

Spontaneous CP violation in the supersymmetric Higgs sector

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Spontaneous CP violation in the minimal supersymmetric standard model with a gauge singlet and a cubic superpotential is examined. Although the tree-level Higgs potential conserves CP , it is shown that with the inclusion of the one-loop top-quark radiative effects CP may be broken spontaneously. The CP -violating minimum requires two neutral (h_1, h_2) and one charged ($H^\pm$) Higgs boson to be relatively light with $m_{h_1} + m_{h_2} \lesssim 100$ GeV and $m_{H^\pm} \lesssim 110$ GeV. The electric dipole moment of the electron is in the observable range of $(\frac{1}{3} \text{ to } 3) \times 10^{-27}$ e cm.

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There are a number of reasons why it is interesting to study CP violation arising from the Higgs sector. Such CP violation could lead to large and soon-observable electric dipole moments for the electron and the neutron [1]. It may have a role to play in baryogenesis in the early Universe [2]. It is even possible that this may be the origin of the CP violation observed in the kaon system [3]. And such Higgs sector effects may be readily studied at present and future colliders.

In this Rapid Communication we explore CP violation phenomenology in the Higgs sector of low-energy supersymmetry, in the simplest model where such effects can appear [4]. That model is the minimal supersymmetric standard model (MSSM) to which a gauge-singlet Higgs superfield N has been added with the requirement that the superpotential contain only cubic terms [5]. There is no significant CP violation in the parameters of the Higgs effective potential (see below), so that any CP violation in the Higgs sector of the model must be spontaneous.

It has been shown by Romao [6] that the potential of this model, both at the tree-level and with certain radiative effects included, has no CP -violating minimum. He proved that at the CP -violating extremum there is always a mode with negative mass-squared. Our main result is that for much of the parameter space the top-quark loop contribution to the effective potential [7] turns the CP -violating saddle point into a minimum, and in

fact the absolute minimum [8]. At this CP -violating true minimum some of the Higgs fields are necessarily very light (these being the modes that have negative mass-squared when the top-effect is turned off), in fact too light generally, ruling out much of parameter space. We argue, however, that if the top-loop effect is large enough a viable CP -violating minimum can result. In such a minimum the sum of the masses of the lightest two neutral scalars is not much above M_Z and the charged Higgs boson mass is less than about 110 GeV. In this scenario the prospects for detecting these Higgs particles at the CERN e^+e^- collider LEP and LEP-200 are very good indeed. Moreover, since two of the neutral Higgs fields are light, the electron electric dipole moment (EDM) comes out in the experimentally accessible range of $(10^{-28} \text{ to } 3 \times 10^{-27})$ e cm (typically).

Only the radiative effects of the top-quark will be significant, so that the relevant terms in the superpotential are

$$W = \lambda H_1 H_2 N + \frac{1}{3} k N^3 + h_t Q H_2 T^c. \quad (1)$$

Here $H_1 = \begin{pmatrix} H_1^{0+} \\ -H_1^- \end{pmatrix}$, $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$, so that $H_1 H_2 = \epsilon^{ab} H_1^a H_2^b = (H_1^{0+} H_2^0 + H_1^- H_2^+)$. The scalar potential for the fields H_1, H_2 , and N is then given by $V = V_0 + V_{\text{top}}$, where

$$V_0 = \frac{1}{8} (g_1^2 + g_2^2) (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g_2^2 (|H_1|^2 |H_2|^2 - |H_1 H_2|^2) + \lambda^2 [|H_1 H_2|^2 + |N|^2 (|H_1|^2 + |H_2|^2)] + k^2 |N|^4 + \lambda k (H_1 H_2 N^{*2} + \text{H.c.}) + \lambda A_\lambda (H_1 H_2 N + \text{H.c.}) + \frac{1}{3} k A_k (N^3 + \text{H.c.}) + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |N|^2, \quad (2)$$

$$V_{\text{top}} = \frac{3}{16\pi^2} [(h_t^2 |H_2|^2 + M_{\text{sq}}^2)^2 \ln\{(h_t^2 |H_2|^2 + M_{\text{sq}}^2)/Q^2\} - \{h_t^2 |H_2|^2 \ln(h_t^2 |H_2|^2/Q^2)\}]. \quad (3)$$

Here $|H_1|^2 \equiv H_1^\dagger H_1$, etc. V_{top} denotes the leading-log top-quark one-loop radiative correction. We have assumed degenerate squarks: $M_{\tilde{t}_L}^2 = M_{\tilde{t}_R}^2 = M_{\text{sq}}^2 \gg (174 \text{ GeV})^2$.

