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Quantum stress tensor in the three-dimensional black hole

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The quantum stress tensor $\langle T_{\mu\nu} \rangle$ is calculated in the (2+1)-dimensional black hole found by Banados, Teitelboim, and Zanelli. The Green's function, from which $\langle T_{\mu\nu} \rangle$ is derived, is obtained by the method of images. For the nonrotating black hole, it is shown that $\langle T_{\mu\nu} \rangle$ is finite on the event horizon, but diverges at the singularity. For the rotating solution, the stress tensor is finite at the outer horizon, but diverges near the inner horizon. This suggests that the inner horizon is quantum mechanically unstable against the formation of a singularity.

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Recently, Banados, Teitelboim, and Zanelli [1] found a black hole solution in 2+1 dimensions which shares many of the features of its (3+1)-dimensional counterpart [2]. In particular, the static solution has a singularity and event horizon, while the rotating Kerr-like black-hole possesses outer and inner horizons and an ergosphere. Asymptotically, however, the 2+1 solution is not flat, but approaches anti–de Sitter space [3]. 2+1 dimensions provides a simpler setting than 3+1 and possibly a more realistic one than 1+1 [4] in which to study the quantum properties of black holes, and specifically, the end point of black hole evaporation. Such an investigation should begin with the quantum stress tensor $\langle T_{\mu\nu} \rangle$ which describes the quantum effects of the black hole on a propagating field in a way that allows one to analyze the back reaction. Provided it can be properly renormalized, $\langle T_{\mu\nu} \rangle$ is a well defined local quantity in contrast with particle number which is not, in general, a meaningful concept in curved spacetime. Another motivation for studying $\langle T_{\mu\nu} \rangle$ in the rotating black hole is to investigate the quantum stability of the inner horizon. The maximally extended Reissner-Nordström and Kerr solutions include an infinite number of asymptotic regions which in principle could be accessed. However, it has been shown that since the inner horizon is an infinite blueshift surface, classical perturbations will diverge there [5], and the associated back reaction will produce a singularity [6]. Quantum effects for the (1+1)-dimensional analogue of the Reissner-Nordström solution were investigated in [7] where it was shown that $\langle T_{\mu\nu} \rangle$ diverges near the inner horizon. Attempts to include quantum corrections in 3+1 dimensions [8] are somewhat inconclusive suggesting that the classical instability either is enhanced or is dampened resulting in a regular spacetime. In this paper, the exact expression for the quantum stress tensor is found for the rotating (2+1)-dimensional black hole and is shown to diverge near the inner horizon. An estimation of the back reaction suggests that the inner horizon will be replaced by a curvature singularity. We use units in which $\hbar = c = G = 1$.

The (2+1)-dimensional black hole solution found by Banados, Teitelboim, and Zanelli [1] is most easily described as three-dimensional anti–de Sitter space (AdS_3) identified under a discrete subgroup of its isometry group. Recall that AdS_3 is the three-dimensional hypersurface

$$-T_1^2+X_1^2-T_2^2+X_2^2=-l^2 \tag{1}$$

embedded in four-dimensional flat space with metric η_{ab}:

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$$ds^2 = -dT_1^2 + dX_1^2 - dT_2^2 + dX_2^2 \tag{2}$$

where $l = (-\Lambda)^{-1/2}$. The hypersurface (1) is a pseudohyperbolic analogue of a three-sphere with radius vector $x^a \equiv (T_1, X_1, T_2, X_2)$, radius $\sqrt{-x^a x_a} = l$, and constant curvature $R = -6/l^2$. We will use lower case Latin indices for the four-dimensional embedding space and lower case Greek indices for AdS_3. The isometry group of AdS_3 is SO(2,2) and corresponds to the subgroup of the isometry group of the embedding space which leaves (1) invariant. Since boosts and rotations in two-dimensional planes generate the isometry group, the simplest coordinate systems for AdS_3 parametrize these symmetries. As we will see, the black hole solution is constructed by identifying the parameters describing boosts in the (T_1, X_1) and (T_2, X_2) planes. Thus, it is in terms of these boost parameters that we wish to express the metric for AdS_3. We view it in terms of two copies of 1+1 Minkowski space, M_1 with coordinates (T_1, X_1) and M_2 with coordinates (T_2, X_2) with the constraint (1) $\rho_1 + \rho_2 = l^2$ where $\rho_i = T_i^2 - X_i^2$. In each space M_i, one can define Rindler coordinates

