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Comments and Addenda

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Some Difficulties with the Use of the Kemmer-Duffin Formalism for Pseudoscalar Mesons*

Nilendra G. Deshpande and Peter C. McNamee†
Northwestern University, Department of Physics, Evanston, Illinois 60201
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While the Kemmer-Duffin formalism for pseudoscalar mesons eliminates some difficulties in the analysis of K_{l3} form factors, it gives rise to difficulties in other areas. It is shown that this formalism leads to: (i) a linear Gell-Mann–Okubo mass formula for mesons; (ii) an incompatibility between SU(3)-symmetric meson-nucleon couplings and experiment; (iii) (3*,3) breaking of chiral symmetry ($H'=u^0+cu^8$) leads to $c=-0.9$ and $f_K/f_\pi=(m_\pi/m_K)^{1/2}$.

In a recent letter,[1] it was argued that the Kemmer-Duffin formalism,[2] when used to describe pseudoscalar mesons, leads to a more satisfactory theory of K_{l3} form factors than does the conventional Klein-Gordon formalism. In particular, the theory successfully accounts for the large negative value of the parameter $\xi=f_-(0)/f_+(0)$ involved in the K_{l3} analysis. It has been further argued that this formalism removes the discrepancy between the value of the Cabibbo angle θ_V obtained from $0^+\to 0^+$ β decays and the value obtained from the Klein-Gordon K_{e3} description.[3]

In this note we show that further consideration of this formalism leads to certain difficulties which cannot be resolved easily. We find: (i) Meson masses satisfy a Gell-Mann–Okubo mass formula which is *linear* in the meson masses. (ii) SU(3)-invariant meson-baryon coupling in this formalism corresponds to an *octet-broken* scheme in the conventional (Klein-Gordon) parametrization; this is inconsistent with experiment. (iii) Incorporation of these mesons in a chiral symmetry with (3*, 3) breaking leads to a linear mass formula, a value of $c=-0.9$, and $f_K/f_\pi=(m_\pi/m_K)^{1/2}$.

Although these difficulties could be overcome by such measures as introducing a large η-η' mixing and giving up the idea of SU(3)-invariant meson-baryon coupling constants in the Kemmer formalism, much of the simplicity which originally recommended the use of the Kemmer formalism[1,3] would be lost.

The Kemmer-Duffin formalism can be derived from the Lagrangian density[4]

$$\mathcal{L}=\tfrac{1}{2}\sum_a \overline{\psi}^a(i\partial_\mu\beta^\mu-m)\psi^a+\mathcal{L}_{\text{int}}\,, \tag{1}$$

where ψ is a real field; $\overline{\psi}=\psi^\dagger\eta=\psi^\dagger(2\beta^0\beta^0-1)$. The 5×5 matrices which describe spin-zero fields satisfy the algebra[5]

$$\beta^\mu\beta^\nu\beta^\rho+\beta^\rho\beta^\nu\beta^\mu=g^{\mu\nu}\beta^\rho+g^{\rho\nu}\beta^\mu\,. \tag{2}$$

In the case of a free field, the correspondence between the Kemmer field and the usual Klein-Gordon field φ is ($\alpha=0,1,2,3,4$)

$$\psi_\alpha=m^{-1/2}\delta_{\alpha\mu}\partial_\mu\varphi+m^{1/2}\delta_{\alpha4}\varphi\,. \tag{3}$$

The vector currents in this formalism are

$$V_\mu^a=\tfrac{1}{2}if^{abc}\overline{\psi}^c\beta_\mu\psi^b \tag{4a}$$

or, in terms of the Klein-Gordon fields,

$$V_\mu^a=f^{abc}(m_b/m_c)^{1/2}\varphi^b\partial_\mu\varphi^c\,. \tag{4b}$$

It is important to note that the essential difference between the formalisms is that in the Kemmer theory it is $m_a^{1/2}\varphi^a$ rather than φ^a that transform as representations of SU(3). It can be shown that the matrix element of the current defined in Eq. (4b) between states of K and π leads to results in Ref. 1 in the lowest order of perturbation theory; and further, the matrix element of the divergence of the current leads to a zero at $q^2=(m_K+m_\pi)^2$; this is a special feature of this model, as has been shown in Ref. 1.

