

## Diffraction dissociation and eikonalization in high energy $pp$ and $p\bar{p}$ collisions

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We show that eikonal corrections imposed on diffraction dissociation processes calculated in the triple Regge limit produce a radical change in the energy dependence of the predicted cross section. The induced correction is shown to be in general agreement with the recent Fermilab Tevatron experimental data.

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Over the past few years, phenomenological investigations of Pomeron exchange processes have been almost exclusively confined to the study of elastic scattering and total cross sections [1–5]. Recently published data from the Fermilab Tevatron [6,7] on single diffraction dissociation (SDD) enables us to evaluate the compatibility of the parametrizations used to describe elastic and diffractive scattering, and whether it is necessary to include screening corrections to obtain a successful description of these processes.

A fundamental problem that must be tackled when one attempts to make a comprehensive analysis of the published high energy data on SDD [6–11] is the fact that there is no unique, agreed upon, experimental definition of SDD. Experimental groups have used different and not always mutually consistent methods of extracting the desired data. In addition, it is difficult to compare the values that the different experimental groups give for  $\sigma_{SD}$ , as in their evaluation of  $d\sigma_{SD}/dM^2 dt$ , they have used diverse integration limits for  $t$  and  $M^2$ . Furthermore, their treatment of the correlations observed between  $M^2$  and  $t$  are entirely different.

With the above limitations in mind, we present in this Rapid Communication a general study of SDD, which is compatible with the analysis of elastic scattering, and at the same time reproduces all the important features of the experimental data measured in SDD at high energies.

Even though the Pomeron was introduced into high energy physics more than 30 years ago, its exact definition and detailed substructure remain an enigma. In contrast with standard Regge trajectories, the Pomeron has no particles on the timelike sector of its trajectory. Nevertheless, it is required both phenomenologically to describe the forward hadron-hadron scattering data and theoretically to ensure that Regge theory is self-consistent. Indeed, in Reggeon field calculus the Pomeron is described as a ladder of Reggeons yielding

[12]  $\alpha(0) = 1$ . We will refer to this as the “soft Pomeron.”

A number of different models have been proposed to account for the rising hadron-hadron cross sections.

(1) Donnachie and Landshoff [1] have advocated an *ad hoc* approach in which the soft Pomeron amplitude keeps its traditional form with  $\alpha(0) = 1 + \Delta \simeq 1.08$ . This simple model reproduces the qualitative features of the experimental data remarkably well.

(2) Alternatively, one may perceive the Pomeron as a two gluon exchange [13], or more generally as a gluon ladder. Lipatov [14] has shown that such a ladder, when calculated within the framework of perturbative QCD, receives its major contribution from high  $p_{\perp}$  gluon exchanges. These give to a series of poles in the complex  $j$  plane above unity. The summation of these poles yields the “hard Pomeron” with  $\Delta = (12/\pi)\alpha_s \ln 2$ . Bjorken has suggested [15] that the generic Pomeron may actually manifest itself in both soft and hard modes, each contributing in a different kinematical domain. Models based on a hybrid Pomeron are very successful in reproducing the data [4,5].

(3) In the QCD inspired model [2,3], the growth of the total cross section is associated with the greater probability of semihard gluons to interact with increasing energy. In this case, the need to describe the data over a wide energy range also requires a hybrid model [3] consisting of a soft  $q$ - $q$  background and semihard  $q$ - $g$  and  $g$ - $g$  interactions.

All the above models of the Pomeron have an intrinsic powerlike  $s^{\Delta}$  rise of the total hadronic cross section. We note [4,16] that the Pomeron amplitude proposed in [1] violates  $s$ -channel unitarity, just above the Tevatron energy range, for small  $b$ . In general, we expect the unitarity bound to induce screening effects which saturate the growth of  $\sigma_{tot}$ , making  $\sigma_{tot} \leq \ln^2 s$ , which is compatible with the Froissart bound. Technically, this is most easily achieved through eikonalization [17], in which the amplitude discussed above serves as the lowest order input to the eikonal expansion. Even though in the eikonal model one only sums over elastic rescattering, ignoring diffraction in the intermediate states, it has the advantage of being simple to apply. In addition, it introduces

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the natural scale of the screening corrections, and allows one to explore different models of the Pomeron.

