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Quark correlation functions in deep-inelastic semi-inclusive processes

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We investigate one-particle semi-inclusive processes in lepton-hadron scattering. In unpolarized scattering order Q^{-1} corrections appear only when transverse momenta are detected. We consider the twist-two and -three matrix elements and calculate the semi-inclusive structure functions in terms of quark correlation functions. We find that at the twist-three level not only the standard quark distribution and fragmentation function contribute, but also two new transverse "profile functions." We discuss the gauge invariance of the hadronic tensor at the twist-three level. The results of our approach are used to calculate expressions for some cross sections for semi-inclusive processes.

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I. INTRODUCTION

In this paper we will investigate unpolarized one-particle semi-inclusive lepton-hadron scattering in the deep-inelastic scattering (DIS) limit. We will present the analysis in terms of quark correlation functions. These objects encode the nonperturbative parts of the process, which we as yet are not able to calculate other than using models. We will present an analysis which is complete up to and including $O(Q^{-1})$ where Q^2 is the negative square of the virtual photon momentum. In this order we will need to introduce four different projections of the quark correlation functions, called "profile functions." The Fourier transform of two of these are, respectively, the quark distribution function and the quark fragmentation function which are known from the naive quark parton model at leading order. The two new functions arise in $O(Q^{-1})$ and do not have simple parton model interpretations. The treatment of an (intrinsic) transverse momentum in this process will turn out to be essential for these new profile functions.

As in all lepton scattering experiments in the one-photon-exchange approximation the semi-inclusive cross section will be essentially given by a contraction between a leptonic tensor and a hadronic tensor. All the interesting physics resides in the hadronic tensor. For the one-particle semi-inclusive cross section the hadronic tensor is given by (see Sec. II for exact definitions and conventions)

$$2M\mathcal{W}_{\mu\nu} = \frac{1}{(2\pi)^4}\int \frac{d^3P_X}{(2\pi)^3 2E_X}\int d^4x\, e^{iq\cdot x} \times \langle P|J_\nu(x)|P_X,p_h\rangle\langle p_h,P_X|J_\mu(0)|P\rangle. \qquad (1)$$

Here q is the virtual photon momentum, P and p_h are nucleon and observed hadron momentum and P_X is shorthand for the ensemble of unobserved hadrons in the final state. Also the phase space integral over P_X should be understood to be a summation over all n-particle states of all types. We denote this tensor by the diagram in Fig. 1(a). In terms of quarks and gluons we know the photon current, which only couples to quarks. This gives us an expression for the hadronic tensor in terms of quark fields

$$2M\mathcal{W}_{\mu\nu} = \frac{1}{(2\pi)^4}\int \frac{d^3P_X}{(2\pi)^3 2E_X}\int d^4x\, e^{iq\cdot x} \times \langle P|\overline{\psi}(x)\gamma_\nu\psi(x)|P_X,p_h\rangle \times\langle p_h,P_X|\overline{\psi}(0)\gamma_\mu\psi(0)|P\rangle, \qquad (2)$$

depicted by the diagram in Fig. 1(b). Flavor degrees of freedom are suppressed throughout. Because of the color and electromagnetic gauge invariance of these quark currents the hadronic tensor is gauge invariant and $q_\mu \mathcal{W}^{\mu\nu} = 0$. We will make a diagrammatic expansion of which the first term is the "Born" diagram, i.e., the naive parton model, and the following diagrams have successively more gluons or quarks participating. After a specific choice

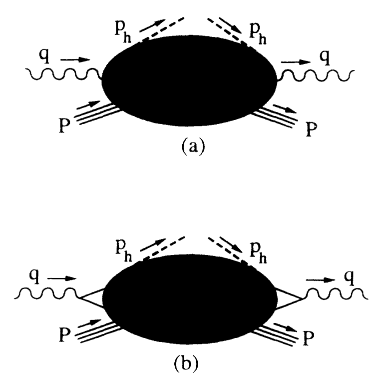

FIG. 1. The general hadronic tensor (a) and the hadronic tensor with quark currents (b).

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of gauge this diagrammatic expansion will be the same as an expansion in Q^{-1}. The relationship between diagrams with successively more gluons or quarks participating and their order in Q^{-1} or "twist" is discussed at great length by Jaffe and Ji in [1]. The separation of a diagram in a hard scattering part and soft correlation functions is discussed by Ellis, Furmanski, and Petronzio (EFP) [2] and by Collins *et al.* in [3]. Qiu [4] and Ji [5] among others discuss this approach for inclusive scatterings. Several authors have used these methods for polarized lepton-hadron and polarized Drell-Yan scattering [1,6]. In the same spirit we will use this method for semi-inclusive lepton-hadron scattering. The Born term, shown in Fig. 2, is given by

$$2M\mathcal{W}^{B\mu\nu} = \frac{1}{(2\pi)^4}\int d^4x\, e^{iq\cdot x}\, \gamma^{\nu}_{ij}\gamma^{\mu}_{kl} \times \Big[\langle P|\overline{\psi}_i(x)\psi_l(0)|P\rangle\langle 0|\psi_j(x)a_h^\dagger a_h\overline{\psi}_k(0)|0\rangle + \langle P|\psi_j(x)\overline{\psi}_k(0)|P\rangle\langle 0|\overline{\psi}_i(x)a_h^\dagger a_h\psi_l(0)|0\rangle\Big]\,. \tag{3}$$

Several comments are in order here. We now have two separate, unobserved final states which are integrated over using completeness, represented by the top and bottom parts of the diagram in Fig. 2. In the top part of the diagram the additional hadron in the final state appears. In Appendix A it is shown how to handle this when integrating over the final state. The result is that we are left with a hadron number operator. In the diagram we see the separation in quark correlation functions representing the soft part and the two γ matrices that denote the hard photon-quark scattering. Furthermore we note that Eq. (3) contains contributions of quarks and antiquarks. In the rest of this paper we will not explicitly account for the antiquarks since they behave exactly like the quarks in a distinct sector. Of course the numerical values of quark and antiquark correlation functions can be quite different. Finally we note that we will be concerned with hadrons produced from the struck quark, i.e., belonging to the (isolated) current jet. This will pose restrictions on the scaling variables x_B and z, as will become clear later in the discussion of kinematics and factorization.

Evaluating Eq. (3) we will find the contributions

$$\mathcal{W}^{B}_{\mu\nu}(Q) = w^{B0}_{\mu\nu} + w^{B1}_{\mu\nu}\frac{1}{Q} + O\left(\frac{1}{Q^2}\right), \tag{4}$$

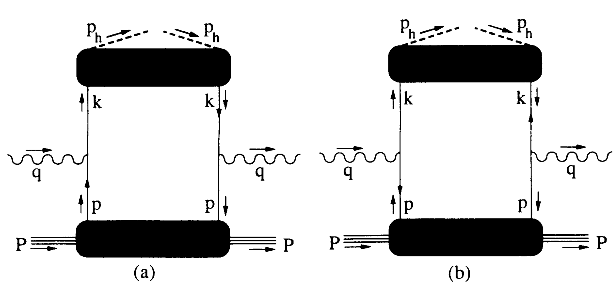

FIG. 2. The Born term in the expansion of the hadronic tensor.

where $q^\mu w^{B0}_{\mu\nu} = 0$ but $q^\mu w^{B1}_{\mu\nu} \neq 0$ so we explicitly lose electromagnetic gauge invariance in $O(Q^{-1})$. This problem arises because at order Q^{-1} there are other contributions involving soft parts that correspond with quark-gluon correlation functions. These are given by the four diagrams of Fig. 3. They have one fermion propagator in the hard scattering part and their leading contribution is $O(Q^{-1})$. Generically

$$\mathcal{W}^{Bg}_{\mu\nu}(Q) = w^{Bg1}_{\mu\nu}\frac{1}{Q} + O\left(\frac{1}{Q^2}\right). \tag{5}$$

Added together we find that for

$$\mathcal{W}_{\mu\nu} = w^{B0}_{\mu\nu} + (w^{B1}_{\mu\nu} + w^{Bg1}_{\mu\nu})\frac{1}{Q} + O\left(\frac{1}{Q^2}\right) \tag{6}$$

one has $q^\mu\mathcal{W}_{\mu\nu} = O(Q^{-2})$, restoring the conservation of the currents.

After explicit evaluation of these five diagrams we want to express the various correlation functions in terms of scalar functions, which we refer to as profile functions. These are Fourier transforms of specific projections of the quark correlation functions.

An important issue is color gauge invariance. The quark correlation functions obtained after factorization of the Born term as in (3) contain quark fields taken at different space-time points, so these matrix elements will not be invariant under local color gauge transformations. For a gauge-invariant definition of the profile functions one needs in the nonlocal matrix elements link operators of the form

$$L(x,x') = \mathcal{P}\exp\left(ig\int d^4y\ \xi^\alpha(x-y,x'-y)A_\alpha(y)\right), \tag{7}$$

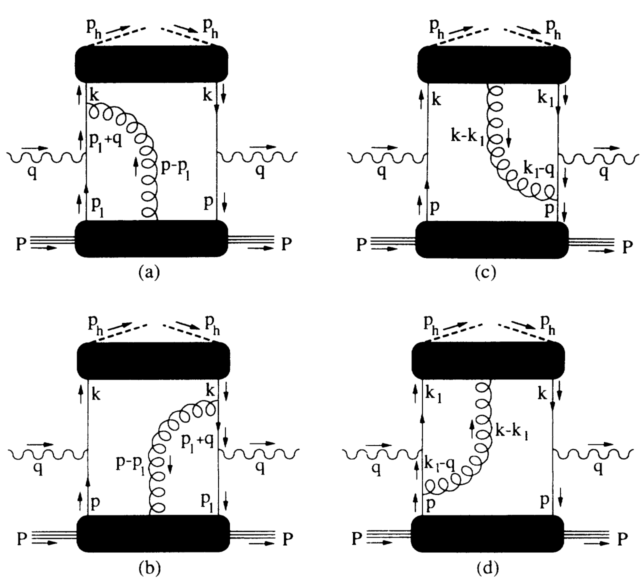

FIG. 3. One-gluon contributions in the expansion of the hadronic tensor. Only quark contributions are shown.

where A is the appropriate gauge field [7]. Gauge invariance is restored when ξ satisfies

$$\frac{\partial}{\partial y^\alpha}\xi^\alpha(x-y,x'-y) = \delta^4(x'-y) - \delta^4(x-y). \quad (8)$$

The path ordering $\mathcal{P}$ is then defined along the flow lines of the "velocity" field ξ^α. A special choice of ξ leads to the well-known line integral

$$L(x,x') = \mathcal{P}\exp\left(ig\int_{x'}^{x} dy^\alpha A_\alpha(y)\right). \quad (9)$$

In the case of hard processes when one integrates over all the transverse momenta, one is left with correlation functions in which the space-time arguments are separated along the light cone [$x^+ - x'^+ = \boldsymbol{x}_\perp - \boldsymbol{x}'_\perp = 0$, where $x^+ = (x^0 + x^3)/\sqrt{2}$]. A proper gauge-invariant correlation function can be obtained by including for the definition of the profile functions a straight link connecting x and x'. The additional matrix elements of the form $\overline{\psi}(x)A^+(y_1)\cdots A^+(y_n)\psi(x')$, that appear and correspond to diagrams with multiple gluons, need not be considered in the gauge $A^+ = 0$. As soon as we consider the dependence on transverse momenta, however, a color link needs to be included explicitly for a gauge-invariant treatment. Because the arguments of the quark fields are separated in the transverse direction, the fields $A_\perp^\alpha$ (α =1,2) will appear. Making an expansion of $\overline{\psi}(x)L(x,0)\psi(0)$ in orders of g, which for the part of the link operator containing the transverse fields $A_\perp^\alpha(y)$ is also an expansion in orders of Q^{-1}, one needs in addition to $\overline{\psi}(x)\psi(0)$ to consider the term

$$ig\overline{\psi}(x)\int d^4y\; \xi^\alpha(x-y,-y)A_\alpha(y)\psi(0). \quad (10)$$

As we will show it is actually possible to cast the results of the diagrams in Fig. 3 in the form of (10) with a specific form of $\xi^\alpha(x-y,-y)$. This shows that the full result up to $O(Q^{-1})$ can be expressed in a form as in Eq. (3) with correlation functions containing a specific color link. Because of the complex form of the link function ξ^α and the fact that the profile functions depend on ξ^α, we will not use this color link for a manifestly (color) gauge-invariant treatment. Rather we choose a special gauge in which $A^+ = 0$ and $A_\perp^\alpha$ are physical gluon fields and present the result of the calculation in terms of the correlation functions $\langle\!\langle\overline{\psi}(x)\psi(0)\rangle\!\rangle$ and $\langle\!\langle\overline{\psi}(x)A_\perp(y)\psi(0)\rangle\!\rangle$. We consider this an important and useful result for phenomenological applications, which will be reported elsewhere. The inclusion of other diagrams with extra gluons related to the construction of manifestly gauge-invariant definitions of our profile functions, mass factorization, and Sudakov effects have not been addressed in this paper.

The rest of the paper is structured as follows. In Sec. II we discuss the generalities of the semi-inclusive cross section, kinematics, structure functions, and inclusive reductions. In Sec. III we will first look at the kinematics of the relevant diagrams and discuss what kind of assumptions are made related to the factorization into hard scattering and correlation functions and make a twist analysis, then we will define the basic objects of which our diagrams are composed and show how they relate to the four profile functions which characterize the process. Then in Sec. IV we will present the actual calculation of the hadronic tensor and demonstrate gauge invariance. Finally in Sec. V we present our results as expressions for various cross sections using these four scalar functions.