$$T_i = \sqrt{\rho_i} \cosh\chi_i \;, \quad X_i = \sqrt{\rho_i} \sinh\chi_i \;, \qquad \rho_i > 0 \;, \quad -\infty < \chi_i < \infty \;,$$
$$T_i = \sqrt{-\rho_i} \sinh\chi_i \;, \quad X_i = \sqrt{-\rho_i} \cosh\chi_i \;, \qquad \rho_i < 0 \;, \quad -\infty < \chi_i < \infty \;, \tag{3}$$

valid in the light-cone interior ($\rho_i > 0$) and exterior ($\rho_i < 0$), respectively. Defining $\chi_1 \equiv \phi$ and $\chi_2 \equiv t$, we see that there are three qualitatively distinct regions: (I) $\rho_1 > l^2$ ($\rho_2 < 0$), (II) $0 < \rho_1, \rho_2 < l^2$, and (III) $\rho_1 < 0$ ($\rho_2 > l^2$), in which the vectors $\partial/\partial\phi$ and $\partial/\partial t$ are spacelike and timelike, spacelike and spacelike, and timelike and spacelike, respectively. It is natural to view I as the asymptotic region of the spacetime. Substituting (3) in (2) with $r^2 \equiv \rho_1 = l^2 - \rho_2$, one obtains the metric for AdS_3:

$$ds^2 = -\left[\frac{r^2}{l^2} - 1\right] dt^2 + \left[\frac{r^2}{l^2} - 1\right]^{-1} dr^2 + r^2 d\phi^2 \;, \qquad t, \phi \in (-\infty, \infty) \tag{4}$$

valid in regions I and II. Since t and ϕ parametrize boosts, they take on all real values.

The black hole solution is now constructed by making some combination of ϕ and t periodic. For the static black hole with mass M, one identifies ϕ with period $2\pi\sqrt{M}$. This is somewhat analogous to the identification which leads to the static cone solution in 2+1 gravity without a cosmological constant [9]. A salient difference, however, is that the cone reduces to flat space as $M \to 0$, while AdS_3, the covering space of the black hole, is recovered as $M \to \infty$. One would expect the event horizon and singularity of the black hole to have a natural geometric interpretation in terms of AdS_3. Indeed, the event horizon is located at ($r = l$) and coincides with the boundary between regions I and II in AdS_3 as well as with the light cone in the 1+1 space M_2. The black hole singularity is located at $r = 0$ corresponding to the boundary between regions II and III and to the light cone in M_1. $r = 0$ is not a curvature singularity since the curvature is bounded and in fact, constant in AdS_3. It is, however, a singularity because there are inextendible incomplete geodesics. $r = 0$ is directly analogous to the Misner space light cone [10] on which incomplete null geodesics pile up. Asymptotically, the black hole solution approaches anti–de Sitter space.

The black hole with nonzero angular momentum J is obtained from (4) by making a linear combination of ϕ and t periodic: $(t, r, \phi) \sim (t - nl\alpha_-, r, \phi + n\alpha_+)$ where

$$\alpha_\pm = \pi(\sqrt{M + J/l} \pm \sqrt{M - J/l}) \;. \tag{5}$$

It is possible to transform to coordinates $(\tilde{t}, \tilde{r}, \tilde{\phi})$:

$$t = \frac{1}{2\pi}(\alpha_+ \tilde{t} - \alpha_- l\tilde{\phi}) \;,$$
$$\phi = \frac{1}{2\pi}(\alpha_+ \tilde{\phi} - \alpha_- \tilde{t}/l) \;, \tag{6}$$
$$r^2 = \frac{(2\pi\tilde{r})^2 - \alpha_-^2 l^2}{\alpha_+^2 - \alpha_-^2} \;,$$

in terms of which the metric (4) becomes

$$ds^2 = -\left[\frac{\tilde{r}^2}{l^2} - M\right] d\tilde{t}^2 - J\, d\tilde{t}\, d\tilde{\phi} + \left[\frac{\tilde{r}^2}{l^2} - M + \frac{J^2}{4\tilde{r}^2}\right]^{-1} d\tilde{r}^2 + \tilde{r}^2 d\tilde{\phi}^2 \tag{7}$$