The main purpose of this Rapid Communication is to examine the role played by eikonalization in SDD. This is investigated utilizing a simple Regge-like Pomeron [1]. Extending the same formalism to include an input Lipatov-type Pomeron is straightforward. As the presently available diffractive data are not sufficiently refined to discriminate between these models of the Pomeron, we shall not discuss it in detail here.

The simplest way to write down the eikonal formulas is to consider the scattering process in impact parameter space. Our amplitude is normalized so that

$$\frac{d\sigma}{dt} = \pi |f(s, t)|^2, \quad (1)$$

$$\sigma_{\text{tot}} = 4\pi \text{Im} f(s, 0). \quad (2)$$

The scattering amplitude in  $b$  space is defined as

$$a(s, b) = \frac{1}{2\pi} \int d\mathbf{q} e^{-i\mathbf{q}\cdot\mathbf{b}} f(s, t) \quad (3)$$

where  $t = -q^2$ .

In this representation,

$$\sigma_{\text{tot}} = 2 \int d\mathbf{b} \text{Im} a(s, b), \quad (4)$$

$$\sigma_{\text{el}} = \int d\mathbf{b} |a(s, b)|^2. \quad (5)$$

$s$ -channel unitarity when written in diagonalized form implies

$$2 \text{Im} a(s, b) = |a(s, b)|^2 + G_{\text{in}}(s, b) \quad (6)$$

where

$$\sigma_{\text{in}} = \int d\mathbf{b} G_{\text{in}}(s, b). \quad (7)$$

We list below several assumptions that we make regarding the eikonal model.

(1) At high energy  $a(s, b)$  is assumed to be pure imaginary and can be reduced to the simple form

$$a(s, b) = i(1 - e^{-\Omega(s, b)}), \quad (8)$$

where  $\Omega(s, b)$  is a real function. Analyticity and crossing symmetry are easily restored to our oversimplified parametrization by substituting  $s^\alpha \rightarrow s^\alpha e^{-i\pi\alpha/2}$ .

(2) From Eq. (6) we can express  $G_{\text{in}}(s, b)$  as

$$G_{\text{in}}(s, b) = 1 - e^{-2\Omega(s, b)}, \quad (9)$$

where  $e^{-2\Omega(s, b)}$  denotes the probability that no inelastic interaction takes place at impact parameter  $b$ .

(3) We write the  $t$ -channel Pomeron exchange as

$$\Omega(s, b) = \nu(s) e^{-b^2/R^2(s)}. \quad (10)$$

In the simple Regge pole model with a trajectory  $\alpha_P(t) =$

$$\frac{M^2 d\sigma_{\text{SD}}}{dM^2} = G_{PPP} \sigma_0^2 \left(\frac{s}{M^2}\right)^{2\Delta} \left(\frac{M^2}{s_0}\right)^\Delta \frac{1}{[\pi \bar{R}_1^2(\frac{s}{M^2})]^2 \pi \bar{R}_1^2(\frac{M^2}{s_0})} \int d\mathbf{b} d\mathbf{b}' \exp\left(-\frac{2(b-b')^2}{\bar{R}_1^2(\frac{s}{M^2})} - \frac{b'^2}{\bar{R}_1^2(\frac{M^2}{s_0})}\right) + G_{PPR} \sigma_0^2 \left(\frac{s}{M^2}\right)^{2\Delta} \left(\frac{M^2}{s_0}\right)^{-1/2} \frac{1}{[\pi \bar{R}_1^2(\frac{s}{M^2})]^2 \pi \bar{R}_2^2(\frac{M^2}{s_0})} \int d\mathbf{b} d\mathbf{b}' \exp\left(-\frac{2(b-b')^2}{\bar{R}_1^2(\frac{s}{M^2})} - \frac{b'^2}{\bar{R}_2^2(\frac{M^2}{s_0})}\right) \quad (16)$$

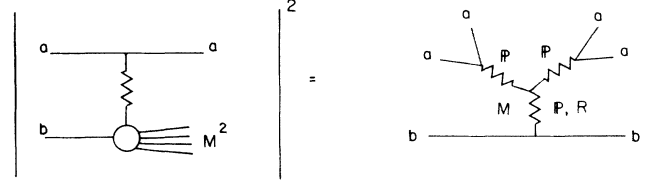


FIG. 1. SDD in the triple Regge approximation.

