PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOLUME 49, NUMBER 3 1 FEBRUARY 1994

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Determining the penguin effect on CP violation in $B^0 \to \pi^+\pi^-$

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(Received 14 September 1993)

A major goal in B physics is measuring the CP-violating asymmetry in the decay $B^0 \to \pi^+\pi^-$. In order to determine one of the phase angles in the CKM matrix from this decay it is necessary to determine the influence of the penguin amplitude. Here we show how, using SU(3) symmetry, the penguin effect can be approximately determined from the ratio of the decay rates of $B^0 \to K^+\pi^-$ and $B^0 \to \pi^+\pi^-$.

PACS number(s): 11.30.Er, 12.15.Ff, 13.25.Hw

Recently evidence has been presented for the decays $\bar{B}^0 \to \pi^+\pi^-$ and $\bar{B}^0 \to \pi^+K^-$ with a combined branching ratio of 2×10^{-5} [1]. The decay $\bar{B}^0 \to \pi^+\pi^-$ is of particular interest since it is one of the prime candidates for the study of CP violation in B decays.

From the study of CP violation in B decays, one can determine the phases γ and β of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2], defined by

$$V_{ub} = A\lambda^3[\rho - i\eta] = A\lambda^3 z e^{-i\gamma} , \quad (1)$$

$$V_{td} = A\lambda^3[(1-\rho) - i\eta] = A\lambda^3 y e^{-i\beta} , \quad (2)$$

where

$$y = \sqrt{(1-\rho)^2 + \eta^2} = (\cos\gamma + \sin\gamma/\tan\beta)^{-1} , \quad (3)$$

$$z = \sqrt{\rho^2 + \eta^2} = (\cos\beta + \sin\beta/\tan\gamma)^{-1} . \quad (4)$$

The decay asymmetry for $B^0 \to \Psi K_S$ determines the angle β and, in the tree approximation, the decay asymmetry for $B^0 \to \pi^+\pi^-$ determines $(\beta+\gamma)$. However, a number of authors [3] have emphasized that there may be a sizable uncertain penguin contribution to the $\bar{B}^0 \to \pi^+\pi^-$ decay, thus making the determination of $(\beta+\gamma)$ uncertain. Here we show that the approximate SU(3) symmetry can be used to estimate this penguin contribution once the ratio of π^+K^- to $\pi^+\pi^-$ is determined.

The effective Hamiltonian responsible for these decays is [4]

$$H_{\text{eff}} = 2\sqrt{2}G_F \left[\xi_u^\alpha (C_1 \mathcal{O}_1^\alpha + C_2 \mathcal{O}_2^\alpha) + \xi_t^\alpha \sum_{k=3}^{6} C_k \mathcal{O}_k^\alpha + \text{H.c.} \right] , \quad (5)$$

where

$$\xi_u^\alpha = V_{ub} V_{u\alpha}^* , \qquad \xi_t^\alpha = V_{tb} V_{t\alpha}^* ,$$

and $\alpha = d$ for the $\pi^+\pi^-$ decay and $\alpha = s$ for the π^+K^- decay. The operators are

$$\mathcal{O}_1^\alpha = \bar{\alpha}\gamma^\mu\gamma_L b \;\; \bar{u}\gamma_\mu\gamma_L u ,$$

$$\mathcal{O}_2^\alpha = \bar{u}\gamma^\mu\gamma_L b \;\; \bar{\alpha}\gamma_\mu\gamma_L u ,$$

$$\mathcal{O}_3^\alpha = \sum_{q=u,d,s} \bar{\alpha}\gamma^\mu\gamma_L b \;\; \bar{q}\gamma_\mu\gamma_L q ,$$

$$\mathcal{O}_4^\alpha = \sum_{q=u,d,s} \bar{q}\gamma^\mu\gamma_L b \;\; \bar{\alpha}\gamma_\mu\gamma_L q ,$$

$$\mathcal{O}_5^\alpha = \sum_{q=u,d,s} \bar{\alpha}\gamma^\mu\gamma_L b \;\; \bar{q}\gamma_\mu\gamma_R q ,$$

$$\mathcal{O}_6^\alpha = -2 \sum_{q=u,d,s} \bar{q}\gamma_L b \;\; \bar{\alpha}\gamma_R q , \quad (6)$$

where $\gamma_{R,L} = (1 \pm \gamma_5)/2$. The coefficients C_k are calculated by the renormalization group equation.