II. SEMI-INCLUSIVE DEEP-INELASTIC LEPTON-HADRON SCATTERING

We will consider semi-inclusive processes of the type $eH \to e'hX$, where H is a hadronic target with mass M, h is a hadron with mass m_h detected in coincidence with the scattered electron, and X is the rest of the final state. The momenta are defined in Fig. 1, the angles in the target rest frame (TRF) are defined in Fig. 4. In a one-hadron semi-inclusive process we can form four independent invariants q^2, $P\cdot q$, $P\cdot p_h$, and $p_h\cdot q$. This includes the invariants that one is familiar with in inclusive scattering:

$$Q^2 = -q^2 \overset{\text{TRF}}{=} 4EE'\sin^2\left(\frac{\theta}{2}\right), \quad (11)$$

$$\nu = \frac{P\cdot q}{M} \overset{\text{TRF}}{=} E - E', \quad (12)$$

and the ratios

$$x_B = \frac{Q^2}{2M\nu},\quad y = \frac{P\cdot q}{P\cdot k} = \frac{\nu}{E}. \quad (13)$$

For semi-inclusive scattering one or more particles (=momenta) in the final state are measured. The additional invariants are fixed by the energy E_h and the component of p_h along $\boldsymbol{q}$, indicated by $p_{h\,\|}$, or equivalently by E_h and $\boldsymbol{p}_{h\perp}^2$. In the TRF they do not depend on the azimuthal angle ϕ_h. Note that if more than one momentum in the final state is measured the relative azimuthal angle appears in the invariant $p_1\cdot p_2$. In particular in the TRF one has $\boldsymbol{p}_{1\perp}\cdot\boldsymbol{p}_{2\perp} = |\boldsymbol{p}_{1\perp}||\boldsymbol{p}_{2\perp}|\cos(\phi_1-\phi_2)$. In DIS processes the ratio

$$z = \frac{P\cdot p_h}{P\cdot q} \overset{\text{TRF}}{=} \frac{E_h}{\nu} \quad (14)$$

will be useful. We will consider in this paper the limit of deep-inelastic scattering, in which $Q^2\to\infty$ and $\nu\to\infty$,

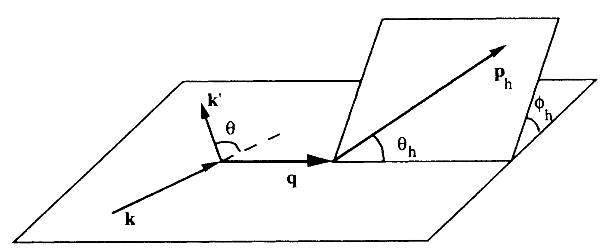

FIG. 4. Definition of scattering plane and momenta in $lH \to l'hX$.

keeping the ratio x_B fixed. In this limit[1] (defining $\hat{z}$ along $-\boldsymbol{q}$), $q^- \to \infty$. One then immediately sees that for a particle produced in the current jet the other invariant involving p_h and q with p_h on mass shell is

$$\frac{2p_h \cdot q}{q^2} \simeq \frac{p_h^- q^+}{q^- q^+} = \frac{p_h^-}{q^-} \simeq z. \tag{15}$$

The cross section for the semi-inclusive process is given by

$$\frac{2E_h d\sigma}{d^3p_h d\Omega dE'} = \frac{\alpha^2}{Q^4}\frac{E'}{E} L_{\mu\nu}\mathcal{W}^{\mu\nu} \tag{16}$$

or

$$\frac{2E_h d\sigma}{d^3p_h dx_B dy} = \frac{\pi\alpha^2 2My}{Q^4} L_{\mu\nu}\mathcal{W}^{\mu\nu}, \tag{17}$$

where $L_{\mu\nu}$ is the spin-averaged lepton tensor

$$L_{\mu\nu} = 2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2 g_{\mu\nu} \tag{18}$$

and the hadronic tensor is the product of nonelementary hadronic currents:

$$2M\mathcal{W}_{\mu\nu} = \frac{1}{(2\pi)^4}\int \frac{d^3P_X}{(2\pi)^3 2E_X}\langle P|J_\nu(0)|P_X, p_h\rangle \times \langle p_h, P_X|J_\mu(0)|P\rangle (2\pi)^4 \delta^4(P + q - p_h - P_X). \tag{19}$$

The integral over P_X indicates a complete summation over all multiparticle states of all hadron types. We have split off the invariant phase space for the detected hadron, which can also be written

$$\frac{d^3p_h}{2E_h} = \frac{dE_h d\boldsymbol{p}_{h\perp}^2 d\phi_h}{4p_{h\parallel}} = \frac{1}{4}\frac{\nu}{p_{h\parallel}} dz d\boldsymbol{p}_{h\perp}^2 d\phi_h. \tag{20}$$

The most general gauge-invariant, Lorentz covariant hadronic tensor for the electromagnetic process (parity and time reversal invariant) can be decomposed in four structure functions $\mathcal{W}_i(x, Q^2, z, \boldsymbol{p}_{h\perp}^2)$ [8]:

$$\begin{aligned}\mathcal{W}_{\mu\nu}(P, p_h, q) = {} & \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\mathcal{W}_1 + \frac{T_\mu T_\nu}{M^2}\mathcal{W}_2 \\ & + \frac{p_{h\perp\mu}T_\nu + T_\mu p_{h\perp\nu}}{Mm_h}\mathcal{W}_3 \\ & + \frac{p_{h\perp\mu}p_{h\perp\nu}}{m_h^2}\mathcal{W}_4,\end{aligned} \tag{21}$$

where

$$T_\mu = P_\mu - \frac{(P\cdot q)}{q^2} q_\mu \tag{22}$$

and

$$p_{h\perp}^\mu \overset{\text{TRF}}{=} (0, \boldsymbol{p}_{h\perp}, 0). \tag{23}$$

[1] $q^\pm = \frac{1}{\sqrt{2}}(q^0 \pm q^3)$.

As in the inclusive case, it is convenient to introduce photon polarizations. The structure functions can be expressed in terms of

$$\begin{aligned}\mathcal{W}_L &= \epsilon_L \cdot \mathcal{W} \cdot \epsilon_L = -\mathcal{W}_1 + \frac{\boldsymbol{q}^2}{Q^2}\mathcal{W}_2, \\ \mathcal{W}_T &= \frac{1}{2}(\epsilon_x \cdot \mathcal{W} \cdot \epsilon_x + \epsilon_y \cdot \mathcal{W} \cdot \epsilon_y) = \mathcal{W}_1 + \frac{\boldsymbol{p}_{h\perp}^2}{2m_h^2}\mathcal{W}_4,\end{aligned}$$

$$\begin{aligned}\mathcal{W}_{LT}\cos(\phi_h) &= -(\epsilon_x \cdot \mathcal{W} \cdot \epsilon_L + \epsilon_L \cdot \mathcal{W} \cdot \epsilon_x) \\ &= -2\frac{|\boldsymbol{q}|}{Q}\frac{|\boldsymbol{p}_{h\perp}|}{m_h}\cos(\phi_h)\,\mathcal{W}_3,\end{aligned}$$

$$\begin{aligned}\mathcal{W}_{TT}\cos(2\phi_h) &= \frac{1}{2}(\epsilon_x \cdot \mathcal{W} \cdot \epsilon_x - \epsilon_y \cdot \mathcal{W} \cdot \epsilon_y) \\ &= \frac{\boldsymbol{p}_{h\perp}^2}{2m_h^2}\cos(2\phi_h)\,\mathcal{W}_4.\end{aligned} \tag{24}$$

The ϵ's here are the standard transversal and longitudinal photon polarization vectors. The tensors multiplying these structure functions in

$$\begin{aligned}\mathcal{W}_{\mu\nu}(P, p_h, q) = {} & P^L_{\mu\nu}\mathcal{W}_L + P^T_{\mu\nu}\mathcal{W}_T - \frac{1}{2}P^{LT}_{\mu\nu}\mathcal{W}_{LT} \\ & + P^{TT}_{\mu\nu}\mathcal{W}_{TT}\end{aligned} \tag{25}$$

are

$$\begin{aligned}P^L_{\mu\nu} &= \frac{T_\mu T_\nu}{T^2}, \\ P^T_{\mu\nu} &= -g^{\mu\nu} + \frac{q_\mu q_\nu}{q^2} + \frac{T_\mu T_\nu}{T^2}, \\ P^{LT}_{\mu\nu} &= \frac{p_{h\perp\mu}T_\nu + T_\mu p_{h\perp\nu}}{T|\boldsymbol{p}_{h\perp}|}, \\ P^{TT}_{\mu\nu} &= g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} - \frac{T_\mu T_\nu}{T^2} + 2\frac{p_{h\perp\mu}p_{h\perp\nu}}{\boldsymbol{p}_{h\perp}^2}.\end{aligned} \tag{26}$$

They can be used to project out the structure functions $\mathcal{W}_i$:

$$\begin{aligned}\mathcal{W}_L &= P^L_{\mu\nu}\mathcal{W}^{\mu\nu}, \\ \mathcal{W}_T &= \frac{1}{2}P^T_{\mu\nu}\mathcal{W}^{\mu\nu}, \\ \mathcal{W}_{LT} &= P^{LT}_{\mu\nu}\mathcal{W}^{\mu\nu}, \\ \mathcal{W}_{TT} &= \frac{1}{2}P^{TT}_{\mu\nu}\mathcal{W}^{\mu\nu},\end{aligned} \tag{27}$$

as they are orthogonal to each other.

In DIS it is convenient to use the combinations $\mathcal{H}_i = \mathcal{H}_i(x, Q^2, z, \boldsymbol{p}_{h\perp}^2)$:

$$\begin{aligned}2z\mathcal{H}_1 &= M(\mathcal{W}_1 + \frac{\boldsymbol{p}_{h\perp}^2}{2m_h^2}\mathcal{W}_4) = M\mathcal{W}_T, \\ 2z\mathcal{H}_2 &= \nu\left(\mathcal{W}_2 + \frac{\boldsymbol{p}_{h\perp}^2 Q^2}{2m_h^2\boldsymbol{q}^2}\mathcal{W}_4\right) = \frac{\nu Q^2}{\boldsymbol{q}^2}(\mathcal{W}_L + \mathcal{W}_T), \\ 2z\mathcal{H}_3 &= -\frac{Q^2}{m_h}\mathcal{W}_3 = \frac{Q^3}{2|\boldsymbol{q}||\boldsymbol{p}_{h\perp}|}\mathcal{W}_{LT}, \\ 2z\mathcal{H}_4 &= Mx_B\frac{Q^2}{m_h^2}\mathcal{W}_4 = \frac{Q^4}{\boldsymbol{q}^2\boldsymbol{p}_{h\perp}^2}\mathcal{W}_{TT}.\end{aligned} \tag{28}$$

For later purposes it is also useful to introduce

$$\begin{aligned} 2z\mathcal{H}_L &= M\mathcal{W}_L \\ &= 2z\left[-\mathcal{H}_1 + \left(1+\frac{Q^2}{\nu^2}\right)\frac{\mathcal{H}_2}{2x_B}\right] \\ &\overset{\text{DIS}}{=} 2z\left[-\mathcal{H}_1 + \frac{\mathcal{H}_2}{2x_B}\right]. \end{aligned} \tag{29}$$

The cross sections are easily obtained from the contraction of leptonic and hadronic tensors. Without any approximations one obtains

$$\begin{aligned} &\frac{2E_h d\sigma}{d^3p_h d\Omega dE'} \\ &= \left(\frac{d\sigma}{d\Omega}\right)_M \frac{Q^2}{\boldsymbol{q}^2}\frac{1}{\epsilon}\left[\mathcal{W}_T + \epsilon\,\mathcal{W}_L + \epsilon\,\mathcal{W}_{TT}\cos(2\phi_h) + \sqrt{\frac{\epsilon(\epsilon+1)}{2}}\,\mathcal{W}_{LT}\cos(\phi_h)\right], \end{aligned} \tag{30}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{4\alpha^2 E'^2}{Q^4}\cos^2\left(\frac{\theta}{2}\right) \tag{31}$$

and

$$\epsilon^{-1} = 1 + 2\frac{\boldsymbol{q}^2}{Q^2}\tan^2\left(\frac{\theta}{2}\right). \tag{32}$$

It is also possible to observe a second particle in the final state. One then finds by a similar analysis two extra structure functions associated with the extra particle, $\mathcal{W}^2_{LT}$ and $\mathcal{W}^2_{TT}$, which enter the cross section in the same way as the one-particle structure functions, (see [9]).