$1 + \Delta + \alpha' t$ . We have

$$\nu(s) = \frac{\sigma_0}{2\pi R^2(s)} \left(\frac{s}{s_0}\right)^\Delta, \quad (11)$$

where

$$R^2(s) = 4 \left( R_0^2 + \alpha' \ln \frac{s}{s_0} \right) \quad (12)$$

and  $\sigma_0 = \sigma(s_0)$ . Agreement with the  $pp$  ( $\bar{p}p$ ) data is obtained with  $R_0^2 = 5.2 \text{ GeV}^{-2}$  and  $\alpha' = 0.25 \text{ GeV}^{-2}$ .

Equations (10)–(12) lead to simple expressions for the total and inelastic cross sections with  $\sigma_{\text{el}} = \sigma_{\text{tot}} - \sigma_{\text{in}}$  [see Fig. 2(a)]:

$$\begin{aligned} \sigma_{\text{tot}} &= 2\pi R^2(s) \{ \ln \nu(s) + C - \text{Ei}[-\nu(s)] \} \\ &\xrightarrow{\nu \gg 1} 2\pi R^2(s) [ \ln \nu(s) + C ], \end{aligned} \quad (13)$$

$$\begin{aligned} \sigma_{\text{in}} &= \pi R^2(s) \{ \ln 2\nu(s) + C - \text{Ei}[-2\nu(s)] \} \\ &\xrightarrow{\nu \gg 1} \pi R^2(s) [ \ln 2\nu(s) + C ], \end{aligned} \quad (14)$$

where  $\text{Ei}(x) = \int_{-\infty}^x (e^t/t) dt$ , and  $C = 0.5773$  is the Euler constant.

The standard approach to evaluate single diffractive dissociation is through the three-body optical theorem [18] leading to the  $PPP$  and  $PPR$  diagrams of interest (see Fig. 1). The appropriate cross section is

$$\begin{aligned} M^2 \frac{d\sigma_{\text{SD}}}{dM^2 dt} &= \sigma_0^2 \left(\frac{s}{M^2}\right)^{2\Delta+2\alpha' t} \\ &\times \left[ G_{PPP}(t) \left(\frac{M^2}{s_0}\right)^\Delta \right. \\ &\left. + G_{PPR}(t) \left(\frac{M^2}{s_0}\right)^{-1/2} \right], \end{aligned} \quad (15)$$

where all of the relevant couplings have been absorbed into  $G_{PPP}(t)$  or  $G_{PPR}(t)$ .  $M^2$  denotes the mass of the diffractive system, and for the Regge trajectory we have taken  $\alpha_R(t) = \frac{1}{2} + t$ .

Equation (15) can be rewritten in the impact parameter representation

where

$$\bar{R}_i^2 \left( \frac{s}{M^2} \right) = 2R_{0i}^2 + r_{0i}^2 + 4\alpha' \ln \left( \frac{s}{M^2} \right). \quad (17)$$

$r_{0i} \leq 1 \text{ GeV}^{-2}$  denotes the radius of the triple vertex [19].  $\bar{R}_1^2(s/M^2) = 2B_{\text{SD}}$ , where  $B_{\text{SD}}$  denotes the slope of the SDD cross section. Upon integrating Eq. (16) we have

$$M^2 \frac{d\sigma_{\text{SD}}}{dM^2} = \frac{\sigma_0^2}{2\pi \bar{R}_1^2 \left( \frac{s}{M^2} \right)} \left( \frac{s}{M^2} \right)^{2\Delta} \times \left[ G_{\text{PPP}} \left( \frac{M^2}{s_0} \right)^\Delta + G_{\text{PPR}} \left( \frac{M^2}{s_0} \right)^{-1/2} \right]. \quad (18)$$

We will now comment on consequences of the above result and its relevance when compared to experimental data [6–11].