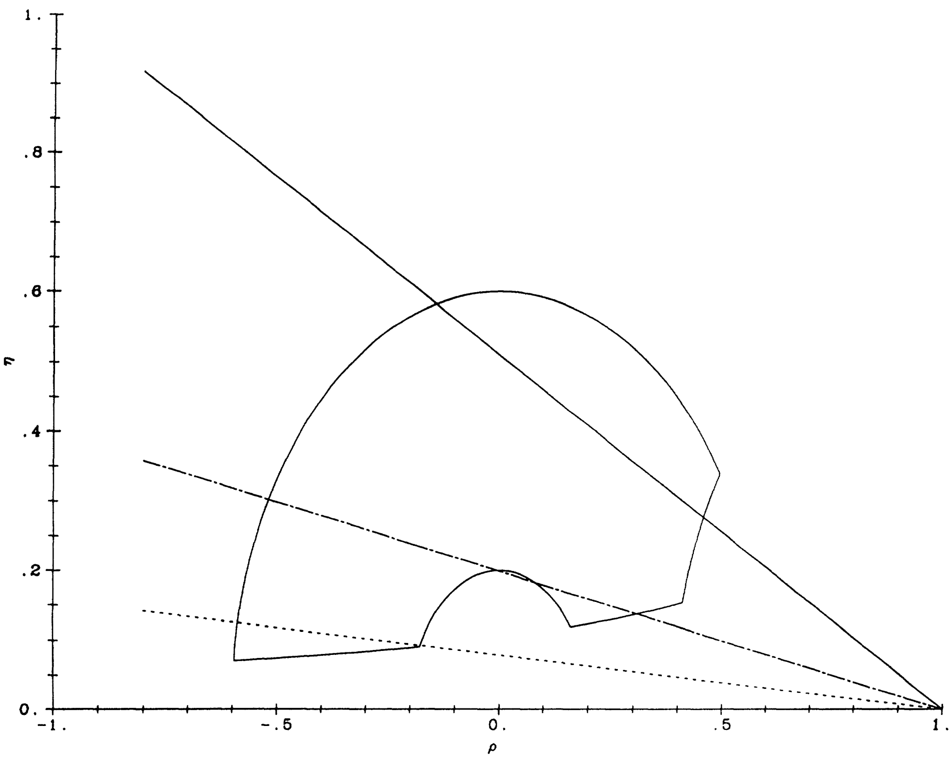

FIG. 1. Circular region defined by the contraints on the CKM parameters ρ and η from the combined measurements of ϵ, $|V_{ub}/V_{cb}|$ and B^0-$\bar{B}^0$ mixing, for $m_t < 180$ GeV. Also shown are presently allowed values of β corresponding to straight lines passing through $(\rho, \eta) = (1, 0)$. The solid line corresponds to $\beta = 0.6\pi/4$, the dotted line to $\beta = 0.1\pi/4$. The dash-dotted line corresponds to $\beta = 0.25\pi/4$ and crosses the allowed region twice.