Up until here all expressions have been completely general. Now we want to specialize to DIS events. Our attention will be the current jet, considering particles produced with bounded transverse momenta, i.e.,

$$m_h^2 + \boldsymbol{p}_{h\perp}^2 \ll p_{h\,\|}^2 \simeq E_h^2. \tag{33}$$

The cross section may be written as (still exact)

$$\begin{aligned} \frac{d\sigma}{dx_B dz dy d\boldsymbol{p}_{h\perp}^2 d\phi_h} = \frac{4\pi\alpha^2 ME}{Q^4}\frac{E_h}{p_{h\,\|}}\Bigg[& x_B y^2\mathcal{H}_1 + \left(1-y-\frac{Mx_B y}{2E}\right)\mathcal{H}_2 + \frac{|\boldsymbol{p}_{h\perp}|}{Q}(2-y)\sqrt{\frac{1-y-\frac{Mx_By}{2E}}{1+\frac{2Mx_B}{yE}}}\cos(\phi_h)\mathcal{H}_3 \\ & + \frac{\boldsymbol{p}_{h\perp}^2}{Q^2}\frac{1-y-\frac{Mx_By}{2E}}{1+\frac{2Mx_B}{yE}}\cos(2\phi_h)\mathcal{H}_4\Bigg] \end{aligned} \tag{34}$$

which becomes, in the DIS limit,

$$\begin{aligned} \frac{d\sigma}{dx_B dz dy d\boldsymbol{p}_{h\perp}^2 d\phi_h} \overset{\text{DIS}}{=} \frac{4\pi\alpha^2 ME}{Q^4}\Bigg[& x_B y^2\,\mathcal{H}_1 + (1-y)\,\mathcal{H}_2 \\ & + \frac{|\boldsymbol{p}_{h\perp}|}{Q}(2-y)\sqrt{(1-y)}\cos(\phi_h)\,\mathcal{H}_3 \\ & + \frac{\boldsymbol{p}_{h\perp}^2}{Q^2}(1-y)\cos(2\phi_h)\,\mathcal{H}_4\Bigg]. \end{aligned} \tag{35}$$

Integrating over the azimuthal angle only leaves the first two terms. Often one is only interested in the z behavior of the structure functions, so defining

$$\begin{aligned} H_i(x_B, Q^2, z) &= \frac{1}{2}\int d\boldsymbol{p}_{h\perp}^2 d\phi_h\,\mathcal{H}_i(x_B, Q^2, z, \boldsymbol{p}_{h\perp}^2) \\ &= \pi\int d\boldsymbol{p}_{h\perp}^2\mathcal{H}_i(x_B, Q^2, z, \boldsymbol{p}_{h\perp}^2), \end{aligned} \tag{36}$$

we are left with

$$\frac{d\sigma}{dx_B dy dz} \overset{\text{DIS}}{=} \frac{8\pi\alpha^2 ME}{Q^4}\left[x_B y^2 H_1 + (1-y)H_2\right]. \tag{37}$$

In the analysis of experimental data, factorization of the structure functions is taken to mean that one can write the structure function H_2 as a flavor sum over products of quark distribution functions and quark fragmentation functions [10]:

$$\begin{aligned} H_2(x_B, Q^2, z) = H_2(x_B, z) = \sum_q e_q^2\, x_B\Big[& f_{q/H}(x_B)D_{h/q}(z) \\ & + f_{\bar{q}/H}(x_B)D_{h/\bar{q}}(z)\Big], \end{aligned} \tag{38}$$

where q indicates quarks of different flavors. In this paper we show the validity of this factorization in leading order in Q, and its extension to $O(Q^{-1})$, and secondly we derive an expression for the fragmentation function in terms of quark correlation functions. Furthermore one obtains

$$2x_B\mathcal{H}_1(x_B, z, \boldsymbol{p}_{h\perp}^2) = \mathcal{H}_2(x_B, z, \boldsymbol{p}_{h\perp}^2), \tag{39}$$

as $\mathcal{H}_L \to 0$ in the DIS limit.

Finally some remarks on the connection to inclusive scattering must be made. The hadronic tensor for inclusive scattering $W_{\mu\nu}(P,q)$ is given by (see Appendix A)

$$\langle n_h(P,q)\rangle W_{\mu\nu}(P,q) = \int\frac{d^3p_h}{2E_h}\mathcal{W}_{\mu\nu}(P,q,p_h), \tag{40}$$

where $\langle n_h(P,q)\rangle$ is the average number of particles produced of type h for given P and q. Combining the definitions we then have

$$\begin{aligned} \langle n_h(x_B)\rangle F_1(x_B) &= \int dz H_1(x_B, z), \\ \langle n_h(x_B)\rangle F_2(x_B) &= \int dz H_2(x_B, z), \end{aligned} \tag{41}$$

where F_i are the standard dimensionless scaling structure functions in inclusive lH.

III. FACTORIZATION AND CORRELATION FUNCTIONS

A. Kinematical factorization and twist analysis

In this section we will discuss the diagrammatic expansion. We start by considering the kinematics. It is convenient to use light cone coordinates for this. With the four vector notation $p = [p^-, p^+, \boldsymbol{p}_\perp]$ with $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ we denote the momenta of the target hadron (P), virtual photon (q), and final-state hadron (p_h) as

$$
\begin{aligned}
P &= \left[\frac{x_B M^2}{A\sqrt{2}}, \frac{A}{x_B\sqrt{2}}, \mathbf{0}_\perp\right], \\
q &= \left[\frac{Q^2}{A\sqrt{2}}, -\frac{A}{\sqrt{2}}, \mathbf{0}_\perp\right], \\
p_h &= \left[\frac{zQ^2}{A\sqrt{2}}, \frac{A(\boldsymbol{p}_{h\perp}^2 + m_h^2)}{zQ^2\sqrt{2}}, \boldsymbol{p}_{h\perp}\right].
\end{aligned} \tag{42}
$$

Here A is a free parameter fixing the frame among those that are connected by boosts along the photon-quark direction. For instance $A = x_B M$ corresponds to the target rest frame (TRF), while for $\boldsymbol{p}_{h\perp} = 0$ the choice $A = (zQ^2)/m_h$ corresponds to the outgoing hadron rest frame. A convenient infinite momentum frame (IMF) is found by choosing $A = Q$. Using the momenta P and q, and the momentum $T \equiv P - (P\cdot q/q^2)\,q$, one can construct the momenta

$$\hat{T}^\mu \equiv \frac{T^\mu}{T} = \left[\frac{Q}{A\sqrt{2}}, \frac{A}{Q\sqrt{2}}, \mathbf{0}_\perp\right], \tag{43}$$

$$\hat{q}^\mu \equiv \frac{q^\mu}{Q} = \left[\frac{Q}{A\sqrt{2}}, -\frac{A}{Q\sqrt{2}}, \mathbf{0}_\perp\right], \tag{44}$$

and the two light cone null vectors

$$\hat{n}_+^\mu = \left(\frac{q^-}{Q}\right)(\hat{T}^\mu - \hat{q}^\mu), \tag{45}$$

$$\hat{n}_-^\mu = \left(-\frac{q^+}{Q}\right)(\hat{T}^\mu + \hat{q}^\mu), \tag{46}$$

with $\hat{n}_+ \cdot \hat{n}_- = 1$. Note that in our results the invariants x_B and z are used as the ratios of light cone coordinates. Thus the replacements

$$
\begin{aligned}
x_B &\longrightarrow -\frac{q^+}{P^+} = \frac{2x_B}{1+\sqrt{1+\frac{4M^2x_B^2}{Q^2}}}, \\
z &\longrightarrow \frac{p_h^-}{q^-} = \frac{z\left(1+\frac{p_{h\parallel}}{E_h}\right)}{1+\sqrt{1+\frac{4M^2x_B^2}{Q^2}}}
\end{aligned} \tag{47}
$$

would provide the necessary so-called hadron mass corrections. Furthermore we assume that for the current jet events we consider, no transverse momenta growing like Q appear. We consistently neglect terms of order transverse momentum divided by Q^2. Also x_B and z should not be too close to zero. To be precise, $x_B Q$ and zQ must be at least larger than some hadronic scale.

Diagrams contributing to the process will be separated into soft hadronic matrix elements and a hard scattering part. The forward matrix elements involving the external particles (momenta P, p_h) are given by untruncated (except for external lines) Green's functions. Given a renormalization scale for these Green's functions, they are calculated as two-particle irreducible in the quark-antiquark channel. The evolution between different renormalization scales is then provided by the logarithmic corrections discussed below. It is assumed that for all of the Green's functions the parton virtualities are restricted to some hadronic scale; i.e., they vanish with some inverse power of the virtualities. The relevant diagrams that are considered in this paper have been given in Figs. 2 and 3. There are other diagrams with one gluon such as those of the type in Fig. 3 with the photon-quark-quark vertex and the gluon-quark-quark vertex interchanged. Those can be absorbed in the soft parts and will not be considered explicitly in our analysis. Diagrams with one gluon which are important are those shown in Fig. 5. They contain in the hard part partons in the final state and will produce logarithmic corrections from the collinear divergencies.[2] Their absorption into Q^2-dependent functions completes the factorization. This leads to the evolution of the profile functions f^+, D^-, also known as the Gribov-Lipatov-Altarelli-Parisi equations, as has been proven explicitly by Ellis, Georgi, Machacek, Politzer, and Ross [11]. The nonlogarithmic terms in their calculation are the gluon jet events which appear in order α_s and which have been calculated in a naive parton model by König and Kroll in Ref. [12]. We expect that from diagrams like those in Fig. 3 which have in addition gluons in the final state one will obtain $Q^{-1}\ln Q^2$ contributions. In this paper however, we will not discuss the logarithmic contributions any further. We do note that in the leading order we can calculate the logarithmic corrections in our approach and that the evolution equations are not altered by the observation of transverse momenta. Finally there are diagrams which have a gluon across between the hadronic parts. With the above assumption on the parton virtualities one immediately sees that they will vanish as some power of Q^{-2}. The general behavior of the contributions of diagrams with any number of partons emerging from the hadronic matrix elements is discussed below.

It is illustrative to first consider the kinematics of the diagrams in Figs. 2 and 3. We assign the four-momenta

$$p = \left[\frac{\xi(p^2+\boldsymbol{p}_\perp^2)}{A\sqrt{2}}, \frac{A}{\xi\sqrt{2}}, \boldsymbol{p}_\perp\right],$$

[2] The quark self-energy and the photon-quark-quark vertex correction provide singular terms needed for the mass factorization.

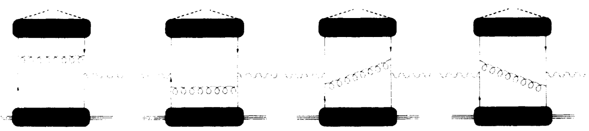

FIG. 5. Gluon contributions giving rise to logarithms.

$$
\begin{aligned}
k &= \left[\frac{\zeta Q^2}{A\sqrt{2}}, \frac{A(k^2+\boldsymbol{k}_\perp^2)}{\zeta Q^2\sqrt{2}}, \boldsymbol{k}_\perp\right], \\
p_1 &= \left[\frac{p_1^2+\boldsymbol{p}_{1\perp}^2}{(1-\xi_1)A\sqrt{2}}, \frac{(1-\xi_1)A}{\sqrt{2}}, \boldsymbol{p}_{1\perp}\right], \\
k_1 &= \left[\frac{(1-\zeta_1)Q^2}{A\sqrt{2}}, \frac{A(k_1^2+\boldsymbol{k}_{1\perp}^2)}{(1-\zeta_1)Q^2\sqrt{2}}, \boldsymbol{k}_{1\perp}\right].
\end{aligned} \tag{48}
$$

Now we apply four-momentum conservation $p+q=k$ to derive $\xi=\zeta=1$ in the limit $Q^2\to\infty$, which is equivalent to $p^+=-q^+=x_B P^+$ and $k^-=q^-=zp_h^-$. For later use we also give expressions for the momenta in the quark propagators in Fig. 3:

$$
\begin{aligned}
\frac{p_1+q}{(p_1+q)^2} &= \left[\frac{-1}{\xi_1 A\sqrt{2}}, \frac{A}{Q^2\sqrt{2}}, \frac{-\boldsymbol{p}_{1\perp}}{\xi_1 Q^2}\right], \\
\frac{k_1-q}{(k_1-q)^2} &= \left[\frac{1}{A\sqrt{2}}, \frac{-A}{\zeta_1 Q^2\sqrt{2}}, \frac{-\boldsymbol{k}_{1\perp}}{\zeta_1 Q^2}\right].
\end{aligned} \tag{49}
$$

It is important to note the suppression of the transverse components relative to the other components.

The objects that enter the diagrams in Figs. 2 and 3 are depicted in Fig. 6. They are partly given by untruncated (except for external particle lines) Green's functions for the blobs and partly by ordinary QCD Feynman rules. Suppressing flavor and color indices we have in the hard part the rules

$$\text{Fig. 6(a)}: i\gamma^\nu_{ij}, \tag{50}$$

$$\text{Fig. 6(b)}: ig\gamma^\alpha_{ij}, \tag{51}$$

$$\text{Fig.6(c)}: \frac{i}{\not p}, \tag{52}$$

while the correlation functions are given by

$$\text{Fig. 6(d)}: T_{ij}(p) = \frac{1}{2(2\pi)^4}\int d^4x\, e^{-ip\cdot x}\langle P|\overline{\psi}_j(x)\psi_i(0)|P\rangle, \tag{53}$$

$$\text{Fig. 6(e)}: D_{ji}(k) = \frac{1}{4z(2\pi)^4}\int d^4x\, e^{ik\cdot x}\langle 0|\psi_j(x)a_h^\dagger a_h\overline{\psi}_i(0)|0\rangle, \tag{54}$$

$$\text{Fig.6(f)}: F^\alpha_{ij}(p,p_1) = \frac{1}{2(2\pi)^8}\int d^4x d^4y\, e^{-ip\cdot x}e^{i(p-p_1)\cdot y}\langle P|\overline{\psi}_j(x)A^\alpha(y)\psi_i(0)|P\rangle, \tag{55}$$

$$\text{Fig. 6(g)}: M^\alpha_{ji}(k,k_1) = \frac{1}{4z(2\pi)^8}\int d^4x d^4y\, e^{ik\cdot x}e^{-i(k-k_1)\cdot y}\langle 0|\psi_j(x)A^\alpha(y)a_h^\dagger a_h\overline{\psi}_i(0)|0\rangle. \tag{56}$$

We note that time ordering is not relevant. Using the QCD equations of motion [13] all correlation functions can ultimately be related to functions containing $\psi_+=(\gamma^-\gamma^+/2)\psi$ and $A^\perp$, which have c-number (anti)commutators at equal light cone time x^+ [14].