(1) We expect the forward SDD differential nuclear slope to be in the range  $\frac{1}{2}B_{\text{el}} < B_{\text{SD}} < B_{\text{el}}$ , where  $B_{\text{el}} = 0.5R^2(s)$  denotes the appropriate elastic scattering slope. In general,  $B_{\text{SD}}$  is  $M^2$  dependent. An explicit logarithmic dependence is implied by the definition of  $\bar{R}_i(s/M^2)$  in Eq. (17). We also note that due to the different  $M^2$  power dependences, the *PPR* contribution is concentrated at lower values of  $M^2$  than the *PPP*. For energies in the CERN Intersecting Storage Rings (ISR) and Tevatron range, where  $\ln^2 s \geq R_0^2$ , we expect qualitatively, that  $B_{\text{SD}} \geq \frac{1}{2}B_{\text{el}}$  with a very moderate  $\ln(s/M^2)$  dependence. This is in agreement with the data. We are unable to make a numerical fit due to strong correlations between  $M^2$  and  $t$ , observed at small values of  $M^2$ .

We strongly urge that measurements of  $B_{\text{SD}}$  be made for higher values of the mass spectrum, say  $M^2 \geq 16 \text{ GeV}^2$ .

(2) The  $M^2$  dependence of the SDD cross section is dominated by

$$G_{\text{PPP}}(M^2)^{-(1+\Delta)} + G_{\text{PPR}}(M^2)^{-(1.5+2\Delta)}.$$

If we express this dependence by  $(M^2)^{-\alpha_{\text{eff}}}$ , we expect  $(\alpha_{\text{eff}} - 1) > \Delta$  and that  $(\alpha_{\text{eff}} - 1)$  approaches  $\Delta$  from above in the limit of very high  $s$  when the importance of the *PPR* term diminishes. This behavior is corroborated by the two recent studies [6,7] of the  $M^2$  distribution at the Tevatron. In passing we note that the experiments at the Fermilab [8] and ISR [9] reported approximate scaling, i.e., a  $(M^2)^{-1}$  behavior. This is most probably due to the much narrower  $M^2$  interval investigated. The approximation in which we only consider the *PPP+PPR* terms is obviously not sufficient to describe data at lower energies, where lower-lying trajectories are important [20].

(3) Equation (18) predicts a strong powerlike  $s^{2\Delta}$  dependence of the differential as well as the integrated SDD cross section. This is a much stronger energy dependence than the predicted  $s^\Delta$  behavior of  $\sigma_{\text{tot}}$ , and clearly not compatible with either theory or data. Indeed, the Collider Detector at Fermilab (CDF) data [7] taken at  $\sqrt{s} = 546$  and 1800 GeV show only a moderate 20% increase of the appropriate cross sections. This should be compared with an 80% increase expected from a  $s^{2\Delta}$  behavior with  $\Delta = 0.125$ , as reported by CDF [7].

Obviously, Eq. (14) violates unitarity. Unitarity is restored, in the eikonal model, by multiplying the integrand of Eq. (16) by  $e^{-2\Omega(s,b)}$  [see Fig. 2(b)]. The resulting cross section is

$$\begin{aligned} \frac{M^2 d\sigma_{\text{SD}}}{dM^2} &= G_{\text{PPP}} \sigma_0^2 \left( \frac{s}{M^2} \right)^{2\Delta} \left( \frac{M^2}{s_0} \right)^\Delta \frac{1}{[\pi \bar{R}_1^2 \left( \frac{s}{M^2} \right)]^2 \pi \bar{R}_1^2 \left( \frac{M^2}{s_0} \right)} \\ &\times \int d\mathbf{b} d\mathbf{b}' e^{-\nu(s)e^{-b^2/R^2(s)}} \exp \left( -\frac{2(b-b')^2}{\bar{R}_1^2 \left( \frac{s}{M^2} \right)} - \frac{b'^2}{\bar{R}_1^2 \left( \frac{M^2}{s_0} \right)} \right) \\ &+ G_{\text{PPR}} \sigma_0^2 \left( \frac{s}{M^2} \right)^{2\Delta} \left( \frac{M^2}{s_0} \right)^{-1/2} \frac{1}{[\pi \bar{R}_1^2 \left( \frac{s}{M^2} \right)]^2 \pi \bar{R}_2^2 \left( \frac{M^2}{s_0} \right)} \\ &\times \int d\mathbf{b} d\mathbf{b}' e^{-\nu(s)e^{-b^2/R^2(s)}} \exp \left( -\frac{2(b-b')^2}{\bar{R}_1^2 \left( \frac{s}{M^2} \right)} - \frac{b'^2}{\bar{R}_2^2 \left( \frac{M^2}{s_0} \right)} \right) \end{aligned} \quad (19)$$