and $\tilde{\phi}$ is periodic in 2π. The rotating solution possesses both an outer and inner horizon at $\tilde{r} = \alpha_+ l/2\pi$ ($r = l$) and $\tilde{r} = \alpha_-/2\pi$ ($r = 0$) corresponding, respectively, to the boundaries between regions I and II and between II from III in AdS_3. In addition, the region $\alpha_+ l/2\pi < \tilde{r} < \sqrt{M}\, l$ defines an ergosphere, in which the asymptotic Killing field $\partial/\partial\tilde{t}$ is spacelike. Finally, one should note that in contrast with the static $J = 0$ black hole, the rotating solution is geodesically complete.

The points identified in the rotating black hole are related by an element of SO(2,2) which as a matrix acting on the embedding space coordinates $x^a \equiv (T_1, X_1, T_2, X_2)$ takes the form

$$\Lambda^a_b \equiv \begin{bmatrix} \cosh\alpha_+ & \sinh\alpha_+ & 0 & 0 \\ \sinh\alpha_+ & \cosh\alpha_+ & 0 & 0 \\ 0 & 0 & \cosh\alpha_- & -\sinh\alpha_- \\ 0 & 0 & -\sinh\alpha_- & \cosh\alpha_- \end{bmatrix} . \tag{8}$$

For $J = 0$ ($\alpha_- = 0$), Λ reduces to a boost in the M_1 space, or equivalently a translation in ϕ, and has fixed points coinciding with the singular surface $r = 0$. For $J \neq 0$, Λ has no fixed points accounting for the nonsingular nature of the rotating solution.

We now introduce a propagating quantum field in the black hole background and calculate its Green's function.

Consider a conformally coupled massless scalar field ϕ governed by the action

$$S=-\int[\tfrac{1}{2}(\nabla\phi)^2+\tfrac{1}{16}R\phi^2]\sqrt{g}\,d^3x \tag{9}$$

with R the scalar curvature. We first review the construction of the Green's function in AdS_3, the covering space of the black hole [11]. AdS_3 is a static spacetime with a globally defined timelike Killing field corresponding to the generator of rotations in the (T_1,T_2) plane in the embedding space. There is therefore a natural vacuum state defined by modes which are positive frequency with respect to this time parameter. Since anti–de Sitter space is not globablly hyperbolic, it is important to address the issue of boundary conditions at infinity. AdS_3 can be conformally mapped to half of the Einstein static universe with infinity mapped to the equator [10]. Therefore, solutions to the equations of motion in one space can be mapped to solutions in the other, and similarly, boundary conditions at infinity correspond to conditions on the fields at the equator. As discussed in [11], there are three natural choices of boundary conditions. The first, which is known as "transparent," simply corresponds to quantizing the field using modes which are smooth on the entire Einstein static universe. The other two boundary conditions are obtained by imposing Dirichlet or Neumann conditions on the field at the equator in the Einstein static universe. The Green's function is given by

$$\bar{G}_\lambda(x,x')=\frac{1}{4\pi}\frac{1}{|x-x'|}+\frac{\lambda}{4\pi}\frac{1}{|x+x'|} \tag{10}$$

with $\lambda=0,1,-1$ for transparent, Neumann, and Dirichlet boundary conditions, respectively. Observe that $|x-x'|\equiv[(x-x')^a(x-x')_a]^{1/2}$ is the chordal distance between x and x' in the four-dimensional embedding space and not the distance in AdS_3. The second term in (10) is obtained from the first by the antipodal transformation $x'\to -x'$, a discrete isometry of AdS_3. In this paper, we will be considering only the $\lambda=0$ Green's function corresponding to transparent boundary conditions

$$\bar{G}(x,x')=\frac{1}{4\pi}\frac{1}{|x-x'|}\ . \tag{11}$$

Note that the Green's function coincides with its form in three-dimensional Minkowski space. This is expected as ϕ is conformally coupled and AdS_3 is conformally flat. We now verify that the Green's function satisfies the ϕ equation of motion as derived from (9):

$$\left[\nabla^\mu\nabla_\mu+\frac{3/4}{l^2}\right]\bar{G}(x,x')=0\ ,\quad x\neq x'\ . \tag{12}$$