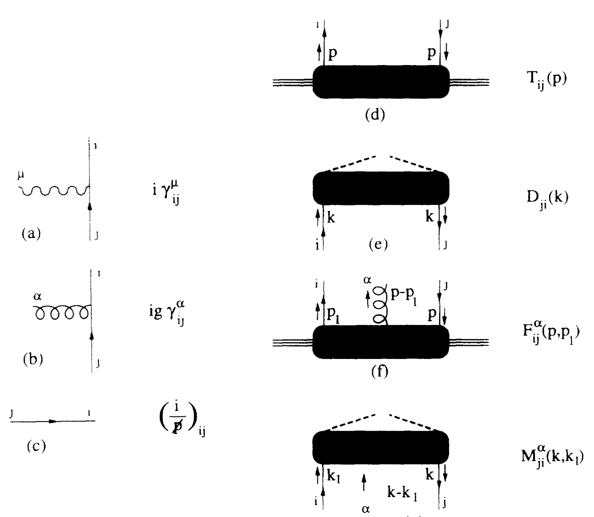

FIG. 6. Rules for the calculation of the diagrams.

We will now discuss the Q dependence and give the general classification of all possible diagrams with more than one parton emerging from the hadronic matrix element according to the powers of Q. We use an extension of the method developed by EFP for inclusive scattering. They have proven that for inclusive scattering this method is equivalent to the operator product expansion (OPE). Because of the hadron in the final state we do not have an OPE for semi-inclusive processes. We can, however, follow the method of EFP for semi-inclusive processes.

Consider an observable, say one of the structure functions $\mathcal{H}_i$ in the expansion of $\mathcal{W}_{\mu\nu}$. Separating the hard scattering from the soft pieces we have schematically

$$O(P,q,p_h) = \sum_{n,m} O_{n,m}(P,q,p_h), \tag{57}$$

where

$$O_{n,m}(P,q,p_h) = \int \prod_i^n \prod_j^m dp_i dk_j\, \hat{T}(P,p_i)\Gamma(q,p_i,k_i) \times \hat{D}(p_h,k_i). \tag{58}$$

Omitting the diagrams with partons in the final state, a general term $O_{n,m}$, with $n+m$ partons participating in the hard scattering is depicted in Fig. 7. To investigate the behavior of the diagrams for high Q^2 it is convenient to work in the particular infinite momentum frame $A=Q$. Under the assumption that all parton virtualities must be restricted to some hadronic scale we can show as seen explicitly for the momenta appearing in Figs. 2 and 3, that k_j^- and $p_i^+ \sim Q$ while k_j^+ and $p_i^- \sim Q^{-1}$. The transverse momenta are all of order unity. For the truncated piece Γ one can convince oneself that in the limit $Q\to\infty$, the Q dependence is determined by dimensional arguments. Assume that the canonical dimensions of the (in general nonlocal) product of operators appearing in $\hat{T}$ and $\hat{D}$ are d_T and d_D. The dominant hard scattering part multiplying $\hat{T}$ and $\hat{D}$ then must behave as

$$\Gamma(p_i,k_j,q) \sim Q^{c-d_T-d_D}, \tag{59}$$

where c is a constant depending on the specific observable. The behavior of $\hat{T}$ is determined by its Lorentz structure and the specific component that one considers. As in the matrix element every Lorentz index contributes as P^μ one finds after collecting the powers of Q in the frame $A=Q$ where $P^+\sim Q$, $P^-\sim M^2/Q$ and $P^\perp \sim M$:

$$\hat{T} \sim M^{d_T-h_T}Q^{h_T}. \tag{60}$$

After determining h_T in this way, the twist of the (nonlocal) operator in $\hat{T}$ is defined as $t_T \equiv d_T - h_T$. Similarly, the behavior of $\hat{D}$, where $p_h^- \sim Q$, $p_h^+ \sim m_h^2/Q$, and $\boldsymbol{p}_{h\perp} \sim m_h$, is given by

$$\hat{D} \sim m_h^{d_D-h_D}Q^{h_D} \tag{61}$$

and the twist is defined as $t_D = d_D - h_D$. Note that we (following Jaffe) discuss twist for an operator which is in general nonlocal. Moreover, different components (defining different profile functions) will have different twist. For example, $\overline{\psi}(x)\gamma^+\psi(0)$ will have $t=2$, but $\overline{\psi}(x)\gamma^-\psi(0)$ has $t=4$. The twist for the $++\cdots+$ component of a symmetric traceless operator is identical to the "twist" as used in the OPE approach for inclusive DIS. The name twist is still used in our context as the above leads for $O_{n,m}$ to the behavior

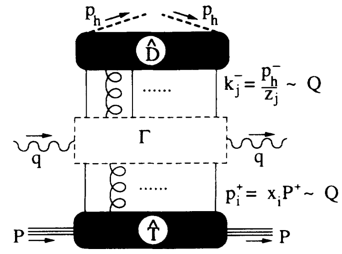

FIG. 7. A general multiparton contribution.

$$O(P,q,p_h) \sim \left(\frac{1}{Q}\right)^{t_T+t_D-c}. \tag{62}$$

The leading Born diagram starts off with bilocal twist-two operators in $\hat{T}$ and $\hat{D}$, but also contains twist-three and twist-four pieces. Diagrams with, for instance, an extra pair of quark fields in $\hat{T}$ or $\hat{D}$ are suppressed by a factor Q^{-2} as t_T or t_D is increased by 2. For gluons the situation is more complicated, since addition of a gluon does not change the twist. By choosing the gauge $A^+=0$ for the lower and $A^-=0$ for the upper blob we force the gluon contribution to add at least one unit of twist. Let it be clear however that one can add in general an arbitrary number of longitudinal gluons to every diagram. Only after a suitable choice of gauge these contributions will vanish.

B. Correlation functions and profile functions

The profile functions will be defined as Fourier transforms of projections of T and D. Explicit definitions of the profile functions are given in Appendix B. With these definitions we find

$$\int dp^-\ \mathrm{Tr}\,[T(p)]_{p^+=x_BP^+} = \frac{q^-}{Q}\frac{2Mx_B}{Q}\,e(x_B,\boldsymbol{p}_\perp), \tag{63}$$

$$\int dp^-\ \mathrm{Tr}\,[T(p)\gamma^\mu]_{p^+=x_BP^+} = \frac{q^-}{Q}\left[(\hat{T}^\mu-\hat{q}^\mu)f^+(x_B,\boldsymbol{p}_\perp)+\frac{2x_Bp_\perp^\mu}{Q}f^\perp(x_B,\boldsymbol{p}_\perp)+O\left(\frac{1}{Q^2}\right)\right], \tag{64}$$

$$\int dk^+\ \mathrm{Tr}\,[D(k)]_{k^-=p_h^-/z} = -\frac{q^+}{Q}\frac{2m_h}{Q}\,E(z,\boldsymbol{p}_{h\perp}-z\boldsymbol{k}_\perp), \tag{65}$$

$$\int dk^+\ \mathrm{Tr}\,[\gamma^\mu D(k)]_{k^-=p_h^-/z} = -\frac{q^+}{Q}\left[\left(\hat{T}^\mu+\hat{q}^\mu\right)D^-(z,\boldsymbol{p}_{h\perp}-z\boldsymbol{k}_\perp)+\frac{2(k_\perp^\mu-p_{h\perp}^\mu/z)}{zQ}D^\perp(z,\boldsymbol{p}_{h\perp}-z\boldsymbol{k}_\perp)\right.$$
$$\left.+\frac{2p_{h\perp}^\mu}{zQ}D^-(z,\boldsymbol{p}_{h\perp}-z\boldsymbol{k}_\perp)+O\left(\frac{1}{Q^2}\right)\right], \tag{66}$$

where f^+ and D^- are twist-two profile functions and e, $f^\perp$, E, and $D^\perp$ are twist-three profile functions. These are all scalar functions. The functions f^+ and e are the same as defined by Jaffe and Ji in [1]. The function D^- is the same as defined by Collins and Soper in [15]. The functions $f^\perp$ and $D^\perp$ are new. Note, however, that we consider the functions without a color gauge link, i.e., there is no manifest color gauge invariance for these functions.

In order to handle the three-parton correlations M and F we define the functions $\mathcal{M}$ and $\mathcal{F}$, completely analogous to M and F but with the replacement $A^\alpha \to D^\alpha = \partial^\alpha - igA^\alpha$. With partial integration we prove the relations

$$\int d^4p_1\ \mathcal{F}^\alpha_{ij}(p,p_1) = -ip^\alpha T_{ij}(p) - ig\int d^4p_1\ F^\alpha_{ij}(p,p_1),$$
$$\int d^4k_1\ \mathcal{M}^\alpha_{ij}(k,k_1) = -ik^\alpha D_{ij}(k) - ig\int d^4k_1\ M^\alpha_{ij}(k,k_1). \tag{67}$$

As argued before, the three-parton correlation functions F and M or $\mathcal{F}$ and $\mathcal{M}$ will be twist three in a suitably chosen gauge. We will now show that the associated leading profile function is in fact expressible in the twist-three profile functions appearing in Eqs. (64) and (66). For this we use the QCD equations of motion [1,2,13]. We define $\psi_\pm = P^\pm\psi$ where $P^\pm = \gamma^\mp\gamma^\pm/2$. It is then straightforward to show that ψ_+ and $\boldsymbol{A}^\perp$ are dynamically independent fields in the gauge $A^+ = 0$ [17]. This means that parts of the three-parton correlation functions corresponding to ψ_- and A^- can be written as correlation functions involving more than three partons and are thereby guaranteed to contribute at twist > 3. From this simple argument we know beforehand, that if we are interested in twist-three contributions (and we are) then we only have to consider $\mathrm{Tr}[\gamma^+F^\alpha(p,p_1)]$, with $\alpha = 1,2$. For the M functions we find twist three for $\mathrm{Tr}[\gamma^-M^\alpha(k,k_1)]$ in the gauge $A^- = 0$. One draws the same conclusion from the twist analysis of the previous section. As $\overline{\psi}\gamma^+\psi = \sqrt{2}\psi_+^\dagger\psi_+$ we conclude that the leading contribution (in twist two) comes from $\psi_+^\dagger A^+\psi_+$. When choosing $A^+ = 0$ the leading contribution is then at twist three and given by $\psi_+^\dagger A^\perp\psi_+$.

In the $A^\pm = 0$ gauge we deduce the equations of motion

$$D^\alpha(x)\gamma^\pm\psi(x) = \partial^\pm\gamma^\alpha\psi(x). \tag{68}$$

This is only true when applied between unpolarized nucleon states. Applying these equations we find, for the relevant transverse components ($\alpha = 1,2$),

$$\frac{1}{2(2\pi)^4}\int dp^- d^4x\ e^{-ip\cdot x}\langle P|\overline{\psi}(x)\gamma^+ iD^\alpha(0)\psi(0)|P\rangle = x_B\boldsymbol{p}^\alpha_\perp f^\perp, \tag{69}$$
$$\frac{1}{4z(2\pi)^4}\int dk^+ d^4x\ e^{ik'\cdot x}\mathrm{Tr}\ \langle 0|\psi(x)iD^\alpha(0)a_h^\dagger a_h\overline{\psi}(0)\gamma^-|0\rangle\Big|_{\boldsymbol{p}_{h\perp}=0} = \frac{\boldsymbol{k}'^\alpha_\perp}{z}D^\perp. \tag{70}$$

Here k' is the quark momentum k obtained after a Lorentz transformation to the frame where $\boldsymbol{p}_{h\perp} = 0$, $\boldsymbol{k}'_\perp = \boldsymbol{k}_\perp - \boldsymbol{p}_{h\perp}/z$ as explained in Appendix B. We have to transform back to the original frame with $\boldsymbol{p}_{h\perp} \neq 0$. Note that because of this transformation

$$\frac{1}{4z(2\pi)^4}\int dk^+ d^4x\ e^{ik'\cdot x}\mathrm{Tr}\ \langle 0|\psi(x)iD^-(0)a_h^\dagger a_h\overline{\psi}(0)\gamma^-|0\rangle\Big|_{\boldsymbol{p}_{h\perp}=0} = k^-D^- \tag{71}$$

will contribute to the transverse twist-three parton piece. This gives us, as the result in leading order which we will use in the next section,

$$\int dp^-\int d^4p_1\ \mathrm{Tr}\,[i\mathcal{F}^\alpha\gamma^\mu]_{p^+=-q^+} = \left(\frac{q^-}{Q}\right)\left\{(\hat{T}^\mu - \hat{q}^\mu)p^\alpha_\perp\ x_Bf^\perp + O\left(\frac{1}{Q^2}\right)\right\}, \tag{72}$$
$$\int dk^+\int d^4k_1\ \mathrm{Tr}\,[i\mathcal{M}^\alpha\gamma^\mu]_{k^-=q^-} = \left(\frac{-q^+}{Q}\right)\left\{(\hat{T}^\mu + \hat{q}^\mu)\left[k'^\alpha_\perp\ \frac{1}{z}D^\perp + \frac{p^\alpha_{h\perp}}{z}D^-\right] + O\left(\frac{1}{Q^2}\right)\right\}. \tag{73}$$

