

## Constraints on black hole remnants

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One possible fate of information lost to black holes is its preservation in black hole remnants. It is argued that a type of effective field theory describes such remnants (generically referred to as informons). The general structure of such a theory is investigated and the infinite pair production problem is revisited. A toy model for remnants clarifies some of the basic issues; in particular, infinite remnant production is not suppressed simply by the large internal volumes as proposed in cornucopion scenarios. Criteria for avoiding infinite production are stated in terms of couplings in the effective theory. Such instabilities remain a problem barring what would be described in that theory as a strong coupling conspiracy. The relation to Euclidean calculations of cornucopion production is sketched, and potential flaws in that analysis are outlined. However, it is quite plausible that pair production of ordinary black holes (e.g., Reissner-Nördstrom or others) is suppressed due to strong effective couplings. It also remains an open possibility that a microscopic dynamics can be found yielding an appropriate strongly coupled effective theory of neutral informons without infinite pair production.

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### I. INTRODUCTION

The physics of black holes has spawned a serious clash between the basic principles of general relativity and quantum mechanics. The problem is illustrated by considering the evolution of a system of mass  $M$  undergoing gravitational collapse to a black hole. In the far past one can take the gravitational interactions to be weak and the system to be in a pure quantum state. The system then collapses to form a horizon, and begins to emit Hawking radiation [1]. This radiation is approximately thermal. If we imagine that the black hole completely evaporates the resulting Hawking radiation is apparently described by a mixed state. One then concludes that in the context of black-hole formation and evaporation, pure states evolve into mixed states: unitary quantum mechanical evolution fails and an amount of quantum information  $\propto M^2/m_{\text{pl}}^2$  (where  $m_{\text{pl}}$  is the Planck mass) is lost.

Following this logic, Hawking [2] proposed that the laws of physics are fundamentally nonunitary, and are described by an operator  $\mathcal{S}$  that linearly maps density matrices to density matrices. However, this proposal suffers several flaws. First, nonunitarity is distasteful and should be contemplated only as a last resort. Second, and more concretely, after careful examination of such evolution, Banks, Peskin, and Susskind [3], and Srednicki [4], have argued that failure of unitarity indicates failure of energy conservation at the same level. This would have dire consequences.

We are therefore confronted with the puzzle of ex-

plaining the fate of information that falls into a black hole.<sup>1</sup> One obvious possibility is that it escapes in the evaporation process; in fact, Hawking's calculation is valid only to the extent that one neglects the back reaction of the radiation and other quantum effects. However, recent investigation of the analogous problem for two-dimensional black holes [9–18,7,8] has allowed greater analytical control. This analysis seems to indicate that information can escape in Hawking radiation only as a result of introduction of drastically new physics; in particular violation of the equivalence principle at low energy and/or weak curvature seems to be required.

A different possibility is that the information is emitted once the black hole reaches the Planck size, where the preceding analysis is no longer trustworthy. This would, however, require emission of an unboundedly large amount of information (corresponding to the possibility of an arbitrarily large initial black hole) in the remaining energy  $E \sim m_{\text{pl}}$ . That must take a long time [2,19,5,6], implying a long-lived black-hole remnant.

Such remnants must be of Planck size and have an infinite number of states to allow them an unbounded information content; since this is their primary characteristic (and to distinguish them from larger remnants) they will be generically referred to as *informons*. They have long been postulated in the abstract [2,19] as one solution to the information problem, but suffer a potentially serious flaw: despite the nearly infinitesimal probability for producing any given informon in ordinary physical processes, their infinite degeneracy would seem to imply an infinite inclusive production rate.

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<sup>1</sup>For recent reviews of the information problem see, e.g., [5–8].

A more concrete realization of such objects in the context of charged black holes has arisen in recent works; one informon candidate is the extremal charged dilatonic black hole of [20,21]. At the extremal value of the mass (determined by the charge) the spatial geometry of the solution is that of an infinitely long throat attached to an asymptotically flat region. A natural conjecture is that the infinite number of states could correspond to excitations of the infinite throat. This possibility was investigated in the two-dimensional reduction of this theory to excitations moving along the throat in [9]. The connection of the full four-dimensional theory was later elaborated in [12,22]; the name *cornucopions* was coined for such objects in [12].

In addition to the generic problem of infinite pair production (to which we will return momentarily), the specific cornucopian proposal has two other potentially serious flaws. First, the basic scenario has suffered difficulties due to semiclassical singularities [11,12]. One may, however, take the attitude that a better understanding of physics in strong coupling could remedy this. Another problem is that cornucopions seem to require a charge to stabilize the throat, so oppositely charged cornucopions could annihilate, resulting in loss of their information. To avoid this a rationale for long-lived neutral remnants must be found.

One attractive rationale is the hypothesis that the underlying quantum theory of gravity places a bound on information content within a given volume, roughly corresponding to one state per Planck volume. Although it is a challenge to find a dynamical implementation of this proposal, it is a plausible feature of a quantum theory of gravity and in particular is hinted at in the properties of string theory. In [5] this assumption was argued to imply either the formation of massive remnants or of Planck-size remnants with large internal volume.<sup>2</sup> The former would seem to require new physics at weak curvatures and runs afoul of causality. If the latter can be made physically acceptable it is thus preferable.

Therefore it is plausible that the problems specific to cornucopions or other related objects could be resolved with an improved understanding of gravity near the Planck scale. That leaves the problem of infinite pair production of informons.

Recent suggestions in this direction have been made by Banks, O’Loughlin, and Strominger [24,25]. They argue that the infinite near-degeneracy of the cornucopions implies that one must be careful in treating them by low-energy effective field theory, casting suspicion on calculations of production rates. In particular [24] gives heuristic arguments for suppression of cornucopian production due to their large volumes; the infinite number of cornucopian states are very far away in the internal space and thus cannot be excited in a finite time by internal causality. Reference [25] furthermore reconsiders Schwinger-Affleck-Manton [26,27] pair production in an electromagnetic field. An approximate instanton is found for the

cornucopian version of this process.<sup>3</sup> In the corresponding Euclidean geometry a horizon forms as a result of the interaction energy required to accelerate the throat. The throat is thus cut off at finite distance and the Euclidean action is finite. Reference [25] advocates that the finite answer results from the failure of the infinite number of states “behind the horizon” to contribute to the calculation.