It is easy to see that all the parameters in Eq. (2) can be made real by field redefinitions, except the ratio $r \equiv A_\lambda/A_k$. We assume that parameter, too, is real. (It would be approximately so in most realistic scenarios [9].)

To examine the CP -violating minimum, let us define

$$D = \lambda k, \quad E = \lambda A_\lambda/|N| = r \lambda A/|N|, \quad (4)$$

$$F = \frac{1}{3} k A |N|/|H_1^0 H_2^0|.$$

Here and throughout $A \equiv A_k$. Let us further define the phases of the fields H_1^0, H_2^0 , and N as

$$\arg(H_1^* H_2 N) = \eta, \quad \arg(N^3) = \zeta. \quad (5)$$

There are three terms in Eq. (2) that depend on η and ζ . Extremizing the potential with respect to these phases gives the CP -violating solution to be

$$\begin{aligned} \cos\eta &= \frac{1}{2} \left(\frac{DF}{E^2} - \frac{D}{F} - \frac{F}{D} \right), \\ \cos\zeta &= \frac{1}{2} \left(\frac{DE}{F^2} - \frac{D}{E} - \frac{E}{D} \right), \\ \cos(\eta - \zeta) &= \frac{1}{2} \left(\frac{EF}{D^2} - \frac{E}{F} - \frac{F}{E} \right), \\ E \sin\eta &= -F \sin\zeta. \end{aligned} \quad (6)$$

These equations can be represented graphically by the triangle in Fig. 1.

Denoting the vacuum expectation values (VEV's) of the neutral fields by $\langle H_1 \rangle = v_1$, $\langle H_2 \rangle = v_2$ and $\langle N \rangle = x$, one can expand the fields around their minimum:

$$\begin{aligned} H_1 &= v_1 + \frac{1}{\sqrt{2}} \frac{v_1}{|v_1|} \left[\Phi_1 - i \frac{|v_2|}{v} \Phi_3 \right], \\ H_2 &= v_2 + \frac{1}{\sqrt{2}} \frac{v_2}{|v_2|} \left[\Phi_2 + i \frac{|v_1|}{v} \Phi_3 \right], \\ N &= x + \frac{1}{\sqrt{2}} \frac{x}{|x|} [\Phi_4 + i \Phi_5], \end{aligned} \quad (8)$$

with $v^2 = |v_1|^2 + |v_2|^2$. In the absence of CP violation, $\Phi_{1,2,4}$ would be scalars and $\Phi_{3,5}$ pseudoscalars. One pseudoscalar gets absorbed by the Z . (We are working in the unitary gauge.)

It is straightforward to expand the potential in Eqs. (2) and (3) about the CP -violating extremum to obtain the mass-squared matrix of Φ_i , $i = 1, 2, \dots, 5$. In doing this it is convenient to adopt a common approach of choosing Q^2 in Eq. (3) so that at the minimum of V_0 the relation $V'_{\text{top}} = 0$ is satisfied. [Then V_{top} does not contribute explicitly to the equations for v_1, v_2 , and x , it only corrects the (22) entry of the Higgs mass-squared matrix.] The mass-squared matrix elements are

$$\begin{aligned} \mathcal{M}_{11}^2 &= \lambda_1 v_1^2 + 3r\lambda^2 v_2^2, \\ \mathcal{M}_{22}^2 &= (1 + \Delta)\lambda_1 v_2^2 + 3r\lambda^2 v_1^2, \\ \mathcal{M}_{33}^2 &= 3r\lambda^2 v^2, \\ \mathcal{M}_{12}^2 &= -\lambda_1 v_1 v_2 + (2 - 3r)\lambda^2 v_1 v_2, \\ \mathcal{M}_{13}^2 &= \mathcal{M}_{23}^2 = 0, \end{aligned}$$

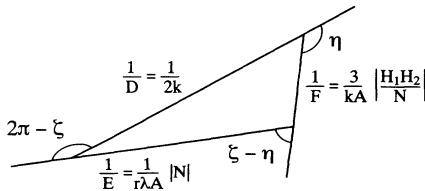


FIG. 1. This triangle shows the relation between the extremal values of the phase angles η and ζ and the magnitudes of the fields and the parameters of the Higgs boson potential. If D, E , and F are not in such a relation that this triangle can be drawn, then there is no CP -violating extremum.