IV. CALCULATION OF THE HADRONIC TENSOR AND GAUGE INVARIANCE

We present the calculation of the hadronic tensor $\mathcal{W}$ from the diagrams of Figs. 2 and 3. It consists of three parts, $\mathcal{W} = \mathcal{W}_B + \mathcal{W}_F + \mathcal{W}_M$, where $\mathcal{W}_B$ is given by the Born diagram of Fig. 2, $\mathcal{W}_F$ by the one-gluon diagrams, Figs. 3(a) and 3(b), and $\mathcal{W}_M$ by Figs. 3(c) and 3(d):

$$\mathcal{W}^{\mu\nu}_B = \frac{4z}{M}\int d^4p\,d^4k\ \delta^4(p+q-k)\mathrm{Tr}\,[\gamma^\mu T(p)\gamma^\nu D(k)]\,, \tag{74}$$

$$\mathcal{W}^{\mu\nu}_F = \frac{-4z}{M}\int d^4p\,d^4k\ \delta^4(p+q-k)\int d^4p_1\Bigg\{\mathrm{Tr}\left[\gamma^\alpha\frac{\not{p}_1+\not{q}}{(p_1+q)^2}\gamma^\mu gF_\alpha(p,p_1)\gamma^\nu D(k)\right] + \mathrm{Tr}\left[\gamma^\nu\frac{\not{p}_1+\not{q}}{(p_1+q)^2}\gamma^\alpha D(k)\gamma^\mu gF_\alpha(p_1,p)\right]\Bigg\}, \tag{75}$$

$$\mathcal{W}_M^{\mu\nu} = \frac{-4z}{M}\int d^4p\, d^4k\; \delta^4(p+q-k)\int d^4k_1\Big\{\mathrm{Tr}\Big[\gamma^\mu \frac{\not k_1 - \not q}{(k_1-q)^2}\gamma^\alpha T(p)\gamma^\nu g M_\alpha(k,k_1)\Big] + \mathrm{Tr}\Big[\gamma^\alpha \frac{\not k_1 - \not q}{k_1 - q^2}\gamma^\nu g M_\alpha(k_1,k)\gamma^\mu T(p)\Big]\Big\}. \tag{76}$$

It is not necessary to consider the color charge operators at this point, as they will disappear again when the gluon fields are eliminated using the QCD equations of motion. As discussed in the Introduction we will show that $\mathcal{W}_F$ and $\mathcal{W}_M$ generate the first order of an expansion in g of the linking exponential. Note that this expansion in g will not be a perturbative QCD expansion, since we have no hard momenta in the soft part, but an expansion in the number of gluon fields. We will discuss only the F part as the M part is completely analogous. The required term for a gauge link is

$$\mathcal{W}_{\mathrm{link},F}^{\mu\nu} = \frac{4z}{M}\int d^4p\, d^4k\; \delta^4(p+q-k)\mathrm{Tr}\Big[\gamma_{kl}^\mu \frac{ig}{2(2\pi)^4}\int d^4x\; e^{-ip\cdot x} \times \int d^4y \langle P|\overline{\psi}_i(x)\,\xi^\alpha_{k'k,jj'}(x-y,-y)A_\alpha(y)\,\psi_l(0)|P\rangle \gamma^\nu_{ij}\frac{1}{4z(2\pi)^4}\int d^4z\; e^{ik\cdot z}\langle 0|\psi_{j'}(z)a_h^\dagger a_h \overline{\psi}_{k'}(0)|0\rangle\Big], \tag{77}$$

where we have the condition on ξ^α:

$$\frac{\partial}{\partial y^\alpha}\xi^\alpha_{k'k,jj'}(x-y,-y) = \left[\delta^4(y) - \delta^4(x-y)\right]\delta_{jj'}\delta_{k'k}. \tag{78}$$

By comparing $\mathcal{W}_F^{\mu\nu}$ with $\mathcal{W}_{\mathrm{link},F}^{\mu\nu}$ we identify

$$\xi^\alpha_{k'k,jj'}(x-y,-y) = \delta_{jj'}\int \frac{d^4p_1}{(2\pi)^4}\left(\gamma^\alpha \frac{i}{\not p_1 + \not q}\right)_{k'k} e^{i(p-p_1)\cdot y} + \delta_{k'k}\int \frac{d^4p_1}{(2\pi)^4}\left(\frac{i}{\not p_1 + \not q}\gamma^\alpha\right)_{jj'} e^{i(p-p_1)\cdot(x-y)}. \tag{79}$$

We calculate the derivative of ξ^α and use the identities

$$(p-p_1)^\alpha \frac{1}{\not p_1 + \not q}\gamma_\alpha = \frac{1}{\not p_1 + \not q}\not k - 1, \tag{80}$$

$$(p-p_1)^\alpha \gamma_\alpha \frac{1}{\not p_1 + \not q} = \not k\frac{1}{\not p_1 + \not q} - 1, \tag{81}$$

which are essentially Ward-Takahashi identities for the hard scattering vertex. In leading order we use the kinematics of Eqs. (48) and (49) to find

$$\frac{\partial}{\partial y^\alpha}\xi^\alpha_{k'k,jj'}(x-y,-y) = \delta_{jj'}P^+_{k'k}\delta^4(y) - \delta_{k'k}P^-_{jj'}\delta^4(x-y) + O\left(\frac{1}{Q}\right). \tag{82}$$

Since for the D correlation function the leading parts are given by $\psi_- = P^-\psi$ fields in the matrix elements and $(P^-)^2 = P^-$, we see that in leading order our ξ^α satisfies condition (78). From this and the analogous reasoning for the other two diagrams we conclude that the four diagrams of Fig. 4 are enough to render our calculation gauge invariant up to $O(Q^{-2})$. Note that both in our derivation of the linking operator in the Introduction as in our determination of the leading diagrams we have not considered the contributions of multiple longitudinal gluons. However, we have proven gauge invariance for the $O(g)$ term which in the gauge $A^+ = 0$, $A^- = 0$, respectively, will be the leading term, since the remaining gluon fields will add at least one unit of twist. In our calculations we will also not need to consider these diagrams as we will choose these gauges.

Now that we have convinced ourselves that our starting expression is gauge invariant we return to the calculation of the diagrams, choosing the gauge where $A^+ = 0$ and $A_\perp^\alpha$ are physical gluon fields. We will expand the correlation functions in the standard basis of Dirac matrices and express them in the profile functions (63)–(66). Because of time reversal and parity invariance together with Hermiticity, only 1 and γ^μ can contribute in unpolarized electromagnetic scattering. Furthermore we can prove with time reversal invariance and Hermiticity that $\mathrm{Tr}\,[\gamma^\mu F_\alpha(p,p_1)]$=$\mathrm{Tr}\,[\gamma^\mu F_\alpha(p_1,p)]$ and the same for $M_\alpha(k,k_1)$. The tensors reduce to

$$\mathcal{W}_B^{\mu\nu} = \frac{4z}{M}\int d^4p\, d^4k\; \delta^4(p+q-k)\frac{1}{16}\mathrm{Tr}\,[\gamma_\sigma T]\,\mathrm{Tr}\,[\gamma_\lambda D]\, S_B^{\mu\sigma\nu\lambda}, \tag{83}$$

$$\mathcal{W}_F^{\mu\nu} = \frac{-4z}{M}\int d^4p\, d^4k\; \delta^4(p+q-k)\int d^4p_1\, \frac{(p_1+q)_\rho}{(p_1+q)^2}\left(\frac{g}{16}\mathrm{Tr}\,[\gamma_\sigma F_\alpha(p,p_1)]\,\mathrm{Tr}\,[\gamma_\lambda D]\left\{S^{\alpha\rho\mu\sigma\nu\lambda} + S^{\nu\rho\alpha\lambda\mu\sigma}\right\}\right), \tag{84}$$

$$\mathcal{W}_M^{\mu\nu} = \frac{-4z}{M}\int d^4p\, d^4k\; \delta^4(p+q-k)\int d^4k_1\, \frac{(k_1-q)_\rho}{(k_1-q)^2}\left(\frac{g}{16}\mathrm{Tr}\,[\gamma_\sigma T]\,\mathrm{Tr}\,[\gamma_\lambda M_\alpha(k,k_1)]\left\{S^{\mu\rho\alpha\sigma\nu\lambda} + S^{\alpha\rho\nu\lambda\mu\sigma}\right\}\right), \tag{85}$$

where

$$S_B^{\mu\sigma\nu\lambda} = \mathrm{Tr}\left[\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^\lambda\right], \tag{86}$$

$$S^{\alpha\rho\mu\sigma\nu\lambda} = \mathrm{Tr}\left[\gamma^\alpha\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^\lambda\right]. \tag{87}$$

Contributions of the Dirac unit matrix either give rise to an odd number of γ matrices or are suppressed by Q^{-1}. Since the fermion propagator in the frame $A = Q$ goes as Q^{-1} we only have to pick out the order 1 contribution of the rest of the integrand. The leading contribution is given by

$$S^{\perp\rho\mu-\nu+} + S^{\nu\rho\perp+\mu-} \tag{88}$$

for $\mathcal{W}_F^{\mu\nu}$ and

$$S^{\mu\rho\perp-\nu+} + S^{\perp\rho\nu+\mu-} \tag{89}$$

for $\mathcal{W}_M^{\mu\nu}$. Because $\gamma^+\gamma^+ = \gamma^-\gamma^- = 0$ and $\{\gamma^\perp, \gamma^\pm\} = 0$ this means that $\rho = -$ in the first case and $\rho = +$ in the second. This in turn implies that the fermion propagators are in fact constants in leading order as can be read off from Eq. (49) in the frame $A = Q$

$$\frac{(p_1+q)^\rho}{(p_1+q)^2} = \frac{1}{2Q}\left(\hat{T} - \hat{q}\right)^\rho, \tag{90}$$

$$\frac{(k_1-q)^\rho}{(k_1-q)^2} = -\frac{1}{2Q}\left(\hat{T} + \hat{q}\right)^\rho. \tag{91}$$

Now we can take the propagators outside the k_1 integration and use relations (67). Furthermore we decompose $\delta^4(p+q-k)$ as $\delta(p^+ + q^+)\,\delta(q^- - k^-)\,\delta^2(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp)$. We finally find

$$\mathcal{W}_B^{\mu\nu} = \frac{z}{4M}\int dp^+dk^-d^2\boldsymbol{p}_\perp\,\frac{1}{16}\mathrm{Tr}\left[\gamma_\sigma T\right]\mathrm{Tr}\left[\gamma_\lambda D\right]S_B^{\mu\sigma\nu\lambda}, \tag{92}$$

$$\begin{aligned}\mathcal{W}_F^{\mu\nu} = \frac{-z}{4M}\int dp^+dk^-d^2\boldsymbol{p}_\perp\,\frac{(\hat{T}-\hat{q})^\rho}{(2Q)^2}\Bigg\{\Bigg(\int d^4p_1\,\mathrm{Tr}\left[i\mathcal{F}_\alpha\gamma_\sigma\right]\,\mathrm{Tr}\left[\gamma_\lambda D\right]\\ -\mathrm{Tr}\left[\gamma_\sigma T\right]\,\mathrm{Tr}\left[\gamma_\lambda D\right]p_{\perp\alpha}\Bigg)\left(S^{\alpha\rho\mu\sigma\nu\lambda} + S^{\nu\rho\alpha\lambda\mu\sigma}\right)\Bigg\},\end{aligned} \tag{93}$$

$$\begin{aligned}\mathcal{W}_M^{\mu\nu} = \frac{-z}{4M}\int dp^+dk^-d^2\boldsymbol{p}_\perp\,\frac{(-\hat{T}-\hat{q})^\rho}{(2Q)}\Bigg\{\Bigg(\mathrm{Tr}\left[T\gamma_\sigma\right]\,\int d^4k_1\mathrm{Tr}\left[i\mathcal{M}_\alpha\gamma_\lambda\right]\\ -\mathrm{Tr}\left[\gamma_\sigma T\right]\,\mathrm{Tr}\left[\gamma_\lambda D\right]k_{\perp\alpha}\Bigg)\left(S^{\mu\rho\alpha\sigma\nu\lambda} + S^{\alpha\rho\nu\lambda\mu\sigma}\right)\Bigg\}.\end{aligned} \tag{94}$$

This is the contribution of the relevant diagrams, expressed in quantities which we have given as tensors in next-to-leading order on the basis $\{\hat{T},\, \hat{q},\, p_\perp, k_\perp, p_{h\perp}\}$ in the previous section. To be precise, we will substitute the relations (64), (66), (72), (73), (90), and (91). Note that $\hat{T}^2 = 1$, $\hat{q}^2 = -1$, $\hat{T}\cdot\hat{q} = 0$, and that they do not have transverse components. The calculation is then very well suited for symbolic manipulation with a computer. We have used FORM [18] to evaluate the total hadronic tensor as

$$\begin{aligned}\widetilde{\mathcal{W}}^{\mu\nu}(P,q,p_h,\boldsymbol{p}_\perp) = &\left(-\hat{q}^\mu\hat{q}^\nu - g^{\mu\nu}\right)\left(\frac{z}{M}f^+D^-\right) + \left(\hat{T}^\mu\hat{T}^\nu\right)\left(\frac{z}{M}f^+D^-\right)\\ &+\left(\hat{T}^\mu p_\perp^\nu + \hat{T}^\nu p_\perp^\mu\right)\left(\frac{2xz}{MQ}f^\perp D^- + \frac{2}{MQ}f^+D^\perp - \frac{2z}{MQ}f^+D^-\right)\\ &+\left(\hat{T}^\mu p_{h\perp}^\nu + \hat{T}^\nu p_{h\perp}^\mu\right)\left(\frac{2}{MQ}f^+D^- - \frac{2}{zQM}f^+D^\perp\right) + O\left(\frac{1}{Q^2}\right),\end{aligned} \tag{95}$$

where

$$\mathcal{W}^{\mu\nu} = \int d^2\boldsymbol{p}_\perp\widetilde{\mathcal{W}}^{\mu\nu}. \tag{96}$$

Note that the result is color gauge invariant, albeit not manifestly. One can convince oneself that $q_\mu\widetilde{\mathcal{W}}^{\mu\nu} = O(Q^{-2})$. We see that the correct treatment of the color gauge invariance also restores the electromagnetic gauge invariance.