(i) Gell-Mann–Okubo mass formula. An octet breaking of the Hamiltonian leads naturally to a linear mass formula, as is evident if a symmetry-breaking term $\frac{1}{2}\Delta m d^{8ab}\overline{\psi}^a\psi^b$ is added to the Lagrangian. More formally, we take the expectation value of $\partial_\mu V^\mu$ between meson states in the limit $q^2 \to 0$. If[5]

$$\langle M^a(p)|V_\mu^c|M^b(p')\rangle = i\sqrt{2}\, f^{cba}\overline{u}^a(p)\left(\beta_\mu g_V(q^2) + \frac{q_\mu g_S(q^2)}{m_a+m_b}\right) \times u^b(p')\left(\frac{m_a m_b}{E_a E_b}\right)^{1/2}(2\pi)^{-3}, \quad (5)$$

then

$$\lim_{q^2\to 0}\langle M^a(p)|\partial\cdot V^c|M^b(p')\rangle = \sqrt{2}\, f^{cba} g_V(0)(m_b - m_a)\overline{u}^a(p)u^b(p')\left(\frac{m_a m_b}{E_a E_b}\right)^{1/2}(2\pi)^{-3}. \quad (6)$$

If the Hamiltonian is octet-broken, then $\partial\cdot V^c$ is a member of the same octet:

$$\partial\cdot V^c = -i[F^c, H^8] = f^{c8d}H^d.$$

If $g_V(0)$ is SU(3)-symmetric as is assumed,[1] use of the Wigner-Eckart theorem gives

$$4m_K = 3m_\eta + m_\pi. \quad (7)$$

If we assume large η-η' mixing, then

$$4m_K = 3(\cos^2\theta m_\eta + \sin^2\theta m_{\eta'}) + m_\pi, \quad (8)$$

resulting in $\theta \cong 24°$.

(ii) Meson-baryon coupling. The SU(3)-invariant Yukawa coupling of the Kemmer fields is[6]

$$\mathcal{L}_{\rm int} = (Af^{abc} + Bd^{abc})\overline{N}^a\gamma_5 N^b\,\overline{\mathfrak{u}}\psi^c, \quad (9a)$$

or, in terms of the Klein-Gordon fields,

$$\mathcal{L}_{\rm int} = (Af^{abc} + Bd^{abc})\overline{N}^a\gamma_5 N^b (m_c/2)^{1/2}\varphi^c. \quad (9b)$$

The presence of the factor $m_c^{1/2}$ in Eq. (9b) corresponds to octet-broken meson-baryon coupling for the conventional Klein-Gordon formalism. One can determine the $D/(F+D)$ ratio either from the known couplings $p\pi^0 p$ and $\Sigma^0\pi^0\Lambda$ or from the couplings $\Sigma^0 K^- p$ and $pK^-\Lambda$; the $D/(F+D)$ ratio that results is 0.75 ± 0.25. Assuming the known πNN coupling constant, the scheme in Eq. (9) leads to a value for the $pK^-\Lambda$ coupling constant of -30 ± 5. This is clearly inconsistent with experiment, the favored value being -15 ± 2.[7]

(iii) Chiral symmetry. Following the work of Gell-Mann, Oakes, and Renner[8] and of Glashow and Weinberg,[9] we assume

$$H = H_0 - u^0 - cu^8, \quad (10)$$

where H_0 is SU(3)×SU(3)-invariant and where u^a (and v^a) transform as members of the $(3^*, 3)$ representation. Using the PCAC (partially conserved axial-vector current) definition[6]

$$\partial\cdot A^a = 2^{-1/2} g_a m_a{}^2\,\overline{\mathfrak{u}}\psi^a \quad (11a)$$

$$= \tfrac{1}{2} g_a m_a{}^{5/2}\varphi^a, \quad (11b)$$

we closely parallel the work of Ref. 7. We first take the limit[10]

$$\lim_{p\to 0}\langle M^a(p)|u^0 + cu^8|M^a(p')\rangle = -m_a\overline{\mathfrak{u}}\,u(p')\left(\frac{m_a{}^2}{E_a E_a'}\right)^{1/2}(2\pi)^{-3}. \quad (12)$$

We assume SU(3) invariance for the matrix element (ignoring η-η' mixing):

$$\langle M^a(p)|u^c|M^b(p')\rangle = (\alpha\delta^{ab}\delta^{c0} + \beta d^{abc})\overline{u}^a(p)u^b(p')\left(\frac{m_a m_b}{E_a E_b}\right)^{1/2}(2\pi)^{-3}, \quad (13)$$

and neglect the variation of α, β with q^2. Taking the $p\to 0$ limit of the left-hand side of Eq. (13), we find

$$\lim_{p\to 0}\langle M^a(p)|u^c|M^b(p')\rangle = \sqrt{2}\, d^{acd}(g_a m_a)^{-1}\langle 0|v^d|M^b(p')\rangle\left(\frac{m_a}{E_a}\right)^{1/2}(2\pi)^{-3/2}. \quad (14)$$

Defining

$$\langle 0|v^d|M^b(p')\rangle = K^{db}\overline{\mathfrak{u}}\,u(p')\left(\frac{m_b}{E_b}\right)^{1/2}(2\pi)^{-3/2} \quad (15)$$

and collecting the information in Eqs. (12)–(14), we find the following results:

(a) A linear mass formula, in agreement with Eq. (7);

$$m_a = m_{\rm av} + d^{8aa}\Delta m.$$

(b) $\alpha = 0$, and hence $m_{\rm av} = \beta(\frac{2}{3})^{1/2}$. Since $\Delta m = c\beta$, we find[11]

$$c = (\tfrac{2}{3})^{1/2}\frac{\Delta m}{m_{\rm av}} \cong -0.9.$$

(c) $g_\pi m_\pi \simeq g_K m_K \simeq g_\eta m_\eta$.