There is an objection to these arguments. If there are an infinite number of “ground states” of the cornucopian, then there should be an infinite number of excited black hole states that result from adding the interaction energy. It is not clear why this infinite degeneracy should not be included in the calculation; instead the calculation appears to include a factor roughly equal to the Hawking-Bekenstein entropy of the resulting excited black hole. To resolve these issues a more thorough analysis is required. In particular one would like to relate the calculation in [25] to standard derivations of pair production.

Such a relation arises through an effective field theory for informons. Indeed, informons appear as pointlike particles to an external observer. Furthermore in general one’s analysis must be carried out in the absence of a detailed knowledge of Planck scale physics. It then seems reasonable to adopt the pragmatic principle that *anything plausible can happen at the Planck scale*, and thus to characterize informon physics only in general terms. In particular, we do not know that informons have large volume interiors. They must, however, have an infinite spectrum below an energy of order  $m_{\text{pl}}$  to solve the information problem. Therefore it is useful to introduce an effective field theory which includes the possibility of infinite informon degeneracy and in which pair creation and other issues can be investigated. A particular case of such a theory is that arising from cornucopions.

In outline, this paper will first describe such an effective theory and its coupling to external fields. Next in Sec. III a toy model for informons will be constructed, and heuristic arguments based on the large internal volume in this model will be offered that infinite production does not occur. These arguments are false, as the more careful analysis of Sec. IV shows. This section also states in specific terms the infinite production problem. The following section (Sec. V) investigates the possibility that finite lifetimes for remnants could change the infinite production rate. Section VI makes a comparison with the finite calculations for pair production of excited cornucopions, as appeared in [25], and a possible source of discrepancy is traced. Also discussed is pair production of ordinary black holes: if black holes do not lose information and thus have infinitely many internal states, one may worry that they are infinitely produced. Conclusions and speculations appear in Sec. VII.

## II. EFFECTIVE THEORIES FOR INFORMONS

Suppose we assume that information is not emitted now new physics encountered during Hawking evapora-

<sup>2</sup>A crude model of the dynamics of possibly similar objects was studied previously in [23].

<sup>3</sup>This is similar to the instanton for pair production of Reissner-Nordström black holes discussed in [28].

tion before the mass and radius of the black hole reach the Planck scale, and also that information is not destroyed. Black holes then must leave remnants with mass comparable to  $m_{\text{pl}}$  and radius of order  $l_{\text{pl}}$ . These must have an infinite number of “internal states” to allow them to store the information from an arbitrarily large initial black hole. These informon states are written

$$|k, A\rangle, \quad (2.1)$$

where  $k$  is the momentum and  $A$  labels the states in the Hilbert space corresponding to a single informon. Standard field theory arguments based on causality and Lorentz invariance at distances  $\gg l_{\text{pl}}$  then imply that antiremnant states must also exist, and that informons are described by a field

$$I_A(x) = \int \frac{d^2k}{(2\pi)^3 2\omega_k} [I_A(k) e^{-k \cdot x} + I_A^\dagger(k) e^{ik \cdot x}]. \quad (2.2)$$

Free informon propagation is then governed by the action<sup>4</sup>

$$S_K = \int d^4x \sum_A \left[ -\frac{1}{2} (\partial I_A)^2 - \frac{1}{2} m_A^2 I_A^2 \right]. \quad (2.3)$$

We also must describe interactions with other fields. For simplicity first consider couplings to an external real scalar field  $\phi(x)$ . The general three-point action is

$$S_\phi = - \int d^4x d^4y d^4z \sum_{AB} I_A(x) \langle A | \tilde{\mathcal{O}}(x, y; z) | B \rangle \times I_B(y) \phi(z), \quad (2.4)$$

where  $\tilde{\mathcal{O}}(x, y; z)$  is an operator acting on the informon Hilbert space as well as on the coordinates  $x, y, z$ . The Planck size of the informon implies that the interaction is local on larger scales, so

$$\tilde{\mathcal{O}}(x, y; z) = \mathcal{O}(x) \delta(x - y) \delta(x - z) \quad (2.5)$$

up to terms falling exponentially at long distances. This gives

$$S_\phi = - \int d^4x \phi(x) \sum_{AB} I_A(x) \langle A | \mathcal{O}(x) | B \rangle I_B(x) \quad (2.6)$$

or, in momentum space,

$$- \int d^4k d^4q \phi(q) \sum_{AB} I_A(k + q) I_B(k) \times \langle A | \mathcal{O}(k^2, q^2, k \cdot q) | B \rangle. \quad (2.7)$$

Equation (2.7) may be extended to higher-order interactions. In general the informon interacts with its surroundings through a sum of such terms.

The generalizations for couplings to gravity or electromagnetism are straightforward. For a weak electromagnetic field the interaction is likewise of the form

$$S_{\text{em}} = - \int d^4x A_\mu \sum_{AB} [I_A^*(x) \langle A | \mathcal{J}^\mu | B \rangle I_B(x) + \text{H.c.}], \quad (2.8)$$

where  $\mathcal{J}^\mu$  is in operator that must reduce to the standard electromagnetic current at zero momentum transfer. This may be supplemented by higher-order couplings, both as required by gauge invariance and otherwise. The general such action for informons of charge  $q$  coupled to electromagnetism is then

$$S[I] = \int d^4x \left\{ \sum_A [ - |(\partial_\mu + iq A_\mu) I_A|^2 - m_A^2 |I_A|^2 ] - \left[ \sum_i \hat{\mathcal{O}}_i[A_\mu] \sum_{AB} I_A^*(x) \langle \bar{A} | \mathcal{O}_i(x^\mu, -i\partial_\nu) | B \rangle I_B(x) + \text{H.c.} \right] \right\}, \quad (2.9)$$

where  $\hat{\mathcal{O}}_i[A_\mu], \mathcal{O}_i$  are local operators acting on the electromagnetic field or internal Hilbert space, respectively. ( $\mathcal{O}_i$  may also include derivatives acting on informon fields, as indicated.) Likewise for a weak gravitational field,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , the coupling is of the form

$$S_g = - \int d^4x h_{\mu\nu} \sum_{AB} I_A(x) \langle A | \mathcal{T}^{\mu\nu} | B \rangle I_B(x), \quad (2.10)$$

where again  $\mathcal{T}^{\mu\nu}$  should reduce to the standard stress tensor at zero momentum transfer; with inclusion of higher-order interactions this becomes

$$S_g[I] = \int d^4x \sqrt{-g} \left[ \sum_A \frac{1}{2} (-\nabla_\mu I_A \nabla^\mu I_A - m_A^2 I_A^2) - \sum_i \hat{\mathcal{O}}_i[g_{\mu\nu}] \sum_{AB} I_A(x) \langle A | \mathcal{O}_i(x^\mu, -i\partial_\nu) | B \rangle I_B(x) \right]. \quad (2.11)$$

<sup>4</sup>For simplicity we take the informons to be scalar, although the labels  $A$  could equally well include spin. The following discussion would then generalize.