$$\begin{aligned} \mathcal{M}_{44}^2 &= r(4r - 1)A^2 + 36r^2\lambda^2 v_1^2 v_2^2 / x^2 \\ &\quad + 4r(6r - 1)\lambda A(v_1 v_2 / x) \cos\eta, \\ \mathcal{M}_{55}^2 &= 3rA^2 + 12r\lambda^2 v_1^2 v_2^2 / x^2 + 12r\lambda A(v_1 v_2 / x) \cos\eta, \\ \mathcal{M}_{45}^2 &= 4r\lambda A(v_1 v_2 / x) \sin\eta, \\ \mathcal{M}_{14}^2 &= -r\lambda A v_2 \cos\eta + 2\lambda^2 x v_1 - 6r\lambda^2 v_2^2 v_1 / x, \\ \mathcal{M}_{24}^2 &= -r\lambda A v_1 \cos\eta + 2\lambda^2 x v_2 - 6r\lambda^2 v_1^2 v_2 / x, \\ \mathcal{M}_{34}^2 &= r\lambda A v \sin\eta, \\ \mathcal{M}_{15}^2 &= -3r\lambda A v_2 \sin\eta, \\ \mathcal{M}_{25}^2 &= -3r\lambda A v_1 \sin\eta, \\ \mathcal{M}_{35}^2 &= -3r\lambda A v \cos\eta - 6r\lambda^2 v_1 v_2 v / x, \end{aligned} \quad (9)$$

where $\lambda_1 = \frac{1}{2}(g_1^2 + g_2^2) = M_Z^2 / v^2$, $A \equiv A_k$, and $rA \equiv A_\lambda$. Here and henceforth v_1, v_2 , and x stand for the absolute values of the VEV's. Δ in the second line of Eq. (9) is the top-quark radiative correction for which a reasonable approximate form is

$$\Delta = \frac{1}{\lambda_1} \frac{3h_t^4}{4\pi^2} \left[\ln \left(\frac{M_{\text{sq}}^2}{m_t^2} \right) + p \right]. \quad (10)$$

The parameter p represents certain nonlogarithmic corrections which may be significant if $\tilde{t}_L - \tilde{t}_R$ mixing is not negligible [10]. The physical charged Higgs boson mass is given by

$$m_{H^\pm}^2 = M_W^2 + (3r - 1)\lambda^2 v^2. \quad (11)$$

Since λ_1 and $v = \sqrt{v_1^2 + v_2^2} = 174$ GeV are known, our parameter set is $\{\tan\beta, x, A, r, \lambda, \cos\eta, \Delta\}$ where $\tan\beta = v_2/v_1$. [We have used Eqs. (4) and (6) to eliminate k in favor of the angle $\cos\eta$.]

Since two of the neutral Higgs bosons (h_1 and h_2) are relatively light in this model, their masses and mixings are constrained by the LEP data on Higgs boson searches. In Fig. 2 is displayed a plot of the experimentally allowed region in the $\lambda - \cos\eta$ plane for fixed values of the other parameters. The first constraint is that h_1 and h_2 have not been produced in the decay

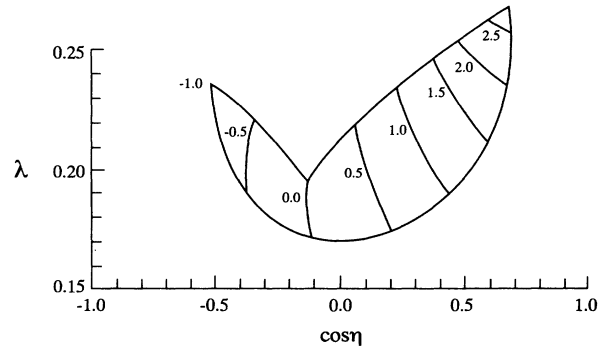


FIG. 2. A plot of the allowed region (inside the closed curves) of the $(\lambda - \cos\eta)$ plane, for $\tan\beta = 1$, $r = 0.8$, $A = 3v$, $x = 3.8v$, and $\Delta = 3.8$. The lower boundary of the region corresponds to $m_{h_1} + m_{h_2} = M_Z$. The upper boundary is the constraint from nonobservation of $Z \rightarrow Z^* + h$. The contours inside the region represent the electron EDM in units of 10^{-27} e cm.

