hypercharge and isospin with $(-1)^{2I}=(-1)^{Y}$]. The resulting groups are,[13] not unexpectedly, $\mathrm{U}(1)\times Z_2$ and $\mathrm{U}(2)\times Z_2$, where Z_2 is the cyclic group of order 2: $P, P^2=I$.

Given $\mathrm{U}(1)\times Z_2$ and $\mathrm{U}(2)\times Z_2$, is P well defined? Mathews[14] analyzed the first case and concluded that P and $e^{i\pi Y}P=(-1)^Y P$ are physically equivalent. Or, the relative parity between states with even and odd Y values is arbitrary. For $U(2)\times Z_2$, following the arguments of Yang and Tiomno and of Mathews, it is not difficult to convince oneself that

$$P \text{ and } P'=(-1)^Y P=(-1)^{2I}P$$

are physically indistinguishable. This is again based on the lack of transition between half-integral- (odd Y) and integral- (even Y) isospin states, under the assumption of isospin symmetry. Consequently, if isospin symmetry is conserved, then the parity of isospinors is not absolute.

VI. AMBIGUITIES IN BROKEN CHIRAL SYMMETRIES

So far, the equivalence of parity operators does not have any operational consequences. A new element is now injected by considering the chiral rotations. It turns out that the operators P and $(-1)^{2I}P$ are actually[1] transformed into each other by the chiral operator $W=(-1)^{2I^-}$ ($W^2=1$):

$$WPW=(-1)^{2I}P=(-1)^Y P. \tag{18}$$

The physical meaning of W is now clear: It reverses the parity of isospinors. If

$$[H,W]=0, \tag{19}$$

then the parity of isospinors is not absolute; while if

$$[H,W]\neq 0, \tag{20}$$

then it is meaningful to speak of "scalar isospinors" and "pseudoscalar isospinors," so that absolute parity may be assigned to isospinors.

In Sec. V, it was concluded that isospinors cannot have absolute parity if isospin is conserved. Consequently, when the chiral symmetry [SU(2) ×SU(2) or SU(3)×SU(3)] is broken down to U(2), the Hamiltonian must obey

$$WHW=H. \tag{19'}$$

A strong-interaction theory based on Eq. (19′) was discussed earlier.[15,16]

Let us now consider the $(3,\bar{3})+(\bar{3},3)$ model[17] of broken SU(3)×SU(3) symmetry, where the Hamiltonian takes the following form:

$$H=H_0+a(u_0-\sqrt{2}\,u_8)+b(u_0+\tfrac{1}{2}\sqrt{2}\,u_8)$$
$$\equiv H_0+H_1'+H_2'. \tag{21}$$

In this case,[1]

$$WHW=H_0+H_1'-H_2', \tag{22}$$

so that H is nonunique[18] when $b\neq 0$. However, considerable controversy[19-21] was generated as to the physical interpretation of this result. In the following we will turn to a critical review of Refs. 19, 20, and 21.

The physical equivalence of P and $(-1)^{2I}P$, hereafter referred to as the W symmetry, were not challenged in these works. However, the condition $WHW=H$ was regarded as unnecessary.

In Refs. 19 and 20, it was suggested that the nonlinear nature of chiral symmetry made Eq. (19′) unnecessary, while for linear models, the W symmetry was automatically satisfied. Indeed, in a linear model,[19,20] when $WHW\neq H$, the isospinors kaon and κ meson have masses so that

$$M_K=f(b),\quad M_\kappa=f(-b), \tag{23}$$

$$M_K\neq M_\kappa,\quad \text{if } b\neq 0. \tag{24}$$

It was argued that, inasmuch as W effects the interchanges $b\leftrightarrow -b$ [Eq. (22)] and kaon $\leftrightarrow$ κ meson, there is no problem with the W symmetry. Actually this argument amounts to using W twice (which is the identity). In fact, as long as $M_K\neq M_\kappa$, the W symmetry is violated, so that P and $(-1)^{2I}P$ are no longer physically equivalent. (Compare with the charge-conjugation symmetry. If the positron and the electron have unequal masses, the equivalence between particle and antiparticle is surely destroyed.)