It is worth emphasizing the extent to which such theories are effective field theories in the conventional sense. First, since informons resulting from evaporation of a neutral black hole should have size  $\sim l_{\text{pl}}$  they have

excitations with Planckian frequencies and wave numbers. These modes are integrated out in writing the effective field theory. However, informons also have a infinite number of low frequency internal modes. These states are not integrated out but instead are explicitly accounted for.

One might object that at least for cornucopions or other similar scenarios where the large number of modes arise due to a large internal volume it is awkward to describe the internal states in this fashion. In that case one has a clear interpretation of the states in terms of modes propagating on the throat, and no other description seems necessary. However, there are two separate reasons to describe the internal states in this general fashion. First, for cornucopions infalling states eventually form black holes, and then the interpretation of the resulting system in terms of quantum states becomes fogged. Although the precise description of the quantum states is not known, it is nonetheless important to make explicit the assumption that there is some spectrum of quantum states being excited. This is in contrast to information loss. Secondly, and more importantly, although it is true that there is a semiclassical description (of limited validity) and thus a partial picture of the states for large cornucopions, we can hardly expect this to be true for Planck-sized informons. Even if they have large volume interiors connected through Planck scale throats this is impossible for a long distance observer to explore directly without squeezing through the Planck scale throat. More generally one may expect a rather complicated Planckian dynamics that does not correspond to large volume but may nonetheless (or may not) have some similar features encoded in the spectrum and interactions. In any case, we make the fundamental assumption that whatever the dynamics of quantum gravity is, it falls into the framework of unitary evolution of quantum states so that information is preserved.

In the absence of an understanding of quantum gravity we must therefore adopt rather modest goals. These are to find whether there is any possible spectrum of informon states and any possible set of interactions of these states with external fields that allows a solution of the black-hole information problem. Without understanding the microscopic dynamics, the best one can do is formulate this question within the framework of an effective theory for informons such as has been described.

### III. A TOY MODEL AND A FALSE ARGUMENT

One can readily construct toy models for the internal Hilbert space of an informon that serve as examples of the above general framework. In particular, the perceived benefits of having an infinite volume informon may be implemented by choosing a spectrum and set of in-

teractions of the appropriate form. It is irrelevant to the outside observer whether the internal space is “real.”

A particularly simple model arises by taking the internal Hilbert space to be the Fock space of a single scalar field  $f$  propagating on the semi-infinite line  $(0, \infty)$ . The action is

$$S_f = -\frac{1}{2} \int d^2\sigma (\nabla f)^2, \quad (3.1)$$

where  $\sigma, \tau$  are internal spacetime coordinates, and some boundary conditions, e.g., Neumann, are specified at  $\sigma=0$ . The states of the informon are labeled by the occupation numbers of the  $f$  eigenmodes,

$$I_{A \leftrightarrow |A\rangle} = |\{n_\kappa\}\rangle = \prod_\kappa \frac{(f_\kappa^\dagger)^{n_\kappa}}{\sqrt{n_\kappa!}} |0\rangle, \quad (3.2)$$

where

$$f(\tau, \sigma) = \int_0^\infty \frac{d\kappa}{4\pi\kappa} (f_\kappa e^{-i\kappa\tau} + f_\kappa^\dagger e^{i\kappa\tau}) \cos\kappa\sigma. \quad (3.3)$$

It is convenient to take the mass of the informon to be  $m_A^2 = m_0^2 + 2H_A$ , where  $m_0$  is a mass of order  $m_{\text{pl}}$  and  $H_A$  is the Hamiltonian of the internal theory,

$$H_A = \int_0^\infty d\kappa \kappa n_\kappa. \quad (3.4)$$

This supplies a spectrum with an infinite number of states below any mass  $> m_0$ . In order to avoid arbitrarily massive informons we may also remove from the theory all states with mass larger than  $m_{\text{max}} > m_0$ .

Next consider interactions with an external field; for concreteness take it to be electromagnetic although other fields could equally well be considered. Suppose that the informon is charged and interacts with the field through the standard minimal coupling terms. These do not affect the internal structure of the remnant: they are diagonal in the informon Hilbert space. Suppose that there are also interactions with the field through other operators, e.g.,

$$F^2(q) = \int d^4x e^{-iq \cdot x} F_{\mu\nu}^2(x), \quad (3.5)$$

and that these are nondiagonal on informon states. They may be incorporated by temporarily adopting a first-quantized description for the informon. The propagator for a free informon on mass  $m$  is given by

$$G(x, x') = \int_0^\infty dT \int_x^{x'} \mathcal{D}X \exp \left\{ i \int_0^T d\tau (\dot{X}^2 - m^2) \right\}, \quad (3.6)$$

where  $\tau$  is interpreted as the proper time and  $\dot{X} = dX/d\tau$ . We may therefore introduce a coupling to the internal theory, corresponding to the diagram of Fig. 1:

$$\begin{aligned} & \int_0^\infty dT \mathcal{D}X e^{i \int_0^T d\tau (\dot{X}^2 - m^2)} e^{ik \cdot X(0) - i(k+q) \cdot X(T)} \int_0^T d\tau' F_{\mu\nu}^2(X(\tau')) \mathcal{O}(\tau', 0) \\ & = \int d^4q F^2(q) \int_0^\infty dT \int_0^T d\tau' e^{i\tau[k^2 + m^2] + i(T-\tau)[(k+q)^2 + m^2]} \mathcal{O}(\tau', 0), \end{aligned} \quad (3.7)$$

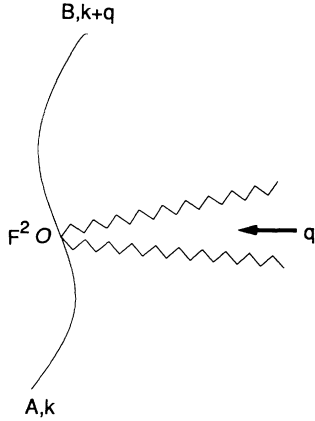


FIG. 1. An incoming informon in state  $A$  and with momentum  $k$  interacts with the electromagnetic field through the operator  $F^2 \mathcal{O}$  to produce an informon in state  $B$  with momentum  $k+q$ .

where  $\mathcal{O}(\tau', 0)$  is a local operator acting at  $(\tau, \sigma) = (\tau', 0)$  in the internal  $f$  theory. [Free propagators must be truncated from (3.7) to define the vertex.] Such a theory is similar to the effective theories of combined dimensions introduced to describe cornucopions in [22, 12].