This is most easily checked by expressing the wave operator $\nabla^\mu\nabla_\mu$ in AdS_3 in terms of derivatives ∂_a in the embedding space. $P^{ab}=\eta^{ab}+x^ax^b/l^2$ satisfies $P^{ab}x_b=0$ and is a projection operator for AdS_3. Applying it to the wave operator $\partial^a\partial_a$, one obtains

$$\begin{aligned}\nabla^\mu\nabla_\mu&=P^{ab}\partial_a(P_b^c\partial_c)\\&=P^{ab}\partial_a\partial_b+3\frac{x^a}{l^2}\partial_a\ .\end{aligned} \tag{13}$$

Using this one verifies that (11) satisfies (12). Since the black hole solution corresponds to AdS_3 with discrete identifications, the Green's function $G(x,x')$ for the black hole can be obtained from the Green's function (11) for its covering space by the method of images [12]. Since the images of x' are Λ^nx' with Λ given in (8), the Green's function is

$$G(x,x')=\sum_{n=-\infty}^{\infty}\bar{G}(x,\Lambda^nx')=\frac{1}{4\pi}\sum_{n=-\infty}^{\infty}\frac{1}{|x-\Lambda^nx'|}\ . \tag{14}$$

The contributions from the nth and $-n$th terms ensure that (14) is symmetric in x and x'.

The quantum stress tensor can now be obtained from $G(x,x')$. Varying the action (9) with respect to $g_{\mu\nu}$ yields

$$\begin{aligned}T_{\mu\nu}=&\tfrac{3}{4}\nabla_\mu\phi\nabla_\nu\phi-\tfrac{1}{4}g_{\mu\nu}(\nabla\phi)^2-\tfrac{1}{4}\phi\nabla_\mu\nabla_\nu\phi\\&+\tfrac{1}{4}g_{\mu\nu}\phi\nabla^\lambda\nabla_\lambda\phi+\tfrac{1}{8}G_{\mu\nu}\phi^2\end{aligned} \tag{15}$$

with $G_{\mu\nu}$, the Einstein tensor for the background spacetime. It follows from the equation of motion for ϕ that $T_{\mu\nu}$ is traceless and conserved. The quantum stress tensor $\langle T_{\mu\nu}\rangle$ is obtained by point splitting (15) and then taking its expectation value. Using the ϕ equation of motion in the fourth term, and substituting in $G_{\mu\nu}=l^{-2}g_{\mu\nu}$ for AdS_3, one obtains

$$\begin{aligned}\langle T_{\mu\nu}\rangle=\lim_{x'\to x}\Bigg[&\frac{3}{4}\nabla_\mu^x\nabla_\nu^{x'}G-\frac{1}{4}g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha^x\nabla_\beta^{x'}G\\&-\frac{1}{4}\nabla_\mu^x\nabla_\nu^xG-\frac{1}{16l^2}g_{\mu\nu}G\Bigg]\end{aligned} \tag{16}$$

in terms of the Green's function (14). The renormalization of the stress tensor, ordinarily a difficult procedure in 3+1 dimensions [13], is achieved here by simply subtracting off the coincident $n=0$ term in the image sum (14) [14]. Substituting (14) into (16) and using $\nabla_\mu\nabla_\nu x^a=g_{\mu\nu}x^a/l^2$, one eventually finds

$$\begin{aligned}\langle T_{\mu\nu}\rangle&=\frac{3}{16\pi}\sum_{n\neq0}(S_{\mu\nu}^n-\tfrac{1}{3}g_{\mu\nu}g^{\lambda\rho}S_{\lambda\rho}^n)\ ,\\S_{\mu\nu}^n&=\partial_\mu x^a\partial_\nu x^bS_{ab}^n\ ,\\S_{ab}^n&=\frac{(\Lambda^n)_{ab}}{|x-\Lambda^nx|^3}\\&\quad+\frac{3(\Lambda^n)_{ac}x^c(\Lambda^{-n})_{bd}x^d-(\Lambda^n)_{ac}x^c(\Lambda^n)_{bd}x^d}{|x-\Lambda^nx|^5}\ .\end{aligned} \tag{17}$$

$S_{\mu\nu}^n$ is the pull back to AdS_3 of S_{ab}^n.