What about nonlinear models? In this case, if $WHW\neq H$, then the time dependences of $F_{4,5,6,7}$ and $F^5_{4,5,6,7}$ are different. This means that the "vector" and "axial-vector" isospinors do not behave the same way; i.e., to use P or $(-1)^{2I}P$ is again physically distinguishable.

In Ref. 21 (also Ref. 20), emphasis was laid on the nonuniqueness of H. In particular, Dashen gave a detailed prescription on how to get rid of the nonuniqueness. Three basic assumptions were made:

(1) The physical vacuum $|0\rangle$ is unique.

(2) There is a one-to-one association of H' and $|0\rangle$.

(3) The unique physical $|0\rangle$ is fixed by requiring that, as $H'\to 0$, $SU(3)|0\rangle=|0\rangle$.

Now, one of the basic implications of the transformation W is the physical equivalence[18] of SU(3) and $\widetilde{SU}(3)$, generated by $F_{1,2,3,8}$ and $F^5_{4,5,6,7}$. [Mathematically, SU(3) and $\widetilde{SU}(3)$ are described by different symbols, and can always be distinguished.] It is clear that, if we take assumption (3) for granted [i.e., somehow SU(3), and not $\widetilde{SU}(3)$, can be chosen], the physical equivalence between SU(3) and $\widetilde{SU}(3)$ is *a priori* ruled out. We believe, therefore, that assumption (3) cannot be implemented, physically. (See also Ref. 22.)

Assumption (2) was suggested in analogy with ferromagnetism. In this case, an external magnetic field (along the z axis) fixes both H' and $|0\rangle$ uniquely. In fact, the symmetry group of both H'

and $|0\rangle$ is SO(2) – rotations around the z axis. However, for chiral symmetry H' is supposed to have the symmetry U(2), and $|0\rangle$, an SU(3) group. The W symmetry corresponds to the fact that, given U(2), there are two SU(3)'s in which U(2) may be embedded. The association of H' and $|0\rangle$ is, therefore, not one-to-one.

Finally, we come to the time-honored assumption (1). In our opinion, after we have been dealing with degenerate vacua in chiral theories for so long, there seems to be no reason to insist on (1). Indeed, the interpretation of W as a hidden symmetry calls for a relaxation of this assumption.

Some further insight on Dashen's prescriptions can be gained if we study, instead of broken SU(3)×SU(3), broken SU(2)×SU(2). Let us consider the $(\frac{1}{2},\frac{1}{2})$model of broken SU(2)×SU(2):

$$H=H_0+H', \quad H'\sim(\tfrac{1}{2},\tfrac{1}{2}). \tag{25}$$

[The $(\frac{1}{2},\frac{1}{2})$ model corresponds to the $(3,\bar{3})+(\bar{3},3)$ model of broken SU(3)×SU(3), when $b\neq 0$. Specifically, $H'\sim H_2'$, as defined in Eq. (21).] Here, the physical $|0\rangle$ is an eigenstate of isospin and parity,

$$\mathrm{SU}(2)|0\rangle=P|0\rangle=|0\rangle. \tag{26}$$

However, in SU(2)×SU(2) there is a finite rotation

$$W=e^{i2\pi(I_3)_-}=(-1)^{2I_-} \tag{27}$$

which renders H' nonunique:

$$WHW=H_0-H', \quad H'\sim(\tfrac{1}{2},\tfrac{1}{2}). \tag{28}$$

Further, unlike SU(3)×SU(3) for which $W\mathrm{SU}(3)W=\widetilde{\mathrm{SU}}(3)$, here W commutes with the entire SU(2)×SU(2) group. [The only effect of W is $WPW=(-1)^{2I}P$.] This shows clearly that the $(\frac{1}{2},\frac{1}{2})$ model of broken SU(2)×SU(2) is nonunique, and that properties of $|0\rangle$ (Dashen's prescriptions) cannot resolve this nonuniqueness.[22,23]