This model shares several features with more fundamental theories of informons with large internal volumes. First, there is an infinite spectrum of low-energy states of the informon arising from the infinite volume. Second, any attempt to use electric or magnetic fields to move one of these objects will inevitably excite the internal state of the informon through the interaction (3.7). However, one feature it does not share with such scenarios is a dynamical geometry. One expects that in models with dynamical geometry the dynamics is nonetheless encoded in the informon spectrum and interactions, and thus would also be described by effective theories as in the preceding section. Although the model of this section does not capture all aspects of such a theory it is useful for illustrating relations between infinite volume and pair production.

Within the context of this model arguments similar to those for finite pair production of cornucopions [12, 24] are easily made. First, in reference to thermodynamics problems with informon spectra, it appears that the infinite “internal volume” and the “causality” of the internal theory imply that equilibration of the thermal ensemble takes a long time. Furthermore, as emphasized in [24], one would expect that the action to create configurations that differ from the vacuum over a large (internal) volume  $V$  should be proportional to  $V$ . This reasoning suggests that it should be difficult to create informon states with excitations arbitrarily far down the internal line. Finally and more concretely we can imagine Schwinger pair production of these in a background

electric field. The instanton arising in the Schwinger process has finite temporal extent; the characteristic scale governing the process is the electric length,  $\sim m_A/qE$ . By “causality” of the internal theory it should not possible for states arbitrarily far down the internal line to be excited by the operator  $\mathcal{O}$  if it only acts for this finite time at  $\sigma=0$ . Since below  $m_{\max}$  there are only finitely many states that are localized in a finite interval in  $\sigma$ , only finitely many remnant states are produced and the total pair production rate is finite. The problem of infinite pair production is solved.

Unfortunately, the latter arguments are false.

#### IV. INFINITE PAIR PRODUCTION

The flaw in the above chain of reasoning can be seen, for example, by revisiting Schwinger’s calculation as described in [29]. The decay rate  $\Gamma$  of an electric field into informons is given by the imaginary part of the Euclidean vacuum-to-vacuum amplitude,

$$V_4 \Gamma = 2 \operatorname{Im} \ln \int \mathcal{D} \tilde{A}_\mu \mathcal{D} I e^{-S[A_\mu] - S[I_A]}, \quad (4.1)$$

where  $V_4$  is the four-volume,  $S[A_\mu]$  is the standard gauge action,  $S[I]$  is the informon action (2.9), and the gauge field is divided into external and fluctuation pieces,  $A_\mu = A_\mu^0 + \tilde{A}_\mu$ . Integrating over  $I_A$  gives

$$V_4 \Gamma = 2 \operatorname{Im} \ln \int \mathcal{D} \tilde{A}_\mu e^{-S[A_\mu] - S_{\text{eff}}[A_\mu]}, \quad (4.2)$$

where

$$S_{\text{eff}}[A_\mu] = \frac{1}{2} \ln \det \{ [ -(\partial_\mu + iq A_\mu)^2 + m_A^2 ] \delta_{AB} + \mathcal{M}_{AB} \} \quad (4.3)$$

and one defines

$$\mathcal{M}_{AB} = \sum_i \hat{\mathcal{O}}_i [A_\mu] \langle \bar{A} | \mathcal{O}_i | B \rangle + \text{H.c.} \quad (4.4)$$

Suppose that electromagnetism is weakly coupled (more on this later); then one may work to leading order in the coupling by simply dropping  $\tilde{A}_\mu$ . Using  $\ln \det = \operatorname{Tr} \ln$  and the standard Schwinger proper time trick, the decay rate becomes

$$V_4 \Gamma = \operatorname{Im} \int_0^\infty \frac{dT}{T} \operatorname{Tr} e^{-HT} \quad (4.5)$$

with “Hamiltonian”

$$\langle \bar{A} | H | B \rangle = \frac{1}{2} [ -(\partial_\mu + iq A_\mu^0)^2 + m_A^2 ] \delta_{AB} + \frac{1}{2} \mathcal{M}_{AB}(x^\mu, -i\partial_\nu). \quad (4.6)$$

The trace over positions in (4.5) can then be rewritten as a functional integral; an integral over momentum must also appear to accommodate the derivative dependence in the  $\mathcal{O}_i$ . This yields

$$V_4 \Gamma = 2 \operatorname{Im} \int_0^\infty \frac{dT}{T} \int \mathcal{D} X \mathcal{D} P \exp \left\{ - \int_0^T d\tau \left[ \frac{1}{2} (P + q A^0)^2 - iP_\mu \dot{X}^\mu \right] \right\} \sum_A \langle \bar{A} | \exp \left\{ - \frac{T}{2} m_A^2 - \frac{1}{2} \int_0^T d\tau \mathcal{M}(X^\mu, P_\nu) \right\} | A \rangle. \quad (4.7)$$

For finite production this expression must be finite.

Now the problem of infinite pair production is manifest. Suppose first that there is no coupling via operators  $\mathcal{O}_i$ . Suppose also that there are an infinite number of internal states  $|A\rangle$  with masses  $m_A < m_{\max}$ , for some  $m_{\max} \sim m_{\text{pl}}$ . Then

$$\sum_A e^{-Tm_A^2} > e^{-m_{\max}^2 T} \sum_A 1 = \infty e^{-m_{\max}^2 T}. \tag{4.8}$$

This is then inserted into (4.7). The  $P$  integral can be explicitly performed, and the  $T$  integral and  $X$  functional integral are given by the saddle point described by the Schwinger instanton. Recall that this corresponds to circular motion of the charged particle in the Euclidean continuation of the electric field. The total production rate is infinite from the overall infinite factor.