The hadronic tensor $\widetilde{\mathcal{W}}^{\mu\nu}$ is in fact the hadronic tensor for the semi-inclusive process where one observes in addition to a hadron h the transverse momentum of the jet, i.e., the transverse momentum of the ejected quark in our approach:

$$\frac{d\sigma^{(e+H\to e'+h+\text{jet})}}{d^3p_h d^2\boldsymbol{p}_\perp d\Omega dE'} = \frac{\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{2E_h}L_{\mu\nu}\widetilde{\mathcal{W}}^{\mu\nu}. \tag{97}$$

As mentioned in Sec. II this gives then rise to two extra structure functions. We project out the leading structure functions (all other structure functions are of order Q^{-2}):

$$\widetilde{\mathcal{W}}_T(x_B, z, \boldsymbol{p}_{h\perp}, \boldsymbol{p}_\perp) = \frac{z}{M}\, f^+(x_B, \boldsymbol{p}_\perp) D^-(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) + O\left(\frac{1}{Q^2}\right),$$

$$\widetilde{\mathcal{W}}_{LT}^1(x_B, z, \boldsymbol{p}_{h\perp}, \boldsymbol{p}_\perp) = \frac{|\boldsymbol{p}_{h\perp}|}{Q}\frac{4}{M}\left[f^+(x_B, \boldsymbol{p}_\perp)\left(\frac{1}{z}D^\perp(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) - D^-(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp)\right)\right],$$

$$\widetilde{\mathcal{W}}_{LT}^2(x_B, z, \boldsymbol{p}_{h\perp}, \boldsymbol{p}_\perp) = \frac{|\boldsymbol{p}_\perp|}{Q}\frac{4}{M}\{z[f^+(x_B, \boldsymbol{p}_\perp) - x_B f^\perp(x_B, \boldsymbol{p}_\perp)]D^-(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) - f^+(x_B, \boldsymbol{p}_\perp) D^\perp(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp)\}. \tag{98}$$

V. RESULTS

A. The quark distribution and fragmentation function

In this last part we will present the cross sections of several processes in lepton-hadron scattering. These cross sections will be expressed in the previously derived quark profile functions. First we will examine the properties of these functions.

The profile function f^+ is just the well-known quark distribution function, see [19]. The function

$$f_{q/H}(x_B, \boldsymbol{p}_\perp) = f^+(x_B, \boldsymbol{p}_\perp) = \frac{1}{2(2\pi)^3}\int dx^-\, d^2\boldsymbol{x}_\perp e^{i(q^+x^- + \boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp)} \times \langle P|\overline{\psi}(x)\gamma^+\psi(0)|P\rangle\Big|_{x^+=0} \tag{99}$$

is interpreted as the probability of finding a quark with light cone momentum fraction $x_B = p^+/P^+ = -q^+/P^+$ and transverse momentum $\boldsymbol{p}_\perp$ in a target with no transverse momentum. This can be seen when one quantizes the quark fields on the light cone with $x^+ = 0$ (see [1]). As the ψ_+ fields are the essentially free fields one can then substitute a free field expansion. The operator in the matrix element then essentially reduces to a quark number operator, counting quarks with momentum fraction x_B and transverse momentum $\boldsymbol{p}_\perp$. The integral over x_B and $\boldsymbol{p}_\perp$ comes out as 1, so f^+ is a probability distribution. Upon integration over the transverse momentum one finds the the quark distribution

$$f_{q/H}(x_B) = \frac{1}{4\pi}\int dx^-\, e^{iq^+x^-} \times \langle P|\overline{\psi}(x)\gamma^+\psi(0)|P\rangle\Big|_{x^+=\boldsymbol{x}_\perp=0}. \tag{100}$$

With this form one can easily prove sum rules expressing probability and momentum conservation at the quark level. As a check on the answer one can consider a quark target. One expects that the probability of finding a quark in a quark reduces to a δ function in x_B and $\boldsymbol{p}_\perp$. An explicit calculation, substituting a free quark state for the target state $|P\rangle$ leads to

$$f_{q/q}(x_B, \boldsymbol{p}_\perp) = \delta(1 - x_B)\delta^2(\boldsymbol{p}_\perp). \tag{101}$$

The fragmentation function

$$D_{h/q}(z, \boldsymbol{p}_{h\perp}) = D^-(z, \boldsymbol{p}_{h\perp}) = \frac{1}{4z(2\pi)^3}\int dx^+\, d^2\boldsymbol{x}_\perp e^{i(q^-x^+ + \frac{\boldsymbol{p}_{h\perp}\cdot\boldsymbol{x}_\perp}{z})} \times \mathrm{Tr}\,\langle 0|\psi(x)\, a_h^\dagger a_h\, \overline{\psi}(0)\gamma^-|0\rangle\Big|_{x^-=0, \boldsymbol{p}_{h\perp}=\boldsymbol{0}}, \tag{102}$$

is interpreted as the multiplicity of hadrons (of a certain type) found in the hadronization products (jet) of a quark with transverse momentum zero, hadrons with momentum fraction $z = p_h^-/k^- = p_h^-/q^-$, and transverse momentum $\boldsymbol{p}_{h\perp}$. Analogous to the case of the distribution function this can be seen by quantizing the quark fields on the light cone, but now for $x^- = 0$. One then finds essentially a hadron number operator acting between two quark states. The sum rules discussed hereafter then confirm that D^- is a multiplicity distribution. If the original quark does have a transverse momentum, one should make the substitution $\boldsymbol{p}_{h\perp} \to \boldsymbol{p}'_{h\perp} = \boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp$, where $\boldsymbol{p}'_{h\perp}$ is then the transverse momentum of the hadron with respect to the quark with momentum $\boldsymbol{k}_\perp$. Upon integration over $\boldsymbol{p}_{h\perp}$ one finds

$$D_{h/q}(z) = \frac{z}{8\pi}\int dx^+\, e^{iq^-x^+} \times \mathrm{Tr}\,\langle 0|\psi(x)\, a_h^\dagger a_h\, \overline{\psi}(0)\gamma^-|0\rangle\Big|_{x^-=\boldsymbol{x}_\perp=0, \boldsymbol{p}_{h\perp}=\boldsymbol{0}}. \tag{103}$$

This is the scaling fragmentation function. Both forms were first defined in [15] and also used in [16]. By integrating over z and using the techniques of Appendix A we can derive the sum rule

$$\int dz\, D_{h/q}(z) = \langle n_h\rangle_q, \tag{104}$$

expressing the fact that $D(z)$ is a multiplicity distribution. In [15] it was already proven that

$$\sum_{\text{hadrons}}\int dz\, zD(z) = 1, \tag{105}$$

which expresses momentum conservation. Finally we can also consider here the production of a quark from a quark. We substitute free quark operators for $a_h^\dagger, a_h$ and find

$$D_{q/q}(z, \boldsymbol{p}_{q\perp}) = \delta(1 - z)\delta^2(\boldsymbol{p}_{q\perp}). \tag{106}$$

The profile functions $f^\perp$ and $D^\perp$ do not have an interpretation as a distribution function, since they depend on interacting quark fields so no free field expansion can be applied. The antiquark contributions can be derived completely analogously and lead to

$$f_{\bar{q}/H}(x_B, \boldsymbol{p}_\perp) = \frac{1}{2(2\pi)^3} \int dx^- \, d^2\boldsymbol{x}_\perp e^{i(q^+x^- + p_\perp \cdot x_\perp)} \mathrm{Tr} \langle P|\psi(x)\overline{\psi}(0)\gamma^+|P\rangle \Big|_{x^+=0}, \tag{107}$$

$$D_{h/\bar{q}}(z, \boldsymbol{p}_{h\perp}) = \frac{1}{4z(2\pi)^3} \int dx^+ \, d^2\boldsymbol{x}_\perp e^{i(q^-x^+ + \frac{p_{h\perp}}{z} \cdot x_\perp)} \langle 0|\overline{\psi}(x)\gamma^- a_h^\dagger a_h \psi(0)|0\rangle \Big|_{x^-=0, \boldsymbol{p}_{h\perp}=0}. \tag{108}$$

Inclusion of different flavors gives rise to a sum over contributing flavours and quark charge factors e_q.

B. Semi-inclusive cross sections

Starting with the hadronic tensor derived in Sec. IV, several cross sections for semi-inclusive lepton-hadron scattering can be expressed in distribution and fragmentation functions. We will present a cross section for the process $e + H \to e' + h + \text{jet}$, i.e., not only a hadron is detected, but also the axis of the jet it belongs to. Other cross sections will be determined from this one by integrating over measured variables. With the structure functions (98) and accounting for antiquarks and flavor we have

$$\begin{aligned}
\frac{d\sigma^{(e+H\to e'+h+\text{jet})}}{dx_B dy dz d^2\boldsymbol{p}_{h\perp} d^2\boldsymbol{p}_\perp} = \frac{8\pi\alpha^2 ME}{Q^4} \sum_{i=q,\bar{q}} e_i^2 \Bigg\{ & \left(\frac{y^2}{2} + 1 - y\right) x_B f_{i/H}(x_B, \boldsymbol{p}_\perp) D_{h/i}(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) \\
& + 2(2-y)\sqrt{1-y}\cos\phi_h \frac{|\boldsymbol{p}_{h\perp}|}{Q} \frac{x_B}{z} \Bigg[f_{i/H}(x_B, \boldsymbol{p}_\perp) \\
& \times \left(\frac{1}{z} D_{h/i}^\perp(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) - D_{h/i}(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) \right) \Bigg] \\
& + 2(2-y)\sqrt{1-y}\cos\phi_j \frac{|\boldsymbol{p}_\perp|}{Q} \frac{x_B}{z} \{ -f_{i/H}(x_B, \boldsymbol{p}_\perp) D_{h/i}^\perp(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) \\
& + z[f_{i/H}(x_B, \boldsymbol{p}_\perp) - x_B f_{i/h}^\perp(x_B, \boldsymbol{p}_\perp)] D_{h/i}(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) \} \Bigg\}.
\end{aligned} \tag{109}$$

Here ϕ_h and ϕ_j are the azimuthal angles of hadron and jet with respect to the virtual photon direction. Note that $\boldsymbol{p}_{h\perp}$ is defined with respect to the z axis defined by the virtual photon, but the transverse momentum entering the fragmentation function $\boldsymbol{p}'_{h\perp} = \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp$ is defined with respect to the jet axis.

In our approach the transverse momentum of the jet immediately reflects the transverse momentum of the quarks within the target. To determine the jet cross section from this one has to first integrate over $\boldsymbol{p}_{h\perp}$. A direct integration over $\boldsymbol{p}_{h\perp}$ yields

$$\begin{aligned}
\frac{d\sigma^{(e+H\to e'+h+\text{jet})}}{dx_B dy dz d^2\boldsymbol{p}_\perp} = \frac{8\pi\alpha^2 ME}{Q^4} \sum_{i=q,q} e_i^2 \Bigg\{ & \left(\frac{y^2}{2} + 1 - y\right) x_B f_{i/H}(x_B, \boldsymbol{p}_\perp) D_{h/i}(z) \\
& - 2(2-y)\sqrt{1-y}\cos\phi_j \frac{|\boldsymbol{p}_\perp|}{Q} x_B^2 f_{i/H}^\perp(x_B, \boldsymbol{p}_\perp) D_{h/i}(z) \Bigg\}
\end{aligned} \tag{110}$$

which gives, for the jet cross section,

$$\begin{aligned}
\frac{d\sigma^{(e+H\to e'+h+\text{jet})}}{dx_B dy d^2\boldsymbol{p}_\perp} = \frac{8\pi\alpha^2 ME}{Q^4} \sum_{i=q,q} e_i^2 \Bigg\{ & \left(\frac{y^2}{2} + 1 - y\right) x_B f_{i/H}(x_B, \boldsymbol{p}_\perp) \\
& - 2(2-y)\sqrt{1-y}\cos\phi_j \frac{|\boldsymbol{p}_\perp|}{Q} x_B^2 f_{i/h}^\perp(x_B, \boldsymbol{p}_\perp) \Bigg\}.
\end{aligned} \tag{111}$$

Note that we pick up a factor $\langle n_h \rangle$ on both sides of the equation when integrating over z, which cancels. With this cross section we can calculate an expectation value for the azimuthal angle ϕ_{jet}:

$$\langle \cos\phi_{\text{jet}} \rangle = -\left(\frac{2|\boldsymbol{p}_\perp|}{Q}\right) \frac{(2-y)\sqrt{1-y}}{2(1-y)+y^2} \left(\frac{x_B f^\perp(x_B, \boldsymbol{p}_\perp)}{f(x_B, \boldsymbol{p}_\perp)}\right), \tag{112}$$

where the structure factor given by the ratio of $x_B f^\perp$ and f comes as an extension of the result previously derived by Cahn in [20] for the case of free quarks. Integrating over $\boldsymbol{p}_\perp$ in Eq. (110) we find the one-particle semi-inclusive cross section. The z dependence is simply given by a factor $D(z)$, and the cosine integrates to zero due to the azimuthal symmetry of $f^\perp(x_B, \boldsymbol{p}_\perp)$,

$$\frac{d\sigma^{(e+H\to e'+h)}}{dx_B dy dz} = \frac{8\pi\alpha^2 ME}{Q^4} \left[1 - y + \frac{y^2}{2}\right] \times \sum_{i=q,\bar{q}} e_i^2 x_B f_{i/H}(x_B) D_{h/i}(z). \tag{113}$$