The stress tensor (17) can be evaluated in a particular set of coordinates y^μ in AdS_3 by substituting in the corresponding embedding $x^a=x^a(y^\mu)$. For the static $J=0$ $(\alpha_-=0)$ black hole in coordinates (t,r,ϕ) (4), (17) takes the form

$$\langle T^{\nu}_{\mu} \rangle = \frac{A(M)}{r^3}\mathrm{diag}(1,1,-2),$$

$$A(M) \equiv \frac{\sqrt{2}}{32\pi} \sum_{n=1}^{\infty} \frac{\cosh 2n\pi\sqrt{M}+3}{(\cosh 2n\pi\sqrt{M}-1)^{3/2}}, \quad (18)$$

where M is the black hole mass. Since the series converges exponentially for all real M, the stress tensor is finite everywhere except near the singularity where it diverges as r^{-3}. The divergence there arises from the fact that since $r=0$ remains invariant under the action of Λ, the denominator in the Green's function (14) vanishes. Even though the coordinates (t,r,ϕ) break down near the event horizon, it is clear that the scalar $\langle T_{\mu\nu}\rangle\langle T^{\mu\nu}\rangle$ is smooth there. For $M \gg 1$, the first term in the series gives the leading order behavior $A(M) \sim e^{-\pi\sqrt{M}}$. Recall that as $M \to \infty$, ϕ becomes unidentified and AdS_3 is recovered. Since $\langle T_{\mu\nu}\rangle$ was renormalized with respect to AdS_3, it vanishes in this limit. For small M, the series can be approximated by an integral yielding $A(M) \sim M^{-3/2}$. From the invariance of the vacuum under the anti–de Sitter group, one would expect $\langle T^{\nu}_{\mu}\rangle \sim \delta^{\nu}_{\mu}$. However, the identification in ϕ breaks the underlying symmetry picking out ϕ as a preferred direction. $\langle T^{\nu}_{\mu}\rangle$ is traceless and conserved. One should note, however, that in analogy to the Casimir effect the energy density is negative.

For the rotating black hole, the stress tensor (17) becomes

$$\langle T^t_t \rangle = \frac{1}{4\pi}\sum_{n=1}^{\infty} [(\cosh n\alpha_+ + 2\cosh n\alpha_- - 3)r^2 - 2(\cosh n\alpha_- - 1)l^2]\frac{c_n}{|d_n|^{5/2}},$$

$$\langle T^r_r \rangle = \frac{1}{4\pi}\sum_{n=1}^{\infty} [(\cosh n\alpha_+ - \cosh n\alpha_-)r^2 + (\cosh n\alpha_- - 1)l^2]\frac{c_n}{|d_n|^{5/2}},$$

$$\langle T^{\phi}_{\phi} \rangle = \frac{1}{4\pi}\sum_{n=1}^{\infty} [-(2\cosh n\alpha_+ + \cosh n\alpha_- - 3)r^2 + (\cosh n\alpha_- - 1)l^2]\frac{c_n}{|d_n|^{5/2}},$$

$$\langle T^{\phi}_{t} \rangle = \frac{3}{4\pi}\sum_{n=1}^{\infty} \sinh n\alpha_+ \sinh n\alpha_- \frac{r^2/l^2-1}{|d_n|^{5/2}} l, \quad (19)$$

$$\langle T^r_t \rangle = \langle T^{\phi}_r \rangle = 0,$$

$$c_n \equiv \cosh n\alpha_+ + \cosh n\alpha_- + 2,$$

$$d_n \equiv |x - \Lambda^n x|^2 = 2(\cosh n\alpha_+ - \cosh n\alpha_-)r^2 + 2(\cosh n\alpha_- - 1)l^2,$$