The infinite production for nontrivial  $\mathcal{O}_i$  can also be seen in (4.7). First consider the case of weakly coupled effective field theory, in which the couplings through the  $\mathcal{O}_i$  to all states are small. In this case the integrals can be evaluated by saddle point techniques. The  $P$  saddle point is given by

$$P_\mu \simeq i\dot{X}_\mu - qA_\mu^0 + \dots, \tag{4.9}$$

where neglected terms arise from the  $P$  dependence of the  $\mathcal{O}_i$ . The resulting integral is of the form

$$V_4 \Gamma = 2 \text{Im} \int_0^\infty \frac{dT}{T} \int \mathcal{D}X e^{-\int_0^T d\tau [(1/2)\dot{X}^2 + iqA_\mu^0 \dot{X}^\mu]} \sum_A \langle \bar{A} | \exp \left\{ -\frac{T}{2} m_A^2 - \frac{1}{2} \int_0^T d\tau \mathcal{M}_{AB}(X^\mu, i\dot{X}_\nu - qA_\nu^0 + \dots) \right\} | A \rangle. \tag{4.10}$$

Under the assumption of weak coupling, the integrals over  $T$  and  $X$  may be approximated by the Schwinger saddle point for each  $A$ . Near these saddle points the contributions of the  $\mathcal{O}_i$  terms are small. Therefore they cannot render the infinite sum finite, and an infinite production rate follows.

This result could be altered in strong coupling if the couplings grew arbitrarily large as the states are enumerated. However, that would violate tree-level unitarity bounds.<sup>5</sup> This amounts to saying that the loop effects reduce the effective couplings to  $\mathcal{O}(1)$ . Therefore the only remaining possibility is that the couplings are of order one but vary with the internal state of the informon such that the terms in the sum (4.7) oscillate and add to a finite total amplitude. This would clearly take precise adjustment of couplings for a given spectrum.

The above result can be explicitly illustrated in the toy model of the preceding section. There the decay rate takes the form

$$\begin{aligned} V_4 \Gamma &= 2 \text{Im} \int_0^\infty \frac{dT}{T} \int \mathcal{D}X e^{-\int_0^T d\tau [(1/2)\dot{X}^2 + iqA_\mu^0 \dot{X}^\mu]} \sum_A \langle \bar{A} | \exp \left\{ -\frac{T}{2} m_A^2 - \int_0^T d\tau F^2(X(\tau)) \mathcal{O}(\tau, 0) \right\} | A \rangle \\ &= \text{Im} \int_0^\infty \frac{dT}{T} \int \mathcal{D}X e^{-\int_0^T d\tau [(1/2)\dot{X}^2 + iqA_\mu^0 \dot{X}^\mu]} e^{-m_0^2(T/2)} \int \mathcal{D}f e^{-S_f^E - \int_0^T d\tau F^2(X(\tau)) \mathcal{O}(\tau, 0)}, \end{aligned} \tag{4.11}$$

where the trace over internal states has been rewritten as a functional integral over the internal  $f$  theory, and where  $S_f^E$  is the Euclidean version of the free action (3.1). Now we see directly the flaw in the previous reasoning. First off, if the coupling to  $F^2$  were turned off, production would be suppressed by  $e^{-H_A T}$ , where  $H_A$  is the Hamiltonian (3.4), or equivalently the mass shift. However, there are infinitely many states below any mass, resulting in an infinite answer. Now, by locality, if we turn the coupling to  $F^2$  back on, it can hardly change this infinite answer since the vast majority of states are far down the line, outside the influence of the operator  $\mathcal{O}(\tau, 0)$ . Production is not suppressed by the large volume.

These problems likewise extend (pursuing the connection discussed in Schwinger's classic paper) to the problem of infinite vacuum polarization and therefore to an infinite and thus nonsensical Lagrangian for electromagnetism.

The same arguments should apply if one considers couplings of informons to gravity or other fields. Pair production is only suppressed by the mass corresponding to the states, so if there are infinitely many states below a given mass, infinite rates should result. Couplings to other operators that affect the internal state of the informon should not alter this unless these couplings become large for infinitely many of the internal states and are carefully

<sup>5</sup>Strictly speaking this requires continuation of couplings from small Euclidean frequency to small Lorentzian frequency.

arranged to oscillate so that they give a finite production rate. This greatly restricts the possibility that remnants give a viable solution to the information problem.

### V. DECAYING INFORMONS

The above calculation, strictly speaking, applies only to absolutely stable informons. It is also conceivable that remnants decay, and that this could alter the results.

First let us estimate the decay rate of an informon arising from the evaporation of a black hole of initial mass  $M$ . Such an informon must have roughly  $\mathcal{N} \sim (M/m_{\text{PI}})^2$  states excited to encode the information. The characteristic energy spacing between these states must therefore be  $\Delta E \sim m_{\text{PI}}/\mathcal{N}$ . The decay time for one of these states is  $\gtrsim 1/\Delta E$ . This may be shown explicitly by considering for example an  $s$ -wave coupling giving decay to massless  $\phi$  quanta,

$$m_{\text{PI}} \int d^4x I_A(x) I_B(x) \phi(x), \quad (5.1)$$

or equivalently follows from the observation that it must take a time  $\Delta t \sim 1/\Delta E$  to emit a quantum of energy  $\Delta E$ . The characteristic decay time between remnant states is therefore  $\sim t_{\text{PI}}/\mathcal{N}$  and typical decay widths are  $\Gamma_A \sim m_{\text{PI}}/\mathcal{N}$ . (For the informon to decay completely it must emit  $\mathcal{N}$  such quanta, taking a time  $\sim t_{\text{PI}}(M/m_{\text{PI}})^4$ , in accord with the estimate of [30,6].)

The resulting mixing between informon states can effectively be represented by including a nontrivial mass matrix,

$$m_A \delta_{AB} \rightarrow m_A \delta_{AB} + \frac{1}{2} \Delta m_{AB}, \quad (5.2)$$

which need not be hermitian. The elements of  $\Delta m_{AB}$  are then of order  $m_{\text{PI}}/\mathcal{N}$  and make contributions of order  $m_{\text{PI}}^2/\mathcal{N}$  to  $m_{AB}^2$ . Reexamining the pair-production rate, (4.7), we see that a rough criterion for when the finite lifetime becomes important is if the Euclidean time over which the instanton process takes place satisfies  $T m_{\text{PI}}^2/\mathcal{N} \gtrsim 1$ . Therefore for arbitrarily large  $\mathcal{N}$  informon decay does not make a significant contribution—the informons decay too slowly.

### VI. CORNUCOPION PAIR PRODUCTION

Although the above discussion sheds some light on the arguments for finite production made in [12,24] it is yet to be compared with [25] which claims to calculate a finite pair production rate for cornucopions. There it is pointed out that it is not possible to pair produce cornucopions in their ground states since any attempt to move them results in excitation of the cornucopion above extremality and thus formation of a horizon. Therefore the problem is that of pair producing the resulting black holes. Reference [25] found an approximate Euclidean instanton geometry that describes this pair production process. The instanton action is finite, and the instanton appeared not to include the infinite number of states behind the horizon that would lead to an infinite degeneracy factor and rate. However, this argument is quite similar to Euclidean arguments that yield the Bekenstein-Hawking entropy of a black hole; such

reasoning suggests that in fact black holes have only finitely many internal states in conflict with the presumption that they may contain infinite information.