The way out of all these difficulties is to abandon the uniqueness of the physical $|0\rangle$ and demand

$$WHW=H, \tag{19'}$$

$$W|0\rangle=|\tilde{0}\rangle\neq|0\rangle. \tag{29}$$

[Since W is a chiral rotation, Eq. (29) is not surprising at all.] In other words, W, which expresses the physical equivalence of P and $(-1)^{2I}P$, is an exact, but hidden symmetry.[24] It is realized by the double degeneracy of the physical vacuum state. Thus, even though SU(3)×SU(3) and SU(2)×SU(2) are broken, a finite chiral rotation $W=(-1)^{2I_-}$ remains invariant. Within the context of broken SU(3)×SU(3), the physical vacua are eigenstates of U(2):

$$\mathrm{U}(2)|0\rangle=|0\rangle, \quad \mathrm{U}(2)|\tilde{0}\rangle=|\tilde{0}\rangle. \tag{30}$$

As $H'\to 0$, one of them becomes an eigenstate of SU(3), and the other, $\widetilde{\mathrm{SU}}(3)$.

Finally, we emphasize that, even though $|0\rangle$ is degenerate, since the degeneracy is finite (twofold), there is no problem of zero-mass (Goldstone) bosons.

VII. FURTHER REMARKS

1. Because of the existence of conflicting symmetries,[2] our discussion would be theoretically inconsistent, unless one takes for granted[13] the sharply distinguishable classes of interactions (the strong, electromagnetic, etc.). For each interaction, there is an exact symmetry group, which gives rise to further exact symmetries from its outer automorphisms.

2. The use of hidden symmetry (or degenerate symmetry) in particle physics is dictated by physical expediency, not by any *a priori* principles. The question at stake is what states are easily prepared (i.e., physically realizable). This "initial condition" dictates whether a symmetry is to be hidden or degenerate.

3. The description of a physical system in quantum mechanics calls for the use of operators and state vectors in a Hilbert space. It is known that the relation between quantum-mechanical symmetry operators and "physical" operators is many-to-one.[12] The use of hidden symmetries suggests that also the quantum-mechanical state vectors and the "physical" states have a many-to-one correspondence. Both possibilities are related to the intrinsic properties of the symmetry group under consideration – the former, its center, and the latter, its outer automorphisms.

4. In our discussions, a very prominent role is played by the global properties of the Lie groups under consideration.[25,26] Much has been learned on continuous symmetries by studying the local (infinitesimal) properties of Lie groups. It is remarkable that the discrete symmetries seem to be equally well understandable in terms of their global properties.

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[1]T. K. Kuo, Phys. Rev. D 4, 3620 (1971).
[2]T. K. Kuo, Phys. Rev. D 4, 3637 (1971).
[3]J. Goldstone, Nuovo Cimento 19, 154 (1961); Y. Nambu, in *Group Theoretical Concepts and Methods in Elementary Particles*, edited by F. Gürsey (Gordon and Breach, New York, 1964).

[4]Given $\mathfrak{D}$, corresponding to any inner automorphism of g: $g \rightarrow g' = \chi g \chi^{-1}$, $\chi \in G$, we also have the representation: $g \rightarrow \mathfrak{D}(g')$. Here $\mathfrak{D}(g)$ and $\mathfrak{D}(g')$ are equivalent since $\mathfrak{D}(g') = \mathfrak{D}(\chi g \chi^{-1}) = \mathfrak{D}(\chi)\, \mathfrak{D}(g)\, \mathfrak{D}(\chi)^{-1}$.

[5]Indeed, if we are only given G, it is rather difficult to do otherwise. If we wish to "break" the physical equivalence of $\mathfrak{D}(g)$ and $\mathfrak{D}(\bar{g})$, something exterior to G must be brought in, so that it might serve to distinguish g from $\bar{g}$.

[6]R. Houtappel, H. Van Dam, and E. P. Wigner, Rev. Mod. Phys. 37, 595 (1965).

[7]If no selection rule exists, then, starting from $|\psi_L\rangle$, it will make transitions to $|\psi_R\rangle$, so that we can always produce the mixed states $|\psi_\pm\rangle$.