of a real Z [11]. Since there is no significant suppression in the Zh_1h_2 coupling, $Z \rightarrow h_1 + h_2$ should be kinematically forbidden. The bottom (small λ) boundary of the allowed region in Fig. 2 corresponds to $m_{h_1} + m_{h_2} = M_Z$. The second constraint is that a light Higgs particle have not been produced in the decay $Z \rightarrow Z^*h$. If $h = \sum_{i=1}^5 \alpha_i \Phi_i$, where $\sum_i |\alpha_i|^2 = 1$, then the cross section for the process is approximately proportional to $|\alpha_1 \cos\beta + \alpha_2 \sin\beta|^2 m_h^{-1}$. The condition we have imposed, that gives the upper (large λ) boundary of the allowed region is [11] $m_h > (60 \text{ GeV})|\alpha_1 \cos\beta + \alpha_2 \sin\beta|^2$. In addition, the “pseudoscalar” should be heavier than 20 GeV [12], which is always satisfied in the fit of Fig. 2 (see Table I).

The allowed region in Fig. 2 corresponds to the fixed parameters having the values $\tan\beta = 1, r = 0.8, A = 3v, x = 3.8v$, and $\Delta = 3.8$. This is a relatively large value for Δ , but is realized if the top Yukawa coupling is large, near its renormalization-group fixed point. For example, if $M_{\text{sq}} = 3 \text{ TeV}$, $h_t = 1.05$ [at $Q^2 = (3 \text{ TeV})^2$] and $p = 1$ in Eq. (10), we obtain $\Delta = 3.8$ at $Q^2 = m_t^2$. [This involves a further running from $Q^2 = M_{\text{sq}}^2$ to m_t^2 assuming nonsupersymmetric (non-SUSY) two-doublet spectrum.] A larger value of h_t will be inconsistent with perturbative unification (in the desert scenario), so a larger Δ seems to be unlikely, unless the squark masses are heavier than 3 TeV. If Δ is taken to be smaller (with the other parameters fixed at the above values), the allowed region rapidly shrinks and vanishes at $\Delta \simeq 3.1$ corresponding to $M_{\text{sq}} \simeq 1.5 \text{ TeV}$ (with the same h_t). Similarly, if $\tan\beta$ is increased above 1, the allowed region shrinks and disappears at $\tan\beta \simeq 1.3$. In this scenario of CP violation, then, the parameters $\Delta(M_{\text{sq}}, h_t)$ and $\tan\beta$ are tightly constrained. The same is seen from Fig. 2 to be true of the parameter λ which must lie in a rather narrow range (around 0.2 for $r = 0.8$). Numerically it is found that as r decreases the allowed range of λ shifts upward, and vice versa. On the other hand, the parameters r, A, x , and

$\cos\eta$ can vary over a rather wide range. For example, we find solutions for $0.5 \leq r \leq 2.7$. Much of the above behavior can be understood fairly well analytically.

Consider certain subdeterminants of the 5×5 mass-squared matrix given in Eq. (9). If the CP -violating extremum is to be a local minimum, this matrix must have no negative eigenvalues and its subdeterminants must be all non-negative:

$$\det_{1235} = 9\lambda^2 r^2 A^2 \sin^2 2\beta \sin^2 \eta v^6 \times \left\{ \frac{1}{2} \lambda_1 \left[\frac{1}{2} \lambda_1 \Delta - 2(3r-1)\lambda^2 \right] - \lambda^4 (3r-1)^2 \right\}, \quad (12)$$

$$\det_{345} = 3\lambda^2 r^3 A^2 x^{-2} (3r-1) \sin^2 \eta \{ 9\lambda^2 \sin^2 2\beta v^6 + 4A^2 x^2 v^2 + 12\lambda A \sin 2\beta \cos \eta x v^4 \}, \quad (13)$$

where $\det_{ij\dots k}$ is the determinant of the submatrix corresponding to the $ij\dots k$ th rows and columns. Since $r \geq 0$ from M_{33}^2 , Eq. (13) implies that $(3r-1) \geq 0$. If one sets $\Delta = 0$, then the right-hand side of Eq. (12) with $(3r-1) \geq 0$ is negative. This is the result of Romao [6], that at tree level spontaneous CP violation does not occur in the model.