In fact the calculation of [25] apparently neglects the source of the infinite number of information-bearing states, and thus sidesteps the issue. To see this, momentarily consider the simpler problem of forming a Schwarzschild black hole by throwing in a large number of quantum particles. The Penrose diagram for this process is shown in Fig. 2. (Temporarily ignore Hawking radiation.) As described by an observer who uses the standard Schwarzschild time slicing, the matter never crosses the horizon and the evolution is unitary. In this observer's description the information in the infalling matter is contained in states near the horizon. We may compare this to a different viewpoint, namely that of ingoing Eddington-Finkelstein time,<sup>6</sup> as shown in Fig. 3. An observer using the Eddington-Finkelstein time slicing would say that the information crosses the horizon and hits the strong-curvature region. In either case the observers count the same number of states (excluding the possibility that information is truly annihilated at the singularity). Similar statements can also be made if Hawking evaporation is allowed, with states piling up on the effective horizon;<sup>7</sup> since a black hole can begin with an arbitrarily large mass and evaporate down, the number of such states is infinite. The moral is that if we choose to describe processes in terms of external Schwarzschild time, the "internal" states are never really internal: instead they are frozen at the horizon.

This behavior can be explicitly illustrated. Consider for example a massless scalar field  $\psi$  in a static spherically symmetric black-hole geometry,

$$ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + R^2(r) d\Omega_2^2. \quad (6.1)$$

For appropriate  $g_{tt}$ ,  $g_{rr}$ , and  $R$  (which we take to asymptote to flat form at infinity) this specializes to Schwarzschild or to the near-extremal dilatonic black holes, i.e., excited cornucopions. Mode propagation is most easily discussed in tortoise coordinates,

$$\frac{dr^*}{dr} = \left[ \frac{g_{rr}}{-g_{tt}} \right]^{1/2}. \quad (6.2)$$

These range from the horizon at  $r^* = -\infty$  to spatial infinity at  $r^* = \infty$ . Expanding in spherical harmonics,

$$\psi = \frac{u(r,t)}{R} Y_{lm}(\theta, \phi), \quad (6.3)$$

the action becomes

$$- \int d^4x \sqrt{-g} (\nabla\psi)^2 \\ \propto \int dr^* dt [(\partial_t u)^2 - (\partial_{r^*} u)^2 - V(r^*) u^2] \quad (6.4)$$

<sup>6</sup>For a concise review of Eddington-Finkelstein coordinates see [31].

<sup>7</sup>The effective horizon is first outgoing null ray that hits strong coupling before reaching  $\mathcal{I}^+$ .

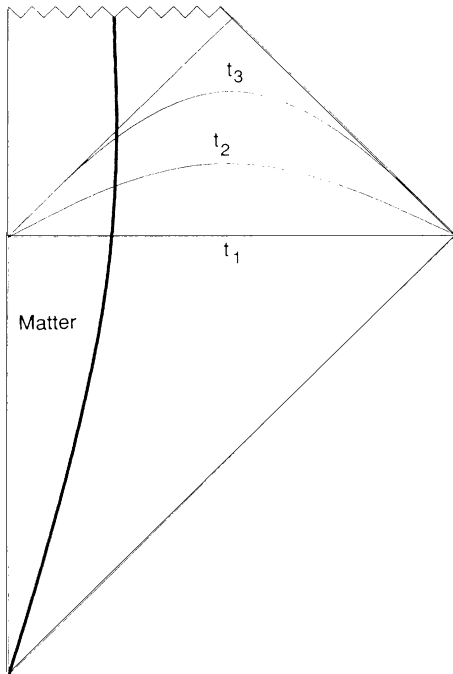


FIG. 2. Shown is the time slicing for a collapsing black hole corresponding to external Schwarzschild time. As seen by the external observer, the infalling states never cross the horizon, but are asymptotically frozen at the horizon.

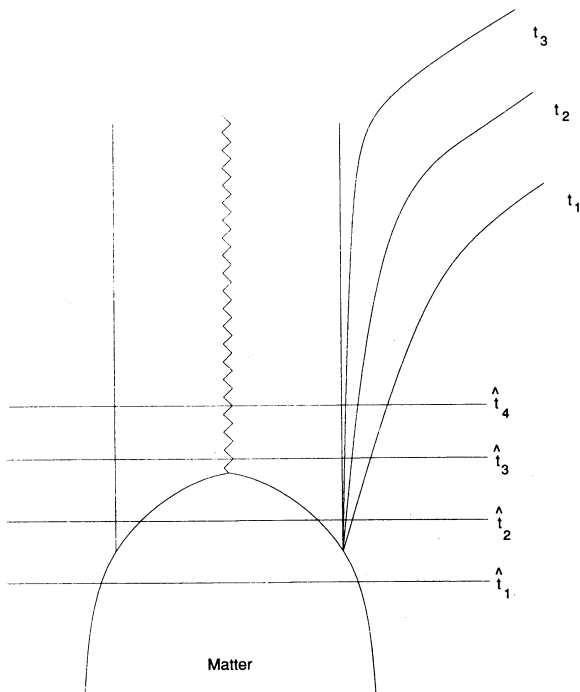


FIG. 3. The geometry of Fig. 2 may equally well be described using an Eddington-Finkelstein time slicing. In this time slicing infalling states that the external observer saw frozen at the horizon instead cross the horizon in finite time to become internal states of the black hole.

with effective potential

$$V(r^*) = -g_{tt} \frac{l(l+1)}{R^2} + \frac{\partial_{r^*}^2 R}{R} . \tag{6.5}$$

This effective potential vanishes at infinity and at the horizon.

An incoming mode of a given frequency  $\omega$  at infinity takes the form  $e^{-i\omega(t+r^*)}$  near the horizon, plus the part reflected from the barrier. A wave packet formed from these modes will reach  $r^* = -\infty$  only at  $t = \infty$ ; as described in  $r$  coordinates it gets asymptotically close to the horizon at infinite time. There are an infinite number of such states frozen at the horizon, arising from the infinite volume in  $r^*$ , or equivalently from the infinite number of past histories a black hole of mass  $M$  may have had.

