PHYSICAL REVIEW D VOLUME 49, NUMBER 9 1 MAY 1994

Cluster structure of disoriented chiral condensate in rapidity distribution

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(Received 8 December 1993)

We study the creation of disoriented chiral condensates with some initial boundary conditions that may be expected in high energy collisions. The equations of motion in the linear σ model are solved numerically with and without a Lorentz-boost invariance. We suggest that a distinct cluster structure of coherent pion production in the rapidity distribution may emerge due to a quench and may be observed in experiments.

PACS number(s): 11.30.Rd, 12.38.Mh, 14.40.Aq, 25.75.+r

Recently much attention has been paid to an interesting proposal to observe a disoriented chiral condensate (DCC) in high-energy collisions [1–4]. Such events can be signaled by a coherent pion emission along some particular isospin direction in the collision domain. A probability distribution of neutral pions is characterized by a nonbinomial function if all isospin directions are equally probable:

$$P(r) = \frac{1}{2}\frac{1}{\sqrt{r}}, \tag{1}$$

where $r = n_{\pi^0}/(n_{\pi^0} + n_{\pi^\pm})$. This behavior may have already been observed in the so-called Centauro events in cosmic-ray collisions [5]. The central theoretical work is to understand the mechanism for creating such a DCC domain in high energy collisions. Rajagopal and Wilczek [6] have suggested that the expansion of highly relativistic debris from collisions "quenches" the high-temperature field configurations such that the long-wavelength modes will grow with time, leading to a correlated domain. It is recently reported, however, that the correlation size may be very small [7].

In this paper, we shall study the creation of the disoriented chiral condensate with a more realistic initial condition that may be expected in high-energy collisions. We shall reexamine the idea of a quench based on a linear σ model and suggest that indeed, following a quench, correlated pion fields will populate near the light cone where the leading collision particles are expanding at the speed of light. The pion condensates will have distinct cluster structures which may be reflected in the final pion production. We also confirm the general picture of a "baked Alaska" sketched by Bjorken, Kowalski, and Taylor [8].

We assume that following a quench the possible DCC is to be described by classical low energy effective interactions of pions at zero temperature:

$$\mathcal{L} = \int d^4x \left\{ \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{\lambda}{4}\left(\phi_i\phi_i - v^2\right)^2 + H\sigma \right\} \tag{2}$$

where $\phi_i \equiv (\sigma, \boldsymbol{\pi})$ stands for a vector in internal space. $H\sigma$ is an explicit chiral-symmetry-breaking term which is responsible for the mass of the pseudo Goldstone bosons, the pions. We look for a solution to the classical equations of motion of ϕ with a given initial condition. Clearly the initial condition is essential to determine whether or not there will be DCC in the system. For example, if the ϕ and $\dot{\phi}$ initially align in the σ direction throughout the space, then the system does not have any pion field at any time. What might be the typical initial configurations? In a quench, the system begins at a temperature well above the chiral phase transition point T_c. There are thermal fluctuations in all internal directions. Therefore, it is appropriate to assume that initially the space averages $\langle\phi\rangle \sim 0$ and $\langle\dot{\phi}\rangle \sim 0$ but $\langle\phi^2\rangle \neq 0$ and $\langle\dot{\phi}^2\rangle \neq 0$ both for σ and π fields.

We are modeling a situation which is more relevant to highly relativistic collisions. The system should satisfy the space-time geometry of such a collision where the incident nuclei (or hadrons) are, shortly after the collision, highly Lorentz-contracted "pancakes" receding in opposite longitudinal directions from the collision point [9]. An approximate $1+1$ Lorentz invariance of the system is indeed inspired by the existence of a central-plateau structure in the rapidity distribution of produced particles in high-energy cosmic-ray events [10] and pp or $p\bar{p}$ collisions [11]. Ideally one should also include the transverse dimension in the equations of motion. Though the results of Ref. [7] may be applied to the evolution in the transverse direction, they may not change qualitatively