[8]Quite generally, as a system becomes more and more complicated, it would be harder and harder to prepare "mixtures." Correspondingly, a given symmetry will start out to be degenerate and pass over into being hidden. For classical systems, most of the familiar symmetries become hidden (for instance, rotational symmetry for an irregular classical object).

[9]Thus, in Ref. 1, the existence of the W symmetry implies that $F_{4,5,6,7}$ and $F^5_{4,5,6,7}$ must exist simultaneously.

[10]C. N. Yang and J. Tiomno, Phys. Rev. 79, 495 (1950).

[11]It should be emphasized that the essentials of a symmetry operator connecting two (nonidentical) physical systems are precisely (a) that the two systems are different, (b) but that they are not intrinsically different, i.e., indistinguishable when isolated from each other.

[12]E. P. Wigner, *Group Theory* (Academic, New York, 1957). See p. 242 and p. 333.

[13]T. D. Lee and G. C. Wick, Phys. Rev. 148, 1385 (1966); T. D. Lee, in *Lectures at the Second Hawaii Topical Conference on Particle Physics*, edited by S. Pakvasa and S. F. Tuan (Hawaii Univ. Press, Honolulu, Hawaii, 1967).

[14]P. T. Mathews, Nuovo Cimento 6, 642 (1957).

[15]T. K. Kuo, Phys. Rev. D 2, 2439 (1970). Note that, in Eq. (34), $2\sqrt{2}$ should read $2\sqrt{3}$.

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[17]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[18]T. K. Kuo, Phys. Rev. D 2, 342 (1970).

[19]S. Okubo and V. Mathur, Phys. Rev. D 2, 394 (1970); S. Okubo, *ibid.* 2, 3005 (1970).

[20]K. Mahanthappa and L. Maiani, Phys. Letters 33B, 499 (1970).

[21]R. Dashen, Phys. Rev. D 3, 1879 (1971).

[22]More explicitly, Dashen argues that one must "match" two SU(3)'s – one for which SU(3) $|0\rangle = |0\rangle$ (as $H' \rightarrow 0$), and the other for which H' transforms as a definite combination of a singlet (u_0) and an octet (u_8). Clearly, such a "matching" does not exist for SU(2) × SU(2). Going back to SU(3) × SU(3), to say that u_8 (and not $\tilde{u}_8 = W u_8 W$) transforms like an octet already assumes a preference of SU(3) over $\widetilde{\text{SU}}(3)$. This, we emphasize, entails a preference of P over $(-1)^{2I} P$.

[23]The reader may wonder about the validity of the theorems on p. 1882 of Ref. 21, which are based implicitly on the uniqueness of the vacuum state. When the vacuum becomes doubly degenerate, it is not difficult to see that $F(\vec{\omega})$ has two equal local minima, one for $\vec{\omega} = 0$, the other at $e^{i\vec{\omega}\cdot\vec{Q}} = W$.

[24]It should be emphasized that the interpretation of Eqs. (19) and (20) is independent of whether W is degenerate or hidden. Thus, in Sec. III, the interpretation of $[P, H] = 0$ or $[P, H] \neq 0$ is the same, whether we are studying a classical system (in which P is hidden) or a quantum system (in which P may be degenerate).

[25]For instance, in Ref. 2 the gauge groups $e^{i\theta F}$ and $e^{i\theta Y}$ are found to be compact (a global property) primarily because SU(2) is compact. [Their compactness is correlated by the global relations $(-1)^{2J} = (-1)^F$ and $(-1)^{2I} = (-1)^Y$.] Thus, the quantization of F and Y is traced back to the compactness of the SU(2) group. [Compare with C. N. Yang, Phys. Rev. D 1, 2360 (1970).]

[26]Since $[H, (I_3)_-] \neq 0$, the (global) commutation relation $[H, W] = [H, e^{i2\pi(I_3)_-}] = 0$ may appear strange. Actually, such behavior (Lie algebra versus Lie group) is by no means exceptional. Thus, while $[H_{\text{em}}, I_{1,2}] \neq 0$, we have $[H_{\text{em}}, e^{i2\pi I_{1,2}}] = 0$.