where  $\nu(s)$  is given by Eq. (11) and  $R^2(s)$  by Eq. (12). After integration we have

$$\frac{M^2 d\sigma_{\text{SD}}}{dM^2} = \frac{\sigma_0^2}{2\pi \bar{R}_1^2 \left( \frac{s}{M^2} \right)} \left( \frac{s}{M^2} \right)^{2\Delta} \left[ G_{\text{PPP}} \left( \frac{M^2}{s_0} \right)^\Delta a_1 \frac{1}{[2\nu(s)]^{a_1}} \gamma[a_1, 2\nu(s)] + G_{\text{PPR}} \left( \frac{M^2}{s_0} \right)^{-1/2} a_2 \frac{1}{[2\nu(s)]^{a_2}} \gamma[a_2, 2\nu(s)] \right] \quad (20)$$

where

$$a_i = \frac{2R^2(s)}{\bar{R}_1^2 \left( \frac{s}{M^2} \right) + 2\bar{R}_i^2 \left( \frac{M^2}{s_0} \right)} \quad (21)$$

and  $\gamma(a, 2\nu)$  denotes the incomplete Euler gamma function  $\gamma(a, 2\nu) = \int_0^{2\nu} z^{a-1} e^{-z} dz$ .

We list below the important consequences of the expression we obtained in Eq. (20).

(1) The  $b$ -space SDD amplitude, which is the integrand of Eq. (19), differs from the intrinsic integrand of Eq. (16) by the corrective multiplicative factor  $e^{-2\Omega(s,b)}$ . Whereas the unabsorbed  $b$ -space SDD amplitude is central and can be approximated by a Gaussian centered at  $b = 0$ , the corrected amplitude has a dip at  $b = 0$ , and its Gaussian approximation is centered at some  $b = b_0 \neq 0$ . This behavior suggests that the generalized unitarity condition [20] is satisfied. This is consistent with the general

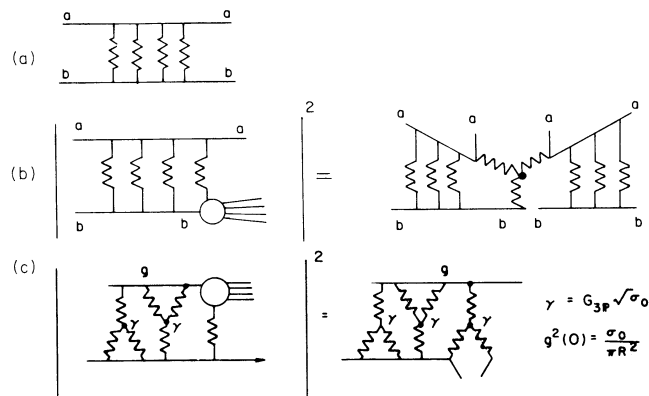


FIG. 2. (a) Screening corrections in the eikonal approximation to elastic scattering. (b) Screening corrections in the eikonal approximation to SDD. (c) Inelastic shadowing (screening) corrections to SDD.

pattern expected of SDD  $b$ -space amplitudes after screening has been included [21,22].

(2) Our qualitative observation that  $B_{SD} \geq \frac{1}{2} B_{el}$  is unchanged. We expect the ratio  $B_{SD}/B_{el}$  to grow with energy, up to a limiting value of 1.

(3) The dominant  $M^2$  dependence of  $\frac{d\sigma_{SD}}{dM^2}$  is identical to that determined from Eq. (18). We stress that the two properties of the triple Regge model, those concerning the  $t$  and  $M^2$  dependence, which are in agreement with experiment are essentially unchanged once the eikonal correction is made to the original SDD amplitude.