Equations (12) and (13) allow us to place bounds on λ^2 if CP is spontaneously broken in the Higgs sector:

$$0 \leq (3r-1)\lambda^2 \leq \frac{1}{2} \lambda_1 (\sqrt{1+\Delta} - 1) \quad (14)$$

One also sees from Eqs. (12) and (13) that the light Higgs boson masses are made largest by making $\sin 2\beta$ large, that is, $\tan\beta \simeq 1$ is preferred. As a consequence of Eq. (14), the charged Higgs boson mass is predicted to be less than about 110 GeV [see Eq. (11)].

More insight can be obtained by considering the small λ limit, which is justified by eq. (14) and by our numerical results. As $\lambda \rightarrow 0$, the mixing between the $\Phi_{1,2,3}$ block and $\Phi_{4,5}$ block goes away. If one block diagonalizes to obtain the 3×3 block to order λ^2 one finds

$$\mathcal{M}^2(\Phi_1, \Phi_2, \Phi_3) = \begin{pmatrix} \lambda_1 v_1^2 & -v_1 v_2 [\lambda_1 + 2(3r-1)\lambda^2] & 0 \\ -v_1 v_2 [\lambda_1 + 2(3r-1)\lambda^2] & \lambda_1 v_2^2 (1 + \Delta) & 0 \\ 0 & 0 & 0 \end{pmatrix} + C \lambda_1 \begin{pmatrix} v_2^2 \cos^2 \eta & v_1 v_2 \cos^2 \eta & -v v_2 \sin \eta \cos \eta \\ v_1 v_2 \cos^2 \eta & v_1^2 \cos^2 \eta & -v v_1 \sin \eta \cos \eta \\ -v v_2 \sin \eta \cos \eta & -v v_1 \sin \eta \cos \eta & v^2 \sin^2 \eta \end{pmatrix}, \quad (15)$$

where $C \equiv [4r(3r-1)/(4r-1)] (\lambda^2/\lambda_1)$. From the eigenvalues of $\mathcal{M}^2(\Phi_1, \Phi_2, \Phi_3)$ a nontrivial lower bound on λ^2 can be derived as follows. Label the eigenvalues to be $M_I^2 \leq M_{II}^2 \leq M_{III}^2$. It is possible by considering the eigenvalues as a function of C (defined above) to show that $M_I^2 \leq C M_{II}^2 = C \lambda_1 v^2$. (This can be done by plotting the eigenvalues of $\mathcal{M}^2(C)/M_{II}^2$ versus C and considering how often they cross the line $y = C$ which can be determined from the equation $\det\{\mathcal{M}^2(C)/M_{II}^2 - CI\} = 0$. This is easily seen to be quadratic (not cubic) in C and to have two real roots, one of which is $C = 0$.) From Eq. (15) one sees that for small λ^2 and $\Delta \geq \cot^2 \beta - 1$ the largest entry of $\mathcal{M}^2$ is greater than $(1 + \Delta)\lambda_1 v_2^2 =$

TABLE I. Some selected points in the $(\lambda - \cos\eta)$ plane for the same parameter values as in Fig. 2. m_{h_1} and m_{h_2} are the masses (in GeV) of the lightest two neutral Higgs bosons h_1 and h_2 . The R_{ij} are defined by $h_i = \sum_{j=1}^5 R_{ij} \Phi_j$. The electron EDM d_e is in units of $10^{-28} e \text{ cm}$.

$\cos\eta$	λ	m_{h_1}	m_{h_2}	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	d_e
-0.5	0.235	34.3	56.9	-0.78	-0.29	0.55	0.53	0.17	0.83	-10.0
0.0	0.17	49.5	41.6	-0.92	-0.29	0.25	0.24	0.08	0.97	1.75
0.0	0.21	42.5	53.3	0.88	0.31	0.35	-0.33	-0.11	0.94	3.0
0.5	0.195	59.2	32.0	-0.82	-0.24	0.52	0.49	0.18	0.85	11.5
0.5	0.225	63.2	30.6	-0.73	-0.21	0.65	0.61	0.23	0.76	15.2
0.5	0.253	68.5	26.2	-0.67	-0.19	0.74	0.68	0.27	0.68	21.5
0.67	0.223	66.7	24.5	-0.81	-0.23	0.55	0.51	0.20	0.84	19.8