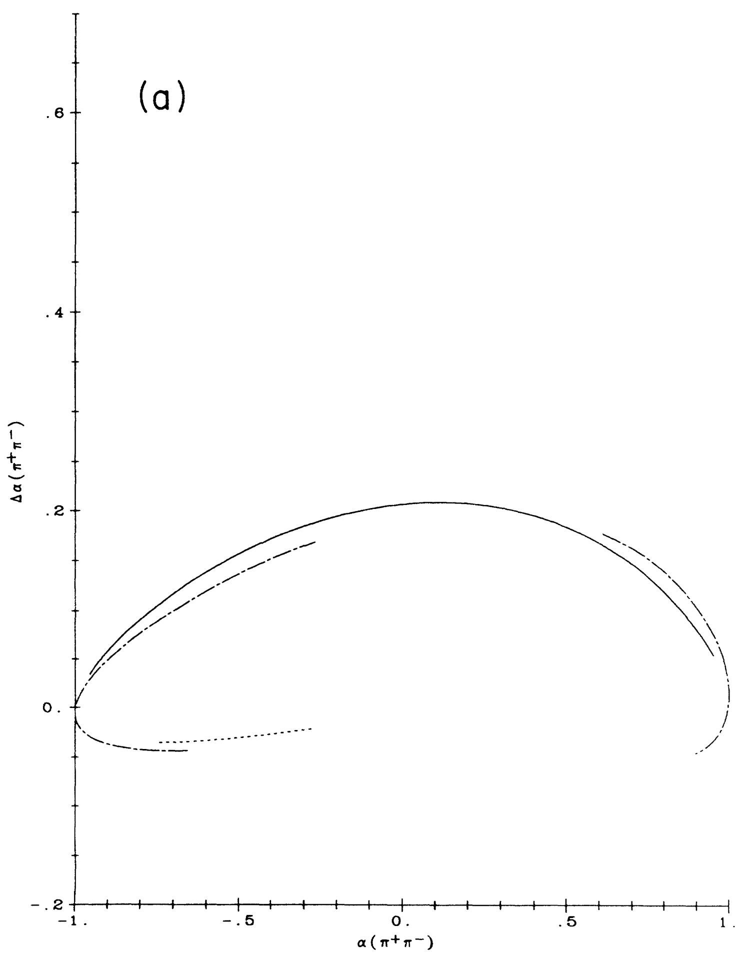

FIG. 2. Curves of constant β in the $(\Delta\alpha(\pi^+\pi^-), \alpha(\pi^+\pi^-))$ plane, for $R = 1/3$ (a), $R = 1$ (b), and $R = 3$ (c). The values of β are those of Fig. 1.

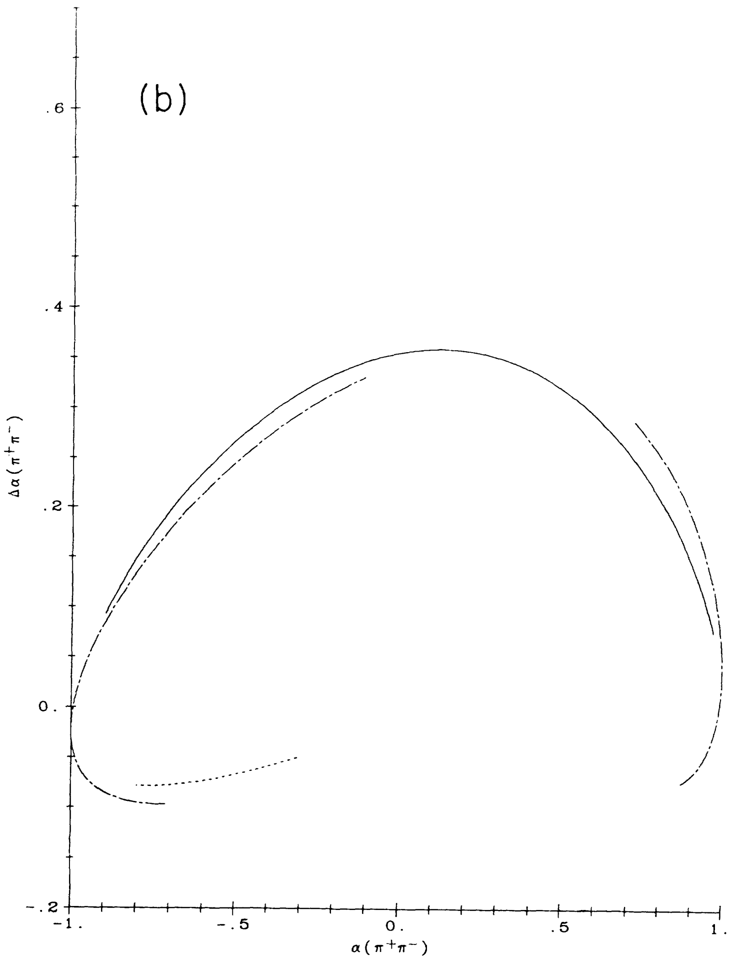

FIG. 2. (*Continued*).

