Approximate flavor symmetries in the lepton sector

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Approximate flavor symmetries in the quark sector have been used as a handle on physics beyond the standard model. Because of the great interest in neutrino masses and mixings and the wealth of existing and proposed neutrino experiments it is important to extend this analysis to the leptonic sector. We show that in the seesaw mechanism the neutrino masses and mixing angles do not depend on the details of the right-handed neutrino flavor symmetry breaking, and are related by a simple formula. We propose several *Ansätze* which relate different flavor symmetry-breaking parameters and find that the MSW solution to the solar neutrino problem is always easily fit. Further, the v_μ-v_τ oscillation is unlikely to solve the atmospheric neutrino problem and, if we fix the neutrino mass scale by the MSW solution, the neutrino masses are found to be too small to close the Universe.

I. INTRODUCTION

The smallness of Yukawa couplings has been related to the approximate flavor symmetries [1,2]. The Yukawa couplings can be understood as being naturally small [3], since by putting them equal to zero new global symmetries appear in the theory. These are the flavor symmetries under which each of the fermions transforms separately but scalars do not. In the low-energy theory these symmetries are broken by different amounts, as is evident from the nonzero masses of fermions. The lack of knowledge of the exact mechanism by which the symmetries were broken was parametrized by a set of small numbers (denoted by ϵ). Each Yukawa coupling is then given approximately as the product of small symmetry-breaking parameters ϵ for all flavor symmetries broken by that coupling. So for example the coupling of a scalar doublet H to the ith generation of left-handed doublet quarks Q_i and the jth generation of right-handed up quarks U_j is given by

$$\lambda_{ij}^U \approx \epsilon_{Q_i} \epsilon_{U_j} . \tag{1}$$

Whereas this gives the right order of magnitude for the couplings, the exact relation of couplings to the ϵ's may be off by a factor of 2 or 3. This is because the underlying theory [possibly a grand unified theory (GUT)] by which the flavor symmetries are broken is unknown. Therefore all estimates of the possible new flavor changing interactions should be taken to have at least the same uncertainty.

The flavor symmetry-breaking parameters ϵ can be estimated in several ways. They can be postulated by

Ansätze [2] which are consistent with the known values of fermion masses and mixings. Alternatively, one may use the experimental results to constrain the ϵ's. This was done in the quark sector [4] using the known values of quark masses and Kobayashi-Maskawa mixing matrix elements.

The important result so obtained was the flavor-changing scalar interactions are possible even for the masses of the new scalars as low as a few hundred GeV to 1 TeV, and numerous estimates for different flavor changing interactions were given [2,4]. Hall and Weinberg [4] also noticed that, allowing for complex Yukawa couplings in this scheme, the observed smallness of *CP* violation constrains the phases to be small (i.e., of the order of 10^{-3}).

In the lepton sector, the masses are small, indicating that the approximate flavor symmetries are preserved to a high degree of accuracy. Therefore lepton flavor changing interactions would be extremely hard to test [2]. Further, since the neutrino masses and mixing angles have not been measured yet, one cannot estimate the corresponding ϵ's without additional assumptions. Nevertheless we find it important to address the question of approximate flavor symmetries in the lepton sector, because many experiments which aim to measure or set better limits on neutrino masses and mixings are under way or being planned for the near future. Indeed, in this paper we find that statements can be made about the lepton sector, regardless of additional assumptions about the ϵ's. For example, while the Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino problem can be easily fit, the predicted neutrino masses are unlikely to close the Universe.

This paper is organized as follows. In Sec. II we express the lepton sector Yukawa couplings in terms of flavor symmetry breaking parameters. Here we study two cases of neutrino masses. In the seesaw mechanism

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[5] case we show that the neutrino masses and mixings are independent of the right-handed neutrino flavor symmetry-breaking mechanism. We also include for completeness the case of Dirac neutrino masses only (as in the case of charged fermions). In Sec. III we provide several *Ansätze* for the lepton flavor symmetry-breaking parameters and list their predictions in terms of ratios of neutrino masses, and mixings. We study their relevance to the solar neutrino problem, atmospheric neutrino problem, closure of the Universe, etc. General features are noted which are independent of the particular *Ansatz* used.