$(1 + \Delta)M_Z^2 \sin^2 \beta$. Therefore $M_{II}^2 \geq (1 + \Delta)M_Z^2 \sin^2 \beta$. The trace of $\mathcal{M}^2$ implies that $M_I^2 + M_{II}^2 + M_{III}^2 = M_Z^2(1 + \Delta \sin^2 \beta + C)$. Thus $M_I^2 + M_{II}^2 \leq M_Z^2(C + \cos^2 \beta)$. This together with $M_I^2 \leq CM_Z^2$ imply that if $C \leq \cos^2 \beta$, then $(M_I + M_{II})/M_Z \leq \sqrt{C + \cos^2 \beta}$, while if $C \geq \cos^2 \beta$, then $(M_I + M_{II})/M_Z \leq \sqrt{2C + 2\cos^2 \beta}$. In order to have $M_I + M_{II} \geq M_Z$ it then must be that $C \geq (1 - \cos^2 \beta)^2$. Using the definition of C one gets a lower bound on λ^2 , which combining with Eq. (14) gives

$$\frac{4r-1}{4r} \lambda_1 (1 - \cos^2 \beta)^2 \leq (3r-1)\lambda^2 \leq \frac{1}{2} \lambda_1 (\sqrt{1 + \Delta} - 1). \quad (16)$$

For the values of $r = 0.8, \Delta = 3.8, \tan \beta = 1$ that are used in Fig. 2 one has $0.13 \leq \lambda \leq 0.41$ which is seen to be satisfied well by the numerical results. One can also derive from Eq. (16) a better lower limit on r than $r \geq 1/3$.

A remark about the small λ limit is in order. From Eq. (4) and Fig. 1 it can be seen that as $\lambda \rightarrow 0, x/A \rightarrow r/k$. (k in Fig. 2 is about 0.63, the fixed point value.) Terms have been dropped in deriving Eq. (15) that are down by $O(\lambda x/A)$. For large r , these terms become important, which tend to decrease the light eigenvalues of $\mathcal{M}^2$. This is why no realistic solution exists for large r ($r \gtrsim 2.7$).

In Fig. 2 we have also plotted contours of the electric dipole moment of the electron. These have been computed using the results of Leigh, Paban, and Xu (LPX) [1]. Only the graphs with the same topology as the top quark graphs (Figs. 1 and 2 of LPX), and only the contributions from the lightest two neutral Higgs bosons have been included. This should be good to about 20%. The typical values of d_e are in the range $(10^{-28} \text{ to } 3 \times 10^{-27})$ e cm, interesting for future atomic experiments. In Table I are presented the masses and mixings of the light-

est Higgs bosons for various λ and $\cos \eta$ in Fig. 2 from which we have computed d_e . The neutron EDM (d_n) arising from the two-loop Higgs exchange graphs [1] is $d_n \sim 10^{-26}$ e cm. The contribution to d_n coming from the one-loop gluino graph is suppressed both by powers of the squark mass [which has to be big, see Eq. (11)] and by small phases. (The gluino phase is itself a loop effect while the squark mass phase ends up being suppressed by $\lambda x/A \sim 1/5$.) Thus there is no conflict with present limits on d_n [10].

It is interesting to ask if the Higgs sector CP violation alone can account for ϵ and ϵ' in the K meson system. Pomarol [3] has shown that this may indeed be possible in a more generalized version of the SUSY singlet model. Since the effects of spontaneous CP violation felt in the fermion sector in our model is identical, we expect the results of Ref. [3] to hold in our case as well.

So far we have said nothing about whether the CP -violating minimum that we have been studying is also the absolute minimum. For fixed $|H_1|, |H_2|$, and $|N|$ it is easy to see that V at the CP -violating extremal angles [Eq. (6)] is lower than at the CP -conserving values $\eta, \zeta = 0, \pi$. Of course the true CP -violating and CP -conserving minima will not be at the same values of H_1, H_2 , and N . However, it can be shown analytically that for small λ at least the CP -violating minimum is the lowest [10]. We have verified this result numerically for some reasonable values of the parameters. It should be emphasized that the existence of this deeper CP -violating minimum should rule out much of the parameter space because of the tendency of the lightest neutral Higgs boson to be too light.

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sumed in the proof of that theorem.

- [9] In supergravity models with a canonical Kahler potential $r = 1$ (and thus real) at the unification scale. r can develop a phase in the process of running only through complex gaugino mass terms, but this effect will be small due to constraints on the gaugino phase from the neutron EDM.
- [10] The radiative corrections are treated more exactly in a forthcoming paper, K.S. Babu and S.M. Barr (unpublished).
- [11] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
- [12] ALEPH Collaboration, D. Decamp *et al.*, Phys. Lett. B **265**, 475 (1991).