The amplitude for the decays may be written as

$$A(\bar{B}^0 \to \pi^+\pi^-) = e^{-i\gamma}T_\pi + \lambda y e^{i\beta}P_\pi \ ,$$
$$A(\bar{B}^0 \to \pi^+K^-) = -P_K + \lambda e^{-i\gamma}T_K \ , \qquad (7)$$

where

$$T_\pi = |V_{ub}V_{ud}|\langle \pi^+\pi^-|C_1\mathcal{O}_1^d + C_2\mathcal{O}_2^d|\bar{B}^0\rangle \ ,$$
$$P_K = |V_{tb}V_{ts}|\left\langle \pi^+K^-\left|\sum_{k=3}^{6} C_k\mathcal{O}_k^s\right|\bar{B}^0\right\rangle \ . \qquad (8)$$

T_K and P_π are obtained by the substitutions $\mathcal{O}^d \rightleftharpoons \mathcal{O}^s$ and $\pi^- \rightleftharpoons K^-$. In the spectator approximation the decays are

$$b(\bar{d}) \to u(\bar{d}) + \pi^- \ ,$$
$$b(\bar{d}) \to u(\bar{d}) + K^- \ . \qquad (9)$$

Since each $\mathcal{O}_k^d$ is related to $\mathcal{O}_k^s$ by an SU(3) transformation (180° rotation around the V-spin axis), it would follow in the spectator and SU(3) invariance approximations that $T_K = T_\pi$ and $P_K = P_\pi$. A formal evaluation of the $\langle \mathcal{O}_k^\alpha \rangle$ matrix elements is typically done using factorization. The results are quite uncertain, both because of the uncertain $B \to \pi$ transition amplitude and the use of the factorization approximation. We expect these two uncertanties to be common to the two transitions so that we will use the factorization result for the *ratio* of the matrix elements:

$$T_K/T_\pi = P_K/P_\pi = f_K/f_\pi \ . \qquad (10)$$

In fact, the s and d quarks are much lighter than the b quark, so that one expects the difference between the two to be negligible. However, these quarks will then hadronize into kaons and pions. These hadronizations are different, as illustrated by the difference between their decay constants, f_K and f_π. Therefore it is natural to use Eq. (10) as an estimate of the SU(3) violation.

Neglecting final state interaction effects, P and T are real. Then, the rate ratio is given by

$$R = \frac{\Gamma(\bar{B}^0 \to \pi^+K^-)}{\Gamma(\bar{B}^0 \to \pi^+\pi^-)}$$
$$= \left(\frac{f_K}{f_\pi}\right)^2 \frac{x^2 - 2\lambda x\cos\gamma + \lambda^2}{1 + 2\lambda xy\cos(\beta+\gamma) + \lambda^2y^2x^2} \ , \qquad (11)$$

where $x = P_K/T_K = P_\pi/T_\pi$ and y is given by Eq. (3). The asymmetry in the decay $\bar{B}^0 \to \pi^+\pi^-$ is given by