with $\alpha_\pm$ given in (5). In the $J=0$ $(\alpha_-=0)$ limit, (19) reduces to (18). Recall that in (t,r,ϕ) coordinates, the outer and inner horizons are located at $r=l$ and $r=0$. Outside the inner horizon, where d_n is positive and the infinite sums converge exponentially, $\langle T_{\mu\nu}\rangle$ is smooth. The inner horizon, in terms of the embedding coordinates, is the surface $r^2 = T_1^2 - X_1^2 = 0$ corresponding to the light cone in the $1+1$ space M_1. Inside the horizon, $\rho = r^2 = T_1^2 - X_1^2$ becomes negative, and the denominators d_n in (19) vanish on a sequence of timelike surfaces:

$$\rho = \rho_n, \quad \rho_n \equiv -\frac{\cosh n\alpha_- - 1}{\cosh n\alpha_+ - \cosh n\alpha_-} l^2, \quad M > J/l. \quad (20)$$

As we now demonstrate, the nth surface in (20) consists of points x^a connected to their image $\Lambda^n x$ by a null geodesic and is known as a polarized hypersurface [15]. Since x and $\Lambda^n x$ are identified in the black hole solution, the connecting null geodesic is self-intersecting. In AdS_3, geodesics are the analogues of great circles on ordinary spheres. In other words, they are curves which also lie on a two-dimensional plane passing through the origin in the four-dimensional embedding space. Two points x and y are connected by a spacelike, lightlike, or timelike geodesic depending on whether $x^a y_a < -l^2$, $x^a y_a = -l^2$, or $-l^2 < x^a y_a < l^2$, respectively [16]. (Points with $x^a y_a > l^2$ lie on different branches of a hyperboloid and, therefore, are not connected by *any* geodesic.) Since a point x on the nth polarized hypersurface satisfies $d_n = |x - \Lambda^n x|^2 = 0$ implying $x^a(\Lambda^n)_{ab}x^b = -l^2$, x and $\Lambda^n x$ are connected by a null geodesic. As one approaches a polarized hypersurface (20) from a geodesic distance s, $\langle T^{\nu}_{\mu}\rangle$ diverges as $s^{-5/2}$. Since these surfaces in the $n \to \infty$ limit approach the inner horizon, $r=0$, the stress tensor will diverge there. (It should be noted that $\langle T_{\mu\nu}\rangle$ is in fact finite at the inner horizon as it is approached from the outside. This is due to the fact that though each of the polarized hypersurfaces contains null geodesics, the inner horizon itself does not and is said to be noncompactly generated.) One can estimate the back reaction due to the diverging stress tensor by substituting $\langle T_{\mu\nu}\rangle$ into the field equation. Integrating twice, one finds that the metric perturbation diverges as $\delta g_{\mu\nu} \sim s^{-1/2}$ on each of the polarized hypersurfaces. This suggests that the inner horizon is quantum mechanically unstable against formation of a curvature singularity:

For the extremal case $(M=J/l)$, the stress tensor (19) becomes

$$\langle T^t_t \rangle = K(3r^2 - 2l^2),$$

$$\langle T^r_r \rangle = Kl^2,$$

$$\langle T^{\phi}_{\phi} \rangle = -K(3r^2 - l^2),$$

$$\langle T^{\phi}_{t} \rangle = \frac{3}{2}K\left[\frac{r^2}{l^2} - 1\right]l,$$

$$\langle T^t_r \rangle = \langle T^{\phi}_r \rangle = 0,$$

$$K \equiv \frac{\sqrt{2}}{16\pi l^5}\sum_{n=1}^{\infty}\frac{\cosh n\pi\sqrt{2M}+1}{(\cosh n\pi\sqrt{2M}-1)^{3/2}}.$$

For $M = J/l \gg 1$, one has $K \approx l^{-5} e^{-\pi\sqrt{M/2}}$. Note that in contrast to the nonextremal case, (21) is smooth everywhere but diverges asymptotically.

In this paper, we studied the stress tensor for a propagating quantum field in the $2+1$ black hole. Considering the relatively simple geometric structure of the black hole solution, one would hope that further investigation would lead to a greater understanding of its quantum properties.

Note added. After completion of this paper, I received two papers, Akita Junior College Report No. AJC-HEP-19, 1993, by K. Shiraishi and T. Maki, and MIT Report No. CTP-2243, 1993, by G. Lifschytz and M. Ortiz, in which the quantum stress tensor for the nonrotating black hole is calculated.

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