II. APPROXIMATE FLAVOR SYMMETRIES IN THE LEPTON SECTOR

By adding the right-handed neutrinos N_i, $i = 1,2,3$ to the particle content of the standard model, we can allow for Dirac-type masses. Under the action of approximate flavor symmetries, whenever an N_i enters a Yukawa interaction, the corresponding coupling must contain the symmetry breaking parameter ϵ_{N_i}.

A natural way to justify the smallness of neutrino masses is to use the *seesaw mechanism* in which the smallness of the left-handed neutrino masses is explained by the new scale of heavy right-handed neutrinos. The mass matrices will have the structure

$$m_{N_{Dij}} \approx \epsilon_{L_i} \epsilon_{N_j} v_{\text{SM}} , \tag{2}$$

$$m_{N_{Mij}} \approx \epsilon_{N_i} \epsilon_{N_j} v_{\text{big}} , \tag{3}$$

$$m_{E_{ij}} \approx \epsilon_{L_i} \epsilon_{E_j} v_{\text{SM}} , \tag{4}$$

where m_{N_D} and m_E are the neutrino and charged lepton Dirac mass matrices, m_{N_M} is the right-handed neutrino Majorana mass matrix, $v_{\text{SM}} = 174$ GeV and v_{big} is the new large mass scale. The generation indices i and j run from 1 to 3. In the following we assume a hierarchy in the ϵ's (i.e., $\epsilon_{L_1} \ll \epsilon_{L_2} \ll \epsilon_{L_3}$, etc.) as suggested by the hierarchy of quark and charged lepton masses. Then the diagonalization of the neutrino mass matrix will give a heavy sector with masses $m_{N_{Hi}} \approx \epsilon_{N_i}^2 v_{\text{big}}$ and a very light sector with the mass matrix

$$m_{N_{Lij}} \approx (m_{N_D} m_{N_M}^{-1} m_{N_D}^T)_{ij} \approx \epsilon_{L_i} \epsilon_{L_j} \frac{v_{\text{SM}}^2}{v_{\text{big}}} , \tag{5}$$

where the number $\text{Tr}[\epsilon_N (\epsilon_N \epsilon_N)^{-1} \epsilon_N]$ is assumed to be of order 1. We have the expected result: the heavy right-handed neutrino decouples from the theory, leaving behind a very light left-handed neutrino. The masses and mixing angles are independent of the right-handed symmetry breaking parameters ϵ_{N_i}:

$$m_i^N \approx \epsilon_{L_i}^2 \frac{v_{\text{SM}}^2}{v_{\text{big}}} ,$$

$$m_i^E \approx \epsilon_{L_i} \epsilon_{E_i} v_{\text{SM}} \quad \text{(no sum on } i\text{)} , \tag{6}$$

$$V_{ij} \approx \frac{\epsilon_{L_i}}{\epsilon_{L_j}} \quad (i < j) .$$

Therefore, in addition to the unknown scale v_{big}, only two sets of ϵ's are needed: ϵ_{L_i} and ϵ_{E_i}. In fact, the neutrino masses and mixings depend only on ϵ_{L_i} and they are approximately related through

$$V_{ij} \approx \left[\frac{m_i^N}{m_j^N} \right]^{1/2} . \tag{7}$$

Equation (7) reduces the number of parameters needed to describe neutrino masses and mixings by three; for example, given two mixing angles and one neutrino mass, we can predict the third mixing angle and the other two neutrino masses. These results are extremely general. They follow simply from the factorization of the Dirac masses, regardless of the specific form of $m_{N_M}^{-1}$, which only contributes to set the scale.