These observations may now be applied to pair production of black holes arising from excitations of conuptions. Whereas one could certainly attempt to find a description of black-hole pair production in Eddington-Finkelstein coordinates, it is much simpler to describe the process in Schwarzschild coordinates. As emphasized above, these coordinates, and their Euclidean continuation, fail to cover the interior of the black hole. However, as described in these coordinates there should be an infinite number of states at the horizon, corresponding to the infinite number of possible histories of the black hole. These would at least naively appear to give an infinite production rate through the fluctuation determinant about the instanton.

It would be useful to actually compute the fluctuation determinant to verify this. In particular one might think a Euclidean momentum cutoff would remove infinitely many of these states since they have large wave numbers near the horizon. However, it is not clear that this is sensible: these modes arise from propagation of perfectly well-behaved physical states into the black hole. Similar outgoing modes are also crucial to the Hawking radiation. Therefore in general they should not be truncated from the theory. If their contribution proves to be negligible that should have a physical explanation. Since these are the information-bearing modes it would be quite interesting to understand the physics modifying their contribution.<sup>8</sup>

From the point of view of the effective field theory the only possible such rationale is carefully arranged strong couplings. However, as pointed out in [22] the only low-energy modes propagating on the throat of the conuption are fermion modes. Their contributions are not expected to make the Schwinger production rate finite since charged fermions are reflected from the throat. Rather the arguments of [24] rely on propagating modes that excite the gauge field corresponding to rotations of

<sup>8</sup>It is conceivable that a class of cutoffs similar to a cutoff in Kruskal momentum renders the answer finite and has a physical interpretation.



the throat. These should then be massive.<sup>9</sup> Their propagation down the throat is exponentially attenuated and thus they cannot produce a strongly coupled effective field theory. One might then ask if this is true why do they nonetheless produce a horizon as described in [25]. This presumably occurs due to the unbounded growth of the coupling constant along the throat; in fact for fixed asymptotic coupling the horizon forms at arbitrarily large coupling for arbitrarily weak external field.

The upshot is that the formation of the horizon and strong coupling of the effective field theory both rely not only on *strong* [ $O(1)$ ] coupling in the underlying theory of the throat but on *unbounded* coupling. This makes the argument of [25] for finite production rather implausible. In particular if a strong coupling modification of cornucopions did not have unbounded couplings but nonetheless had infinitely many states then the effective field theory should be weakly coupled and infinite production results.

On the other hand similar logic can be applied to investigate a potentially more serious concern. If it is true that information is retained in a black hole as it evaporates, then a black hole of mass (say) 1 gm should have an infinite number of internal states. Therefore when we describe these in an effective field theory at energies  $E \ll 1$  1 gm, we might expect to encounter infinite pair production.

For real black holes, however, there are possible outs. First, the modes with energies and angular momenta satisfying  $EM \gtrsim 1$  have amplitude of order one to penetrate the black hole and thus are strongly coupled as viewed in the effective theory, in contrast to cornucopions. Thus what would be described from the effective theory as a strong coupling conspiracy is quite plausible. If we considered for example pair production of extremal Reissner-Nordström black holes [28] this could also be directly connected to the apparent finiteness of the fluctuation determinant with an appropriate momentum cutoff. Furthermore, the calculation is modified by the Hawking decay of the black hole, following arguments of the previous section. The characteristic time for a black-hole state of mass  $M$  to decay to a lower state is given by  $t_{\text{PI}} M / m_{\text{PI}}$ , in accord with the fact that the radiation has characteristic energy  $m_{\text{PI}}^2 / M$ . Thus from the discussion of the preceding section  $\Delta m^2 \sim m_{\text{PI}}^2$  and the rough criterion for the decay to be important is  $T m_{\text{PI}}^2 \gtrsim 1$  where  $T$  is the proper time for the appropriate instanton. This criterion will be satisfied for an instanton producing a black hole larger than the Planck mass. Therefore strong effective couplings and/or the decay can substantially suppress the pair production rate. These arguments then offer a plausible explanation for infinite production of cornucopions yet finite production of ordinary black holes.<sup>10</sup>

<sup>9</sup>The former point was made by Thorlacius, the latter by Banks and Strominger.

<sup>10</sup>They also lend credence to the possibility that Reissner-Nordström black holes are viable charged remnants [32].

It is, however, worth emphasizing the possibility that strong couplings and/or short decay times do not prevent infinite production. This is a concern since the rate may be arbitrarily small for a given black-hole state yet infinite due to the infinite number of possible black-hole states. That raises the disturbing possibility of infinite production of black holes in slowly varying macroscopic fields. If this were the case it would be difficult to see another convincing way out of the catastrophe. The possibilities are as follows.

(1) Although information is not lost by black holes, and black holes can be assigned internal states, black holes cannot be treated by the type of effective field theories being discussed. However, it is difficult to see how this could be so, from the reasoning in Sec. II. To reiterate: black holes are localized, and if they are allowed to have internal states and momenta the only known description of them that is Lorentz invariant and causal at long distances is in terms of effective fields. One would need a loophole in these arguments.

(2) Information does not pass the horizon and is reradiated in some form. This would likely require new physics and in particular a breakdown of the equivalence principle at weak curvatures.

(3) Information is in fact annihilated. However, this would be expected to happen only at the singularity. This phenomenon could therefore presumably not be discovered by an external observer until the black hole reached the Planck size. Such an external observer would thus still attribute an infinite number of states to the black hole, and therefore would still apparently confront infinite pair production of black holes. One escape would be annihilation of information at the horizon, but this would again require a major breakdown of known physics at arbitrarily weak curvatures.

## VII. CONCLUSION AND SPECULATIONS

If black holes leave remnants with internal states of characteristic size and mass  $r_{\text{remnant}}, m_{\text{remnant}}$ , then these remnants should be describable in terms of an effective field theory on scales  $E < m_{\text{remnant}}, 1/r_{\text{remnant}}$ . This follows from the basic principles of quantum mechanics, macroscopic Lorentz invariance, and causality. There should be one field  $I_A$  for each allowed remnant internal state  $|A\rangle$ . Couplings of slowly varying external fields to such informons are also easily incorporated in such an effective field theory using the basic technology of form factors. If the spectrum of informons is infinite and if this effective theory is weakly coupled then informons will be infinitely produced in processes like that of Schwinger. Thus there is no viable weakly coupled informon (hence cornucopion) scenario. It may be possible that this conclusion is altered for a strongly coupled effective theory of informons, but only if the couplings are carefully fixed through what would be described in that theory as a strong coupling conspiracy. In particular it is far from clear that the cornucopion scenario consistently arranges such a conspiracy; the finite rates computed in [25] neglected the probable origin of the infinite

number of states and rely on unbounded coupling in the underlying field theory. The same logic does not, however, apply to ordinary black holes since they are strongly coupled in the effective sense and also have relatively short decay times.