If we compare this with Eq. (37) we see that we have recovered the factorization and scaling of the semi-inclusive structure function

$$H_2(x_B, Q^2, z) = H_2(x_B, z) = \sum_q e_q^2 x_B \Big[f_{q/H}(x_B) D_{h/q}(z) + f_{\bar{q}/H}(x_B) D_{h/\bar{q}}(z) \Big], \quad (114)$$

and the semi-inclusive Callan-Gross relation

$$2x_B H_1(x_B, z) = H_2(x_B, z). \quad (115)$$

We recover the Callan-Gross relation because the corrections coming from the twist-three diagrams are only entering in $\mathcal{H}_3$ and $\mathcal{H}_4$. Finally we will consider the cross section (35) from which we started for $l + H \to l' + h + X$. To this end we have to integrate over the jet transverse momentum. For the functions $\mathcal{H}_i$ occurring in Eq. (35) we find

$$\mathcal{H}_1(x_B, z, Q^2, \boldsymbol{p}_{h\perp}^2) = \frac{1}{2} \sum_{i=q,\bar{q}} e_i^2 \int d^2\boldsymbol{p}_\perp f_{i/H}(x_B, \boldsymbol{p}_\perp) D_{h/i}(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp),$$

$$\mathcal{H}_2(x_B, z, Q^2, \boldsymbol{p}_{h\perp}^2) = x_B \sum_{i=q,\bar{q}} e_i^2 \int d^2\boldsymbol{p}_\perp f_{i/H}(x_B, \boldsymbol{p}_\perp) D_{h/i}(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp),$$

$$\begin{aligned}\mathcal{H}_3(x_B, z, Q^2, \boldsymbol{p}_{h\perp}^2) = \frac{2x_B}{z} \sum_{i=q,\bar{q}} e_i^2 \int d^2\boldsymbol{p}_\perp \Bigg\{ & \frac{\boldsymbol{p}_\perp \cdot \boldsymbol{p}_{h\perp}}{\boldsymbol{p}_{h\perp}^2} \{ -f_{i/h}(x_B, \boldsymbol{p}_\perp) D_{i/h}^\perp(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) \\ & + z[f_{i/h}(x_B, \boldsymbol{p}_\perp) - x_B f_{i/h}^\perp(x_B, \boldsymbol{p}_\perp)] D_{i/h}(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) \} \\ & + f_{i/H}(x_B, \boldsymbol{p}_\perp) \left(\frac{1}{z} D_{h/i}^\perp(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) - D_{h/i}(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) \right) \Bigg\}.\end{aligned} \quad (116)$$

To find the last expression we used

$$\int d^2\boldsymbol{p}_\perp\, p_\perp^\mu f(x_B, \boldsymbol{p}_\perp) D(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp) = \frac{p_{h\perp}^\mu}{\boldsymbol{p}_{h\perp}^2} \int d^2\boldsymbol{p}_\perp\, (\boldsymbol{p}_\perp \cdot \boldsymbol{p}_{h\perp}) f(x_B, \boldsymbol{p}_\perp) D(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{p}_\perp). \quad (117)$$

We conclude that these structure functions do not factorize without extra assumptions on their transverse momentum dependence. However we do read off an extension of the Callan-Gross relation:

$$\mathcal{H}_2(x_B, z, \boldsymbol{p}_{h\perp}^2) = 2x_B \mathcal{H}_1(x_B, z, \boldsymbol{p}_{h\perp}^2). \quad (118)$$

VI. CONCLUSIONS

In this paper we have analyzed the structure functions in semi-inclusive deep-inelastic lepton-hadron scattering. We have considered all four structure functions, two of which ($\mathcal{W}_{LT}$ and $\mathcal{W}_{TT}$) can only be measured by considering explicitly the momentum component of the produced hadron perpendicular to the virtual photon momentum, or by considering explicitly the perpendicular momentum of the produced jet. In all cases we have only considered the contribution of one (forward or current) jet being produced and particles therein. In the analysis we have included the twist-two and twist-three contributions.

The twist-two pieces in the structure functions can be expressed in terms of the well-known quark distribution and fragmentation functions which can be considered as specific projections of quark-quark correlation functions (profile functions). The twist-three pieces, which constitute the main contributions in $\mathcal{W}_{LT}$ and $\mathcal{W}_{TT}$, involve two new profile functions that appear as projections of quark-quark correlation functions. Up to order $1/Q$, however, one must also include quark-quark-gluon correlation functions in order to obtain a gauge-invariant result. By virtue of the QCD equations of motion the contributing pieces do not introduce new profile functions. The inclusion of the $1/Q$ contributions does not affect the evolution of the twist-two profile functions (the quark distribution and fragmentation functions). We have not considered the evolution of the new profile functions, or the $O(\alpha_s)$ contributions in the process. An extension of the analysis to $O(Q^{-2})$ along the same lines as discussed here seems to be very hard to us. In that situation not only several new profile functions will appear but one has also lost the separation between kinematic quantities appearing in the distribution region from those appearing in the fragmentation region.

Results in the on-shell parton model such as those previously derived by Cahn [20] follow immediately as a special case by defining the profile functions with respect to a free quark target, i.e., the amplitudes in (B3) are then $\mathcal{A}_1 = \mathcal{A}_2 = 4\pi^4\delta^4(P-p)$ and $\mathcal{B} = 8\pi^4 m_q \delta^4(P-p)$. In this case the azimuthal asymmetry $\langle \cos\phi \rangle$ is of purely kinematical origin.

The analysis in terms of quark correlation functions is of interest as it provides a convenient method to relate cross sections of hard processes directly to matrix elements of (nonlocal) quark and gluon operators. The latter could in principle be calculated when we knew how to solve QCD, or they can be calculated in a model. Although some data for semi-inclusive muon scattering exist [21] we will not discuss them here. Nor have we presented any model as this would require extra assumptions. In the presented analysis a minimal amount of assumptions was made, namely those needed for the factorization of the hard scattering part. We also want to point out that one can do exactly the same analysis for Drell-Yan processes and e^+e^- annihilation, thereby testing the validity of this approach and extending the experimental possibilities of measuring the profile functions.

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APPENDIX A: INTERMEDIATE STATES AND COMPLETENESS

The cross sections we are considering describe the detection of one hadron in correlation with a scattered electron. In an experiment often many hadrons of the same type are produced in one event. This means that we have to pay special attention not only when we are integrating over the hadronic variables when we want to find an inclusive result, but also when we isolate a hadronic operator. In this appendix we discuss how to treat these situations, based on an example situation with only one type of scalar particle. Extensions to flavor and spin are obvious. An n-particle state is denoted as $|p_1,\ldots,p_n\rangle$ and we abbreviate the phase space integrations as $d\tilde{p} = \frac{d^3p}{(2\pi)^3 2E_p}$. The order of the particles appearing in the states has no meaning. Therefore the identity in this space looks like [22]

$$\mathcal{I} = \sum_{n=0}^{\infty} \mathcal{I}_n, \tag{A1}$$

where

$$\mathcal{I}_n = \frac{1}{n!}\int d\tilde{p}_1\cdots d\tilde{p}_n\, a^\dagger(p_1)\cdots a^\dagger(p_n)|0\rangle\langle 0|a(p_1)\cdots a(p_n) \tag{A2}$$

and

$$\mathcal{I}_0 = |0\rangle\langle 0|. \tag{A3}$$

We first consider the situation with a multiparticle state where we integrate over all particles except one, the situation as we encounter it in our expressions. We denote with P_X this complete system minus one particle:

$$\begin{aligned}\int \frac{d^3P_X}{(2\pi)^3 2E_X}|P_X,p_h\rangle\langle P_X,p_h| &= |p_h\rangle\langle p_h| + \int d\tilde{p}_1|p_1,p_h\rangle\langle p_1,p_h| + \frac{1}{2!}\int d\tilde{p}_1 d\tilde{p}_2|p_1,p_2,p_h\rangle\langle p_1,p_2,p_h| + \cdots\\ &= a_h^\dagger \mathcal{I} a_h = a_h^\dagger a_h,\end{aligned} \tag{A4}$$

thus we can replace the sum minus one particle by the hadronic number operator. Then we want to integrate over p_h to reconstruct an inclusive formula:

$$\begin{aligned}\int \frac{d^3p_h}{(2\pi)^3 2E_h}\frac{d^3P_X}{(2\pi)^3 2E_X}|P_X,p_h\rangle\langle P_X,p_h| &= \int d\tilde{p}_h|p_h\rangle\langle p_h| + \int d\tilde{p}_h d\tilde{p}_1|p_1,p_h\rangle\langle p_1,p_h|\\ &\quad+\frac{1}{2!}\int d\tilde{p}_h d\tilde{p}_1 d\tilde{p}_2|p_1,p_2,p_h\rangle\langle p_1,p_2,p_h| + \cdots .\end{aligned} \tag{A5}$$

This is the identity operator, but with every term separately multiplied with the number of particles. The summation over all states then yields just the average number of particles h produced in the processes considered:

$$\int \frac{d^3p_h}{(2\pi)^3 2E_h}\frac{d^3P_X}{(2\pi)^3 2E_X}|P_X,p_h\rangle\langle P_X,p_h| = \sum_n n\mathcal{I}_n. \tag{A6}$$

Evaluated between particle states and reexpressed as cross sections this relation reads

$$\int d\Omega_h dE_h \frac{d\sigma}{d\Omega_h dE_h d\Omega_e dE_e} = \langle n_h(\Omega_e,E_e)\rangle \frac{d\sigma}{d\Omega_e dE_e}. \tag{A7}$$

APPENDIX B: STRUCTURE OF THE CORRELATION FUNCTIONS

In this appendix we study the projections of the correlation functions defined as

$$f_A(P;x_B,\boldsymbol{p}_\perp) = \frac{1}{2(2\pi)^3}\int dx^-\, d^2\boldsymbol{x}_\perp e^{i(q^+x^- + \boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp)}\, \langle P|\overline{\psi}(x)\Gamma_A\psi(0)|P\rangle\Big|_{x^+=0}, \tag{B1}$$

$$4zD_B(p_h;z,\boldsymbol{k}_\perp) = \frac{1}{(2\pi)^3}\int dx^+\, d^2\boldsymbol{x}_\perp e^{i(q^-x^+ - \boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp)}\ \mathrm{Tr}\,\langle 0|\psi(x)a_h^\dagger a_h\overline{\psi}(0)\,\Gamma_B|0\rangle\Big|_{x^-=0}, \tag{B2}$$

(with $q^+ = -x_B P^+$ and $q^- = p_h^-/z$). Using parity and time reversal invariance as well as Hermiticity one proves that the most general structure of the matrix elements is

$$\langle P|\overline{\psi}_i(x)\psi_j(0)|P\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot x}\ [\mathcal{A}_1(P,p)\not{p} + \mathcal{A}_2(P,p)\not{P} + \mathcal{B}(P,p)]_{ji}, \tag{B3}$$

$$\langle 0|\psi_i(x)\, a_h^\dagger a_h\, \overline{\psi}_j(0)|0\rangle = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot x}\ [\mathcal{C}_1(p_h,k)\not{k} + \mathcal{C}_2(p_h,k)\not{p}_h + \mathcal{D}(p_h,k)]_{ij}\,. \tag{B4}$$

At this point one immediately sees that for unpolarized distributions f_α and D_β are zero except for $\Gamma_{A,B} = \gamma^\mu$ or $\Gamma_{A,B} = 1$.