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our conclusion on the longitudinal evolution. Suppose that the domain size in the transverse direction can only reach about 1–2 fm as demonstrated in Ref. [7]. We can then divide the whole transverse area in heavy ion collisions into small domains of size 1–2 fm. Our study then can be applied to the longitudinal evolution of such a small transverse area. Since there could be many domains of such small transverse area in a heavy ion collision, the distribution of the total neutral to charged pion ratio may not have the form of Eq. (1), even if each individual domain has a disoriented chiral condensate. Our model study in this paper may not lead to Centauro events in heavy ion collisions. To study the formation of many small domains with disoriented chiral condensates in heavy ion collisions, one has to look for some alternative signals. However, our study of 1+1 system may be relevant to high-energy *pp* collisions, where chiral symmetry is initially restored by the violent collisions (as proposed by Bjorken *et al.* [8]) and evolution occurs early through a rapid longitudinal expansion.

Neglecting the transverse dimension, ϕ is only a function of t and z. In terms of more convenient coordinates, the proper time $\tau = \sqrt{t^2 - z^2}$ and the (spatial) rapidity variable $\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$, the equations of motion read

$$\left[\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\frac{\partial}{\partial\tau}\right) - \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2}\right]\sigma = -\lambda\sigma\left(\sigma^2 + \boldsymbol{\pi}^2 - v^2\right) + H \ , \tag{3}$$

$$\left[\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\frac{\partial}{\partial\tau}\right) - \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2}\right]\boldsymbol{\pi} = -\lambda\boldsymbol{\pi}\left(\sigma^2 + \boldsymbol{\pi}^2 - v^2\right) \ . \tag{4}$$

Given an initial condition, Eqs. (3) and (4) completely determine the space-time evolution of the system.

The simplest case to solve Eqs. (3) and (4) is when the system has an initial Lorentz-boost invariance; i.e.,

$$\phi(\tau_0, \eta) = \phi_0, \tag{5}$$

where ϕ_0 is independent of η. The lack of $\frac{\partial}{\partial\tau}\frac{\partial}{\partial\eta}$ in the Klein-Gordon operator guarantees that Eq. (5) is sufficient to maintain the boost invariance at any time [there is no need to impose $\frac{\partial}{\partial\tau}\phi(\tau_0,\eta) = \text{const}$]. To model a quench, we assume that the system is initially in a symmetric phase and lies on the top of the "Mexican hat" of the potential:

$$\boldsymbol{\pi}(\tau_0) = 0, \quad \sigma(\tau_0) \simeq 0. \tag{6}$$

To obtain a nontrivial solution for π field, we allow a small initial "kick" of the system, i.e., a nonzero velocity $\frac{\partial\phi}{\partial\tau}(\tau_0) \neq 0$. In this case, since the system starts from one point $z \sim 0$ at $t \sim 0$ (the rapidity dependence of ϕ decouples and the motion is zero dimensional), the correlation in isospin direction is automatically 100%. That is, if the initial kick is in the π^0 direction, then $\pi^\pm = 0$ at all times such that there will be only π^0 production. A numerical solution is plotted in Fig. 1 where we have used the following standard parameter input: $v = 87.4$ MeV, $H = (119 \text{ MeV})^3$, and $\lambda = 19.97$ so that $f_\pi = 92.5$ MeV, $m_\pi = 135$ MeV, and $m_\sigma = 600$ MeV. The initial condition we used in Fig. 1 is $\phi(\tau_0) = 0$, $\partial\phi(\tau_0)/\partial\tau = (1,5,0,0)$ MeV/fm, at $\tau_0 = 1$ fm/c. The general feature of the solution is following. The σ field grows from zero and takes about 1 fm/c proper time to reach the true vacuum expectation value $\langle\sigma\rangle \sim f_\pi$, while the π field oscillates around zero rather slowly and eventually tends to zero when the proper time gets large: $\tau = \tau_{\max} \sim 30$ fm/c. In terms of the real space-time variables t and z, the disoriented chiral condensate develops in the interior region of the receding pancakes, surrounded by the normal vacuum in the exterior; as time (t) evolves, the DCC will be squeezed toward the light cone, leaving behind a normal vacuum where $\langle\boldsymbol{\pi}\rangle = 0$ in the central region inside the light cone. For $t \gg \tau_{\max}$, the π field is found only in a small region $\Delta z \simeq \frac{\tau_{\max}^2}{2t}$ near the light front while the σ field occupies most of the space inside the light cone, surrounded by a thin shell of DCC. This picture supports a space-time geometry of DCC sketched in [8].