For completeness, we note that in the case of *Dirac masses only*, the neutrinos and the charged leptons mass matrices become

$$m_{N_{Dij}} \approx \epsilon_{L_i} \epsilon_{N_j} v_{\text{SM}} , \tag{8}$$

$$m_{E_{ij}} \approx \epsilon_{L_i} \epsilon_{E_j} v_{\text{SM}} . \tag{9}$$

Their diagonalization yields

$$m_i^N \approx \epsilon_{L_i} \epsilon_{N_i} v_{\text{SM}} \quad \text{(no sum on } i\text{)} ,$$

$$m_i^E \approx \epsilon_{L_i} \epsilon_{E_i} v_{\text{SM}} \quad \text{(no sum on } i\text{)} , \tag{10}$$

whereas the lepton mixing matrix is approximately diagonal with off-diagonal elements of the order of

$$V_{ij} \approx \frac{\epsilon_{L_i}}{\epsilon_{L_j}} \quad (i < j) . \tag{11}$$

Therefore in this case we need three sets of ϵ's to explain masses and mixing [cf. Eqs. (10) and (11)]: ϵ_{L_i}, ϵ_{N_i}, and ϵ_{E_i}.

In addition to having one more set of (unknown) ϵ's than the *seesaw* case, the *Dirac only* case has no natural explanation for the very low scale associated with the neutrino masses, i.e., that the flavor symmetries of right-handed neutrinos are much better preserved than for other fermions. It turns out that one typically gets the ϵ_{N_i} of order 10^{-12} or so, compare to quark and charged lepton ϵ's which are typically 10^{-3} and larger. Therefore, in the following we will mostly concentrate on the *seesaw* case.

III. ANSÄTZE AND PREDICTIONS

While it was possible to determine the flavor symmetry breaking parameters in the quark sector from quark masses and mixings [4], in the lepton sector the situation is much more difficult. At this direct laboratory experiments provide only upper limits on neutrino masses and mixings, although some indirect sources such as solar neutrinos or cosmology point to some specific allowed ranges. Therefore to estimate the sizes of lepton flavor symmetry-breaking parameters ϵ's additional assumptions are needed.

Our strategy is as follows: we list several plausible or GUT motivated *Ansätze* which relate ϵ's of different fields. This will enable us to estimate ratios of neutrino masses and mixings. If the mixings are consistent with the allowed range for the MSW solution [6] of the solar neutrino problem, we take this as a hint to fix the mass scale and predict all neutrino masses. We then look at further predictions. As is the case with any calculation based on these approximate flavor symmetries [1,2,4], the factors of 2 or 3 may contribute coherently factors of an order of magnitude or so. In addition, the numerical results depend on the specific *Ansatz*. What we seek are the general features of those results rather than the detailed numerical results themselves.

As our first *Ansatz* (*Ansatz* I in Table I) we assume that $\epsilon_{L_i} = \epsilon_{E_i}$ [2]. This *Ansatz* can be justified as follows [7]. Assume that in the lepton sector the only combination of symmetry which is broken is the axial flavor symmetry. This means that, instead of breaking separately left (L) and right (R) flavor symmetries, only the combination $L - R$ is broken. Therefore we need only one set of ϵ's, which are then determined from $\epsilon_{L_i} \approx \sqrt{m_i^E}/v_{\mathrm{SM}}$. The v_e-v_μ mixing found in this way is consistent with the small-mixing-angle region [8] of the MSW explanation for the solar neutrino problem (SNP). The mixing angles for this and the other *Ansätze* can be found in Table I. We checked that for these mixing angles the third flavor does in fact decouple (see [9]). If this is indeed the solution for that problem, the mass of the muon neutrino must be around 3×10^{-3} eV. This then sets the scale for the new physics and the other neutrino masses at

$$v_{\mathrm{big}} \approx \frac{\epsilon_{L_2}^2 v_{\mathrm{SM}}^2}{m_{v_\mu}} \approx 10^{13} \text{ GeV} ,$$ (12)

$$m_{v_e} \approx \frac{\epsilon_{L_1}^2}{\epsilon_{L_2}^2} m_{v_\mu} \approx 10^{-5} \text{ eV} ,$$

$$m_{v_\tau} \approx \frac{\epsilon_{L_3}^2}{\epsilon_{L_2}^2} m_{v_\mu} \approx 10^{-2} \text{ eV} .$$ (13)

Taking into account the excluded regions due to the IMB experiment [10], the predicted muon to tau neutrino mixing [$\Delta m^2 \approx 10^{-3}$ eV^2, $\sin^2(2\theta_{\mu\tau}) \approx 0.2$] is around a factor of five away from the parameters consistent with a v_μ-v_τ oscillation explanation of the atmospheric neutrino problem (ANP) [11], and still two orders of magnitude away from the laboratory limits for this mixing. Finally, the smallness of al three neutrino masses in this *Ansatz* suggests that neutrinos cannot be responsible for closing the Universe.