(4) Equation (20) exhibits a weak  $s$  dependence. This is best seen if we examine our result in the high energy limit, where we have  $a_i \rightarrow 2$ , and  $\gamma[a_i, 2\nu(s)] \rightarrow \Gamma(2)$ . Thus the factor  $s^{2\Delta}$  is compensated by  $[1/\nu(s)]^{a_i}$  and Eq. (20) reduces to

$$M^2 \frac{d\sigma_{SD}}{dM^2} = \pi \Gamma(2) R^2(s) \left[ G_{PPP} \left( \frac{M^2}{s_0} \right)^{-\Delta} + G_{PPR} \left( \frac{M^2}{s_0} \right)^{-(1/2+2\Delta)} \right]. \quad (22)$$

Since  $\sigma_{SD}$  is not very sensitive to the high  $M^2$  integration limit, we find that  $\sigma_{SD}$  depends on  $s$  only through  $R^2(s)$ . Our result indicated that the changes induced by eikonalization on  $\sigma_{tot}$  and  $\sigma_{SD}$  are quite different. For  $\sigma_{tot}$  the input  $s^\Delta$  power behavior is modified to  $\ln^2 s$ ; the energy scale at which this change becomes appreciable is at  $\sqrt{s} \approx 3$  TeV [4]. For  $\sigma_{SD}$  the input  $s^{2\Delta}$  power behavior is modified to  $\ln s$ ; this occurs at an energy scale which is considerably lower, i.e.,  $\sqrt{s} \approx 300$  GeV. In addition we expect that  $\sigma_{SD}/\sigma_{tot} \xrightarrow{s \rightarrow \infty} 0$ .

We utilize the Pomeron and Regge parameters which were determined from the analysis of the total and elastic cross sections, i.e.,  $\sigma_0$ ,  $\alpha(0)$ ,  $\alpha'$ , and  $R_0^2$  to numerically evaluate our theoretical model. The values employed are summarized in Table I. In addition we need to fit  $G_{PPP}$  and  $G_{PPR}$  or, alternatively, an overall normalization and the ratio  $G_{PPR}/G_{PPP}$ . Some caution is required as  $\Delta$  and  $G_{PPR}/G_{PPP}$  are correlated in SDD analysis. Donnachie and Landshoff [1] suggest a global fit with  $\Delta = 0.08$ . This choice is compatible with  $M^2$  distribution measured by the CDF collaboration, if the

$G_{PPP} = 5.5$	
$\sigma_{0P} = 70 \text{ GeV}^{-2}$	
$\Delta = 0.08$	(fixed)
$R_{0P}^2 = 5.2 \text{ GeV}^{-2}$	(fixed)
$\alpha' = 0.25 \text{ GeV}^{-2}$	(fixed)
$G_{PPR} = 0.5 G_{PPP}$	(fixed)
$R_{0R}^2 = 3.75 \text{ GeV}^{-2}$	(fixed)
$\alpha_R(t) = 0.5 + t$	(fixed)
$\sigma_{0R} = 4.4 \text{ GeV}^{-2}$	(fixed)

PPR contributes 40% of the integrated  $\sigma_{SD}$  at 546 GeV. This value of  $\Delta$ , which was proposed in Ref. [1], used the E710 measurement [23] of  $\sigma_{tot}$  at  $\sqrt{s} = 1800$  GeV. A recent CDF measurement [24] at the same energy suggests a considerably higher value for  $\sigma_{tot}$ , which is consistent with a value of  $\Delta = 0.11$ . The PPR contribution to  $\sigma_{SD}$  at 546 GeV is now reduced to 15%. The above two sets of parameters have a somewhat different extrapolation to energies much higher than 1800 GeV, but provide similar results in the ISR-Tevatron energy range [6–11].

As we state previously, the main new result of our analysis is that the scale at which the SDD cross section shows sensitivity to eikonalization is about 300 GeV, which is an order of magnitude smaller than the appropriate scale for the total cross sections. A comparison of our prediction with the complete set of published data is shown in Fig. 3. As can be seen, we are unable to find an adequate overall fit to  $\sigma_{SD}$  as measured over the entire ISR-Tevatron range [6–11]. Although we obtain an excellent fit for the  $\sigma_{SD}$  data above the ISR energy range, we considerably underestimate the lower energy  $\sigma_{SD}$  data measured at the ISR, no doubt due to the fact that the lower-lying trajectories have been excluded [20]. As we mentioned, the different experimental groups have used diverse constraints and algorithms to define SDD. The  $M^2$  integration limits differ for the various experiments (we have corrected for this in our calculation by adjusting our  $M^2$  integration at each point according to the experimental specifications). In addition, the  $M^2$  and  $t$  distributions observed at lower  $M^2$  values are correlated, and these correlations were handled differently by each group.