While the essential feature of the above solution may be generic, the real situation is more complicated. One complication is that the system may not start from one point shortly after the collision. In this case, the initial condition such as Eq. (5) is not appropriate. If the system initially has fluctuations in rapidities and is not correlated in isospin directions, the crucial question would be whether a quench can yield a coherent pion field at a later time.

We assume that, shortly after the collision, the typical configuration is that of a thermal random fluctuation at high temperature. The fluctuation is governed by the temperature. Similarly to Ref. [6], we choose $\phi(\tau_0)$ and $\partial\phi(\tau_0)/\partial\tau$ randomly according to Gaussian distributions so that

$$\langle\phi(\tau_0,\eta)\rangle_\eta = \left\langle\frac{\partial}{\partial\tau}\phi(\tau_0,\eta)\right\rangle_\eta = 0 \tag{7}$$

but $\langle\phi^2(\tau_0,\eta)\rangle_\eta = v^2/4$ and $\langle(\frac{\partial}{\partial\tau}\phi(\tau_0,\eta))^2\rangle_\eta = v^2\ /\text{fm}^2$. By solving Eqs. (3) and (4), we find that as the proper time evolves, the average value $\langle\sigma\rangle_\eta$ grows from zero quickly and takes its final value (unlike the case with boost-invariant initial condition, the oscillation of $\langle\sigma\rangle_\eta$ around f_π is quickly damped at large τ); $\langle\boldsymbol{\pi}\rangle_\eta$ grows from zero and oscillates around zero slowly in the same way as in Fig. 1. Again, $\langle\boldsymbol{\pi}\rangle_\eta$ decreases to zero at about $\tau \simeq 20 \sim 30$ fm/c. This clearly indicates a strong correlation among the isospin directions in rapidities. Indeed, we can define a correlation function at a given τ:

$$C(\tau,\eta-\eta') = \frac{2\boldsymbol{\pi}_2(\tau,\eta)\cdot\boldsymbol{\pi}(\tau,\eta')}{\boldsymbol{\pi}^2(\tau,\eta)+\boldsymbol{\pi}^2(\tau,\eta')} \ . \tag{8}$$

We find that after about 1 fm/c evolution in proper time (which is the time scale that it takes for the average value of σ field to reach its final value $\langle\sigma\rangle_\eta \sim f_\pi$), $C(\tau,\eta-\eta')$ changes from a zero value corresponding to a typical random distribution to an exponential distribution with nonvanishing width as shown in Fig. 2. For small $\Delta\eta = \eta - \eta'$, $C(\tau,\Delta\eta)$ can be fit by an exponential function

$$C(\tau,\Delta\eta) \propto \exp(-c_0 m_\pi \tau|\Delta\eta|) \qquad (\tau - \tau_0 \geq 1 \text{ fm}/c) \tag{9}$$

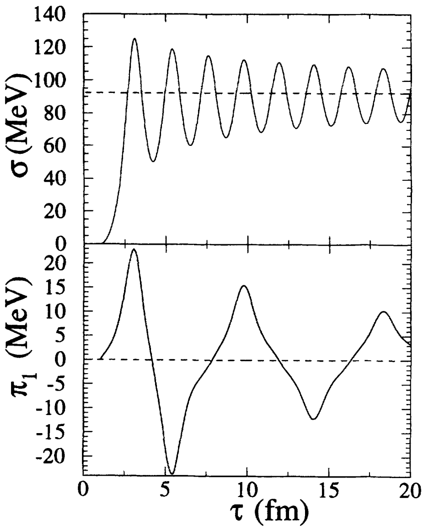

FIG. 1. Proper time evolution of σ and π_1 fields following a Lorentz-boost invariant initial condition at $\tau_0 = 1$ fm/c.