(d) $K^{aa} = (gm)\Delta m/c$, independent of a; $K^{08} = 0$.

Result (b) indicates that if the Kemmer formalism describes the pseudoscalar mesons, the SU(2)×SU(2) limit (i.e., $c = -\sqrt{2}$) is not very close to the physical world. Result (c) is more readily compared with data. Using the usual Klein-Gordon definition of PCAC,

$$\partial \cdot A^a = 2^{-1/2} f_a m_a^{\,2} \varphi^a , \tag{16}$$

and taking normalizations into account, we find

$$f_a = g_a (m_a/2)^{1/2} . \tag{17}$$

Hence

$$f_K/f_\pi = (m_\pi/m_K)^{1/2} \simeq 0.53 , \tag{18}$$

which is in decided disagreement[12] with the experimental value of ~1.28. It should be noted, however, that the Callan-Treiman relation derived in the Kemmer formalism by Ref. 1,

$$\hat{f}_+(m_K^{\,2}) + \hat{f}_-(m_K^{\,2}) = (1/\sqrt{2}) f_K/f_\pi , \tag{19}$$

is considerably improved. If it is assumed that at $p_\pi^{\,2}=0$ the form factors $\hat{f}_\pm$ vary slowly with q^2 $[\hat{f}_\pm(m_K^{\,2}) \simeq \hat{f}_\pm(0)]$, then the left-hand side of the equation is $(1/\sqrt{2})(0.39 \pm 0.20)$, while the right-hand side of the equation is $(1/\sqrt{2})(0.53)$, and not the usual value of $(1/\sqrt{2})(1.28 \pm 0.06)$.

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†Present address: Instituut voor Theoretische Fysica, Universiteit Leuven, Louvain, Belgium.

[1]E. Fischbach *et al.*, Phys. Rev. Letters 26, 1200 (1971).

[2]R. J. Duffin, Phys. Rev. 54, 1114 (1938); N. Kemmer, Proc. Roy. Soc. (London) A173, 91 (1939); F. J. Belinfante, Physica 6, 849 (1939).

[3]E. Fischbach *et al.*, Phys. Rev. Letters 27, 1403 (1971).

[4]The notation has been changed slightly from that of Ref. 1 in order to accommodate the use of the metric $g^{00}=1$, $g^{11}=g^{22}=g^{33}=-1$. A fuller description of the formalism can be found in N. Deshpande and P. McNamee, Phys. Rev. D (to be published).

[5]A convenient representation is

$$\beta^\mu_{\alpha\beta} = -i[\delta_{\alpha\mu}\delta_{\beta 4} - \epsilon(\mu)\delta_{\alpha 4}\delta_{\beta\mu}] ,$$

$$\eta_{\alpha\beta} = 2\delta_{\alpha 0}\delta_{\beta 0} + 2\delta_{\alpha 4}\delta_{\beta 4} - \delta_{\alpha\beta} ,$$

where $\alpha, \beta = 0,1,2,3,4$ and $\mu, \nu = 0,1,2,3$; $\epsilon(\mu) = g^{\mu\mu}$ (μ not summed). The plane-wave solution of the Kemmer equation

$$(i\partial_\mu \beta_\mu - m)\psi = 0$$

is

$$\psi_p(x) = u(p) e^{-ipx} (m/p^0)^{1/2} (2\pi)^{-3/2} ,$$

with

$$u_\alpha(p) = (-ip_\mu \delta_{\alpha\mu} + m\delta_{\alpha 4})(2m^2)^{-1/2} .$$

[6]The "spinor" $\mathfrak{U} = u(p=0)$ is Lorentz-invariant and hence $\overline{\mathfrak{U}}\psi$ is a pseudoscalar; $\mathfrak{U}_\alpha = 2^{-1/2}\delta_{\alpha 4}$.

[7]For a complete analysis of the $D/(F+D)$ ratio, see R. D. Field and J. D. Jackson, Phys. Rev. D 4, 693 (1971). See also J. K. Kim, Phys. Rev. Letters 19, 1079 (1967); C. H. Chan and F. T. Meiere, *ibid.* 20, 588 (1968), and Phys. Letters 28B, 125 (1968); N. Zovko, *ibid.* 23, 143 (1966).

[8]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[9]S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[10]The reduction formula for Kemmer fields is

$$\langle BM^a(p)|J|A\rangle = i\bar{u}(p)(m - p_\mu \beta^\mu) \times \int d^4x\, e^{ip\cdot x} \langle B|T(\psi^a(x)J)|A\rangle (m/p^0)^{1/2}(2\pi)^{-3/2} .$$

[11]Δm and $m_{\rm av}$ are calculated from the π and K masses.

[12]Disagreement is even worse with the value of 1.45 ±0.05 advocated in Ref. 3.