Regardless of these reservations, we can minimize the

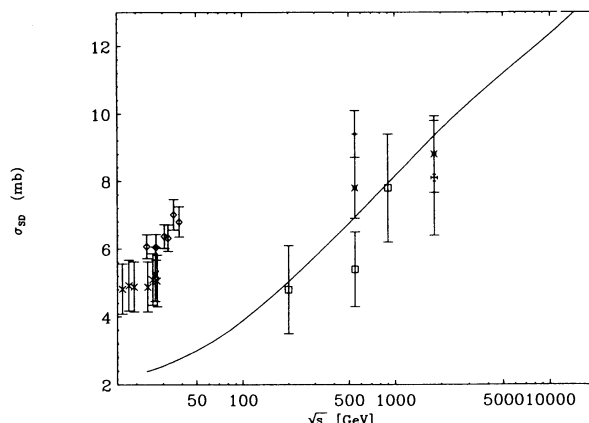


FIG. 3. The single diffractive cross section using parameters given in Table I, compared to data summarized in Refs. [6–11].

experimental uncertainties and consider the two CDF measurements at  $\sqrt{s} = 546$  and 1800 GeV, where they find [7]  $R = \sigma_{SD}(1800)/\sigma_{SD}(546) = 1.20 \pm 0.06$ , with  $1.4 \leq M^2 \leq 0.15$  s. If we take  $\Delta = 0.08$  we predict a ratio of  $R_{PPP} = 1.35$  for the *PPP* term, and for the *PPR* term  $R_{PPR} = 1.25$ . Assuming the *PPR* contribution to account for 40% of the SDD cross section at 546 GeV, we have a theoretical prediction of  $R = 1.31$ . For  $\Delta = 0.11$ , we obtain  $R_{PPP} = 1.35$  and  $R_{PPR} = 1.20$ . This gives us a prediction for  $R = 1.33$ , assuming the *PPR* to account for 15% of the SDD cross section at 546 GeV.

The CDF group start their  $M^2$  integration at  $M_{\min}^2 = 1.4$  GeV<sup>2</sup>, which is much too low for any triple Regge analysis. To eliminate the region of low diffractive masses, we compare with the experimental ratio quoted by CDF [25] of  $R = 1.24 \pm 0.10$  obtained with  $M_{\min}^2 = 16$  GeV<sup>2</sup>. For  $\Delta = 0.08$  we obtain  $R = 1.34$ , while for  $\Delta = 0.11$  we have  $R = 1.37$ .

Extrapolation of our model to ISR energies (using values of the parameters normalized to the CDF data) underestimates the measured values of  $\sigma_{SD}$ . This is not unexpected, as our simple model with only *PPP* + *PPR* contributions is clearly not sufficient at ISR energies, where a more detailed analysis [20] demonstrates the importance of lower-lying trajectories at these energies. Examining SDD data over the whole energy range [6–11], it appears that screening corrections become important at energies lower than that predicted by our eikonal model. This is not surprising, as in our treatment of

eikonalization we have only included elastic rescattering effects in the intermediate states, while completely ignoring diffractive effects or so-called inelastic shadowing correction [see Fig. 2(c)] [26]. Such corrections cannot be considered to be small as the ratio  $\sigma_{SD}/\sigma_{el}$  is of the order of  $\frac{1}{2}$  at the Tevatron energies. It means that dimensionless triple Pomeron vertex introduced in Eq. (19) is about  $\frac{1}{8}$  and diagrams of Fig. 2(c) should be taken into account at the next stage of our approach.

In contrast with the previous point we expect the extrapolation of our results to extremely high energies to be trustworthy. Integrating over  $1.4$  GeV<sup>2</sup>  $\leq M^2 \leq 0.15$  s, we predict that  $\sigma_{SD} = 13.3$  and 13.9 mb at  $\sqrt{s} = 16$  and 40 TeV, respectively, demonstrating the very weak  $s$  dependence predicted by our model.

In conclusion, we wish to emphasize that our model does reproduce the main features of SDD above 300 GeV, in particular the exceedingly moderate dependence of  $\sigma_{SD}$  on  $s$ . The model which does not include lower-lying Regge trajectories is too simple to successfully describe the SDD data at lower energies.

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