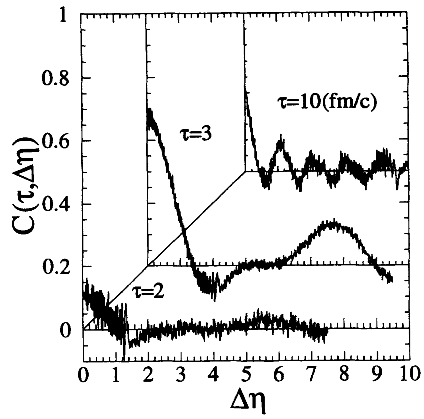

FIG. 2. The correlation function $C(\tau, \Delta\eta)$ of DCC. The initial configuration is that of a thermal fluctuation at $\tau_0 = 1$ fm/c.

where $c_0 \sim 0.4$. At $\tau > 20 \sim 30$ fm/c, the correlation disappears as indicated by the average value $\langle \boldsymbol{\pi} \rangle_\eta$. A similar result is derived in [12] based on a nonlinear σ model in $1+1$ dimensions where the pion fields are the phases of the order parameter. However, in our model, we find that if initially $\langle \phi \rangle_\eta^2 \simeq f_\pi^2$, a correlated distribution does not occur. This clearly shows that the emergence of a correlation in isospin orientation is mainly due to the unstable long-wavelength modes of the pion fields when $\langle \phi \rangle^2 < f_\pi^2$. Our result unambiguously points to the success of the "quench" mechanism for creating DCC (i.e., a high-temperature initial condition plus zero-temperature equations of motion). Equation (9) also suggests that the correlation occurs mainly in the region near the light cone where τ is small. At a given large t, a region of ordinary vacuum where $\langle \boldsymbol{\pi} \rangle_\eta$ vanishes effectively is found inside a shell of coherent pion field. However, the cluster structure of pions radiated from the coherent pion field may occur anywhere in the whole rapidity region.

We should emphasize that the results shown here are obtained with the initial time $\tau_0 = 1$ fm/c when quenching begins. With this initial time, a rapidity correlation length is of 1 unit. As is shown in Fig. 2, this correlation length $\Delta\eta$ decreases with time τ. It also decreases if we take a large value of initial time τ_0. In heavy ion collisions, especially when a quark-gluon plasma is formed, the initial time scale might be very large. Consequently, the sizes of the cluster structure one might see in the rapidity distribution become extremely small. Furthermore, fluctuations of many domains in the transverse direction will wipe out the characteristics of each domain structure. One therefore has to find some alternative signals of small domains other than cluster structures in phase space. We will discuss this in a separate paper [13]. For hadron-hadron collisions, the initial time τ_0 might be close to 1 fm/c. The cluster size might become visible. However, in such a small system, the question whether thermalization can be achieved in the first place still remains.

Although the size of correlated DCC domains may be small in space, we conclude that DCC production may be characterized by the cluster structures in rapidities whose size depends strongly on the initial time scale. What is less clear is whether there is indeed a quench after the collision. Strictly speaking, the quench condition is an idealization of the more complex situation where the temperature relaxes to zero only gradually. For example, the hydrodynamics suggests a temperature drop according to $T = T_0(\frac{\tau_0}{\tau})^{1/3}$ [9]. In this case, Eqs. (3) and (4) may not be appropriate. Clearly, more theoretical work is needed to examine if DCC can be created in the process of slow cooling.

We wish to thank M. Asakawa, J. Bjorken, M. Suzuki, and Dandi Wu for very useful discussions. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Divisions of High Energy Physics and Nuclear Physics of the U.S. Department of Energy, under Contract No. DE-AC03-76SF00098, and by the Natural Sciences and Engineering Research Council of Canada.

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