Consider as the first case $\Gamma_A = \gamma^\mu$ for the f-profile functions. One obtains (choosing a frame where $\boldsymbol{P}_\perp = 0$)

$$\begin{aligned}
f^{[\gamma^+]}(P;x_B,\boldsymbol{p}_\perp) &= \frac{1}{2(2\pi)^3}\int dx^-\, d^2\boldsymbol{x}_\perp e^{i(q^+x^- + \boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp)}\ \langle P|\overline{\psi}(x)\gamma^+\psi(0)|P\rangle\Big|_{x^+=0}\\
&= \frac{1}{2(2\pi)^3}\int \frac{dp^-}{2\pi}\ \mathrm{Tr}\left[\gamma^+(\mathcal{A}_1\not{p} + \mathcal{A}_2\not{P} + \mathcal{B})\right]\Big|_{p^+=-q^+}\\
&= \frac{1}{(2\pi)^4}\int dp^2\left(\mathcal{A}_1(P,p) + \frac{1}{x_B}\mathcal{A}_2(P,p)\right)\Big|_{p^+=-q^+} \equiv\ f^+(x_B,\boldsymbol{p}_\perp),
\end{aligned} \tag{B5}$$

$$\begin{aligned}
f^{[\gamma^-]}(P;x_B,\boldsymbol{p}_\perp) &= \frac{1}{2(2\pi)^3}\int dx^-\, d^2\boldsymbol{x}_\perp e^{i(q^+x^- + \boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp)}\ \langle P|\overline{\psi}(x)\gamma^-\psi(0)|P\rangle\Big|_{x^+=0}\\
&= \frac{1}{2(2\pi)^3}\int \frac{dp^-}{2\pi}\ \mathrm{Tr}\left[\gamma^-(\mathcal{A}_1\not{p} + \mathcal{A}_2\not{P} + \mathcal{B})\right]\Big|_{p^+=-q^+}\\
&= \frac{M^2}{2(2\pi)^4(q^+)^2}\int dp^2\left(\frac{p^2+\mathbf{p}_\perp^2}{M^2}\mathcal{A}_1(P,p) + x_B\mathcal{A}_2(P,p)\right)\Big|_{p^+=-q^+}\\
&\equiv\ \frac{M^2x_B^2}{2(q^+)^2}\,f^-(x_B,\boldsymbol{p}_\perp),
\end{aligned} \tag{B6}$$

$$\begin{aligned}
f^{[\gamma_\perp]}(P;x_B,\boldsymbol{p}_\perp) &= \frac{1}{2(2\pi)^3}\int dx^-\, d^2\boldsymbol{x}_\perp e^{i(q^+x^- + \boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp)}\ \langle P|\overline{\psi}(x)\gamma_\perp\psi(0)|P\rangle\Big|_{x^+=0}\\
&= \frac{1}{2(2\pi)^3}\int \frac{dp^-}{2\pi}\ \mathrm{Tr}\left[\gamma_\perp(\mathcal{A}_1\not{p} + \mathcal{A}_2\not{P} + \mathcal{B})\right]\Big|_{p^+=-q^+}\\
&= -\frac{\boldsymbol{p}_\perp}{(2\pi)^4q^+}\int dp^2\,\mathcal{A}_1(P,p) \equiv -\frac{\boldsymbol{p}_\perp x_B}{q^+}f^\perp(x_B,\boldsymbol{p}_\perp).
\end{aligned} \tag{B7}$$

Here f^+, f^- and $f^\perp$ are the profile functions. We assume that the functions $\mathcal{A}_i$ and $\mathcal{B}$ have no singularities, so that the profile functions are all of order 1. In the frame $A = Q$ we also see at which twist these functions enter the calculations. When f^+, i.e., the normal parton distribution function, enters at order 1, then $f^\perp$ enters at $O(Q^{-1})$ and f^- at $O(Q^{-2})$, as we expected from the discussion in Sec. III A. It is straightforward to express $f^{[\gamma^\mu]}$ as

$$f^{[\gamma^\mu]}(P;x_B,\boldsymbol{p}_\perp) = \frac{q^-}{Q}\left[(\hat{T}^\mu - \hat{q}^\mu)f^+ + \frac{2x_Bp_\perp^\mu}{Q}f^\perp + O\left(\frac{1}{Q^2}\right)\right]. \tag{B8}$$

The function $f^{[1]}$ introduces another profile function $e(x_B,\boldsymbol{p}_\perp)$,

$$\begin{aligned}
f^{[1]}(P;x_B,\boldsymbol{p}_\perp) &= \frac{1}{2(2\pi)^3}\int dx^-\, d^2\boldsymbol{x}_\perp e^{i(q^+x^- + \boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp)}\langle P|\overline{\psi}(x)\psi(0)|P\rangle\Big|_{x^+=0}\\
&= \frac{1}{2(2\pi)^3}\int \frac{dp^-}{2\pi}\ \mathrm{Tr}\left[\mathcal{A}_1\not{p} + \mathcal{A}_2\not{P} + \mathcal{B}\right]\Big|_{p^+=-q^+}\\
&= -\frac{M}{(2\pi)^4q^+}\int dp^2\,\frac{\mathcal{B}(P,p)}{M} \equiv -\frac{Mx_B}{(q^+)}\,e(x_B,\boldsymbol{p}_\perp),
\end{aligned} \tag{B9}$$

thus leading to the result

$$f^{[1]}(P;x_B,\boldsymbol{p}_\perp) = \frac{q^-}{Q}\,\frac{2Mx_B}{Q}\,e(x_B,\boldsymbol{p}_\perp). \tag{B10}$$

Although the profile function e appears to enter at twist three, it will only give contributions at $O(Q^{-2})$ to the hadronic tensor. This is because e is always connected with the analogous $D^{[1]}$ function, which is also twist three.

In the case of a quark target we can explicitly calculate $\mathcal{A}_i$ and $\mathcal{B}$ and find

$$\begin{aligned} f^+(x_B,\boldsymbol{p}_\perp) = f^-(x_B,\boldsymbol{p}_\perp) = f^\perp(x_B,\boldsymbol{p}_\perp) &= e(x,\boldsymbol{p}_\perp) \\ &= \delta(1-x_B)\,\delta^2(\boldsymbol{p}_\perp). \end{aligned} \tag{B11}$$

The analysis of the D-profile functions can be performed along similar lines. First again consider $\Gamma_A = \gamma^\mu$. In this case we want to consider the profile functions in the rest system of the produced hadron h. For that purpose we perform the Lorentz transformation

$$\left[p_h^-,\frac{m_{h\perp}^2}{2p_h^-},\boldsymbol{p}_{h\perp}\right] \longrightarrow \left[p_h^-,\frac{m_h^2}{2p_h^-},\boldsymbol{0}_\perp\right], \tag{B12}$$

$$\left[k^-,k^+,\boldsymbol{k}_\perp\right] \longrightarrow \left[k^-,k^+ - \frac{\boldsymbol{p}_{h\perp}\cdot\boldsymbol{k}_\perp}{p_h^-} + \frac{\boldsymbol{p}_{h\perp}^2\,k^-}{2(p_h^-)^2},\boldsymbol{k}_\perp - \frac{k^-\,\boldsymbol{p}_{h\perp}}{p_h^-}\right], \tag{B13}$$

$$\begin{aligned} \left[D^-,D^+,\boldsymbol{D}_\perp\right] &\longrightarrow \left[D^{-\prime},D^{+\prime},\boldsymbol{D}'_\perp\right] \\ &= \left[D^-,D^+ - \frac{\boldsymbol{p}_{h\perp}\cdot\boldsymbol{D}_\perp}{p_h^-} + \frac{\boldsymbol{p}_{h\perp}^2}{2(p_h^-)^2}\,D^-,\boldsymbol{D}_\perp - \frac{\boldsymbol{p}_{h\perp}}{p_h^-}\,D^-\right], \end{aligned} \tag{B14}$$

where $m_{h\perp}^2 = m_h^2 + \boldsymbol{p}_{h\perp}^2$. The profile functions in the primed system ($\boldsymbol{p}'_{h\perp} = \boldsymbol{0}_\perp$) are then given by (note that the hadron operators now have as argument p'_h)

$$\begin{aligned} 4zD^{[\gamma^-]\prime}(p'_h;z,\boldsymbol{k}'_\perp) &= \frac{1}{(2\pi)^3}\int dx^+\,d^2\boldsymbol{x}_\perp e^{i(q^-x^+ - \boldsymbol{k}'_\perp\cdot\boldsymbol{x}_\perp)}\ \mathrm{Tr}\,\langle 0|\psi(x)\,a_h^\dagger a_h\,\overline{\psi}(0)\gamma^-|0\rangle\Big|_{x^-=0} \\ &= \frac{1}{(2\pi)^4}\int dk^+\ \mathrm{Tr}\left[\gamma^-(\mathcal{C}_1\not{k}' + \mathcal{C}_2\not{p}'_h + \mathcal{D})\right]\Big|_{k^-=q^-} \\ &= \frac{2}{(2\pi)^4}\int dk^2\,\left[\mathcal{C}_1(p'_h,k') + z\mathcal{C}_2(p'_h,k')\right]\Big|_{k^-=q^-} \equiv 4zD^-(z,-z\boldsymbol{k}'_\perp), \end{aligned} \tag{B15}$$

$$\begin{aligned} 4zD^{[\gamma^+]\prime}(p'_h;z,\boldsymbol{k}'_\perp) &= \frac{1}{(2\pi)^3}\int dx^+\,d^2\boldsymbol{x}_\perp e^{i(q^-x^+ - \boldsymbol{k}'_\perp\cdot\boldsymbol{x}_\perp)}\ \mathrm{Tr}\,\langle 0|\psi(x)\,a_h^\dagger a_h\,\overline{\psi}(0)\gamma^+|0\rangle\Big|_{x^-=0} \\ &= \frac{1}{(2\pi)^4}\int dk^+\ \mathrm{Tr}\left[\gamma^+(\mathcal{C}_1\not{k}' + \mathcal{C}_2\not{p}'_h + \mathcal{D})\right]\Big|_{k^-=q^-} \\ &= \frac{m_h^2}{(2\pi)^4(q^-)^2}\int dk^2\left(\frac{k^2+\boldsymbol{k}'^2_\perp}{m_h^2}\,\mathcal{C}_1(p'_h,k') + \frac{1}{z}\mathcal{C}_2(p'_h,k')\right)\Big|_{k^-=q^-} \\ &\equiv \frac{2m_h^2}{z(q^-)^2}\,D^+(z,-z\boldsymbol{k}'_\perp), \end{aligned} \tag{B16}$$

$$\begin{aligned} 4zD^{[\gamma_\perp]\prime}(p'_h;z,\boldsymbol{k}'_\perp) &= \frac{1}{(2\pi)^3}\int dx^+\,d^2\boldsymbol{x}_\perp e^{i(q^-x^+ - \boldsymbol{k}'_\perp\cdot\boldsymbol{x}_\perp)}\ \mathrm{Tr}\,\langle 0|\psi(x)\,a_h^\dagger a_h\,\overline{\psi}(0)\boldsymbol{\gamma}_\perp|0\rangle\Big|_{x^-=0} \\ &= \frac{1}{(2\pi)^4}\int dk^+\ \mathrm{Tr}\left[\boldsymbol{\gamma}_\perp(\mathcal{C}_1\not{k}' + \mathcal{C}_2\not{p}'_h + \mathcal{D})\right]|_{k^-=q^-} \\ &= \frac{2\boldsymbol{k}'_\perp}{(2\pi)^4 q^-}\int dk^2\,\mathcal{C}_1(p'_h,k') = 4\frac{\boldsymbol{k}'_\perp}{q^-}D^\perp(z,-z\boldsymbol{k}'_\perp). \end{aligned} \tag{B17}$$

From this we can immediately find the values in the original frame:

$$D^{[\gamma^-]}(p_h;z,\boldsymbol{k}_\perp) = D^-(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp), \tag{B18}$$

$$\begin{aligned} D^{[\gamma^+]}(p_h;z,\boldsymbol{k}_\perp) &= \frac{m_h^2}{2z(q^-)^2}\,D^+(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp) + \frac{\boldsymbol{p}_{h\perp}^2}{2z^2(q^-)^2}\,D^-(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp) \\ &\quad + \frac{\boldsymbol{p}_{h\perp}\cdot(\boldsymbol{k}_\perp - \boldsymbol{p}_{h\perp}/z)}{z^2(q^-)^2}\,D^\perp(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp), \end{aligned} \tag{B19}$$

$$D^{[\gamma_\perp]}(p_h;z,\boldsymbol{k}_\perp) = \frac{\boldsymbol{k}_\perp - \boldsymbol{p}_{h\perp}/z}{zq^-}\,D^\perp(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp) + \frac{\boldsymbol{p}_{h\perp}}{zq^-}D^-(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp). \tag{B20}$$

In terms of $\hat{T}$, $\hat{q}$ and the transverse momenta one has

$$D^{[\gamma^\mu]}(p_h;z,\boldsymbol{k}_\perp) = -\frac{q^+}{Q}\left[\left(\hat{T}^\mu + \hat{q}^\mu\right) D^- + \frac{2(k_\perp^\mu - p_{h\perp}^\mu/z)}{zQ}\, D^\perp + \frac{2p_{h\perp}^\mu}{zQ}\, D^- + O\left(\frac{1}{Q^2}\right)\right]. \tag{B21}$$

The scalar function $D^{[1]}$ introduces the additional profile function $E(z, \boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp)$,

$$\begin{aligned} 4zD^{[1]}(p_h;z,\boldsymbol{k}_\perp) &= \frac{1}{(2\pi)^3}\int dx^+\, d^2\boldsymbol{x}_\perp e^{i(q^-x^+ + \boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp)}\ \mathrm{Tr}\ \langle 0|\psi(x)\, a_h^\dagger a_h\, \overline{\psi}(0)|0\rangle\Big|_{x^-=0} \\ &= \frac{1}{(2\pi)^4}\int dk^+\ \mathrm{Tr}\ [\mathcal{C}_1 \not{k} + \mathcal{C}_2 \not{p}_h + \mathcal{D}]\Big|_{k^-=q^-} \\ &= \frac{2m_h}{(2\pi)^4 q^-}\int dk^2\, \frac{\mathcal{D}(p_h,k)}{m_h} \ \equiv\ \frac{4m_h}{q^-}\, E(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp), \end{aligned} \tag{B22}$$

leading to the result

$$D^{[1]}(p_h;z,\boldsymbol{k}_\perp) = -\frac{q^+}{Q}\,\frac{2m_h}{Q}\, E(z,\boldsymbol{p}_{h\perp} - z\boldsymbol{k}_\perp). \tag{B23}$$

We find, for the quark→quark profile functions,

$$D^-(z,\boldsymbol{k}_\perp) = D^+(z,\boldsymbol{k}_\perp) = D^\perp(z,\boldsymbol{k}_\perp) = E(z,\boldsymbol{k}_\perp) = \delta(1-z)\,\delta^2(\boldsymbol{k}_\perp). \tag{B24}$$

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