Where does this leave us with the information problem? Strictly speaking informons are not yet ruled out since strong coupled effective theories leave us a little room for maneuvering. Given difficulties with other proposed resolutions of the information conundrum it is certainly worth investigating the possibility of viable scenarios. It would seem that at a minimum all fields that can pair produce informons should be effectively strongly coupled to avoid infinite production. This requires that all such fields couple strongly to light modes of the informon. One way to imagine such a possibility is if informons consist of internal regions that are not just large in one direction but in all directions. In this case there would indeed be light modes internal to the informon. The large volume could be connected through a Planck-sized throat, and with appropriate dynamics the couplings might be arranged to be sufficiently strong. Perhaps such a picture (see Fig. 4) would yield a finite production rate. The challenge is to find a viable dynamics that describes such an object, or other realizations of informon scenarios that evade infinite production. If quantum gravity indeed includes such states with Planck masses and infinite spectra this may be a deep clue towards its structure.

*Note added.* Recent investigations [33] have suggested the possibility of remnant theories that sidestep the constraints of this paper and have finite production rates. These may in particular occur in describing Reissner-Nordström black holes. Other recent investigations [34]

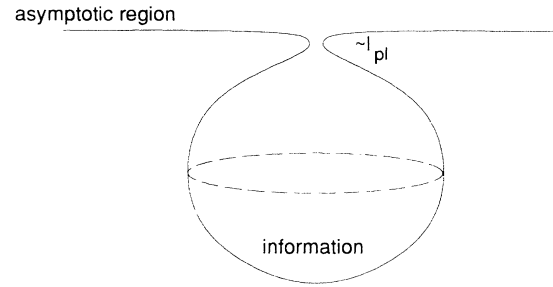


FIG. 4. A (speculative) picture of an informon with a large internal volume in all dimensions. Such an object would have light “internal” modes for all external fields, and possibly could be arranged to have appropriate couplings at the throat to prevent infinite pair production.

have also shown the existence of instantons for pair production of dilatonic black holes that at the classical level have infinite throats. However, a complete calculation of the pair production rate also requires incorporating quantum corrections, and at present no systematic method for doing this exists.

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- [1] S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975).
- [2] S. W. Hawking, *Commun. Math. Phys.* **87**, 395 (1982).
- [3] T. Banks, M. E. Peskin, and L. Susskind, *Nucl. Phys.* **B244**, 125 (1984).
- [4] M. Srednicki, “Is purity eternal?” UCSB Report No. UCSBTH-92-22, hep-th/9206056 (unpublished).
- [5] S. B. Giddings, *Phys. Rev. D* **46**, 1347 (1992).
- [6] J. Preskill, “Do black holes destroy information?” Caltech Report No. CALT-68-1819, hep-th/9209058 (unpublished).
- [7] J. A. Harvey and A. Strominger, in *Recent Directions in Particle Theory—From Superstrings and Black Holes to the Standard Model*, Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, 1992, edited by J. Harvey and J. Polchinski (World Scientific, Singapore, 1993).
- [8] S. B. Giddings, in *String Quantum Gravity and Physics at the Planck Energy Scale*, Proceedings of International Workshop on Theoretical Physics, Erice, Sicily, 1992, edited by N. Sanchez (World Scientific, Singapore, 1993).
- [9] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, *Phys. Rev. D* **45**, R1005 (1992).
- [10] H. Verlinde, in *Sixth Marcel Grossmann Meeting on General Relativity*, Proceedings, Kyoto, Japan, 1991, edited by A. Sato and T. Nakamura (World Scientific, Singapore, 1992).
- [11] J. G. Russo, L. Susskind, and L. Thorlacius, *Phys. Lett. B* **292**, 13 (1992).
- [12] T. Banks, A. Dabholkar, M. R. Douglas, and M. O’Loughlin, *Phys. Rev. D* **45**, 3607 (1992).
- [13] L. Susskind and L. Thorlacius, *Nucl. Phys.* **B382**, 123 (1992).
- [14] S. W. Hawking, *Phys. Rev. Lett.* **69**, 406 (1993).
- [15] B. Birnir, S. B. Giddings, J. A. Harvey, and A. Strominger, *Phys. Rev. D* **46**, 638 (1992).
- [16] S. P. de Alwis, *Phys. Lett. B* **289**, 278 (1992); **300**, 330 (1993).
- [17] A. Bilal and C. Callan, *Nucl. Phys.* **B394**, 73 (1993).
- [18] J. G. Russo, L. Susskind, and L. Thorlacius, *Phys. Rev. D* **46**, 3444 (1992).
- [19] Y. Aharonov, A. Casher, and S. Nussinov, *Phys. Lett. B* **191**, 51 (1987).
- [20] G. W. Gibbons and K. Maeda, *Nucl. Phys.* **B298**, 741 (1988).
- [21] D. Garfinkle, G. Horowitz, and A. Strominger, *Phys. Rev. D* **43**, 3140 (1991); **45**, 3888(E) (1992).
- [22] S. B. Giddings and A. Strominger, *Phys. Rev. D* **46**, 627 (1992).
- [23] D. Morgan, *Phys. Rev. D* **43**, 3144 (1991).

- [24] T. Banks and M. O’Loughlin, *Phys. Rev. D* **47**, 540 (1993).
- [25] T. Banks, M. O’Loughlin, and A. Strominger, *Phys. Rev. D* **47**, 4476 (1993).
- [26] J. Schwinger, *Phys. Rev.* **82**, 664 (1951).
- [27] I. K. Affleck and N. S. Manton, *Nucl. Phys.* **B194**, 38 (1982).
- [28] D. Garfinkle and A. Strominger, *Phys. Lett. B* **256**, 146 (1991).
- [29] I. K. Affleck, O. Alvarez, and N. S. Manton, *Nucl. Phys.* **B197**, 509 (1982).
- [30] R. D. Carlitz and R. S. Willey, *Phys. Rev. D* **36**, 2327 (1987); **36**, 2336 (1987).
- [31] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), pp. 828–832.
- [32] A. Strominger and S. Trivedi, *Phys. Rev. D* **48**, 5778 (1993).
- [33] S. B. Giddings, “Comments on information loss and remnants,” Report No. UCSBTH-93-35, hep-th/9310101 (unpublished).
- [34] F. Dowker, J. Gauntlett, S. B. Giddings, and G. Horowitz (unpublished).