$$\alpha(\pi^+\pi^-) = \sin 2(\beta + \gamma - \delta) \ , \qquad (12)$$

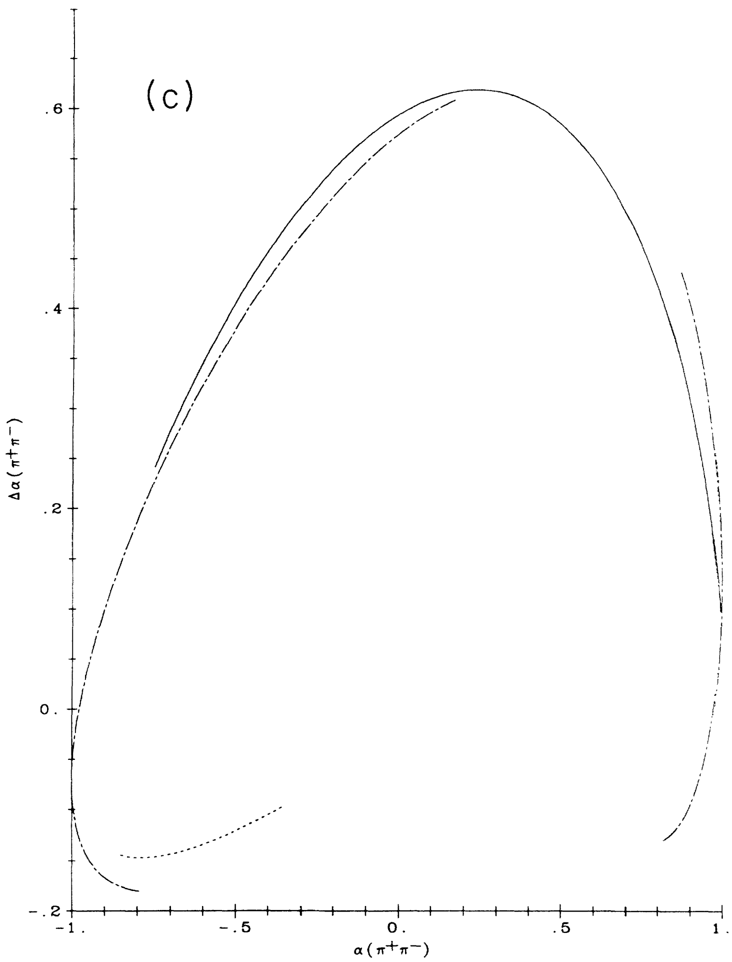

FIG. 2. (*Continued*).

where δ, the deviation of the phase of $A(\bar{B}^0 \to \pi^+\pi^-)$ from its tree value, is given by

$$\tan\delta = \frac{\lambda yx \sin(\beta+\gamma)}{1+\lambda yx \cos(\beta+\gamma)} . \qquad (13)$$

Assuming the angle β is determined from the asymmetry of the decay into ΨK_S and that R is measured, then for any value of γ, we can use Eq. (11) to determine x, Eq. (13) to determine δ, and Eq. (12) to determine $\alpha(\pi^+\pi^-)$. There are two solutions for x; in what follows we choose the solution with positive x, as given by factorization.

Results are shown in Figs. 1 and 2. Figure 1 shows the allowed region [5] in the (ρ,η) plane for $m_t < 180$ GeV. Lines illustrating three fixed values of β ($\beta = 0.1\pi/4$, $0.25\pi/4$, and $0.6\pi/4$) are shown crossing this region. While there are now no definitive data on R, the CLEO results [1] favor a value between 1/3 and 3. Therefore, we choose the values $R = 1/3$, 1, and 3 in our illustrations. Figure 2 shows the deviation of the asymmetry parameter $\alpha(\pi^+\pi^-)$ from its tree value, that is, $\Delta\alpha(\pi^+\pi^-) = \alpha(\pi^+\pi^-) - \sin 2(\beta+\gamma)$, as a function of $\alpha(\pi^+\pi^-)$, for our selected values of β and R.

For small and moderate values of the magnitude of $\alpha(\pi^+\pi^-)$ there are significant deviations for values of R of order 1 or greater. It is interesting to note that the size of these deviations depends very little on the value of β. For values of $\alpha(\pi^+\pi^-)$ in the neighborhood of $+1$ (-1) there are two solutions for γ for fixed β, even in the tree approximation. These solutions correspond to values of $(\beta+\gamma)$ either greater or smaller than $\pi/4$ $(3\pi/4)$. The sign of the deviation is seen to depend on which of the two solutions is chosen.

We have ignored in these calculations final state interactions which could produce a strong phase factor δ between the penguin and tree terms. It is generally believed that δ is not very large. The first order effect of δ is a term [6] proportional to $\sin\delta$ that shows up as a difference in the decay rates of B^0 and $\bar{B}^0$. If we assume that R is calculated for the sum of the decays of B^0 and $\bar{B}^0$, this effect cancels out. Thus, the only effect of δ is proportional to $(1-\cos\delta)$ and should be unimportant for our results.

It has been pointed out by Gronau and others [7] that detailed studies of the time dependence of several B^0 decays could rigorously "trap" the penguin contribution. Here we point out that, long before such measurements can be made, one can obtain a reasonable approximate value for the penguin effect simply from the measurement of branching ratios.

This work was supported by the United States Department of Energy, under Contract No. DE-FG02-91ER-40682. The work of J.P.S. was partially supported by the Portuguese JNICT, under CIÊNCIA Grant No. BD/374/90-RM.

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