TABLE I. Neutrino mixing angle predictions in the three *Ansätze* introduced. As noted in the text, these results are meant as estimates rather than precise calculations.

Ansatz	$\sin^2(2\theta_{e\mu})$	$\sin^2(2\theta_{e\tau})$	$\sin^2(2\theta_{\mu\tau})$
I	2×10^{-2}	10^{-3}	0.2
II	0.2	0.1	0.8
III	2×10^{-3}	8×10^{-6}	2×10^{-2}

Another interesting *Ansatz* (*Ansatz* II in Table I) is suggested by the fact that in the quark sector $\epsilon_{Q_i} \approx \epsilon_{U_i}, i = 1,2,3$, as found by Hall and Weinberg [4]. Inspired by an SU(5)-type unification we are led to look at an ansatz in which

$$\epsilon_{L_i} \propto \epsilon_{D_i}, \quad \epsilon_{E_i} \propto \epsilon_{Q_i} \approx \epsilon_{U_i} .$$ (14)

In this *Ansatz* we *predict* (using the numerical values of ϵ_Q, ϵ_U, and ϵ_D from [4]) the ratios of charged lepton masses to be within factors of three of the measured values. We consider this an interesting result. Further, the mixing angles are consistent with the three flavor mixing explanations of the SNP [9] for squared masses of v_μ and v_τ of order 10^{-4} eV^2. Therefore, the v_μ-v_τ oscillation explanation of the ANP is unlikely. In addition this mass scale cannot be tested in the laboratory nor provide an explanation for dark matter.

Finally, one might look for inspiration in the breaking of SO(10) into SU(4)$\otimes$SU(2)$_L \otimes$SU(2)$_R$. We know that at the renormalization scale of 1 GeV we have been working at, the SU(2)$_R$ symmetry must be badly broken since $m_t \gg m_b$ and $m_e \gg m_{v_e}$. Further, assuming the *Ansatz*, $\epsilon_{L_i} \propto \epsilon_{Q_i}$, $\epsilon_{E_i} \propto \epsilon_{D_i}$, and $\epsilon_{N_i} \propto \epsilon_{U_i}$ would lead to $m_e/m_\mu \approx 4.8 \times 10^{-2}$, which is wrong by an order of magnitude. Assuming that the SU(4) might still provide some useful information for the SU(2)$_L$ singlets we look at the *Ansatz* (*Ansatz* III in Table I)

$$\epsilon_{E_i} \propto \epsilon_{D_i}, \quad \epsilon_{N_i} \propto \epsilon_{U_i} .$$ (15)

In this *Ansatz* we again predict a v_e-v_μ mixing angle consistent with the small-angle MSW solution to the SNP. Again, assuming that this is indeed the solution for that problem fixes the seesaw neutrino masses at

$$m_{v_\mu} \approx 10^{-3} \text{ eV} ,$$

$$m_{v_e} \approx \left[\frac{\epsilon_{L_1}}{\epsilon_{L_2}} \right]^2 m_{v_\mu} \approx 10^{-6} \text{ eV} ,$$ (16)

$$m_{v_\tau} \approx \left[\frac{\epsilon_{L_3}}{\epsilon_{L_2}} \right]^2 m_{v_\mu} \approx 0.5 \text{ eV} .$$

We again find it unlikely that the values obtained can close the Universe or solve the ANP.

In conclusion, we extended the concept of approximate flavor symmetries to the lepton sector. In particular we considered the see-saw mechanism as a source of the neutrino masses and showed that the predictions do not depend on the neutrino flavor symmetry breaking parameters. This yields a simple relation [cf. Eq. (7)] between neutrino masses and mixing angles which reduces the number of parameters needed to describe the lepton sector. The lack of information of the neutrino masses and mixing angles led us to propose several *Ansätze*. These exhibit the following common features. They are consistent with the MSW solution of the SNP. The ANP is unlikely to be explained through v_μ-v_τ oscillations, and

the scale of neutrino masses is too small to close the Universe.

ACKNOWLEDGMENTS

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